6 SAMPLE PAPER

[Average Concept]

Time Allowed: 3 Hours] [*Maximum Marks:* 80

General Instructions:

Read the following instructions very carefully and strictly follow them:

(*i*) *This Question paper contains* ***38*** *questions.* ***All*** *questions are* ***compulsory.***

(*ii*) *This Question paper is divided into* ***five*** *Sections –* ***A, B, C, D*** *and* ***E.***

(*iii*) *In* ***Section A,*** *Questions no.* ***1*** *to* ***18*** *are* ***multiple choice questions*** *(MCQs) and Questions no.* ***19*** *and* ***20*** *are* ***Assertion-Reason based*** *questions of* ***1 mark each.***

(*iv*) *In* ***Section B,*** *Questions no.* ***21*** *to* ***25*** *are* ***Very Short Answer (VSA)-type*** *questions, carrying* ***2 marks each.***

(*v*) *In* ***Section C,*** *Questions no.* ***26*** *to* ***31*** *are* ***Short Answer (SA)-type*** *questions, carrying* ***3 marks each.***

(*vi*) *In* ***Section D,*** *Questions no.* ***32*** *to* ***35*** *are* ***Long Answer (LA)-type*** *questions, carrying* ***5 marks each.***

(*vii*) *In* ***Section E,*** *Questions no.* ***36*** *to* ***38*** *are* ***Case study-based questions,*** *carrying* ***4 marks each.***

(*viii*) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(*ix*) *Use of calculators is* ***not*** *allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. Which of these relations on set A where $A = \{1, 2, 3, 4\}$ are equivalence relation? **[NCERT Part-I, Page 2]**

(*a*) $R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 4)\}$

(*b*) $R_2 = \{(1, 4), (2, 2), (3, 3), (4, 1), (4, 2), (4, 4)\}$

(*c*) $R_3 = \{(1, 1), (1, 2), (1, 3)\}$

(*d*) $R_4 = \{(1, 1), (1, 2), (1, 4), (2, 2), (2, 4), (3, 3), (4, 1), (4, 4)\}$

2. The value of $\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is **[Conceptual Application]**

(*a*) $\frac{1}{\sqrt{2}}$ (*b*) $\frac{1}{\sqrt{3}}$ (*c*) $\frac{1}{2\sqrt{2}}$ (*d*) $\frac{1}{3\sqrt{3}}$

3. The principal value of $\operatorname{cosec}^{-1}(4)$ will lie between **[NCERT Part-I, Page 2]**

(*a*) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$ (*b*) $[0, \pi] - \{0\}$ (*c*) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \{0\}$ (*d*) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

4. For the matrix diag $(-2, -2, -2)$ which one is not true? **[NCERT Part-I, Page 40]**

(*a*) Diagonal matrix (*b*) Scalar matrix (*c*) Unit matrix (*d*) Square matrix

5. If $A = \begin{bmatrix} x & -3 & 1 \\ 2 & y & 1 \\ 1 & 1 & z \end{bmatrix}$ and $xyz = 7, x + y - 6z = 11$, then $A.\ adj\ A$ is equal to **[NCERT Part-I, Page 88]**

(*a*) $-5I$ (*b*) $5I$ (*c*) $4I$ (*d*) $-4I$

6. The function *f*, defined by $f(x) = \begin{cases} \dfrac{k(x^2-1)}{x-1}, & if\ x > 1 \\ 4, & if\ x \le 1 \end{cases}$ is continuous at $x = 1$, then $k =$ **[NCERT Part-I, Page 105]**

(*a*) 4 (*b*) –4 (*c*) 2 (*d*) –3

7. For function *f*, defined by $f(x) = |x + 3|, x \in R$, sort out the incorrect statement. **[Conceptual Application]**

(*a*) Continuous at $x = -3$.
(*b*) Differentiable at $x = -3$.
(*c*) Not differentiable at $x = -3$.
(*d*) Continuous for $x \in R$.

8. If $\dfrac{x}{a} + \dfrac{y}{b} = 5$, then $\dfrac{dy}{dx}$ is equal to **[NCERT Part-I, Page 122-123]**

(*a*) $\dfrac{a}{b}$ (*b*) $-\dfrac{a}{b}$ (*c*) $\dfrac{5-a}{b}$ (*d*) $-\dfrac{b}{a}$

9. $\int_0^{\frac{\pi}{2}} \log(\tan x)dx$ equals to **[NCERT Part-II, Page 273-274]**

(*a*) 0 (*b*) $\dfrac{1}{2}$ (*c*) $\dfrac{\pi}{2}$ (*d*) π

10. $\int e^{\sqrt{x}} dx$ is equal to **[NCERT Part-II, Page 259-260]**

(*a*) $2(\sqrt{x}-1)e^{\sqrt{x}} + C$ (*b*) $e^{\sqrt{x}} + C$ (*c*) $\dfrac{e^{\sqrt{x}}}{2\sqrt{x}} + C$ (*d*) $2\sqrt{x} \cdot e^{\sqrt{x}} + C$

11. The value of $\hat{i} \cdot (\hat{j} \times \hat{k}) + 3\hat{j} \cdot (\hat{k} \times \hat{i}) + x\hat{k} \cdot (\hat{i} \times \hat{i})$ is **[Integrated Question]**

(*a*) 2 (*b*) 4 (*c*) $-2 + x$ (*d*) $4 + x$

12. Equation of *x*-axis is **[NCERT Part-II, Page 382]**

(*a*) $\dfrac{x}{0} = \dfrac{y}{1} = \dfrac{z}{1}$
(*b*) $\dfrac{x-1}{1} = \dfrac{y}{0} = \dfrac{z}{0}$
(*c*) $\dfrac{x-1}{1} = \dfrac{y-1}{0} = \dfrac{z}{0}$
(*d*) $\dfrac{x}{1} = \dfrac{y-1}{0} = \dfrac{z-1}{0}$

13. Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 9$ and the lines $x = 0$ and $x = 3$ is **[Conceptual Application]**

(*a*) $\dfrac{9\pi}{2}$ sq units (*b*) $\dfrac{\pi}{3}$ sq units (*c*) $\dfrac{9\pi}{4}$ sq units (*d*) $\dfrac{3\pi}{4}$ sq units

14. $\int \dfrac{\tan\sqrt{x}}{\sqrt{x}} dx$ is equal to **[NCERT Part-II, Page 235-236]**

