4 SAMPLE PAPER

[Easy Concept]

Time Allowed: 3 Hours] [*Maximum Marks:* 80

General Instructions:

Read the following instructions very carefully and strictly follow them:

(*i*) *This Question paper contains* ***38*** *questions.* ***All*** *questions are* ***compulsory.***

(*ii*) *This Question paper is divided into* ***five*** *Sections –* ***A, B, C, D*** *and* ***E.***

(*iii*) *In* ***Section A,*** *Questions no.* ***1*** *to* ***18*** *are* ***multiple choice questions*** *(MCQs) and Questions no.* ***19*** *and* ***20*** *are* ***Assertion-Reason based*** *questions of* ***1 mark each.***

(*iv*) *In* ***Section B,*** *Questions no.* ***21*** *to* ***25*** *are* ***Very Short Answer (VSA)-type*** *questions, carrying* ***2 marks each.***

(*v*) *In* ***Section C,*** *Questions no.* ***26*** *to* ***31*** *are* ***Short Answer (SA)-type*** *questions, carrying* ***3 marks each.***

(*vi*) *In* ***Section D,*** *Questions no.* ***32*** *to* ***35*** *are* ***Long Answer (LA)-type*** *questions, carrying* ***5 marks each.***

(*vii*) *In* ***Section E,*** *Questions no.* ***36*** *to* ***38*** *are* ***Case study-based questions,*** *carrying* ***4 marks each.***

(*viii*) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(*ix*) *Use of calculators is* ***not*** *allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. The value of $|A|$, if $A = \begin{bmatrix} 0 & 2x-1 & \sqrt{x} \\ 1-2x & 0 & 2\sqrt{x} \\ -\sqrt{x} & -2\sqrt{x} & 0 \end{bmatrix}$, where $x \in \mathbb{R}^+$, is **[NCERT Part-I, Page 79]**

(*a*) $(2x + 1)^2$ (*b*) 0 (*c*) $(2x + 1)^3$ (*d*) $(2x - 1)^2$

2. Given that A is a square matrix of order 3 and $|A| = -2$, then $|adj\ (2A)|$ is equal to **[NCERT Part-I, Page 90]**

(*a*) -2^6 (*b*) $+4$ (*c*) -2^8 (*d*) 2^8

3. If the area of the triangle with vertices (–3, 0), (3, 0) and (0, k) is 9 sq units, then the value(s) of k will be **[NCERT Part-I, Page 82]**

(*a*) 9 (*b*) ±3 (*c*) –9 (*d*) 6

4. If $f(x) = \begin{cases} \dfrac{kx}{|x|}, & \text{if } x < 0 \\ 3, & \text{if } x \geq 0 \end{cases}$ is continuous at $x = 0$, then the value of k is **[NCERT Part-I, Page 105]**

(*a*) –3 (*b*) 0 (*c*) 3 (*d*) any real number

5. If $\vec{a} = 4\hat{i} + 6\hat{j}$ and $\vec{b} = 3\hat{j} + 4\hat{k}$, then the vector form of the component of $\vec{a}$ along $\vec{b}$ is **[NCERT Part-II, Page 357-358]**

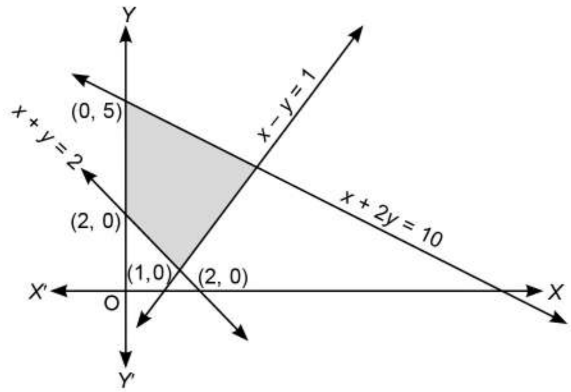
(*a*) $\frac{18}{5}(3\hat{i} + 4\hat{k})$ (*b*) $\frac{18}{25}(3\hat{j} + 4\hat{k})$ (*c*) $\frac{18}{5}(3\hat{i} + 4\hat{k})$ (*d*) $\frac{18}{25}(4\hat{i} + 6\hat{j})$

6. The general solution of the differential equation $ydx - xdy = 0$; (Given $x, y > 0$) is of the form

(*a*) $xy = C$ (*b*) $x = Cy^2$ (*c*) $y = Cx$ (*d*) $y = Cx^2$

(Where 'C' is an arbitrary positive constant of integration) **[NCERT Part-II, Page 306-307]**

7. The feasible region corresponding to the linear constraints of a Linear Programming Problem is given below. **[Conceptual Application]**



Which of the following is not a constraint to the given Linear Programming Problem?

(*a*) $x + y \geq 2$ (*b*) $x + 2y \leq 10$ (*c*) $x - y \geq 1$ (*d*) $x - y \leq 1$

8. The value of λ for which two vectors $2\hat{i} - \hat{j} + 2\hat{k}$ and $3\hat{i} + \lambda\hat{j} + \hat{k}$ are perpendicular is **[NCERT Part-II, Page 356]**

(*a*) 2 (*b*) 4 (*c*) 6 (*d*) 8

9. For any integer n, the value of $\int_{-\pi}^{\pi} e^{\cos^2 x} \sin^3(2n+1)x \, dx$ is **[NCERT Part-II, Page 274]**

(*a*) –1 (*b*) 0 (*c*) 1 (*d*) 2

10. If $A = [a_{ij}]$ is a square matrix of order 2 such that $a_{ij} = \begin{cases} 1, \text{when } i \neq j \\ 0, \text{when } i = j \end{cases}$, then A^2 is **[Conceptual Application]**

(*a*) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}_{2\times2}$ (*b*) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}_{2\times2}$ (*c*) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}_{2\times2}$ (*d*) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2}$

11. The corner points of the bounded feasible region determined by a system of linear constraints are (0, 3) (1, 1) and (3, 0). Let $Z = px + qy$, where $p, q > 0$. The condition on p and q so that the minimum of Z occurs at (3, 0) and (1, 1) is **[Conceptual Application]**

(*a*) $p = 2q$ (*b*) $p = \frac{q}{2}$ (*c*) $p = 3q$ (*d*) $p = q$

12. The lines $\vec{r} = \hat{i} + \hat{j} - \hat{k} + \lambda(2\hat{i} + 3\hat{j} - 6\hat{k})$ and $\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu(6\hat{i} + 9\hat{j} - 18\hat{k})$; (where λ and μ are scalars) are **[Conceptual Application]**

(*a*) coincident (*b*) skew (*c*) intersecting (*d*) parallel

13. If A and B are invertible square matrices of the same order, then which of the following is not correct? **[Conceptual Application]**

(*a*) $|AB^{-1}| = \frac{|A|}{|B|}$ (*b*) $|(AB)^{-1}| = \frac{1}{|A||B|}$

(*c*) $(AB)^{-1} = B^{-1}A^{-1}$ (*d*) $(A + B)^{-1} = B^{-1} + A^{-1}$

14. A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. If the events of their solving the problem are independent then the probability that the problem will be solved, is **[Conceptual Application]**

