5 SAMPLE PAPER

[Easy Concept]

Time Allowed: 3 Hours] [*Maximum Marks:* 80

General Instructions:

Read the following instructions very carefully and strictly follow them:

(*i*) *This Question paper contains **38** questions. **All** questions are **compulsory**.*

(*ii*) *This Question paper is divided into **five** Sections – **A**, **B**, **C**, **D** and **E**.*

(*iii*) *In **Section A**, Questions no. **1** to **18** are **multiple choice questions** (MCQs) and Questions no. **19** and **20** are **Assertion-Reason based** questions of **1 mark each**.*

(*iv*) *In **Section B**, Questions no. **21** to **25** are **Very Short Answer (VSA)-type** questions, carrying **2 marks each**.*

(*v*) *In **Section C**, Questions no. **26** to **31** are **Short Answer (SA)-type** questions, carrying **3 marks each**.*

(*vi*) *In **Section D**, Questions no. **32** to **35** are **Long Answer (LA)-type** questions, carrying **5 marks each**.*

(*vii*) *In **Section E**, Questions no. **36** to **38** are **Case study-based questions**, carrying **4 marks each**.*

(*viii*) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(*ix*) *Use of calculators is **not** allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. If for a square matrix A, $A^2 - A + I = O$, then A^{-1} equals **[Conceptual Application]**

(*a*) A (*b*) $A + I$ (*c*) $I - A$ (*d*) $A - I$

2. If $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and $A = B^2$, then x equals **[NCERT Part-I, Page 41, 51]**

(*a*) ± 1 (*b*) -1 (*c*) 1 (*d*) 2

3. Let $A = \{3, 5\}$. Then number of reflexive relations on A is **[Conceptual Application]**

(*a*) 2 (*b*) 4 (*c*) 0 (*d*) 8

4. $\sin\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to **[NCERT Part-I, Page 19]**

(*a*) 1 (*b*) $\frac{1}{2}$ (*c*) $\frac{1}{3}$ (*d*) $\frac{1}{4}$

5. If $\begin{vmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 0$, then the value of α is **[NCERT Part-I, Page 78-79]**

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

6. The interval in which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing, is **[NCERT Part-I, Page 153]**

(*a*) $(-1, \infty)$ (*b*) $(-2, -1)$ (*c*) $(-\infty, -2)$ (*d*) $[-1, 1]$

7. If $x = A \cos 4t + B \sin 4t$, then $\frac{d^2x}{dt^2}$ is equal to **[NCERT Part-I, Page 137]**

(*a*) x (*b*) $-x$ (*c*) $16x$ (*d*) $-16x$

8. The function $f(x) = [x]$, where $[x]$ denotes the greatest integer less than or equal to x is continuous at **[NCERT Part-I, Page 105]**

(*a*) $x = 1$ (*b*) $x = 1.5$ (*c*) $x = -2$ (*d*) $x = 4$

9. The derivative of x^{2x} w.r.t. x is **[NCERT Part-I, Page 130]**

(*a*) x^{2x-1} (*b*) $2x^{2x} \log x$ (*c*) $2x^{2x}(1 + \log x)$ (*d*) $2x^{2x}(1 - \log x)$

10. $\int \frac{\sec x}{\sec x - \tan x} dx$ equals **[NCERT Part-II, Page 241]**

(*a*) $\sec x - \tan x + C$ (*b*) $\sec x + \tan x + C$ (*c*) $\tan x - \sec x + C$ (*d*) $-(\sec x + \tan x) + C$

11. If a line makes angles of 90°, 135° and 45° with the x, y and z axes respectively, then its direction cosines are **[NCERT Part-II, Page 377-378]**

(*a*) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (*b*) $-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$ (*c*) $\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}$ (*d*) $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

12. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is **[NCERT Part-II, Page 383-384]**

(*a*) 0° (*b*) 30° (*c*) 45° (*d*) 90°

13. Two vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are collinear if **[NCERT Part-II, Page 349]**

(*a*) $a_1b_1 + a_2b_2 + a_3b_3 = 0$ (*b*) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

(*c*) $a_1 = b_1, a_2 = b_2, a_3 = b_3$ (*d*) $a_1 + a_2 + a_3 = b_1 + b_2 + b_3$

14. The magnitude of the vector $6\hat{i} - 2\hat{j} + 3\hat{k}$ is **[NCERT Part-II, Page 348]**

(*a*) 1 (*b*) 5 (*c*) 7 (*d*) 12

15. $\int_{-1}^{1} \frac{|x-2|}{x-2} dx, x \neq 2$ is equal to **[NCERT Part-II, Page 268]**

(*a*) 1 (*b*) -1 (*c*) 2 (*d*) -2

16. The sum of the order and the degree of the differential equation $\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^3\right) = 0$ is **[NCERT Part-II, Page 302]**

(*a*) 2 (*b*) 3 (*c*) 5 (*d*) 0

17. If for any two events A and B, $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then $P\left(\frac{B}{A}\right)$ is equal to **[NCERT Part-II, Page 408]**

(*a*) $\frac{1}{10}$ (*b*) $\frac{1}{8}$ (*c*) $\frac{7}{8}$ (*d*) $\frac{17}{20}$

18. Five fair coins are tossed simultaneously. The probability of the events that at least one head comes up is **[Conceptual Application]**

(*a*) $\frac{27}{32}$ (*b*) $\frac{5}{32}$ (*c*) $\frac{31}{32}$ (*d*) $\frac{1}{32}$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (*a*), (*b*), (*c*) and (*d*) as given below.)

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true but R is not the correct explanation of A.

(*c*) A is true but R is false.

(*d*) A is false but R is true.

19. Assertion (*A*) : $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\,dx = 3$ **[NCERT Part-II, Page 273-274]**

Reason (*R*) : $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

20. Assertion (*A*) : Two coins are tossed simultaneously. The probability of getting two heads, if it is known that at least one head comes up, is $\frac{1}{3}$. **[NCERT Part-II, Page 408]**

Reason (*R*) : Let E and F be two events with a random experiment, then $P\left(\frac{F}{E}\right) = \frac{P(E \cap F)}{P(E)}$.

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Sketch the region bounded by the lines $2x + y = 8$, $y = 2$, $y = 4$ and the y-axis. Hence, obtain its area using integration. **[Conceptual Application]**

22. If the vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3$, $|\vec{b}| = \frac{2}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$. **[Integrated Question]**

OR

Find the area of a parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$. **[NCERT Part-II, Page 365]**

23. Write the domain and range (principal value branch) of the following function:

$f(x) = \tan^{-1}x$ **[NCERT Part-I, Page 24]**

24. If $f(x) = \begin{cases} x^2, & \text{if } x \geq 1 \\ x, & \text{if } x < 1 \end{cases}$, then show that f is not differentiable at $x = 1$. **[NCERT Part-I, Page 119]**

OR

Find the value(s) of 'λ', if the function **[NCERT Part-I, Page 105]**

$$f(x) = \begin{cases} \frac{\sin^2 \lambda x}{x^2}, & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$ is continuous at $x = 0$.