(*a*) $\cot\sqrt{x} + C$
(*b*) $2\log|\sec\sqrt{x}| + C$
(*c*) $-2\log|\sec\sqrt{x}| + C$
(*d*) $\sqrt{x}\sec\sqrt{x} + C$

15. The domain of the function $\sin^{-1}(5x-3)$ is **[NCERT Part-I, Page 19]**

(*a*) $[2, 4]$ (*b*) $\left(-\dfrac{3}{5}, \dfrac{3}{5}\right)$ (*c*) $\left[\dfrac{2}{5}, \dfrac{4}{5}\right]$ (*d*) $[-1, 1]$

16. The solution of the equation $(2y-1)dx - (2x+3)dy = 0$ is **[NCERT Part-II, Page 306-307]**

(*a*) $\dfrac{2x-1}{2y+3} = C$ (*b*) $\dfrac{2y+1}{2x-3} = C$ (*c*) $\dfrac{2x+3}{2y-1} = C$ (*d*) $\dfrac{2x-1}{2y-1} = C$

17. If A and B are two independent events, then which of the following is not true? **[NCERT Part-II, Page 418]**

(*a*) $\overline{A}, B$ are also independent.
(*b*) $A, \overline{B}$ are also independent.
(*c*) $\overline{A}, \overline{B}$ are also independent.
(*d*) $P(A \cap B) = 0$.

18. Let A and B be two events such that $P(A) = \frac{3}{8}$, $P(B) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$. Then $P(A'/B). P(A/B')$ is equal to **[NCERT Part-II, Page 408]**

(*a*) $\frac{3}{5}$ (*b*) $\frac{1}{5}$ (*c*) $\frac{3}{20}$ (*d*) $\frac{3}{25}$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (*a*), (*b*), (*c*) and (*d*) as given below.)

(*a*) Both *A* and *R* are true and *R* is the correct explanation of *A*.
(*b*) Both *A* and *R* are true but *R* is not the correct explanation of *A*.
(*c*) *A* is true but *R* is false.
(*d*) *A* is false but *R* is true.

19. **Assertion (*A*):** The cofactor of (–2) in the determinant $\begin{vmatrix} 2 & 5 & 8 \\ 0 & 3 & 7 \\ -1 & -2 & 6 \end{vmatrix}$ is (–14). **[NCERT Part-I, Page 84]**

Reason (*R*): If A_{ij} and M_{ij} represent cofactor and minor of an element a_{ij} of a given determinant, then $A_{ij} = (-1)^{i+j} M_{ij}$.

20. **Assertion (*A*):** If $\vec{a} = \hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = -\hat{i} + 3\hat{j} + 5\hat{k}$ are represented along one side and diagonal of a parallelogram respectively, then area of parallelogram is $3\sqrt{14}$ square units. **[Conceptual Application]**

Reason (*R*): $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$.

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Draw the graph of $y = \sin^{-1}x, x \in [-1, 1]$. **[NCERT Part-I, Page 20]**

22. Construct a matrix $A = [a_{ij}]$ of order 2 × 3, where **[NCERT Part-I, Page 36-37]**

$$a_{ij} = \begin{cases} i + j, \text{ if } (i + j) \text{ is odd} \\ 2i - j, \text{ if } (i + j) \text{ is even} \end{cases}$$

OR

If $A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ 4 & 0 \\ 3 & 6 \end{bmatrix}$ then find $A - 3B'$. **[NCERT Part-I, Page 45-46, 61]**

23. Find the general solution of the differential equation $x\,dy - y\,dx = \sqrt{x^2 + y^2}\,dx$.
[NCERT Part-II, Page 313-314]

24. Find the value of '*a*' for which the vector $\vec{r} = (a^2 - 4)\hat{i} + 2\hat{j} - (a^2 - 9)\hat{k}$ makes acute angles with coordinate axes. **[NCERT Part-I, Page 356]**

OR

Find the angle between the following pairs of lines:

$\frac{5-x}{-2} = \frac{y+3}{1} = \frac{1-z}{3}$ and $\frac{x}{3} = \frac{1-y}{-2} = \frac{z+5}{-1}$ **[NCERT Part-I, Page 383]**

25. A coin is tossed thrice and all eight outcomes are assumed equally likely. Let the event *E* be "the third throw results in head" and event *F* be "the second throw results in tail". Find whether events *E* and *F* are independent. **[NCERT Part-II, Page 417]**

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. Let A and B be two given non empty sets. Show that the function

$f : A \quad B \to B \quad A$ defined as $f(a, b) = (b, a)$ is a bijective function. **[NCERT Part-I, Page 7]**

27. A rectangle is inscribed in a semicircle of radius r with one of its sides on the diameter of the semicircle. Find the dimensions of the rectangle, so that its area is maximum. Also, find maximum area.

[NCERT Part-I, Page 166]

28. Determine the constant a, b and c such that the function **[NCERT Part-I, Page 105]**

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x} & , \text{ if } x < 0 \\ c & , \text{ if } x = 0 \\ \dfrac{\sqrt{x + bx^2} - \sqrt{x}}{b\sqrt{x^3}} & , \text{ if } x > 0 \end{cases}$$ is continuous at $x = 0$.

29. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, for $-1 < x < 1$, show that $\dfrac{dy}{dx} = \dfrac{-1}{(1+x)^2}$. **[NCERT Part-I, Page 122-123]**

OR

If $y = \sin(\sin x)$, prove that $\dfrac{d^2y}{dx^2} + \tan x \dfrac{dy}{dx} + y\cos^2 x = 0$ **[NCERT Part-I, Page 137]**

30. Sketch the graph $y = |x - 1|$. Evaluate $\int_{-2}^{4} |x-1|dx$. What does the value of this integral represent on the graph? **[Conceptual Application]**

OR

Find $\int_0^4 |x - 1|dx$ **[NCERT Part-II, Page 273]**

31. Find $\int \sqrt{\dfrac{x}{1-x^3}}dx, x \in (0, 1)$ **[NCERT Part-II, Page 235-236]**

OR

Evaluate $\int \sin(\log x)dx$ **[NCERT Part-II, Page 259-260]**

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. Find matrix A if, $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} A \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 3 & -2 \end{bmatrix}$. **[NCERT Part-I, Page 51]**

OR

Find A^{-1}, where $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$. Hence solve the system of equations **[NCERT Part-I, Page 94-95]**