(*a*) $\frac{1}{4}$ (*b*) $\frac{1}{3}$ (*c*) $\frac{1}{2}$ (*d*) $\frac{3}{4}$

15. The degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$ is **[NCERT Part-II, Page 302]**

(*a*) 4 (*b*) $\frac{3}{2}$ (*c*) 2 (*d*) Not defined

16. *ABCD* is a rhombus whose diagonals intersect at *E*. Then $\overrightarrow{EA}+\overrightarrow{EB}+\overrightarrow{EC}+\overrightarrow{ED}$ equals to **[Conceptual Application]**

(*a*) $\vec{0}$ (*b*) $\overrightarrow{AD}$ (*c*) $2\overrightarrow{BD}$ (*d*) $2\overrightarrow{AD}$

17. The set of all points where the function $f(x) = x + |x|$ is differentiable, is **[NCERT Part-I, Page 118-119]**

(*a*) $(0, \infty)$ (*b*) $(-\infty, 0)$ (*c*) $(-\infty, 0) \cup (0, \infty)$ (*d*) $(-\infty, \infty)$

18. If the direction cosines of a line are $<\frac{1}{c}, \frac{1}{c}, \frac{1}{c}>$, then **[NCERT Part-II, Page 378]**

(*a*) $0 < c < 1$ (*b*) $c > 2$ (*c*) $c = \pm\sqrt{2}$ (*d*) $c = \pm\sqrt{3}$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (*a*), (*b*), (*c*) and (*d*) as given below.)

(*a*) Both *A* and *R* are true and *R* is the correct explanation of *A*.
(*b*) Both *A* and *R* are true but *R* is not the correct explanation of *A*.
(*c*) *A* is true but *R* is false.
(*d*) *A* is false but *R* is true.

19. **Assertion (*A*):** The relation $f : \{1, 2, 3, 4\} \rightarrow \{x, y, z, p\}$ defined by $f = \{(1, x), (2, y), (3, z)\}$ is a bijective function.

Reason (*R*): The function $f : \{1, 2, 3\} \rightarrow \{x, y, z, p\}$ such that $f = \{(1, x), (2, y), (3, z)\}$ is one-one. **[NCERT Part-I, Page 7]**

20. Let $f(x)$ be a polynomial function of degree 6 such that $\frac{d}{dx}(f(x)) = (x-1)^3(x-3)^2$, then

Assertion (*A*): $f(x)$ has a minimum at $x = 1$. **[Integrated Question]**

Reason (*R*): When $\frac{d}{dx}(f(x)) < 0, \forall\, x \in (a-h, a)$ and $\frac{d}{dx}(f(x)) > 0, \forall\, x \in (a, a+h)$; where '*h*' is an infinitesimally small positive quantity, then $f(x)$ has a minimum at $x = a$, provided $f(x)$ is continuous at $x = a$.

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. If $f(x) = \frac{1}{4x^2+2x+1}$; $x \in \mathbb{R}$, then find the maximum value of $f(x)$. **[Conceptual Application]**

OR

Find the maximum profit that a company can make, if the profit function is given by $P(x) = 72 + 42x - x^2$, where *x* is the number of units and *P* is the profit in rupees. **[NCERT Part-I, Page 166]**

22. Evaluate: $\int_{-1}^{1} \log\left(\frac{2-x}{2+x}\right)dx$. **[NCERT Part-II, Page 273-274]**

23. Find the value of $\sin^{-1}\left(\cos\left(\frac{33\pi}{5}\right)\right)$. **[Conceptual Application]**

OR

Find the domain of $\sin^{-1}(x^2 - 4)$. **[NCERT Part-I, Page 19]**

24. Find the interval/s in which the function $f:\mathbb{R} \to \mathbb{R}$ defined by $f(x) = x\,e^x$, is increasing. **[NCERT Part-I, Page 153]**

25. Check whether the function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3 + x$, has any critical point/s or not ? If yes, then find the point/s. **[NCERT Part-I, Page 164]**

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. The probability of variable x in an experiment in shown where 'k' is some real number:

$$P(x) = \begin{cases} k, \text{ if } x = 0 \\ 2k, \text{ if } x = 1 \\ 3k, \text{ if } x = 2 \\ 0, \text{ otherwise} \end{cases}$$

[Conceptual Application]

If $\Sigma P(x) = 1$

(*i*) Determine the value of k.

(*ii*) Find $P(X < 2)$.

(*iii*) Find $P(X \geq 2)$.

27. Evaluate: $\int \frac{2x^2+3}{x^2(x^2+9)}dx;\ x \neq 0$ **[NCERT Part-II, Page 253]**

28. Solve the differential equation: $ye^{\frac{x}{y}}dx = \left(xe^{\frac{x}{y}} + y^2\right)dy,\ (y \neq 0)$. **[NCERT Part-II, Page 313-314]**

OR

The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When x = 8 cm and y = 6 cm, find the rate of change of (a) the perimeter, and (b) the area of the rectangle. **[NCERT Part-I, Page 147-148]**

29. Find: $\int \sqrt{\frac{x}{1-x^3}}\,dx;\ x \in (0, 1)$. **[NCERT Part-II, Page 235-236]**

OR

Evaluate: $\int_0^{\frac{\pi}{4}} \log(1+\tan x)\,dx$. **[NCERT Part-II, Page 273-274]**

30. Solve the following Linear Programming Problem graphically: **[NCERT Part-II, Page 397-398]**

Minimize $Z = x + 2y$,

subject to the constraints: $x + 2y \geq 100,\ 2x - y \leq 0,\ 2x + y \leq 200,\ x, y \geq 0$.

OR

Solve the following Linear Programming Problem graphically: **[NCERT Part-II, Page 397-398]**

Maximize $Z = -x + 2y$,

subject to the constraints: $x \geq 3,\ x + y \geq 5,\ x + 2y \geq 6,\ y \geq 0$.