25. Find the vector and the cartesian equations of a line that passes through the point A(1, 2, –1) and parallel to the line $5x - 25 = 14 - 7y = 35z$. **[Conceptual Application]**

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. Find the area of the following region using integration: **[Conceptual Application]**

$\{(x, y) : y^2 \le 2x \text{ and } y \ge x - 4\}$

27. Find the coordinates of the foot of the perpendicular drawn from the point $P(0, 2, 3)$ to the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. **[Conceptual Application]**

OR

Three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Evaluate the quantity $\mu = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$, if $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{c}| = 2$. **[Conceptual Application]**

28. Evaluate: $\int_0^{2\pi} \frac{1}{1 + e^{\sin x}} dx$ **[NCERT Part-II, Page 273-274]**

OR

Find: $\int \frac{x^4}{(x-1)(x^2+1)} dx$ **[NCERT Part-II, Page 253]**

29. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$, then show that $A^3 - 23A - 40I = O$. **[Conceptual Application]**

30. Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$. **[Integrated Question]**

OR

If $y = \tan x + \sec x$, then prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1 - \sin x)^2}$. **[NCERT Part-I, Page 137]**

31. Find the distance between the lines: **[NCERT Part-II, Page 386-387]**

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$;

$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. What is the probability that the student knows the answer, given that he answered it correctly? **[NCERT Part-II, Page 425]**

OR

A shopkeeper sells three types of seeds A_1, A_2 and A_3. They are sold as a mixture in proportion 3 : 3 : 4 respectively. The germination rate of these types of seeds are 85%, 70% and 50% respectively. Find the probability (*i*) that seed will germinate (*ii*) It is of type A_3 given randomly chosen seed does not germinate.

[NCERT Part-II, Page 425]

33. Solve the following Linear Programming Problem graphically: **[NCERT Part-II, Page 397-398]**

Maximise: $P = 70x + 40y$

subject to:

$$3x + 2y \leq 9,$$
$$3x + y \leq 9,$$
$$x \geq 0, y \geq 0$$

34. Evaluate : $\int_0^{\pi/2} \sin 2x \tan^{-1}(\sin x)\, dx$ **[Integrated Question]**

35. The median of an equilateral triangle is increasing at the rate of $2\sqrt{3}$ cm/s. Find the rate at which its side is increasing. **[NCERT Part-I, Page 147-148]**

OR

Sum of two numbers is 5. If the sum of the cubes of these numbers is least, then find the sum of the squares of these numbers. **[Conceptual Application]**

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of ₹ 160. From the same shop, Vikram buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of ₹ 190. Also Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of ₹ 250. **[Conceptual Application]**

Based on the above information, answer the following questions:

(*i*) Convert the given above situation into a matrix equation of the form AX = B.

(*ii*) Find $|A|$.

(*iii*) Find A^{-1}.

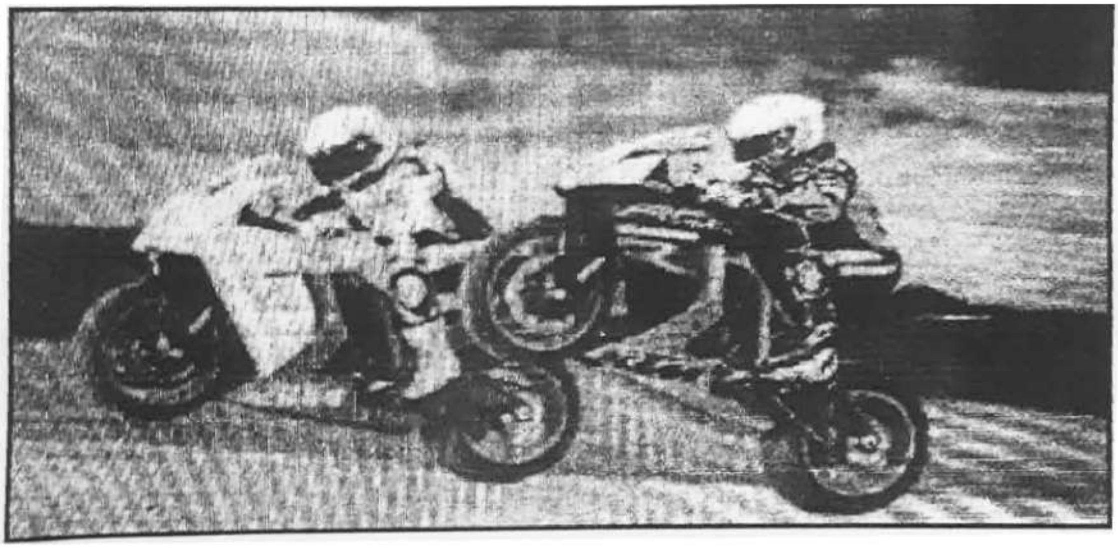
OR

(*iii*) Determine $P = A^2 - 5A$.

Case Study - 2

37. An organization conducted bike race under two different categories – Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race. **[Conceptual Application]**



Based on the above information, answer the following questions :

(*i*) How many relations are possible from B to G?

(*ii*) Among all the possible relations from B to G, how many functions can be formed from B to G ?

(*iii*) Let R : B $\rightarrow$ B be defined by R = $\{(x, y) : x$ and y are students of the same sex). Check if R is an equivalence relation.

OR

(*iii*) A function f : B $\rightarrow$ G be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$.
Check if f is bijective. Justify your answer.

Case Study - 3

38. An equation involving derivatives of the dependent variable with respect to the independent variables is called a differential equation. A differential equation of the form $\frac{dy}{dx} = F(x, y)$ is said to be homogeneous if $F(x, y)$ is a homogeneous function of degree zero, whereas a function $F(x, y)$ is a homogeneous function of degree n if $F(\lambda x, \lambda y) = \lambda^n F(x, y)$. To solve a homogeneous differential equation of the type $\frac{dy}{dx} = F(x, y) = g\left(\frac{y}{x}\right)$, we make the substitution $y = vx$ and then separate the variables.

[NCERT Part-II, Page 312-314]

Based on the above, answer the following questions:

(*i*) Show that $(x^2 - y^2)dx + 2xy\, dy = 0$ is a differential equation of the type $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$.

(*ii*) Solve the above equation to find its general solution.