$x + 2y - 3z = -4,\ 2x + 3y + 2z = 2,\ 3x - 3y - 4z = 11.$

33. Find the foot of the perpendicular from point (1, 2, –3) to the line

$$\frac{x+1}{2}=\frac{y-3}{-2}=\frac{z}{-1}.$$ **[Conceptual Application]**

OR

Find the shortest distance between the following pairs of lines whose vector equations are:

$\vec{r} = (a-1)\hat{i} + (a+1)\hat{j} - (1+a)\hat{k}$ and $\vec{r} = (1-b)\hat{i} + (2b-1)\hat{j} + (b+2)\hat{k}$

[NCERT Part-II, Page 386-387]

34. Solve the LPP graphically **[NCERT Part-II, Page 397-398]**

Maximise $Z = 15x + 2y$, subject to constraints:

$x - 2y \le 2,\ 3x + 2y \le 12,\ -3x + 2y \le 3,\ x \ge 0,\ y \ge 0.$

35. Find the general solution of the differential equation $\frac{dy}{dx} = (3x + y + 5)^2$.

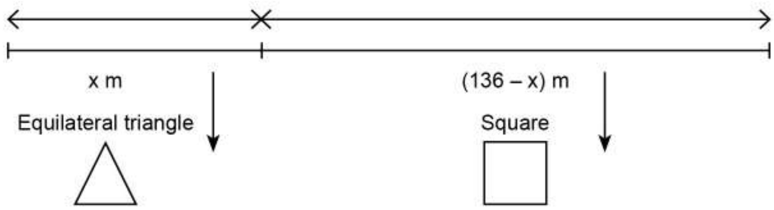
[NCERT Part-II, Page 306-307]

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. An advertising company wants to decorate advertisement boards using boundary of illuminating wire. An illuminating wire of length 136 m is taken and cut into two pieces. One of the pieces is to be put around a square and another around an equilateral triangle. A piece of length x m is put around an equilateral triangle and other around a square as shown. **[Conceptual Application]**



Based on above information, answer the following questions.

(*i*) Find the area of an equilateral triangle in terms of x.

(*ii*) Find the area of square in terms of x.

(*iii*) What length of wire is used around an equilateral triangle, so that combined area of square and an equilateral triangle is minimum?

OR

(*iii*) Find the length of the side of square, when combined area of a square and equilateral triangle is minimum.

Case Study - 2

37. Three students Mehul, Tanya and Charvi appear in an examination and their chances of passing the examination are in the ratio of 1 : 4 : 7. **[Conceptual Application]**

Answer the questions based on above information.

(*i*) What is the probability that all the three pass the examination?

(*ii*) What is the probability that only Tanya passes the examination?

(*iii*) What is the probability that only Mehul passes the examination?

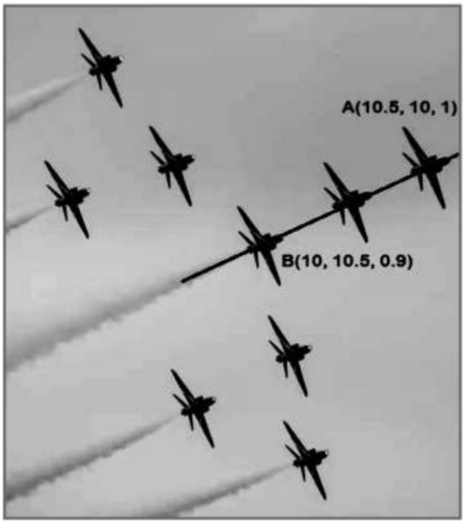
OR

(*iii*) Find the probability that only one of them passes the examination.

Case Study - 3

38. Fighter jets are flying in a formation for an aero show as shown in the figure. Taking their control tower as the reference point and reference point being origin, the coordinates of two fighters in their flight path are $A(10.5 \text{ km}, 10 \text{ km}, 1 \text{ km})$ and $B(10 \text{ km}, 10.5 \text{ km}, 0.9 \text{ km})$. They are moving along the straight line joining A and B at that point as seen in the figure. **[Conceptual Application]**

Based on the above information, answer the following questions:



(*i*) What is the angle made by the line $\overleftrightarrow{AB}$ with the positive direction of z-axis?

(*ii*) What is the Cartesian and vector equation of the line passing through A and B?

SOLUTIONS

1. (*a*) as, R_1 is an equivalence relation since it is reflexive, symmetric and transitive.

R_2 is not reflexive as $(1, 1) \notin R$. Also, it is not symmetric and not transitive. So R_2 is not an equivalence relation.

Clearly R_3 is not symmetric as well as not reflexive.

$\therefore R_3$ is not an equivalence relation.

R_4 is not symmetric as well as not transitive.

$\therefore R_4$ is not an equivalence relation.

2. (*c*) Let $\theta = \sin^{-1}\frac{\sqrt{63}}{8} \Rightarrow \sin\theta = \frac{\sqrt{63}}{8} \Rightarrow \cos\theta = \frac{1}{8}$

$$\sin\left[\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right] = \sin\frac{\theta}{4} = \sqrt{\frac{1-\cos\frac{\theta}{2}}{2}} = \sqrt{\frac{1-\sqrt{\frac{1+\cos\theta}{2}}}{2}}$$

$$= \sqrt{\frac{1-\sqrt{\frac{1+\frac{1}{8}}{2}}}{2}} = \sqrt{\frac{1-\sqrt{\frac{9}{16}}}{2}} = \sqrt{\frac{1-\frac{3}{4}}{2}} = \sqrt{\frac{1}{8}} = \frac{1}{2\sqrt{2}}$$

3. (*d*) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$. As range of principal value for $\text{cosec}^{-1} x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

4. (*c*) as diag. $(-2, -2, -2) = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

and this does not represent a unit matrix.

5. (*a*)

$$\text{A. Adj. A} = |\text{A}|\,\text{I}$$

Now,

$$|\text{A}| = x(yz-1) + 3(2z-1) + 1(2-y)$$
$$= xyz - x + 6z - 3 + 2 - y$$
$$= xyz - (x + y - 6z) - 1 = 7 - 11 - 1 = -5$$
$$\text{A. (Adj. A)} = -5\text{I}$$

6. (*c*) as

$$\lim_{x\to1^-} f(x) = \lim_{x\to1^+} f(x) = f(1)$$

$$\Rightarrow \lim_{x\to1^-}(4) = \lim_{x\to1^+}\frac{k(x^2-1)}{x-1} = 4$$

$$\Rightarrow 4 = 2k = 4 \Rightarrow k = 2$$

7. (*b*) as

$$|x+3| = \begin{cases} x+3, & x \ge -3 \\ -x-3, & x < -3 \end{cases}$$

$$\text{L.H.D (at } x = -3) = \lim_{h\to0}\frac{f(-3-h)-f(-3)}{-h} = \lim_{h\to0}\frac{3+h-3-0}{-h} = -1$$

$$\text{R.H.D (at } x = -3) = \lim_{h\to0}\frac{f(-3+h)-f(-3)}{h} = \lim_{h\to0}\frac{-3+h+3-0}{h} = 1$$

$$\text{L.H.D} \ne \text{R.H.D for } x = -3$$

So, $f(x)$ is not differentiable at $x = -3$.