31. If $(a+bx)e^{\frac{y}{x}} = x$ then prove that $x\frac{d^2y}{dx^2} = \left(\frac{a}{a+bx}\right)^2$. **[NCERT Part-I, Page 137]**

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. Using the matrix method, solve the following system of linear equations: **[NCERT Part-I, Page 94-95]**

$\frac{2}{x}+\frac{3}{y}+\frac{10}{z} = 4,\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z} = 1,\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z} = 2.$

33. Find the coordinates of the image of the point (1, 6, 3) with respect to the line $\vec{r} = (\hat{j}+2\hat{k})+\lambda(\hat{i}+2\hat{j}+3\hat{k})$; where '$\lambda$' is a scalar. Also, find the distance of the image from the y-axis. **[Conceptual Application]**

OR

An aeroplane is flying along the line $\vec{r} = \lambda(\hat{i}-\hat{j}+\hat{k})$; where '$\lambda$' is a scalar and another aeroplane is flying along the line $\vec{r} = \hat{i}-\hat{j}+\mu(-2\hat{j}+\hat{k})$; where '$\mu$' is a scalar. At what points on the lines should they reach, so that the distance between them is the shortest? Find the shortest possible distance between them. **[NCERT Part-II, Page 386-387]**

34. Make a rough sketch of the region $\{(x, y): 0 \le y \le x^2 + 1, 0 \le y \le x + 1, 0 \le x \le 2\}$ and find the area of the region, using the method of integration. **[Conceptual Application]**

35. Let $\mathbb{N}$ be the set of all natural numbers and R be a relation on $\mathbb{N} \times \mathbb{N}$ defined by $(a, b)\ R\ (c, d) \Leftrightarrow ad = bc$ for all $(a, b), (c, d) \in \mathbb{N} \times \mathbb{N}$. Show that R is an equivalence relation on $\mathbb{N} \times \mathbb{N}$. Also, find the equivalence class of (2,6), i.e., [(2, 6)]. **[NCERT Part-I, Page 2, 4]**

OR

Show that the function $f: \mathbb{R} \to \{x \in \mathbb{R}: -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}, x \in \mathbb{R}$ is one-one and onto function. **[NCERT Part-I, Page 7]**

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. Teams A, B, C went for playing a tug of war game. Teams A, B, C have attached a rope to a metal ring and is trying to pull the ring into their own area. **[Conceptual Application]**

Team A pulls with force $F_1 = 6\hat{i} + 0\hat{j}\ kN$,

Team B pulls with force $F_2 = -4\hat{i} + 4\hat{j}\ kN$,

Team C pulls with force $F_3 = -3\hat{i} - 3\hat{j}\ kN$,

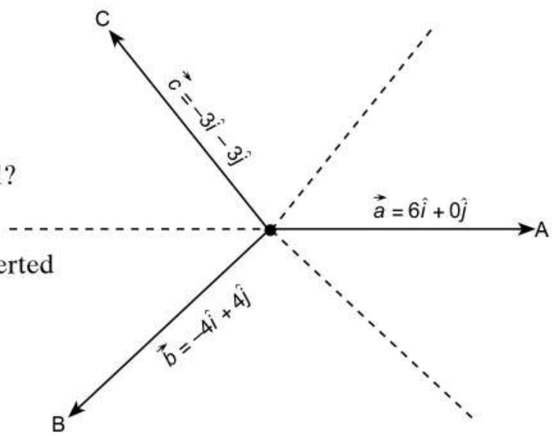
(*i*) What is the magnitude of the force of team A?

(*ii*) Which team will win the game?

(*iii*) Find the magnitude of the resultant force exerted by the teams.

OR

(*iii*) In what direction is the ring getting pulled?



Case Study - 2

37. In an office three employees Jayant, Sonia and Oliver process incoming copies of a certain form. Jayant processes 50% of the forms, Sonia processes 20% and Oliver the remaining 30% of the forms. Jayant has an error rate of 0.06, Sonia has an error rate of 0.04 and Oliver has an error rate of 0.03.

[Conceptual Application]

(*i*) Find the probability that Sonia processed the form and committed an error.

(*ii*) Find the total probability of committing an error in processing the form.

(*iii*) The manager of the Company wants to do a quality check. During inspection, he selects a form at random from the days output of processed form. If the form selected at random has an error, find the probability that the form is not processed by Jayant.

OR

(*iii*) Let E be the event of committing an error in processing the form and let E_1, E_2, and E_3 be the events that Jayant, Sonia and Oliver processed the form. Find the value of $\sum_{i=1}^{3} P(E_i/E)$.

Case Study - 3

38. The relation between the height of the plant ('y' in cm) with respect to its exposure to the sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$, where 'x' is the number of days exposed to the sunlight, for $x \leq 3$.

[Conceptual Application

(*i*) Find the rate of growth of the plant with respect to the number of days exposed to the sunlight.

(*ii*) Does the rate of growth of the plant increase or decrease in the first three days?
What will be the height of the plant after 2 days?

SOLUTIONS

1. (*b*) Matrix is skew symmetric matrix of odd order.

$$A' = -A \Rightarrow |A'| = -|A| \Rightarrow |A| = -|A|$$

$$\Rightarrow \quad 2|A| = 0 \Rightarrow |A| = 0$$

2. (*d*) $$|adj\ (2A)| = |(2A)|^2 = (|2A|)^2$$

$$= (2^3|A|)^2 = 2^6 \cdot (-2)^2 = 2^8$$

3. (*b*) $$\frac{1}{2}\begin{vmatrix} -3 & 0 & 1 \\ 3 & 0 & 1 \\ 0 & k & 1 \end{vmatrix} = \pm 9$$

$$\Rightarrow \quad -3(-k) + 1(3k) = \pm 18$$

$$\Rightarrow \quad 6k = \pm 18 \quad \Rightarrow \quad k = \pm 3$$

4. (*a*) If *f* is continuous at $x = 0$, then

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$$

$$\Rightarrow \quad \lim_{x \to 0^-} \frac{kx}{|x|} = \lim_{x \to 0^+} (3) = 3$$

$$\Rightarrow \quad k \cdot \lim_{x \to 0^-} \frac{x}{-x} = 3 = 3$$

$$\Rightarrow \quad k \cdot \lim_{x \to 0^-} (-1) = 3 \Rightarrow -k = 3 \Rightarrow k = -3$$

5. (*b*), Vector form of component of $\vec{a}$ along $\vec{b}$

= Vector projection of $\vec{a}$ along $\vec{b}$

$$= \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2}\right)\vec{b} = \left[\frac{6 \times 3}{\left(\sqrt{(3^2 + 4^2)}\right)^2}\right] \cdot \left(3\hat{j} + 4\hat{k}\right)$$

$$= \frac{18}{25}(3\hat{j} + 4\hat{k})$$

6. (*c*) $$y\ dx - x\ dy = 0 \Rightarrow \frac{dx}{x} - \frac{dy}{y} = 0$$

$$\Rightarrow \quad \int \frac{dx}{x} - \int \frac{dy}{y} = k \Rightarrow \log|x| - \log|y| = k$$

$$\Rightarrow \quad \log\left|\frac{x}{y}\right| = k \Rightarrow \frac{x}{y} = e^k = C'$$

$$\Rightarrow \quad x = C'y \ [e^k = C']$$

$\Rightarrow \quad y = \frac{1}{C'}x = Cx$, where $C = \frac{1}{C'}$ is constant of integration.