SOLUTIONS

1. (*c*), $$A^2 - A + I = O$$

Premultiplying both sides with A^{-1},

$$A^{-1}(A^2 - A + I) = A^{-1}.O$$

$$\Rightarrow \quad A^{-1}A^2 - A^{-1}A + A^{-1}I = O$$

$$\Rightarrow \quad (A^{-1}\cdot A)\cdot A - I + A^{-1} = O$$

$$\Rightarrow \quad IA - I + A^{-1} = O \qquad [A^{-1}\cdot A = A\cdot A^{-1} = I]$$

$$A - I + A^{-1} = O \qquad [I\cdot A = A\cdot I = A]$$

$$A^{-1} = I - A$$

2. (*c*), $$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$$

Given $$A = B^2$$

Now, $$B^2 = B\cdot B = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}\cdot\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} x^2 & 0 \\ x+1 & 1 \end{bmatrix}$$

Compare with A, we get

$$\begin{bmatrix} x^2 & 0 \\ x+1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$x^2 = 1 \quad \Big| \quad x + 1 = 2$$

$$\Rightarrow \quad x = \pm 1 \quad \Big| \quad x = 1$$

Hence $x = 1$

3. (*b*), n = number of elements in a set

Number of reflexive relations = $2^{n^2 - n}$

Here $n = 2$

Number of reflexive relations = $2^{2^2 - 2} = 2^{4-2} = 2^2 = 4$

4. (*a*), $\sin\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right] = \sin\left[\frac{\pi}{3} + \frac{\pi}{6}\right] = \sin\left(\frac{2\pi + \pi}{6}\right) = \sin\left(\frac{3\pi}{6}\right) = \sin\left(\frac{\pi}{2}\right) = 1$

5. (*d*), $\begin{vmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 0$

$\Rightarrow \alpha\,(2 \times 1 - 4 \times 1) - 3(1 \times 1 - 1 \times 1) + 4(1 \times 4 - 2 \times 1) = 0$

$$\Rightarrow \quad \alpha \times (-2) - 3 \times 0 + 4 \times 2 = 0$$

$$\Rightarrow \quad -2\alpha - 0 + 8 = 0$$

$$\Rightarrow \quad -2\alpha = -8$$

$$\Rightarrow \quad \alpha = \frac{8}{2} = 4$$

6. (*b*), $$f(x) = 2x^3 + 9x^2 + 12x - 1$$

$$f'(x) = 6x^2 + 18x + 12$$

For critical points,

$$f'(x) = 0$$

$$6x^2 + 18x + 12 = 0$$

$$\Rightarrow \quad x^2 + 3x + 2 = 0$$

$$\Rightarrow \quad (x + 2)(x + 1) = 0$$

$$\Rightarrow \quad x = -1, -2$$

Interval	sign of $f'(x)$	Nature of 'f'
$(-\infty, -2)$	+	Increasing
$(-2, -1)$	–	Decreasing
$(-1, \infty)$	+	Increasing

So, $f(x)$ is decreasing at $(-2, -1)$.

7. (*d*), $$x = A\cos 4t + B\sin 4t$$

Differentiating with respect to t, we get

$$\frac{dx}{dt} = -A \cdot 4(\sin 4t) + B \cdot 4\cos 4t$$

Again differentiating w.r.t. t, we get

$$\frac{d^2x}{dt^2} = -A \cdot 16(\cos 4t) + B \cdot 16(-\sin 4t)$$

$$= -16\{A\cos 4t + B\sin 4t)$$

$$= -16x$$

8. (*b*), The function $f(x) = [x]$ is continuous for all x except the integral values of x.

Hence it is continuous at $x = 1.5$ which is not an integer.

9. (*c*) Let $$p = x^{2x}$$

Taking log on both sides, we get

$$\log p = \log x^{2x}$$

$$\Rightarrow \quad \log p = 2x\log x \qquad [\because \log m^n = n\log m]$$

Differentiating with respect to 'x', we get

$$\frac{1}{p}\frac{dp}{dx} = 2\log x + 2x \times \frac{1}{x}$$

$$\Rightarrow \quad \frac{1}{p}\frac{dp}{dx} = 2\log x + 2$$

$$\Rightarrow \quad \frac{dp}{dx} = p \cdot 2(1 + \log x)$$

$$\Rightarrow \quad \frac{dp}{dx} = 2 \cdot x^{2x}(1 + \log x)$$

Required answer is $2x^{2x}(1 + \log x)$.

10. (*b*), $\int \frac{\sec x}{\sec x - \tan x}dx = \int \frac{\frac{1}{\cos x}}{\frac{1}{\cos x} - \frac{\sin x}{\cos x}}dx = \int \frac{1}{1-\sin x}dx$

On multiplying numerator and denominator of integrand by $(1 + \sin x)$, we get

$$\int\left(\frac{1}{1-\sin x} \times \frac{1+\sin x}{1+\sin x}\right)dx$$

$$= \int\left(\frac{1+\sin x}{1-\sin^2 x}\right)dx = \int\left(\frac{1+\sin x}{\cos^2 x}\right)dx$$

$$= \int\left(\frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}\right)dx$$

$$= \int (\sec^2 x + \sec x \tan x)\,dx$$

$$= \int \sec^2 x\,dx + \int \sec x \tan x\,dx$$

$$= \tan x + \sec x + C$$

11. (*a*),

$$\alpha = 90°$$

$$\beta = 135°$$

$$\gamma = 45°$$

$$\text{Direction cosines} = \langle\cos\alpha, \cos\beta, \cos\gamma\rangle$$

$$= \langle\cos 90°, \cos 135°, \cos 45°\rangle$$

$$= \left\langle 0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\rangle$$

12. (*d*), Direction ratios of line $2x = 3y = -z$ which can be written as $\frac{x}{1/2} = \frac{y}{1/3} = \frac{z}{-1}$ are $\left\langle\frac{1}{2}, \frac{1}{3}, -1\right\rangle = \langle a_1, b_1, c_1\rangle$ and direction ratios of line $6x = -y = -4z$ which can be written as $\frac{x}{1/6} = \frac{y}{-1} = \frac{z}{-1/4}$ are $\left\langle\frac{1}{6}, -1, \frac{-1}{4}\right\rangle = \langle a_2, b_2, c_2\rangle$

Now, $a_1a_2 + b_1b_2 + c_1c_2 = \frac{1}{2}\times\frac{1}{6} + \frac{1}{3}\times(-1) + (-1)\left(\frac{-1}{4}\right) = \frac{1}{12} + \frac{-1}{3} + \frac{1}{4} = 0$

So, angle between the lines is 90°.

13. (*b*),

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

If $\vec{a}$ and $\vec{b}$ are collinear, then

$$\vec{a} \parallel \vec{b}$$

$$\Rightarrow \quad \frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$

14. (*c*), Let $\vec{v} = 6\hat{i} - 2\hat{j} + 3\hat{k}$

Magnitude of given vector is

$$|\vec{v}| = \sqrt{a^2 + b^2 + c^2}$$
$$|\vec{v}| = \sqrt{(6)^2 + (-2)^2 + (3)^2}$$
$$= \sqrt{36 + 4 + 9}$$
$$= \sqrt{49} = 7$$

15. (*d*), $\int_{-1}^{1} \frac{|x-2|}{x-2} dx$

For $-1 \le x \le 1$, $|x - 2| = 2 - x$

$$\int_{-1}^{1} \frac{2-x}{x-2} dx = -\int_{-1}^{1} 1 dx = -[x]_{-1}^{1}$$
$$= -[1 - (-1)] = -2$$

16. (*b*), $$\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^3\right) = 0$$

$\Rightarrow$ $$3\left(\frac{dy}{dx}\right)^2 \cdot \frac{d^2y}{dx^2} = 0$$

So, order = 2; degree = 1. Required sum = 3

17. (*c*), $$P(A) = \frac{4}{5}$$
$$P(A \cap B) = \frac{7}{10}$$
$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$
$$= \frac{7/10}{4/5} = \frac{7}{10} \times \frac{5}{4} = \frac{7}{8}$$

18. (*c*) Sample space = $2^5 = 32$

Total outcomes = 32

A is the event that at least one head turns up.