8. (*d*) $$\frac{x}{a}+\frac{y}{b} = 5$$

Differentiating w.r.t. x,

$$\frac{1}{a}+\frac{1}{b}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{b}{a}$$

9. (*a*) $$I = \int_0^{\pi/2} \log(\tan x)\, dx \quad ...(i)$$

Using property $\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$

So, $$I = \int_0^{\pi/2} \log(\cot x)\, dx \quad ...(ii)$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2} (\log \tan x + \log \cot x)\, dx$$

$$\Rightarrow \quad 2I = \int_0^{\pi/2} \log(\tan x \,.\, \cot x)\, dx$$

$$\Rightarrow \quad 2I = \int_0^{\pi/2} \log 1\, dx$$

$$\Rightarrow \quad I = 0 \quad [\because \log 1 = 0]$$

10. (*a*) Let $I = \int e^{\sqrt{x}} dx$, Put $\sqrt{x} = t \Rightarrow x = t^2$

$$\Rightarrow \quad dx = 2t\,dt$$

$$\because \quad I = 2\int t\,.\,e^t dt = 2\left[t.e^t - \int 1e^t dt\right]$$

$$= 2[te^t - e^t] + C$$

$$= 2\left[\sqrt{x}\,e^{\sqrt{x}} - e^{\sqrt{x}}\right] + C$$

$$= 2(\sqrt{x}-1)\,e^{\sqrt{x}} + C$$

11. (*b*) as $\hat{i}.\hat{i} + 3\hat{j}\,.\,\hat{j} + x\,\hat{k}\,.\,\vec{0} = 1 + 3 + 0 = 4$

12. (*b*) As DR's of x-axis are 1, 0, 0 and it passes through point (1, 0, 0).

13. (*c*) $$\text{Area} = \int_0^3 \sqrt{9-x^2}\, dx = \left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3$$

$$= \left[\left(0 + \frac{9}{2}\times\frac{\pi}{2}\right) - 0\right] = \frac{9\pi}{4} \text{ sq units}$$

14. (*b*) Let $\sqrt{x} = t$

$$\Rightarrow \quad \frac{1}{2\sqrt{x}}\, dx = dt \Rightarrow \frac{1}{\sqrt{x}} dx = 2dt$$

So, $$\int \frac{\tan\sqrt{x}}{\sqrt{x}}\, dx = 2\int \tan t\, dt = 2\log|\sec t| + C$$

$$= 2\log|\sec\sqrt{x}| + C$$

15. (*c*) For the domain of $\sin^{-1}(5x-3)$,

$$-1 \le 5x - 3 \le 1$$

$$\Rightarrow \quad 2 \le 5x \le 4$$

$$\Rightarrow \quad \frac{2}{5} \le x \le \frac{4}{5}$$

So, domain of $\sin^{-1}(5x-3)$ is $\left[\frac{2}{5}, \frac{4}{5}\right]$.

16. (*c*) As, $(2y-1)dx-(2x+3)dy=0$

$$\Rightarrow \quad \int \frac{dx}{2x+3}-\int \frac{dy}{2y-1} = k$$

$$\Rightarrow \quad \frac{1}{2}\log|2x+3|-\frac{1}{2}\log|2y-1| = k$$

$$\Rightarrow \quad \log\left|\frac{2x+3}{2y-1}\right| = 2k$$

$$\Rightarrow \quad \frac{2x+3}{2y-1} = e^{2k}=C$$

17. (*d*) $P(A\cap B)=0 \Rightarrow A$ and B are mutually exclusive

18. (*b*)

$$P(A\cap B) = P(A)+P(B)-P(A\cup B)$$

$$= \frac{3}{8}+\frac{5}{8}-\frac{3}{4}=\frac{1}{4}$$

$$P(A'/B).P(A/B') = \frac{P(A'\cap B)}{P(B)}\cdot\frac{P(A\cap B')}{P(B')}$$

$$= \frac{[P(B)-P(A\cap B)][P(A)-P(A\cap B)]}{P(B)[1-P(B)]}=\frac{\left[\frac{3}{8}\right]\left[\frac{1}{8}\right]}{\frac{5}{8}\times\frac{3}{8}}=\frac{1}{5}$$

19. (*a*) A : cofactor of $-2=-\begin{vmatrix}2 & 8\\ 0 & 7\end{vmatrix}=-14$, true

R : true, correct reason.

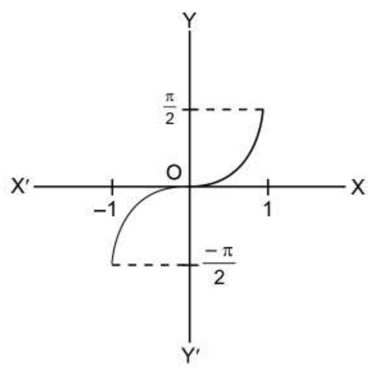
20. (*b*) A : Area $=|\vec{a}\times\vec{b}|$, $\quad \vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 3 & 4\\ -1 & 3 & 5\end{vmatrix}=3\hat{i}-9\hat{j}+6\hat{k}=3(\hat{i}-3\hat{j}+2\hat{k})$

So, $\quad \left|\vec{a}\times\vec{b}\right| = 3\sqrt{1+9+4}=3\sqrt{14}$ sq units, True

R : true, not correct explanation of A.

21. (*b*)

x	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
y	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$



The required graph has been shown.