7. (*c*) As (0, 0) does not satisfy inequation

$x - y \geq 1$ *i.e.*, $0 - 0 \geq 1$ or $0 \geq 1$ (false)

8. (*d*) If perpendicular, then

$$\left(2\hat{i} - \hat{j} + 2\hat{k}\right) \cdot \left(3\hat{i} + \lambda\hat{j} + \hat{k}\right) = 0$$

$$\Rightarrow \quad 6 - \lambda + 2 = 0 \Rightarrow \lambda = 8$$

9. (*b*) Let $f(x) = e^{\cos^2 x} \sin^3(2n+1)x$

Now, $$f(-x) = e^{\cos^2(-x)} \cdot \sin^3(2n+1)(-x)$$
$$= e^{\cos^2 x} \cdot -\sin^3(2n+1)x$$
$$= -e^{\cos^2 x} \cdot \sin^3(2n+1)x$$
$$= -f(x)$$

So, f is an odd function.

Hence, $\int_{-\neq}^{\neq} e^{\cos^2 x} \cdot \sin^3(2n+1)x\, dx = 0$

10. (*d*) $a_{11} = 0, \quad a_{12} = 1, \quad a_{21} = 1, \quad a_{22} = 0$

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$$

11. (*b*) $Z_{(3,\,0)} = Z_{(1,\,1)}$

$\Rightarrow \quad 3p + 0q = p + q \quad \Rightarrow \quad 2p = q$

12. (*d*) parallel as directions are same/proportional.

Also point with p.v. $\left(2\hat{i} - \hat{j} - \hat{k}\right)$ does not lie on line

$\vec{r} = \hat{i} + \hat{j} - \hat{k} + \lambda\left(2\hat{i} + 3\hat{j} - 6\hat{k}\right)$.

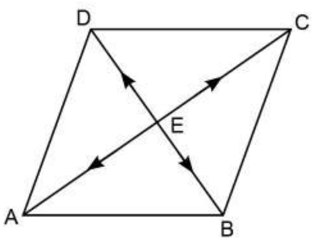
13. (*d*)

14. (*d*) P(Problem solved) = 1 – P(none solved)

$$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = 1 - \frac{1}{4} = \frac{3}{4}$$

15. (*c*)

16. (*a*)



Diagonals of a rhombus bisect each other at E as shown in the figure.

$$\overrightarrow{EA} = -\overrightarrow{EC} \text{ and } \overrightarrow{EB} = -\overrightarrow{ED}$$

$\therefore \ \overrightarrow{EA} + \overrightarrow{EB} + \overrightarrow{EC} + \overrightarrow{ED}$

$$= -\overrightarrow{EC} - \overrightarrow{ED} + \overrightarrow{EC} + \overrightarrow{ED}$$
$$= \vec{0}$$

17. (*c*) $f(x) = x + |x|$

As $|x|$ is differentiable for each $x \in R - \{0\}$ and x is differentiable for each $x \in R$. So, $|x| + x$ is differentiable for each $x \in R - \{0\}$.

18. (*d*) Sum of squares of direction cosines of a line = 1

$$\Rightarrow \quad \frac{1}{c^2} + \frac{1}{c^2} + \frac{1}{c^2} = 1$$

$$\Rightarrow \quad c^2 = 3 \Rightarrow c = \pm\sqrt{3}.$$

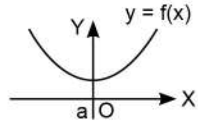
19. (*d*) Assertion is false as f is not a function for domain {1, 2, 3, 4}.

Reason is correct as $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$.

20. (*a*) $f'(x) < 0$ for $x < a$

$f'(x) > 0$ for $x > a$

'f' is continuous



then function attains "local minimum" at $x = a$. Both assertion and reason are true and reason is the correct explanation of assertion.

21. $$f(x) = \frac{1}{4x^2 + 2x + 1} = \frac{1}{\left(2x + \frac{1}{2}\right)^2 + \frac{3}{4}}$$

$f(x)$ is maximum when $\left(2x + \frac{1}{2}\right)^2 + \frac{3}{4}$ is minimum. Minimum value of $\left(2x + \frac{1}{2}\right)^2 + \frac{3}{4} = \frac{3}{4}$.

$\therefore$ Maximum value of $f(x)$ is $\frac{4}{3}$.

OR

$$P(x) = 72 + 42x - x^2$$

$$P'(x) = 42 - 2x$$

For maximum profit, $P'(x) = 0$

$$\Rightarrow \quad 42 - 2x = 0 \Rightarrow x = 21$$

$$P''(x) = -2,$$

$$P''(21) < 0$$

Hence profit is maximum for $x = 21$

$\therefore$ Maximum profit, $P(21) = 72 + 42 \times 21 - (21)^2$

$$= 72 + 882 - 441$$

$$= 72 + 441 = 513$$

Maximum profit $=$ ₹ 513.

22. Let $$f(x) = \log_e\left(\frac{2-x}{2+x}\right)$$

$$f(-x) = \log_e\left(\frac{2+x}{2-x}\right) = \log_e\left(\frac{2-x}{2+x}\right)^{-1}$$

$$= -\log_e\left(\frac{2-x}{2+x}\right) = -f(x)$$

$\Rightarrow$ 'f' is odd function

$$\therefore \quad \int_{-1}^{1} \log_e\left(\frac{2-x}{2+x}\right)dx = 0 \qquad \left[\text{As } \int_{-a}^{a} f(x)\,dx = 0 \text{ if } f(-x) = -f(x)\right]$$

23. $$\sin^{-1}\left[\cos\frac{33\pi}{5}\right] = \sin^{-1}\left[\cos\left(6\pi + \frac{3\pi}{5}\right)\right]$$

$$= \sin^{-1}\left[\cos\frac{3\pi}{5}\right]$$

$$= \sin^{-1}\left[\cos\left(\frac{\pi}{2} + \frac{\pi}{10}\right)\right]$$

$$= \sin^{-1}\left[-\sin\frac{\pi}{10}\right]$$

$$= \sin^{-1}\left[\sin\left(\frac{-\pi}{10}\right)\right]$$

$$= \frac{-\pi}{10} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$$

OR

For domain $-1 \le x^2 - 4 \le 1$

$\Rightarrow$ $3 \le x^2 \le 5$

$\Rightarrow$ $\sqrt{3} \le x \le \sqrt{5}$ or $-\sqrt{5} \le x \le -\sqrt{3}$

$\Rightarrow$ $x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

24. $f(x) = x \cdot e^x$

$f'(x) = x \cdot e^x + e^x \cdot 1 = e^x(x + 1)$

As $e^x > 0$ for $x \in R$, sign of $f'(x)$ depends upon $(x + 1)$.

For increasing $(x + 1) \ge 0 \Rightarrow x \ge -1$, *i.e.* $x \in [-1, \infty)$.

25. Given $f(x) = x^3 + x$

$f'(x) = 3x^2 + 1$

For critical point(s), $f'(x) = 0$

$\Rightarrow$ $3x^2 + 1 = 0$, no real values of x.

So, no critical point.