Each outcomes having probability of occurrence as $\frac{1}{32}$.

Then A^c is the event that no head turns up thus A^c consist of only one outcome.

Hence $P(A^c) = \frac{1}{32}$

Now, $P(A) = 1 - P(A^c) = 1 - \frac{1}{32} = \frac{31}{32}$

19. (*a*), $$I = \int_{2}^{8} \frac{\sqrt{10-x}}{\sqrt{x} + \sqrt{10-x}} dx \quad ...(i)$$

$$= \int_{2}^{8} \frac{\sqrt{10-(2+8-x)}}{\sqrt{10-x} + \sqrt{10-(2+8-x)}} dx \qquad \left[\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right]$$

$$= \int_{2}^{8} \frac{\sqrt{10-10+x}}{\sqrt{10-x} + \sqrt{10-10+x}} dx$$

So, $$I = \int_{2}^{8} \frac{\sqrt{x}}{\sqrt{10-x} + \sqrt{x}} dx \quad ...(ii)$$

Adding (*i*) and (*ii*), we get

$$2I = \int_2^8 \frac{\sqrt{10-x}+\sqrt{x}}{\sqrt{10-x}+\sqrt{x}}dx$$

$$\Rightarrow \quad 2I = \int_2^8 1 \cdot dx$$

$$\Rightarrow \quad 2I = [x]_2^8$$

$$\Rightarrow \quad 2I = 8 - 2$$

$$\Rightarrow \quad 2I = 6$$

$$\Rightarrow \quad I = \frac{6}{2} = 3$$

20. (*a*), $S = \{(H, H), (T, T), (T, H), (H, T)\}$

Let us define the events.

A : getting at least one head.

B : getting both heads.

Now, $A = \{(H, T), (T, H), (H, H)\}$,

$B = \{H, H\}, A \cap B = \{H, H\}$

$$P(\text{at least 1 head}) = \frac{3}{4} = P(A)$$

$$P(A \cap B) = \frac{1}{4}$$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

21. The given equations are:

$$2x + y = 8 \quad ...(i)$$

$$y = 2 \quad ...(ii)$$

$$y = 4 \quad ...(iii)$$

$$x = 0 \quad ...(iv)$$

Consider equation (*i*),

$$2x + y = 8$$

$$\Rightarrow \quad \frac{x}{4} + \frac{y}{8} = 1$$

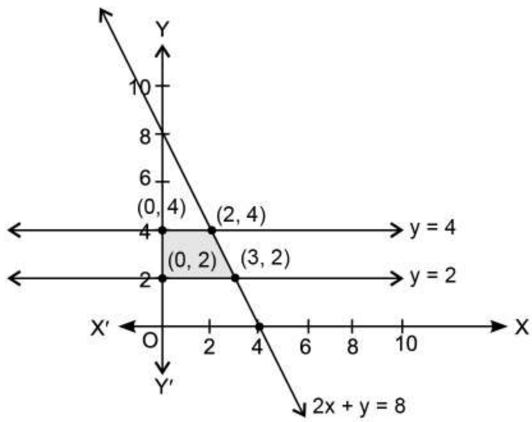
It represents a straight line intersecting x-axis at (4, 0) and y-axis at (0, 8).

Now, equations (*ii*) and (*iii*) represent a straight line parallel to x-axis at a distance of 2 units and 4 units respectively above x-axis.

Equation (*iv*) represents y-axis.

Now, points of intersection are:

(0, 4), (0, 2), (2, 4) and (3, 2).



$$\text{Required area} = \int_2^4 \left(\frac{8-y}{2}\right) dy$$

$$= \frac{1}{2}\left[8y - \frac{y^2}{2}\right]_2^4$$

$$= \frac{1}{2}\left[32 - \frac{16}{2} - 16 + \frac{4}{2}\right]$$

$$= \frac{1}{2}[10] = 5 \text{ sq units}$$

22. We have, $|\vec{a}| = 3 \quad ; \quad |\vec{b}| = \frac{2}{3}$

Now, $|\vec{a} \times \vec{b}| = 1$

Also, $|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \sin\theta$ [θ = angle between $\vec{a}$ and $\vec{b}$]

$\Rightarrow \quad 1 = 3 \times \frac{2}{3} \sin\theta$

$\Rightarrow \quad \sin\theta = \frac{1}{2}$

$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow \quad \theta = \frac{\pi}{6}$

Angle between $\vec{a}$ and $\vec{b}$ is 30°.

OR

Adjacent sides of parallelogram are given by the vectors

$\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$$

$$= 20\hat{i} + 5\hat{j} - 5\hat{k}$$

Now, $|\vec{a} \times \vec{b}| = \sqrt{(20)^2 + (5)^2 + (-5)^2}$

$$= \sqrt{450} = 15\sqrt{2}$$

Hence, area of parallelogram is $15\sqrt{2}$ sq units.

23. Let $y = \tan x$

Domain of function $y = \tan x$ is

$$x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$$

Range of function of $y = \tan x$ is

$$y \in (-\infty, \infty)$$

The function $y = \tan^{-1}x$ is symmetric to the function $y = \tan x$ with respect to line $y = x$.

Therefore, the domain of $\tan^{-1} x$ is R and range is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

24.

$$f'(x) = \begin{cases} 2x, & x \geq 1 \\ 1, & x < 1 \end{cases}$$

$$\text{LHD (at } x = 1) = \lim_{x \to 1^-} 1 = 1$$

$$\text{RHD (at } x = 1) = \lim_{x \to 1^+} 2x = 2 \times 1 = 2$$

So, $\quad \text{LHD} \neq \text{RHD at } x = 1$

$\therefore$ Function is not differentiable at $x = 1$.

OR

As, f is continuous at $x = 0$,

then, $$f(0) = \lim_{x \to 0} \frac{\sin^2 \lambda x}{x^2} \quad ...(i)$$

Now, $$\lim_{x \to 0} \frac{\sin^2 \lambda x}{x^2} = \lim_{x \to 0} \left[\frac{\sin(\lambda x)}{(\lambda x)}\right]^2 \times \lambda^2$$

$$= 1 \times \lambda^2 = \lambda^2 \quad \left\{\because \lim_{x \to 0} \frac{\sin x}{x} = 1\right\}$$

Now, from equation (i), we get

$$\lambda^2 = 1 \quad [\because f(0) = 1]$$

$$\Rightarrow \quad \lambda = \pm 1$$

25. The equation of the given line is,

$$5x - 25 = 14 - 7y = 35z$$

$$\Rightarrow \quad \frac{x-5}{\frac{1}{5}} = \frac{y-2}{\frac{-1}{7}} = \frac{z}{\frac{1}{35}}$$

$$\Rightarrow \quad \frac{x-5}{7} = \frac{y-2}{-5} = \frac{z}{1}$$

So, dr's of given line = <7, –5, 1>

Now, the required line is parallel to given line, so dr's of required line will become proportional to the dr's of given line.