22. $a_{11}=2-1=1, \quad a_{12}=1+2=3, \quad a_{13}=2-3=-1$

$a_{21}=2+1=3, \quad a_{22}=4-2=2, \quad a_{23}=2+3=5$

$$A=\begin{bmatrix}1 & 3 & -1\\ 3 & 2 & 5\end{bmatrix}$$

OR

$$A-3B' = \begin{bmatrix}3 & -1 & 0\\ 4 & 2 & 1\end{bmatrix}-3\begin{bmatrix}2 & -1\\ 4 & 0\\ 3 & 6\end{bmatrix}'=\begin{bmatrix}3 & -1 & 0\\ 4 & 2 & 1\end{bmatrix}-3\begin{bmatrix}2 & 4 & 3\\ -1 & 0 & 6\end{bmatrix}$$

$$= \begin{bmatrix}3 & -1 & 0\\ 4 & 2 & 1\end{bmatrix}-\begin{bmatrix}6 & 12 & 9\\ -3 & 0 & 18\end{bmatrix}=\begin{bmatrix}3-6 & -1-12 & 0-9\\ 4+3 & 2-0 & 1-18\end{bmatrix}=\begin{bmatrix}-3 & -13 & -9\\ 7 & 2 & -17\end{bmatrix}$$

23. $x\,dy = (y + \sqrt{x^2 + y^2})dx$

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{y + \sqrt{x^2 + y^2}}{x}$ ...(*i*)

Put $y = vx \Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

$\therefore$ From (*i*), we get $v + x\dfrac{dv}{dx} = \dfrac{vx + \sqrt{x^2 + v^2x^2}}{x} = v + \sqrt{1+v^2}$

$\Rightarrow \quad \int \dfrac{1}{\sqrt{1+v^2}}dv = \int \dfrac{1}{x}dx$

$\Rightarrow \quad \log|v + \sqrt{1+v^2}| = \log|x| + \log C$

$\Rightarrow \quad \log|v + \sqrt{1+v^2}| = \log Cx \Rightarrow \dfrac{y}{x} + \sqrt{1 + \dfrac{y^2}{x^2}} = Cx$

$\Rightarrow \quad y + \sqrt{x^2 + y^2} = Cx^2$ is the required solution.

24. For acute angles with coordinate axes, components of $\vec{r}$ should be positive

$\Rightarrow \quad a^2 - 4 > 0, \quad 2 > 0, \quad a^2 - 9 < 0, \quad \Rightarrow \quad a^2 > 4 \quad \Rightarrow \quad a > 2$ or $a < -2$ ...(*i*)

and $a^2 < 9 \quad \Rightarrow \quad -3 < a < 3$ ...(*ii*)

From (*i*) and (*ii*), value of a lies between $(-3, -2) \cup (2, 3)$.

OR

Given lines : $\dfrac{x-5}{2} = \dfrac{y+3}{1} = \dfrac{z-1}{-3}$ and $\dfrac{x}{3} = \dfrac{y-1}{2} = \dfrac{z+5}{-1}$

Let 'θ' be angle between given lines.

So, $\cos\theta = \dfrac{(2\times3)+(1\times2)+(-3)\times(-1)}{\sqrt{4+1+9}\,\sqrt{9+4+1}}$

$= \dfrac{6+2+3}{14} = \dfrac{11}{14} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\dfrac{11}{14}\right)$

25. Sample space, $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

E: third throw results in head $= \{HHH, HTH, THH, TTH\}$

F: second throw results in tail $= \{HTH, HTT, TTH, TTT\}$

$E \cap F = \{HTH, TTH\}$

$P(E) = \dfrac{4}{8} = \dfrac{1}{2}, P(F) = \dfrac{4}{8} = \dfrac{1}{2}, P(E \cap F) = \dfrac{2}{8} = \dfrac{1}{4}$

As, $P(E \cap F) = P(E)\,P(F)$ as $\dfrac{1}{4} = \dfrac{1}{2} \times \dfrac{1}{2}$

Hence, events E and F are independent.

26. $f : A \times B \to B \times A$ is defined as

$f(a, b) = (b, a)$

For one-one: Let $(a_1, b_1), (a_2, b_2) \in A \times B$

such that $f(a_1, b_1) = f(a_2, b_2)$

$\Rightarrow \quad (b_1, a_1) = (b_2, a_2) \Rightarrow b_1 = b_2$ and $a_1 = a_2$

$\Rightarrow \quad (a_1, b_1) = (a_2, b_2) \;\therefore\; f$ is one-one

For onto: Now, let $(b, a) \in B \times A$ be any element in the codomain.

Let $(x, y) \in A \times B$, such that $f(x, y) = (b, a)$

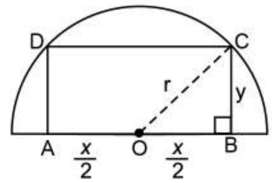
$\Rightarrow$ $(y, x) = (b, a) \Rightarrow y = b, x = a$

$\therefore$ For $(b, a) \in B \times A$, there is $(a, b) \in A \times B$ such that $f(a, b) = (b, a)$

$\therefore$ f is onto.

Since f is one-one and onto, hence, f is bijective.

27. Let x and y be the sides of a rectangle $ABCD$. Let O be the centre of semicircle of radius r. Join OC. In ΔOBC,



by Pythagoras Theorem, $\frac{x^2}{4} + y^2 = r^2$ ...(*i*)

Area of rectangle $A = xy$

$\Rightarrow$ $A^2 = B = x^2y^2$

[if area A is maximum, then A^2 is maximum]

$$B = x^2\left(r^2 - \frac{x^2}{4}\right) \quad \text{[from (i)] ...(ii)}$$

$$\Rightarrow \frac{dB}{dx} = x^2 \cdot \left(-\frac{2x}{4}\right) + \left(r^2 - \frac{x^2}{4}\right)2x$$

$$\Rightarrow \frac{dB}{dx} = 2x\left[r^2 - \frac{x^2}{2}\right]$$

For maxima or minima, $\frac{dB}{dx} = 0$

$\Rightarrow$ $x = \sqrt{2}\,r$ or $x = 0$ (rejected)

Now,
$$\frac{d^2B}{dx^2} = 2x(-x) + \left(r^2 - \frac{x^2}{2}\right)\cdot 2$$

$$\left.\frac{d^2B}{dx^2}\right]_{x=\sqrt{2}r} = -2(\sqrt{2}\,r)^2 + \left(r^2 - \frac{2r^2}{2}\right)\cdot 2 = -4r^2 < 0$$

$\therefore$ for $x = \sqrt{2}\,r$, area is maximum. Now, $y = \sqrt{r^2 - \frac{x^2}{4}} = \sqrt{r^2 - \frac{r^2}{2}} = \frac{r}{\sqrt{2}}$

So, area of rectangle is maximum when $x = \sqrt{2r}$ and $y = \frac{r}{\sqrt{2}}$.