26. $P(x) = \begin{cases} k, & x = 0 \\ 2k, & x = 1 \\ 3k, & x = 2 \end{cases}$

(*i*) We have $\sum P(x) = 1$

$\Rightarrow P(0) + P(1) + P(2) = 1$

$\Rightarrow$ $k + 2k + 3k = 1 \Rightarrow 6k = 1 \Rightarrow k = \frac{1}{6}$

(*ii*) $P(X < 2) = P(0) + P(1) = k + 2k$

$= \frac{1}{6} + 2 \times \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

(*iii*) $P(X \ge 2) = P(2) = 3k = \frac{3}{6} = \frac{1}{2}$

27. Consider $\int \frac{2x^2 + 3}{x^2(x^2 + 9)} dx$

$$= 2\int \frac{x^2}{x^2(x^2+9)} dx + 3\int \frac{1}{x^2(x^2+9)} dx$$

$$= 2\int \frac{1}{x^2+9} dx + \frac{3}{9}\int \frac{(x^2+9) - x^2}{x^2(x^2+9)} dx$$

$$= 2\int \frac{1}{x^2+9} dx + \frac{1}{3}\int \frac{1}{x^2} dx - \frac{1}{3}\int \frac{1}{x^2+9} dx$$

$$= \frac{5}{3}\int \frac{1}{x^2+9} dx + \frac{1}{3}\int \frac{1}{x^2} dx$$

$$= \frac{5}{3} \times \frac{1}{3} \tan^{-1}\frac{x}{3} + \frac{1}{3}\left(-\frac{1}{x}\right) + C$$

$= \frac{5}{9}\tan^{-1}\frac{x}{3} - \frac{1}{3x} + C$, C is constant of integration.

28. Consider equation $$ye^{x/y}\,dx = (xe^{x/y} + y^2)dy$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{xe^{x/y} + y^2}{ye^{x/y}} \qquad ...(i)$$

Let $$x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$$

So, from (*i*), we get

$$v + y\frac{dv}{dy} = \frac{vy \cdot e^v + y^2}{ye^v} = v + \frac{y}{e^v}$$

$$\Rightarrow \quad y\frac{dv}{dy} = \frac{y}{e^v} \Rightarrow e^v\,dv = dy$$

Integrating both sides, we get

$$\int e^v\,dv = \int dy$$

$$\Rightarrow \quad e^v = y + C, \ C \text{ is constant of integration}$$

$$\Rightarrow \quad e^{x/y} = y + C \text{ is solution}$$

OR

$\frac{dx}{dt} = -5$ cm/min, $\frac{dy}{dt} = 4$ cm/min

(*a*) Perimeter of the rectangle $P = 2(x + y)$

$$\therefore \quad \frac{dP}{dt} = 2\frac{d}{dt}(x+y) = 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$$

$$= 2(-5 + 4) = -2 \text{ cm/min.}$$

Hence, perimeter is decreasing at the rate of 2 cm/min.

(*b*) Area of the rectangle $A = xy$

$$\therefore \quad \frac{dA}{dt} = \frac{d}{dt}(xy) = x\frac{dy}{dt} + y\frac{dx}{dt}$$

$$= 4x - 5y$$

$$\left.\frac{dA}{dt}\right]_{x=8 \text{ and } y=6} = 32 - 30 = 2 \text{ cm}^2\text{/min}$$

Hence, area is increasing at the rate of 2 cm^2/min.

29. Consider $$\int \sqrt{\frac{x}{1-x^3}}\,dx = \int \frac{x^{1/2}}{\sqrt{1-\left(x^{3/2}\right)^2}}\,dx$$

$$\left|\begin{array}{l}\text{Let } \quad x^{3/2} = t \\ \Rightarrow \frac{3}{2}x^{1/2}dx = dt\end{array}\right.$$

$$= \frac{2}{3}\int \frac{1}{\sqrt{1-t^2}}\,dt$$

$$= \frac{2}{3}\sin^{-1}t + C = \frac{2}{3}\sin^{-1}\left(x^{3/2}\right) + C, \ C \text{ is constant of integration.}$$

OR

Let $$I = \int_0^{\frac{\pi}{4}} \log(1+\tan x)dx \qquad ...(i)$$

$$I = \int_0^{\frac{\pi}{4}} \log\left\{1 + \tan\left(\frac{\pi}{4} - x\right)\right\}dx$$

$$\left[\text{Using property } \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$= \int_0^{\frac{\pi}{4}} \log\left(1 + \frac{1-\tan x}{1+\tan x}\right)dx$$

$$\Rightarrow \quad I = \int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1+\tan x}\right)dx \qquad ...(ii)$$

Adding (*i*) and (*ii*), we get $2I = \log 2\int_0^{\frac{\pi}{4}} 1\,.\,dx = \log 2[x]_0^{\frac{\pi}{4}} = \frac{\pi}{4}\log 2 \Rightarrow I = \frac{\pi}{8}\log 2$

30. Given LPP is

Minimize $Z = x + 2y$

Subject to the constraints

$x + 2y \geq 100,\ 2x - y \leq 0,\ 2x + y \leq 200,\ x, y \geq 0.$

Consider equation $x + 2y = 100$ corresponding to inequation $x + 2y \geq 100$

Some points on graph of $x + 2y = 100$ are

x	100	0	60
y	0	50	20

On substituting (0, 0) in $x + 2y \geq 100$, we get false statement, so we shade portion not containing (0, 0).

Consider equation $2x - y = 0$ corresponding to inequation $2x - y \leq 0$.

Some points on graph of $2x - y = 0$ are

x	0	20	50
y	0	40	100

On substituting (10, 0) in $2x - y \leq 0$, we get false statement, we shade the portion not containing (10, 0).

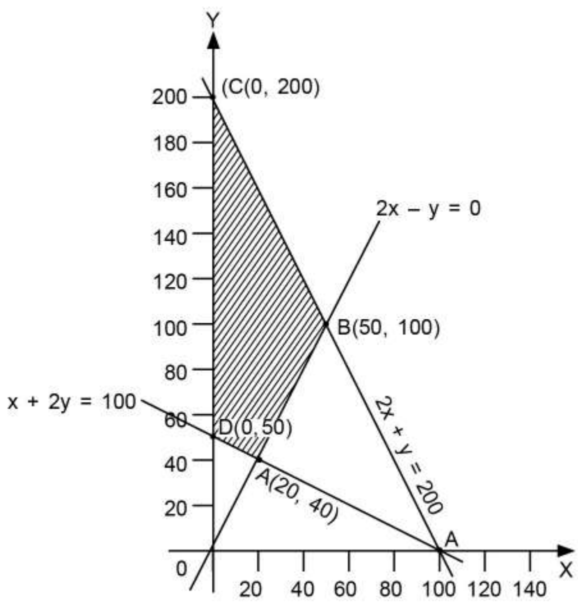
Consider equation $2x + y = 200$ corresponding to inequation $2x + y \leq 200$.

Some points on graph of $2x + y = 200$ are

x	0	100	50
y	200	0	100

On substituting (0, 0) in $2x + y \leq 200$, we get true statement so we shade portion containing (0, 0).

$x, y \geq 0 \Rightarrow$ we work in first quadrant. Plotting the above information on graph we notice shaded portion is feasible solution.