Now, cartesian equation of a line passing through the point (x_1, y_1, z_1) and having dr's proportional to $<a, b, c>$ is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

∴ Cartesian equation of required line is,

$$\frac{x-1}{7} = \frac{y-2}{-5} = \frac{z+1}{1} \qquad [\because (x_1, y_1, z_1) = (1, 2, -1)]$$

In vector form:

$$\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(7\hat{i} - 5\hat{j} + \hat{k})$$

26. The given equations are: $y^2 = 2x$ and $y = x - 4$

Points of intersection:

$$(x-4)^2 = 2x$$

$$\Rightarrow \quad x^2 + 16 - 8x = 2x$$

$$\Rightarrow \quad x^2 - 10x + 16 = 0$$

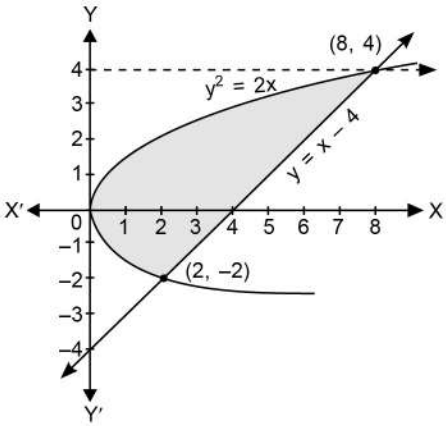
$$\Rightarrow \quad (x-8)(x-2) = 0$$

$$\Rightarrow \quad x = 8, 2$$

So, $y = 8 - 4$ or $2 - 4$

$$y = 4, -2$$

Points of intersection are (8, 4) and (2, –2).



$$\text{Required area} = \int_{-2}^{4}\left(y + 4 - \frac{1}{2}y^2\right)dy$$

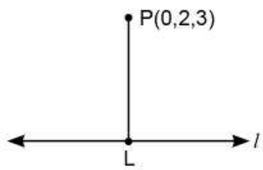
$$= \frac{1}{2}\int_{-2}^{4}(2y + 8 - y^2)\,dy$$

$$= \frac{1}{2}\left[y^2 + 8y - \frac{y^3}{3}\right]_{-2}^{4}$$

$$= \frac{1}{2}\left[16 + 32 - \frac{64}{3} - 4 + 16 - \frac{8}{3}\right]$$

$$= \frac{1}{2}\left[60 - \frac{72}{3}\right] = \frac{1}{2} \times 36 = 18 \text{ sq units}$$

27. Let L be the foot of the ⊥ drawn from the point $P(0, 2, 3)$ on the line 'l' given by $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$



Let $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} = t$

$\Rightarrow \quad x = 5t - 3, y = 2t + 1, z = 3t - 4$

Suppose the coordinates of any general point on line 'l' is

$(5t - 3, 2t + 1, 3t - 4)$

Let coordinates of L be $(5t - 3, 2t + 1, 3t - 4)$

DR's of PL = <5t – 3 – 0, 2t + 1 – 2, 3t – 4 – 3>

= <5t – 3, 2t – 1, 3t – 7>

DR's of line 'l' are <5, 2, 3>.

Since $PL \perp l$, then

$5(5t - 3) + 2(2t - 1) + 3(3t - 7) = 0$

$\Rightarrow 25t - 15 + 4t - 2 + 9t - 21 = 0$

$\Rightarrow \quad 38t = 38 \quad \Rightarrow \quad t = 1$

∴ Coordinates of $L(5 - 3, 2 + 1, 3 - 4) = (2, 3, -1)$

OR

As, $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $\mu=\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$

Now, $$(\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}+\vec{b}+\vec{c})=\vec{0}\cdot\vec{0}$$

$$\Rightarrow \quad |\vec{a}|^2+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{a}+|\vec{b}|^2+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}+\vec{c}\cdot\vec{b}+|\vec{c}|^2=0$$

$$\Rightarrow \quad 9+2\vec{a}\cdot\vec{b}+2\vec{a}\cdot\vec{c}+2\vec{b}\cdot\vec{c}+16+4=0 \qquad [\because |\vec{a}|=3, |\vec{b}|=4, |\vec{c}|=2]$$

$$\Rightarrow \quad 2\vec{a}\cdot\vec{b}+2\vec{a}\cdot\vec{c}+2\vec{b}\cdot\vec{c}=-16-4-9 \qquad \left[\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}, \vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{b}, \vec{a}\cdot\vec{c}=\vec{c}\cdot\vec{a}\right]$$

$$\Rightarrow \quad \vec{a}\cdot\vec{b}+\vec{c}\cdot\vec{a}+\vec{b}\cdot\vec{c}=-\frac{29}{2}=\mu$$

28. $I=\int_0^{2\pi}\frac{1}{1+e^{\sin x}}dx$ ...(*i*)

Apply property:

$$\int_0^a f(x)dx=\int_0^a f(a-x)dx$$

$$I=\int_0^{2\pi}\frac{dx}{1+e^{\sin(2\pi-x)}}=\int_0^{2\pi}\frac{dx}{1+e^{-\sin x}}$$

$$I=\int_0^{2\pi}\frac{dx}{1+\frac{1}{e^{\sin x}}}$$

$$I=\int_0^{2\pi}\frac{e^{\sin x}}{e^{\sin x}+1}dx \qquad ...(ii)$$

Adding (*i*) and (*ii*), we get

$$2I=\int_0^{2\pi}\frac{dx}{1+e^{\sin x}}+\int_0^{2\pi}\frac{e^{\sin x}}{1+e^{\sin x}}dx=\int_0^{2\pi}\frac{1+e^{\sin x}}{1+e^{\sin x}}dx=\int_0^{2\pi}dx=[x]_0^{2\pi}$$

$$\Rightarrow \quad 2I=2\pi$$

$$\Rightarrow \quad I=\frac{2\pi}{2} \Rightarrow I=\pi$$

OR

$$I=\int\frac{x^4}{(x-1)(x^2+1)}dx$$

As numerator is having a degree greater than denominator we need to divide the fraction taking denominator as division.

$$(x-1)(x^2+1)=x^3-x^2+x-1$$

Therefore, $$\frac{x^4}{x^3-x^2+x-1}=(x+1)+\frac{1}{x^3-x^2+x-1}=(x+1)+\frac{1}{(x^2+1)(x-1)}$$