Maximum area,
$$A = \sqrt{B} = x\sqrt{r^2 - \frac{x^2}{4}} = \sqrt{2}\,r\sqrt{r^2 - \frac{r^2}{2}} = \sqrt{2}\,r \cdot \frac{r}{\sqrt{2}}$$

$$= r^2 \text{ sq units.}$$

28.
$$\underset{(x=0)}{\text{LHL}} = \lim_{x \to 0^-}\left\{\frac{\sin(a+1)x}{x} + \frac{\sin x}{x}\right\}$$

$$= \lim_{h \to 0}\left\{\frac{\sin(a+1)(0-h)}{0-h} + \frac{\sin(0-h)}{0-h}\right\}$$

$$= \lim_{h \to 0}\left\{\frac{\sin(a+1)h}{(a+1)h}\cdot(a+1) + \frac{\sin h}{h}\right\}$$

$$\Rightarrow \underset{(x=0)}{\text{LHL}} = a + 1 + 1 = a + 2 \quad \text{...(i)}$$

$$\underset{(x=0)}{\text{RHL}} = \lim_{x \to 0^+}\left[\frac{\sqrt{x + bx^2} - \sqrt{x}}{b\sqrt{x^3}}\right]$$

$$= \lim_{h \to 0} \frac{\sqrt{h + bh^2} - \sqrt{h}}{b\sqrt{h^3}}$$

$$= \lim_{h \to 0} \frac{h + bh^2 - h}{bh\sqrt{h}\left[\sqrt{h + bh^2} + \sqrt{h}\right]}$$

$$= \lim_{h \to 0} \frac{bh^2}{bh\sqrt{h}\sqrt{h}\left[\sqrt{1 + bh} + 1\right]} = \lim_{h \to 0} \frac{1}{\sqrt{1 + bh} + 1} = \frac{1}{2}$$

For continuity at $x = 0$, $\underset{(x=0)}{\text{LHL}} = \underset{(x=0)}{\text{RHL}} = f(0)$

$\Rightarrow \quad a + 2 = \frac{1}{2} = c \Rightarrow c = \frac{1}{2}, a = -\frac{3}{2}$ [from (*i*)]

$\therefore$ Numbers are $a = -\frac{3}{2}$, b any real value ($\neq 0$) and $c = \frac{1}{2}$.

29. $x\sqrt{1+y} = -y\sqrt{1+x}$

Squaring both sides, we get

$$x^2(1 + y) = y^2(1 + x)$$

$\Rightarrow \quad (x^2 - y^2) + (x^2y - xy^2) = 0$

$\Rightarrow \quad (x - y)(x + y + xy) = 0$

$\Rightarrow \quad x - y = 0$ or $x + y + xy = 0$

$\Rightarrow \quad x = y$ or $x + y + xy = 0$

$\Rightarrow \quad x + y + xy = 0$ [As $x \neq y$]

$\Rightarrow \quad y(1 + x) = -x$

$\Rightarrow \quad y = \frac{-x}{1+x}$

$$\frac{dy}{dx} = \frac{(1+x)(-1) + x \cdot 1}{(1+x)^2} = \frac{-1}{(1+x)^2}$$

OR

$y = \sin(\sin x)$ ...(*i*)

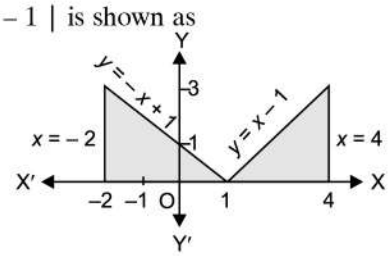
$\Rightarrow \quad \frac{dy}{dx} = \cos(\sin x) . \cos x$ ...(*ii*)

$\Rightarrow \quad \frac{d^2y}{dx^2} = -\sin(\sin x) . \cos x . \cos x - \sin x . \cos(\sin x)$

$\Rightarrow \quad \frac{d^2y}{dx^2} = -y \cdot \cos^2 x - \sin x \cdot \frac{1}{\cos x} . \frac{dy}{dx}$ [from (*i*) and (*ii*)]

$\Rightarrow \quad \frac{d^2y}{dx^2} = -y \cdot \cos^2 x - \tan x \cdot \frac{dy}{dx} \Rightarrow \frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y\cos^2 x = 0$

30. Graph of the curve $y = |x - 1|$ is shown as



$$\int_{-2}^{4} |x-1|\, dx = \int_{-2}^{1} -(x-1)\, dx + \int_{1}^{4} (x-1) dx$$

$$= \left[\frac{-x^2}{2} + x\right]_{-2}^{1} + \left[\frac{x^2}{2} - x\right]_{1}^{4}$$

$$= \left(-\frac{1}{2} + 1\right) - \left(-\frac{4}{2} - 2\right) + \left(\frac{16}{2} - 4\right) - \left(\frac{1}{2} - 1\right)$$

$= 9$ sq units

Value of this integral represents area bounded by the curve $y = |x - 1|$, the x-axis and between $x = -2$ to $x = 4$.

OR

$$\int_0^4 |x-1|\,dx = \int_0^1 -(x-1)\,dx + \int_1^4 (x-1)\,dx$$

$$= \left[-\frac{x^2}{2}+x\right]_0^1 + \left[\frac{x^2}{2}-x\right]_1^4 = \left[\left(-\frac{1}{2}+1\right)-0\right] + \left[(8-4)-\left(\frac{1}{2}-1\right)\right] = \frac{1}{2}+4+\frac{1}{2}=5$$

31. Let $I = \int \sqrt{\frac{x}{1-x^3}}\,dx$

Let $x^{3/2} = t \quad \Rightarrow \quad \frac{3}{2}\sqrt{x}\,dx = dt$

Now, $I = \frac{2}{3}\int \frac{1}{\sqrt{1-t^2}}\,dt = \frac{2}{3}\sin^{-1} t + C = \frac{2}{3}\sin^{-1}(x^{3/2}) + C$

OR

Let $I = \int \underset{\text{(I)}}{\sin(\log x)}\,.\,\underset{\text{(II)}}{1}\,dx$

$= x\,.\,\sin(\log x) - \int \cos(\log x)\,.\,\frac{1}{x}\,.\,x\,dx$

$= x\,.\,\sin(\log x) -$

$\left[\cos(\log x).x - \int -\sin(\log x).\frac{1}{x}.x.dx\right]$

$\Rightarrow \quad I = x\sin(\log x) - x\cos(\log x) - I$

$\Rightarrow \quad I = \frac{x}{2}[\sin(\log x) - \cos(\log x)] + C$

Alternatively:

$\int \sin(\log x)\,dx$

Let $\log x = t$ $\Rightarrow$ $x = e^t$ $\Rightarrow$ $dx = e^t dt$

$= \int \underset{\text{I}}{e^t}\,\underset{\text{II}}{\sin t}\,dt = e^t\cdot(-\cos t) - \int e^t\cdot(-\cos t)\,dt$

$= -e^t\cos t + \int \underset{\text{I}}{e^t}\,\underset{\text{II}}{\cos t}\,dt$

$= -e^t\cdot\cos t + e^t\sin t - \int e^t\sin t\,dt$

$\Rightarrow \quad 2\int e^t\sin t\,dt = e^t(\sin t - \cos t)$

$\Rightarrow \quad \int e^t\sin t\,dt = \frac{e^t}{2}(\sin t - \cos t) + C$

$\Rightarrow \quad \int \sin(\log x)\,dx = \frac{e^{\log x}}{2}[\sin(\log x) - \cos(\log x)] + C$

$= \frac{x}{2}[\sin(\log x) - \cos(\log x)] + C$

32. Let $B = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}, C = \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & 6 \\ 3 & -2 \end{bmatrix}$

Then, $BAC = D \Rightarrow A = B^{-1}DC^{-1}$ ...(*i*)

$B = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}; |B| = \begin{vmatrix} 2 & 4 \\ 1 & 3 \end{vmatrix} = 6 - 4 = 2;$

$\text{adj } B = \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$

$\therefore \quad B^{-1} = \frac{1}{|B|}\text{adj } B = \frac{1}{2}\begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$

$C = \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix}; |C| = \begin{vmatrix} 0 & 2 \\ 1 & 3 \end{vmatrix} = 0 - 2 = -2$

$\text{adj } C = \begin{bmatrix} 3 & -2 \\ -1 & 0 \end{bmatrix}$

$\therefore \quad C^{-1} = \frac{1}{|C|}\text{adj } C = -\frac{1}{2}\begin{bmatrix} 3 & -2 \\ -1 & 0 \end{bmatrix}$

From (i), $$A = B^{-1}DC^{-1}$$

$$= -\frac{1}{4}\begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 6 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ -1 & 0 \end{bmatrix}$$

$$= -\frac{1}{4}\begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3-6 & -2+0 \\ 9+2 & -6-0 \end{bmatrix}$$

$$= -\frac{1}{4}\begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -3 & -2 \\ 11 & -6 \end{bmatrix}$$

$$= -\frac{1}{4}\begin{bmatrix} -9-44 & -6+24 \\ 3+22 & 2-12 \end{bmatrix}$$

$$= -\frac{1}{4}\begin{bmatrix} -53 & 18 \\ 25 & -10 \end{bmatrix}$$

OR

$$|A| = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{vmatrix}$$

$$= 1(-12+6) - 2(-8-6) - 3(-6-9)$$

$$= -6 + 28 + 45 = 67$$

Let A_{ij} be cofactors of a_{ij} in |A|.

$$A_{11} = -6;\ A_{12} = 14;\ A_{13} = -15$$

$$A_{21} = 17;\ A_{22} = 5;\ A_{23} = 9$$

$$A_{31} = 13;\ A_{32} = -8;\ A_{33} = -1$$

Now, $$\text{Adj}\cdot A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}'$$

$$\text{Adj. } A = \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix} \quad ...(i)$$

Given equations:

$$x + 2y - 3z = -4$$

$$2x + 3y + 2z = 2$$

$$3x - 3y - 4z = 11$$

Matrix equation is $$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

i.e. $$AX = B$$

Its solution is $$X = A^{-1}B$$

$\Rightarrow$ $$X = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix} \quad \text{[From } (i)\text{]}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{67}\begin{bmatrix} 24+34+143 \\ -56+10-88 \\ 60+18-11 \end{bmatrix} = \frac{1}{67}\begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix}$$

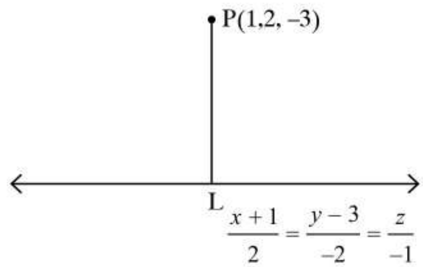
$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

$\Rightarrow$ $x = 3, \quad y = -2, z = 1$ is the solution.

33. Let L be foot of $\perp$ from point P(1, 2, –3) to the line $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$.

Let $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1} = \lambda$

General point on line is $L(2\lambda - 1, -2\lambda + 3, -\lambda)$ ...(*i*)



DR's of PL:

$2\lambda - 1 - 1, -2\lambda + 3 - 2, -\lambda + 3$

i.e. $2\lambda - 2, -2\lambda + 1, -\lambda + 3$

Since PL is perpendicular to the given line, then

$2(2\lambda - 2) - 2(-2\lambda + 1) - 1(-\lambda + 3) = 0$

$\Rightarrow 4\lambda - 4 + 4\lambda - 2 + \lambda - 3 = 0$

$\Rightarrow 9\lambda = 9 \Rightarrow \lambda = 1$

Substituting in (*i*), coordinates of foot of $\perp$ is L(1, 1, – 1).

OR

Lines are: $\vec{r} = (-\hat{i} + \hat{j} - \hat{k}) + a(\hat{i} + \hat{j} - \hat{k})$

$\vec{a_1} = -\hat{i} + \hat{j} - \hat{k}; \vec{b_1} = \hat{i} + \hat{j} - \hat{k}$

and $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + b(-\hat{i} + 2\hat{j} + \hat{k})$

$\vec{a_2} = \hat{i} - \hat{j} + 2\hat{k}, \vec{b_2} = -\hat{i} + 2\hat{j} + \hat{k}$

$\vec{a_2} - \vec{a_1} = \hat{i} - \hat{j} + 2\hat{k} + \hat{i} - \hat{j} + \hat{k} = 2\hat{i} - 2\hat{j} + 3\hat{k}$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ -1 & 2 & 1 \end{vmatrix} = 3\hat{i} + 3\hat{k}$$

$$\text{Shortest distance} = \left| \frac{(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

$$= \left| \frac{(2\hat{i} - 2\hat{j} + 3\hat{k}).(3\hat{i} + 3\hat{k})}{\sqrt{9+9}} \right|$$