Possible points for minimum Z are $A(20, 40)$, $B(50, 100)$, $C(0, 200)$, $D(0, 50)$

Points	$Z = x + 2y$	Values	
$A(20, 40)$	20 + 80	100	← Minimum
$B(50, 100)$	50 + 200	250	
$C(0, 200)$	0 + 400	400	
$D(0, 50)$	0 + 100	100	← Minimum

Z is minimum at $A(20, 40)$ and $D(0, 50)$.

Z is minimum at all the points of line segment joining points $A(20, 40)$ and $D(0, 50)$.

OR

Given LPP is

Maximize $Z = -x + 2y$

Subject to the constraints $x \geq 3$, $x + y \geq 5$, $x + 2y \geq 6$, $y \geq 0$

$x = 3$ is equation corresponding to inequation $x \geq 3$ and represents line parallel to y-axis and $x \geq 3$ represents region not containing (0, 0).

Now, $x + y = 5$ is equation corresponding to inequation $x + y \geq 5$

Some points on graph of $x + y = 5$ are

x	0	5	2
y	5	0	3

On substituting (0, 0) in $x + y \geq 5$, we get false statement, so we shade portion not containing (0, 0).

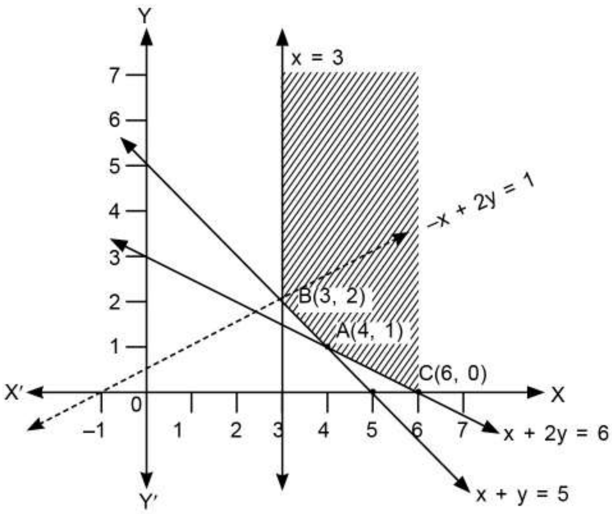
$x + 2y = 6$ is equation corresponding to inequation $x + 2y \geq 6$. Some points on $x + 2y = 6$ are

x	6	0	4
y	0	3	1

On substituting (0, 0) in $x + 2y \geq 6$, we get false statement, we shade the portion not containing (0, 0).

$y \geq 0$ represents portion above x-axis.

On ploting the above information we get the shaded portion as feasible solution.



We notice area is unbounded, consider function to be maximised $Z = -x + 2y$

Corner points	Value of Z	
$A(4, 1)$	$Z = -2$	
$B(3, 2)$	$Z = 1$	← Maximum
$C(6, 0)$	$Z = -6$	

Now, open half plane determined by $-x + 2y > 1$ has points common with the feasible region, hence no maximum value of Z occurs.

31. Given $(a + bx)\, e^{y/x} = x$

Taking log on both sides, we get

$$\log(a + bx) + \frac{y}{x}\log e = \log x$$

$$\Rightarrow \quad \frac{y}{x} = \log x - \log(a + bx) \quad ...(i)$$

$$\Rightarrow \quad y = x[\log x - \log(a + bx)]$$

$$\Rightarrow \quad \frac{dy}{dx} = x\left[\frac{1}{x} - \frac{b}{a+bx}\right] + [\log x - \log(a + bx)]$$

$$= x\left[\frac{a+bx-bx}{x(a+bx)}\right] + \log x - \log(a + bx)$$

$$= \frac{a}{a+bx} + \log x - \log(a + bx)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{a}{a+bx} + \frac{y}{x} \quad \text{[from } (i)]$$

$$\Rightarrow \quad x\frac{dy}{dx} = \frac{ax}{a+bx} + y$$

Differentiating w.r.t. x we get

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{(a+bx)a-(ax)b}{(a+bx)^2} + \frac{dy}{dx}$$

$$\Rightarrow \quad x\frac{d^2y}{dx^2} = \frac{a^2}{(a+bx)^2} = \left(\frac{a}{a+bx}\right)^2$$

32. Matrix equation is

$$\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix} \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} \Rightarrow AX = B$$

[we can change symbols also as $\frac{1}{x} = a, \frac{1}{y} = b, \frac{1}{z} = c$]

Solution is $\quad X = A^{-1}B \quad ...(i)$

$$|A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}$$

$$= 2(120 - 45) - 3(-80 - 30) + 10(36 + 36)$$

$$= 150 + 330 + 720 = 1200 \neq 0.$$

Hence A^{-1} exists.

Cofactors of elements of $|A|$ are

$A_{11} = 75, \quad A_{12} = 110, \quad A_{13} = 72$

$A_{21} = 150, \quad A_{22} = -100, \quad A_{23} = 0$

$A_{31} = 75, \quad A_{32} = 30, \quad A_{33} = -24$

$$\text{adj } A = \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}'$$

$$= \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}\text{ Adj }(A) = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

From (i),
$$\begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 300+150+150 \\ 440-100+60 \\ 288+0-48 \end{bmatrix}$$

$$= \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$\Rightarrow$ $$\frac{1}{x} = \frac{1}{2}, \frac{1}{y} = \frac{1}{3}, \frac{1}{z} = \frac{1}{5}$$

$\Rightarrow$ $$x = 2, y = 3, z = 5$$

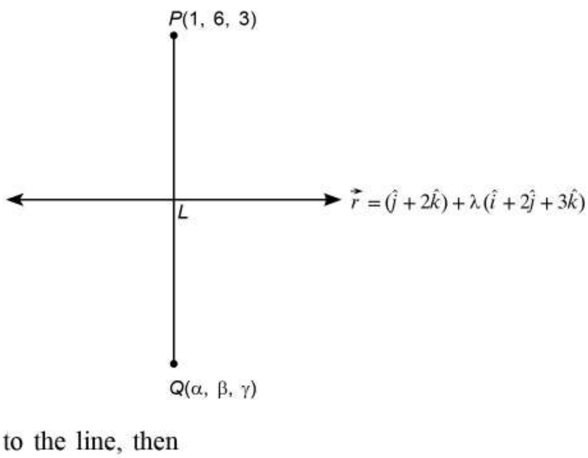
33. Let $Q(\alpha, \beta, \gamma)$ be image of $P(1, 6, 3)$ in line
$$\vec{r} = (\hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$$

Let PL is perpendicular to given line.

Position vector of general point on the line is
$$\vec{r} = \lambda\hat{i} + (1 + 2\lambda)\hat{j} + (2 + 3\lambda)\hat{k} \quad ...(i)$$

Suppose the position vector of point L is given by (i).