Let $$\frac{1}{(x-1)(x^2+1)}=\frac{A}{(x-1)}+\frac{Bx+C}{x^2+1}$$

$$\Rightarrow \quad 1=A(x^2+1)+(Bx+C)(x-1) \qquad ...(i)$$

Put $x=1$, in (*i*), we get

$$1=2A \Rightarrow A=\frac{1}{2}$$

Put $x=-1$ in (*i*), we get

$$1 = 2A - 2(-B + C)$$

$\Rightarrow$ $$1 = 1 - 2(C - B) \qquad \left(\because A = \frac{1}{2}\right)$$

So, $$B = C$$

Put $x = 0$ in (*i*), we get

$$1 = A - C$$

$\Rightarrow$ $$1 = \frac{1}{2} - C \Rightarrow C = \frac{-1}{2}$$

So, $$B = C = \frac{-1}{2}$$

Now, $$\frac{1}{(x^2+1)(x-1)} = \frac{\frac{1}{2}}{x-1} + \frac{\frac{-1}{2}x - \frac{1}{2}}{x^2+1}$$

$$= \frac{1}{2}\left(\frac{1}{x-1}\right) - \frac{1}{2}\left(\frac{x+1}{x^2+1}\right) = \frac{1}{2}\left(\frac{1}{x-1}\right) - \frac{1}{2}\left(\frac{x}{x^2+1}\right) - \frac{1}{2}\left(\frac{1}{x^2+1}\right)$$

$\therefore$ $$I = \int\left\{(x+1) + \frac{1}{(x-1)(x^2+1)}\right\}dx$$

$$= \int(x+1)\,dx + \frac{1}{2}\int\frac{1}{x-1}dx - \frac{1}{2}\int\frac{x}{x^2+1}dx - \frac{1}{2}\int\frac{1}{x^2+1}dx$$

$$= \frac{x^2}{2} + x + \frac{1}{2}\log|x-1| - \frac{1}{2}\frac{\log|x^2+1|}{2} - \frac{1}{2}\tan^{-1}x + C$$

$$= \frac{x^2}{2} + x + \frac{1}{2}\log|x-1| - \frac{1}{4}\log|x^2+1| - \frac{1}{2}\tan^{-1}x + C$$

29. $$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

$$A^2 = A \cdot A$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}$$

$$A^2 \cdot A = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$$

$$A^3 - 23A - 40I = O$$

$$\text{LHS} = A^3 - 23A - 40I = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - 23\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} - 40\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - \begin{bmatrix} 23 & 46 & 69 \\ 69 & -46 & 23 \\ 92 & 46 & 23 \end{bmatrix} - \begin{bmatrix} 40 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 40 \end{bmatrix}$$

$$= \begin{bmatrix} 63-23-40 & 46-46 & 69-69 \\ 69-69 & -6+46-40 & 23-23 \\ 92-92 & 46-46 & 63-23-40 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O = \text{RHS}$$ Hence proved

30. Let $u = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$,

put $x = \sin\theta$

$\therefore$ $u = \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right) = \sec^{-1}\left(\frac{1}{\cos\theta}\right) = \sec^{-1}(\sec\theta) = \theta$

So, $u = \sin^{-1} x$

$\Rightarrow$ $\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$

Let $v = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

put $x = \sin\theta$

So, $v = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$

$= \sin^{-1}(2\sin\theta\cos\theta) = \sin^{-1}(\sin 2\theta) = 2\theta$

$\therefore$ $v = 2\sin^{-1} x$

$\Rightarrow$ $\frac{dv}{dx} = \frac{2}{\sqrt{1-x^2}}$

Now, $\frac{du}{dx} \times \frac{dx}{dv} = \frac{1}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{2}$

$\frac{du}{dv} = \frac{1}{2}$

OR

We have, $y = \tan x + \sec x$

$\Rightarrow$ $y = \frac{\sin x}{\cos x} + \frac{1}{\cos x}$

$\Rightarrow$ $y = \frac{1+\sin x}{\cos x}$

Differentiating both sides with respect to x, we get

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1+\sin x}{\cos x}\right)$

$\Rightarrow$ $\frac{dy}{dx} = \frac{\cos x \frac{d}{dx}(1+\sin x) - (1+\sin x)\frac{d}{dx}(\cos x)}{\cos^2 x}$

$= \frac{\cos^2 x + \sin x + \sin^2 x}{\cos^2 x} = \frac{1+\sin x}{\cos^2 x}$

$$= \frac{1+\sin x}{1-\sin^2 x} = \frac{1}{(1-\sin x)}$$

Again differentiating both sides w.r.t. x, we get

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{1}{1-\sin x}\right)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{(1-\sin x)\frac{d}{dx}(1) - 1\frac{d}{dx}(1-\sin x)}{(1-\sin x)^2}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$$

Hence proved

31. We have, $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

and $\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$

$\Rightarrow$ $\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu'(2\hat{i} + 3\hat{j} + 6\hat{k})$

Where $\mu' = 2\mu$

Now, $\vec{a_1} = \hat{i} + 2\hat{j} - 4\hat{k};\ \vec{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$

$\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$\vec{a_2} - \vec{a_1} = 2\hat{i} + \hat{j} - \hat{k}$

Since, given lines are parallel, then shortest distance between them is given by,

S.D., $$d = \left|\frac{\vec{b} \times (\vec{a_2} - \vec{a_1})}{|\vec{b}|}\right| = \left|\frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 2 & 1 & -1 \end{vmatrix}}{\sqrt{4+9+36}}\right|$$

$$= \left|\frac{-9\hat{i} + 14\hat{j} - 4\hat{k}}{\sqrt{49}}\right| = \frac{\sqrt{293}}{\sqrt{49}} = \frac{\sqrt{293}}{7} \text{ units}$$

32. Let us define the events:

A : Student knows answer

G : Student guesses answer

B : Student answers correctly

$$P(A) = \frac{3}{5},\ P(G) = \frac{2}{5},\ P(B/G) = \frac{1}{3}$$

$$P(B/A) = 1$$

$$P(A|B) = ?$$

According to Bayes' theorem

$$P(A/B) = \frac{P(B/A)\cdot P(A)}{P(B/A)\cdot P(A) + P(B/G)\cdot P(G)} = \frac{1\times(3/5)}{1\times\left(\frac{3}{5}\right)+\left(\frac{1}{3}\right)\left(\frac{2}{5}\right)}$$

$$= \frac{3/5}{\frac{1}{5}\times\left(3+\frac{2}{3}\right)} = \frac{3}{11/3} = \frac{9}{11}$$

So, $P(A/B) = \frac{9}{11}$

OR

$P(A_1) = \frac{3}{10}, P(A_2) = \frac{3}{10}, P(A_3) = \frac{4}{10}$

A : seed germinates

$P\left(\frac{A}{A_1}\right) = \frac{85}{100}, P\left(\frac{A}{A_2}\right) = \frac{70}{100}, P\left(\frac{A}{A_3}\right) = \frac{50}{100}$

(*i*) $P(\text{chosen seed germinates}) = P(A_1)\, P\left(\frac{A}{A_1}\right) + P(A_2)\, P\left(\frac{A}{A_2}\right) + P(A_3)\, P\left(\frac{A}{A_3}\right)$

$$= \frac{3}{10} \times \frac{85}{100} + \frac{3}{10} \times \frac{70}{100} + \frac{4}{10} \times \frac{50}{100}$$

$$= \frac{255 + 210 + 200}{1000} = \frac{665}{1000} = 0.665$$

(*ii*) $P\left(\frac{A_3}{\overline{A}}\right) = \frac{P(A_3 \cap \overline{A})}{P(\overline{A})} = \frac{P(P_3)\, P\left(\frac{\overline{A}}{A_3}\right)}{P(\overline{A})}$

$$= \frac{P(A_3)\,[1 - P\left(\frac{A}{A_3}\right)]}{1 - P(A)}$$

$$= \frac{\frac{4}{10}\left[1 - \frac{5}{10}\right]}{1 - 0.665} = \frac{0.2}{0.335} = \frac{200}{335} = \frac{40}{67}.$$

33. $3x + 2y \leq 9$

$3x + y \leq 9$

$x \geq 0$ and $y \geq 0$

Objective function P = $70x + 40y$ we have to maximise P.

Consider $3x + 2y \leq 9$ ...(*i*)

Let $3x + 2y = 9$

x	0	3
y	9/2	0

Put $x = 0$ and $y = 0$ in above inequality (*i*),

$0 \leq 9$, true

Region containing origin represents solution set of $3x + 2y \leq 9$.

Consider $3x + y \leq 9$ ...(*ii*)

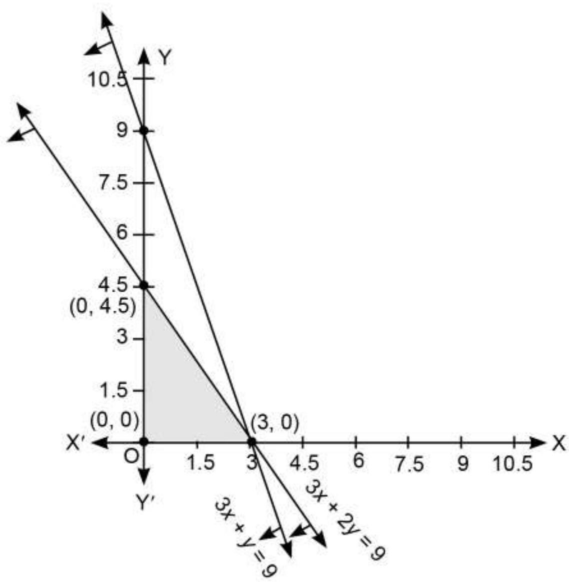
Let $3x + y = 9$

x	0	3
y	9	0

Put $x = 0$ and $y = 0$ in above inequality (*ii*),

$0 \leq 9$, true

Region containing origin represents solution set of $3x + y \leq 9$.



Corner points	Value of $P = 70x + 40y$
(0, 0)	0
(0, 4.5)	180
(3, 0)	210 ← Maximum

Maximum occurs when $x = 3$ and $y = 0$, *i.e.,* at (3, 0).

34. Let $$I = \int_0^{\frac{\pi}{2}} \sin 2x \cdot \tan^{-1}(\sin x) dx$$

$$= \int_0^{\frac{\pi}{2}} 2\sin x \cdot \cos x \cdot \tan^{-1}(\sin x) dx$$

put $\sin x = t \Rightarrow \cos x\, dx = dt$

As $x \to 0, t \to 0$

As $x \to \frac{\pi}{2}, t \to 1$

Now, $\int_0^{\pi/2} 2\sin x \cos x \tan^{-1}(\sin x) dx = \int_0^1 2t \tan^{-1}(t)\, dt$

$$= \left[2\tan^{-1}(t)\frac{t^2}{2}\right]_0^1 - \int_0^1 \left[\frac{1}{1+t^2} \times \frac{2t^2}{2}\right] dt$$

$$= \left[\frac{\pi}{4} - 0\right] - \int_0^1 \frac{t^2}{1+t^2} dt = \frac{\pi}{4} - \int_0^1 \frac{t^2}{1+t^2} dt$$

$$= \frac{\pi}{4} - \left[\int_0^1 \left(\frac{t^2+1}{t^2+1}\right) dt - \int_0^1 \frac{1}{1+t^2} dt\right]$$

$$= \frac{\pi}{4} - \left[\int_0^1 1\, dt - \left[\tan^{-1}(t)\right]_0^1\right]$$

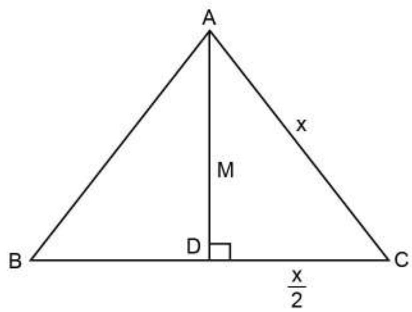
$$= \frac{\pi}{4} - \left[[t]_0^1 - \left(\frac{\pi}{4} - 0\right)\right] = \frac{\pi}{4} - 1 + \frac{\pi}{4} = \frac{\pi}{2} - 1$$

35. Let ABC be an equilateral Δ and let AD be median to side BC.

Now, altitude and median of an equilateral Δ always coincide.

Let 'x' be the side and 'M' be the length of median AD.

The perpendicular drawn from vertex of the equilateral Δ to the opposite side always bisect the side. So, $BD = DC = \frac{x}{2}$



In ΔADC, by Pythagoras Theorem

$$M^2 = (x)^2 - \left(\frac{x}{2}\right)^2$$

So, $M = \frac{\sqrt{3}x}{2}$

Differentiating with respect to 't', we get

$$\frac{dM}{dt} = \frac{\sqrt{3}}{2}\frac{dx}{dt}$$

$$\Rightarrow \quad \frac{dx}{dt} = \frac{2}{\sqrt{3}}\frac{dM}{dt} = \frac{2}{\sqrt{3}} \times 2\sqrt{3} = 4$$

So, side is increasing at the rate of 4 cm/s.

OR

Let the numbers be x and $5 - x$ (as sum of two numbers is 5).

According to question,

We have to minimise the sum of cubes of these numbers.

$$x^3 + (5-x)^3 = y \text{ (say)}$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = 3x^2 + 3(5-x)^2(-1)$$
$$= 3x^2 - 3(25 + x^2 - 10x)$$
$$= 3x^2 - 75 - 3x^2 + 30x$$
$$= 30x - 75$$

For maximum or minimum value,

$$\frac{dy}{dx} = 0 \Rightarrow -75 + 30x = 0$$

$$\Rightarrow \quad 30x = 75$$

$$\Rightarrow \quad x = \frac{75}{30} = \frac{5}{2}$$

$$\frac{d^2y}{dx^2} = 30$$

$$\Rightarrow \quad \left[\frac{d^2y}{dx^2}\right]_{x=\frac{5}{2}} = 30 > 0$$

As, $30 > 0$, then the minima occurs at $x = \frac{5}{2}$ and the other number is also $\frac{5}{2}$.