Now, $$\overrightarrow{PL} = (\lambda - 1)\hat{i} + (2\lambda - 5)\hat{j} + (3\lambda - 1)\hat{k}$$



If $\overrightarrow{PL}$ perpendicular to the line, then

$$1(\lambda - 1) + 2(2\lambda - 5) + 3(3\lambda - 1) = 0$$

$\Rightarrow$ $$\lambda - 1 + 4\lambda - 10 + 9\lambda - 3 = 0$$

$\Rightarrow$ $$14\lambda = 14 \Rightarrow \lambda = 1$$

Position vector of foot of perpendicular drawn from P to line is

$$\vec{r} = \hat{i} + 3\hat{j} + 5\hat{k} \quad ...(ii)$$

Also L is mid point of PQ.

$\therefore$ Position vector of L is

$$\vec{r} = \left(\frac{\alpha+1}{2}\right)\hat{i} + \left(\frac{\beta+6}{2}\right)\hat{j} + \left(\frac{\gamma+3}{2}\right)\hat{k} \quad ...(iii)$$

From (*ii*) and (*iii*) we get

$$\frac{\alpha+1}{2} = 1, \ \frac{\beta+6}{2} = 3, \ \frac{\gamma+3}{2} = 5$$

$$\Rightarrow \quad \alpha = 1, \beta = 0, \gamma = 7$$

Coordinates of image are $Q(1, 0, 7)$

Distance of image from y-axis $= \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$ units.

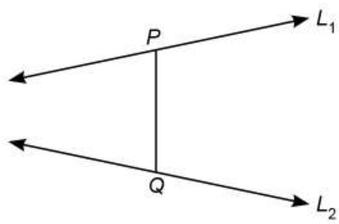
OR

Position vector of point P on line $L_1 : \vec{r} = \lambda(\hat{i} - \hat{j} + \hat{k})$ is $\vec{r} = \lambda\hat{i} - \lambda\hat{j} + \lambda\hat{k}$ ...(*i*)

Position vector of point Q on line $L_2 : \vec{r} = \hat{i} - \hat{j} + \mu(-2\hat{j} + \hat{k})$ is

$$\vec{r} = \hat{i} + (-1-2\mu)\hat{j} + \mu k \quad ...(ii)$$

$$\overrightarrow{PQ} = (1-\lambda)\hat{i} + (-1-2\mu+\lambda)\hat{j} + (\mu-\lambda)\hat{k} \quad ...(iii)$$



If $\overrightarrow{PQ}$ is line of shortest distance then PQ is perpendicular to L_1 and L_2.

As, $PQ \perp L_1$, then

$$\Rightarrow \quad 1(1-\lambda) - 1(-1-2\mu+\lambda) + 1(\mu-\lambda) = 0$$

$$\Rightarrow \quad 1 - \lambda + 1 + 2\mu - \lambda + \mu - \lambda = 0$$

$$\Rightarrow \quad 3\lambda - 3\mu = 2 \quad ...(iv)$$

As, $PQ \perp L_2$, then

$$\Rightarrow 0(1-\lambda) - 2(-1-2\mu+\lambda) + 1(\mu-\lambda) = 0$$

$$\Rightarrow \quad 2 + 4\mu - 2\lambda + \mu - \lambda = 0$$

$$\Rightarrow \quad 3\lambda - 5\mu = 2 \quad ...(v)$$

From (*iv*) and (*v*), we get $\lambda = \frac{2}{3}, \mu = 0$.

From (*i*), for shortest distance position vector of P is $\frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$ or $P\left(\frac{2}{3}, \frac{-2}{3}, \frac{2}{3}\right)$

Similarly, from (*ii*), position vector of Q is $\hat{i} - \hat{j}$ i.e. $Q(1, -1, 0)$

$$\text{Shortest distance} = \sqrt{\left(\frac{2}{3}-1\right)^2+\left(\frac{-2}{3}+1\right)^2+\left(\frac{2}{3}-0\right)^2}$$
$$= \sqrt{\frac{1}{9}+\frac{1}{9}+\frac{4}{9}}$$
$$= \sqrt{\frac{6}{9}} = \sqrt{\frac{2}{3}} \text{ units}$$

34. Given region $\{(x, y): 0 \le y \le x^2 + 1, 0 \le y \le x + 1, 0 \le x \le 2\}$

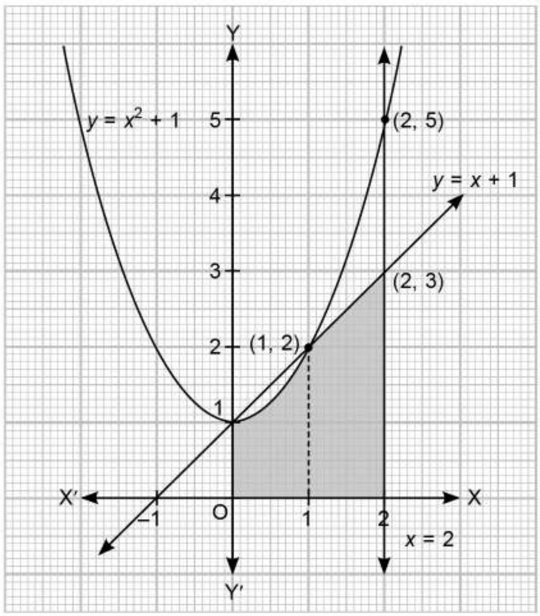
We notice we have to take common region for $y \le x^2 + 1$, $y \le x + 1$, $x \le 2$, $x \ge 0$, $y \ge 0$.

Curve $y = x^2 + 1$ represents parabola symmetrical to y-axis with vertex at (0, 1) and (0, 0) lies in region of $y \le x^2 + 1$.

Curve $y = x + 1$ represents straight line and (0, 0) lies in the region of $y \le x + 1$,

$x = 2$ is a line parallel to y-axis and region of $x \le 2$ contains origin.

Plotting the above information we notice we have to find shaded area.



$$\text{Area} = \int_0^1 (x^2+1)dx + \int_1^2 (x+1)\,dx$$
$$= \left[\frac{x^3}{3}+x\right]_0^1 + \left[\frac{x^2}{2}+x\right]_1^2$$
$$= \left[\left(\frac{1}{3}+1\right)-0\right]+\left[\left(\frac{4}{2}+2\right)-\left(\frac{1}{2}+1\right)\right]$$
$$= \frac{23}{6} \text{ sq units}$$

35. Relation R is defined by $(a, b)\ R\ (c, d) \Leftrightarrow ad = bc$ for all $(a, b), (c, d) \in \mathbb{N} \times \mathbb{N}$.

For reflexive: Let $(a, b) \in N \times N$. Now,$(a, b)\ R\ (a, b) \Rightarrow ab = ba$, which is true in $\mathbb{N}$. Hence, reflexive.

For symmetric: Let $(a, b), (c, d) \in N \times N$, s.t. $(a, b)\ R\ (c, d)$.

Now, $(a, b)\ R\ (c, d)$

$\Rightarrow ad = bc$

$\Rightarrow cb = da \Rightarrow (c, d)\ R\ (a, b)$.