Sum of squares $= x^2 + y^2 = \left(\frac{5}{2}\right)^2 + \left(\frac{5}{2}\right)^2 = \frac{25}{4} + \frac{25}{4} = \frac{50}{4} = \frac{25}{2}$

36. (*i*) Let the purchase of pens, bags and instrument boxes be represented by matrix A. So,

$$A = \begin{matrix} & \text{Pen} & \text{Bags} & \text{Instrument Boxes} & \\ & \left[\begin{matrix} 5 \\ 2 \\ 1 \end{matrix}\right. & \begin{matrix} 3 \\ 1 \\ 2 \end{matrix} & \left.\begin{matrix} 1 \\ 3 \\ 4 \end{matrix}\right] & \begin{matrix} \rightarrow \text{Gautam} \\ \rightarrow \text{Vikram} \\ \rightarrow \text{Ankur} \end{matrix} \end{matrix}$$

Let $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

where x = price of 1 pen

y = price of 1 bag

z = price of 1 instrument box

$$\text{Now,} \quad B = \begin{bmatrix} 160 \\ 190 \\ 250 \end{bmatrix} \begin{matrix} \rightarrow \text{Gautam} \\ \rightarrow \text{Vikram} \\ \rightarrow \text{Ankur} \end{matrix} \quad \textbf{Money spent (in ₹)}$$

Now, $AX = B$

$$\Rightarrow \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 160 \\ 190 \\ 250 \end{bmatrix}$$

(*ii*) Det $A = |A| = 5(-2) - 3(5) + 3$

$$= -2 \times 5 - 15 + 3$$
$$= -10 - 15 + 3$$
$$= -22$$

(*iii*) Let A_{ij} be the cofactors of a_{ij} in $|A|$. Then,

$A_{11} = -2, \quad A_{12} = -5, \quad A_{13} = 3,$

$A_{21} = -10, \quad A_{22} = 19, \quad A_{23} = -7,$

$A_{31} = 8, \quad A_{32} = -13, \quad A_{33} = -1,$

$$Adj\,(A) = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$$

Now, $A^{-1} = \dfrac{1}{|A|} adjA$

$$A^{-1} = \frac{-1}{22} \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$$

OR

(*iii*) $$A^2 = A \cdot A = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 25+6+1 & 15+3+2 & 5+9+4 \\ 10+2+3 & 6+1+6 & 2+3+12 \\ 5+4+4 & 3+2+8 & 1+6+16 \end{bmatrix} = \begin{bmatrix} 32 & 20 & 18 \\ 15 & 13 & 17 \\ 13 & 13 & 23 \end{bmatrix}$$

$$A^2 - 5A = \begin{bmatrix} 32 & 20 & 18 \\ 15 & 13 & 17 \\ 13 & 13 & 23 \end{bmatrix} - \begin{bmatrix} 25 & 15 & 5 \\ 10 & 5 & 15 \\ 5 & 10 & 20 \end{bmatrix}$$

$$\therefore P = \begin{bmatrix} 7 & 5 & 13 \\ 5 & 8 & 2 \\ 8 & 3 & 3 \end{bmatrix} \qquad [\because P = A^2 - 5A]$$

37. (*i*) We have, $B = \{b_1, b_2, b_3\} \Rightarrow n(B) = 3$

$G = \{g_1, g_2\} \Rightarrow n(G) = 2$

Number of possible relations from B to G

$$= 2^{n(B) \times n(G)}$$
$$= 2^{(3\times2)} = 2^6 = 64$$

(*ii*) Number of functions from B to G

$$= 2 \times 2 \times 2$$
$$= 2^3$$

(*iii*) **For reflexive:**

x and x are of same sex.

Therefore $(x, x) \in R$

Hence, R is reflexive.

For symmetric:

If x and y are in same sex then y and x should also from same sex.

Therefore $(x, y) \in R$ and $(y, x) \in R$.

Hence, R is symmetric.

For transitive:

$(x, y) \in R$, $(y, z) \in R$ then $(x, z) \in R$

If (x, y) are of same sex, (y, z) are of same sex then (x, z) should also be of same sex.

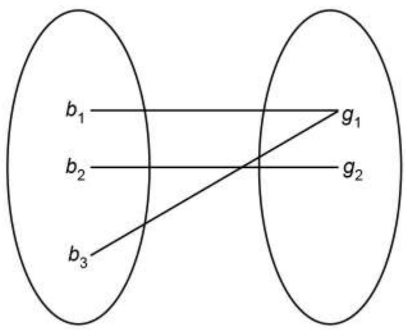
$\therefore$ R is transitive also.

As R is reflexive, symmetric and transitive, hence, R is an equivalence relation.

OR

(*iii*) **Injective (one-one function):** A function $f : X \to Y$ is said to be injective if the images of distinct elements of X are distinct in Y under f. (It is also known as one to one)

$f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$



f is not injective, as b_1 and b_3 have same image g_1.

Surjective (onto function):

All elements in co-domain has at least one pre-image in domain.

From the above diagram, f is surjective.

As 'f' is not one-one but onto, so 'f' is not bijective function.

38. (*i*) We have,

$$(x^2 - y^2)\,dx + 2xy\,dy = 0$$

$$\Rightarrow \quad 2xy\,dy = (y^2 - x^2)dx$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y^2 - x^2}{2xy} \qquad ...(i)$$

Let $$F(x, y) = \frac{y^2 - x^2}{2xy}$$

Now, $$F(\lambda x, \lambda y) = \frac{(\lambda y)^2 - (\lambda x)^2}{2(\lambda x)(\lambda y)}$$

$$\Rightarrow \quad F(\lambda x, \lambda y) = \frac{\lambda^2\left(y^2 - x^2\right)}{2\lambda^2\,xy}$$

$$\Rightarrow \qquad F(\lambda x, \lambda y) = \lambda^\circ F(x, y)$$

So, $F(x, y)$ is a homogeneous function of degree 0. So, equation (i) represents a homogeneous differential equation.

Now, from (i), we have

$$\frac{dy}{dx} = \frac{\frac{y^2}{x^2} - 1}{2\left(\frac{y}{x}\right)}$$

$$\Rightarrow \qquad \frac{dy}{dx} = \frac{\left(\frac{y}{x}\right)^2 - 1}{2\left(\frac{y}{x}\right)} = g\left(\frac{y}{x}\right)$$

(ii)

$$\frac{dy}{dx} = \frac{\left(\frac{y}{x}\right)^2 - 1}{2\left(\frac{y}{x}\right)} \qquad ...(ii)$$

Let $y = vx$

Differentiating both sides with respect to x, we get

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Now, from (ii), $\left(\frac{v^2 - 1}{2v}\right) = v + x\frac{dv}{dx}$

$$\Rightarrow x\frac{dv}{dx} = \frac{-1 - v^2}{2v}$$

$$\Rightarrow \frac{-dx}{x} = \frac{2v}{1 + v^2}dv \quad \Rightarrow \quad \frac{2v\,dv}{1 + v^2} = \frac{-dx}{x}$$

Integrating both sides, we get

$$\int \frac{2v\,dv}{1 + v^2} = -\int \frac{dx}{x}$$

$$\Rightarrow \qquad \log\left|(1 + v^2)\right| = -\log|x| + \log C$$

$$\Rightarrow \qquad \log\left|\left(1 + \frac{y^2}{x^2}\right)\right| = -\log x + \log C$$

$$\Rightarrow \qquad \log\left|\left(1 + \frac{y^2}{x^2}\right) \times x\right| = \log C$$

$$\Rightarrow \qquad \log\left|\frac{x^2 + y^2}{x}\right| = \log C$$

$$\Rightarrow \qquad x^2 + y^2 = Cx$$