Hence, symmetric.

For transitive: Let $(a, b), (c, d), (e, f) \in N \times N$ such that $(a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f)$.

Now, $(a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f) \Rightarrow ad = bc$ and $cf = de$

$\Rightarrow ad \cdot cf = bc \cdot de \Rightarrow af = be$

$\Rightarrow (a, b)\ R\ (e, f)$. Hence, transitive.

Since relation R is reflexive, symmetric and transitive, hence, relation R is an equivalence relation.

For equivalence class $[(2, 6)]$

Let $(a, b) \in \mathbb{N} \times \mathbb{N}$ is related to $(2, 6)$ i.e. $(a, b)\ \mathrm{R}(2, 6) \Rightarrow 6a = 2b \Rightarrow 3a = b$

$\therefore \quad [(2, 6)] = \{(1, 3), (2, 6), (3, 9), ...]$.

OR

The function $f : \mathbb{R} \to \{x \in \mathbb{R} : -1 < x < 1\}$ is defined as

$$f(x) = \frac{x}{1+|x|},\ x \in \mathbb{R}.$$

For one-one:

Case I: If x is positive and y is negative, then we have:

$x \neq y \Rightarrow |x| \neq |y|\ (|x| = |y|$ if $x = -y)$

$\Rightarrow \dfrac{x}{1+|x|} \neq \dfrac{y}{1+|y|} \Rightarrow f(x) \neq f(y)$

Similarly if x is negative and y is positive. We get same result.

Hence, one-one.

Case II: When x and y are both positive, we have:

$f(x) = f(y) \Rightarrow \dfrac{x}{1+x} = \dfrac{y}{1+y} \Rightarrow x = y$

Hence, one-one.

Case III: When x and y are both negative, we have

$f(x) = f(y) \Rightarrow \dfrac{x}{1-x} = \dfrac{y}{1-y} \Rightarrow x = y$

Hence, one-one.

In all the three cases, we notice f is one-one.

For onto: Now, let $y \in R$ such that $-1 < y < 1$.

If $y < 0$ (from co-domain) then there exists $x \in \mathbb{R}$ (domain) such that

$$y = f(x) \Rightarrow y = \frac{x}{1-x}$$

$$y - xy = x \Rightarrow x(1 + y) = y$$

$$\Rightarrow \quad x = \frac{y}{1+y} \in R \text{ (domain)}$$

Now, $$f\left(\frac{y}{1+y}\right) = \frac{\left(\frac{y}{1+y}\right)}{1-\left(\frac{y}{1+y}\right)} = y.$$

If $y > 0$ (from co-domain) then there exists $x \in \mathbb{R}$ (domain)

such that $$y = f(x) \Rightarrow y = \frac{x}{1+x} \Rightarrow y + xy = x$$

$$\Rightarrow \quad x(1 - y) = y \Rightarrow x = \frac{y}{1-y}$$

$$f\left(\frac{y}{1-y}\right) = \frac{\left(\frac{y}{1-y}\right)}{1+\left(\frac{y}{1-y}\right)} = y$$

$\therefore\ f$ is onto. Hence, f is one-one and onto.

36. (*i*) Magnitude of force of team $A = \sqrt{36+0}$

$= 6$ kN.

(*ii*) $\vec{F_1} + \vec{F_3} = 3\hat{i} - 3\hat{j}\,kN$

$|\vec{F_1} + \vec{F_3}| = \sqrt{(3)^2 + (-3)^2} = 3\sqrt{2}\ kN$

Now, $|\vec{F_2}| = \sqrt{(-4)^2 + (4)^2} = 4\ \sqrt{2}\ kN$

So, $|\vec{F_2}| > |\vec{F_1} + \vec{F_3}|$

We can check similarly that,

$|\vec{F_1}| < \left|\vec{F_2} + \vec{F_3}\right|$

and $|\vec{F_3}| < \left|\vec{F_1} + \vec{F_2}\right|$

Now, $\left|\vec{F_2}\right| > \left|\vec{F_1} + \vec{F_3}\right|$ and also $\vec{F_2}$ and $\left(\vec{F_1} + \vec{F_3}\right)$ are unlike.

So, team B will win the game.

(*iii*) $\vec{r}$ (resultant force) $= 6\hat{i} + 0\hat{j} - 4\hat{i} + 4\hat{j} - 3\hat{i} - 3\hat{j} = -\hat{i} + \hat{j}$

$\therefore \quad |\vec{r}| = \sqrt{1+1} = \sqrt{2}$ kN

OR

$\vec{r} = -\hat{i} + \hat{j},\ \hat{r} = -\dfrac{1}{\sqrt{2}}\hat{i} + \dfrac{1}{\sqrt{2}}\hat{j}$

$\cos\theta = -\dfrac{1}{\sqrt{2}},\ \sin\theta = \dfrac{1}{\sqrt{2}}$

$\Rightarrow \theta = 135°$

So, ring is getting pulled at 135° to horizontal line (+ve x-axis)

37. Consider the following events,

J : selected form is processed by Jayant

S : selected form is processed by Sonia

O : selected form is processed by Oliver

E: error has occured

$$P(J) = \frac{50}{100} = \frac{1}{2},\ P(S) = \frac{20}{100} = \frac{1}{5},\ P(O) = \frac{30}{100} = \frac{3}{10}$$

$$P\left(\frac{E}{J}\right) = 0.06,\ P\left(\frac{E}{S}\right) = 0.04,\ P\left(\frac{E}{O}\right) = 0.03$$

(*i*) $P(S)\cdot P\left(\dfrac{E}{S}\right) = 0.2 \times 0.04 = 0.008$

(*ii*) $P(E) = P(J)P\left(\dfrac{E}{J}\right) + P(S)P\left(\dfrac{E}{S}\right) + P(O)\ P\left(\dfrac{E}{O}\right)$

$= 0.5 \times 0.06 + 0.2 \times 0.04 + 0.3 \times 0.03$

$= 0.03 + 0.008 + 0.009 = 0.047$

(*iii*) $P(\text{not } J) = 1 - \dfrac{P(J)P\left(\frac{E}{J}\right)}{0.047}$

$= 1 - \dfrac{0.03}{0.047} = \dfrac{17}{47}$

OR

(*iii*) $\sum_{i=1}^{3} P\left(\frac{E_i}{E}\right) = P\left(\frac{E_1}{E}\right) + P\left(\frac{E_2}{E}\right) + P\left(\frac{E_3}{E}\right) = 1$

38. (*i*) Rate of growth (y) with respect to number of days exposed (x) is $\dfrac{dy}{dx} = 4 - x$.

(*ii*) For increase or decrease of rate of growth we find $\dfrac{d^2y}{dx^2} = -1 < 0$, rate of growth decreases.

As $y = 4x - \dfrac{1}{2}x^2$

Now, height of plant after 2 days is given by, $y]_{x=2} = 4 \times 2 - \dfrac{1}{2}(2)^2 = 8 - 2 = 6$ cm