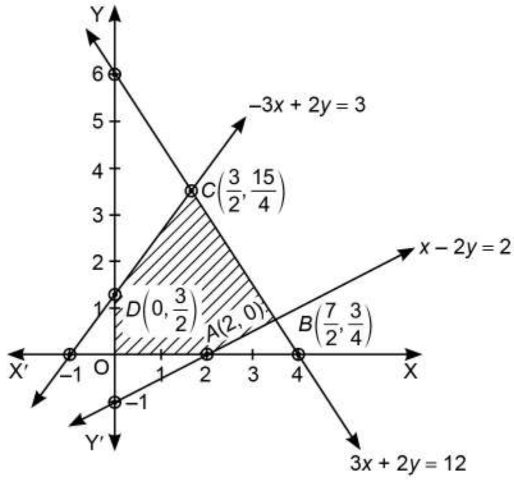
$$= \left| \frac{6-0+9}{3\sqrt{2}} \right| = \frac{5}{\sqrt{2}} \text{ units} = \frac{5\sqrt{2}}{2} \text{ units}$$

34. Maximise $Z = 15x + 2y$

Subject to constraints

$$\begin{aligned} x - 2y &\le 2 \\ 3x + 2y &\le 12 \\ -3x + 2y &\le 3 \\ x &\ge 0, \quad y \ge 0 \end{aligned}$$

Plotting graph of inequations we get the following.



From graph, shaded portion is the feasible solution.

Possible points for maximum Z are $A(2, 0)$, $B\left(\frac{7}{2}, \frac{3}{4}\right)$, $C\left(\frac{3}{2}, \frac{15}{4}\right)$, $D\left(0, \frac{3}{2}\right)$

Points	$Z = 15x + 2y$	Values	
$A(2, 0)$	$30 + 0$	30	
$B\left(\frac{7}{2}, \frac{3}{4}\right)$	$\frac{105}{2} + \frac{3}{2}$	$\frac{108}{2} = 54$	← Maximum
$C\left(\frac{3}{2}, \frac{15}{4}\right)$	$\frac{45}{2} + \frac{15}{2}$	$\frac{60}{2} = 30$	
$D\left(0, \frac{3}{2}\right)$	$0 + 3$	3	

Maximum value is 54 at $x = \frac{7}{2}, y = \frac{3}{4}$.

35. Consider equation, $\frac{dy}{dx} = (3x + y + 5)^2$ ...(*i*)

Let $3x + y + 5 = t$

$\Rightarrow$ $3 + \frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{dy}{dx} = \frac{dt}{dx} - 3$

From (*i*), $\frac{dt}{dx} - 3 = t^2 \Rightarrow \frac{dt}{dx} = t^2 + 3$

$\Rightarrow$ $\frac{dt}{t^2 + 3} = dx \Rightarrow \int \frac{dt}{t^2 + 3} = \int dx$

$\Rightarrow$ $\frac{1}{\sqrt{3}} \tan^{-1} \frac{t}{\sqrt{3}} = x + C$

$$\Rightarrow \quad \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{3x+y+5}{\sqrt{3}}\right) = x + C \text{ is general solution.}$$

36. (*i*) Perimeter $= x \Rightarrow 3a = x$, a is side of equilateral triangle $\Rightarrow a = \frac{x}{3}$ m

$$\therefore \quad \text{Area} = \frac{\sqrt{3}}{4}\left(\frac{x}{3}\right)^2 = \frac{\sqrt{3}}{36}x^2 \text{ sq m}$$

(*ii*) Perimeter of square $= (136 - x)$ m, side of square $= \frac{136-x}{4}$ m

$$\therefore \quad \text{Area of square} = \frac{1}{16}(136-x)^2 \text{ sq m}$$

Combined area, $$A = \frac{\sqrt{3}}{36}x^2 + \frac{1}{16}(136-x)^2$$

$$\frac{dA}{dx} = \frac{\sqrt{3}}{18}x + \frac{1}{16} \times 2(136-x)(-1) = \frac{\sqrt{3}}{18}x - \frac{1}{8}(136-x)$$

For minimum or maximum area, $\frac{dA}{dx} = 0$

$$\Rightarrow \quad \frac{\sqrt{3}}{18}x = \frac{1}{8}(136-x) \Rightarrow 4\sqrt{3}x = 9(136-x)$$

$$\Rightarrow \quad 4\sqrt{3}x + 9x = 1224 \Rightarrow x = \frac{1224}{(4\sqrt{3}+9)}$$

Now, $$\frac{d^2A}{dx^2} = \frac{\sqrt{3}}{18} + \frac{1}{8} > 0$$

So, A is minimum for $x = \frac{1224}{4\sqrt{3}+9}$

Length of wire used to put around an equilateral Δ is $\frac{1224}{4\sqrt{3}+9}$ m, for combined area to be minimum.

OR

As, combined area of equilateral Δ and square is minimum when $x = \frac{1224}{4\sqrt{3}+9}m$.

So, length of side of square $= \left(\frac{136-x}{4}\right) = \left(\frac{136\sqrt{3}}{4\sqrt{3}+9}\right)m$

37. Consider the following events.

M : Mehul passes the examination;

T : Tanya passes the examination;

C : Charvi passes the examination.

$P(M) = \frac{1}{12}$, $P(T) = \frac{4}{12}$, $P(C) = \frac{7}{12}$; $P(\overline{M}) = \frac{11}{12}$, $P(\overline{T}) = \frac{8}{12}$, $P(\overline{C}) = \frac{5}{12}$,

(*i*) P(all passing) $= 1 - $ P(none passing) $= 1 - \frac{11}{12} \times \frac{8}{12} \times \frac{5}{12} = 1 - \frac{110}{432} = \frac{322}{432} = \frac{161}{216}$

(*ii*) P(Only Tanya passes) $= P(T\,\overline{M}\,\overline{C}) = \frac{4}{12} \times \frac{11}{12} \times \frac{5}{12} = \frac{55}{432}$

(*iii*) P(Only Mehul passes) = $P(M\,\overline{T}\,\overline{C}) = \frac{1}{12}\times\frac{2}{3}\times\frac{5}{12}=\frac{5}{216}$

OR

P(only one passes) = $P(M\overline{T}\,\overline{C}) + P(\overline{M}\,T\,\overline{C}) + P(\overline{M}\,\overline{T}\,C)$

$$=\frac{1}{12}\times\frac{2}{3}\times\frac{5}{12}+\frac{11}{12}\times\frac{1}{3}\times\frac{5}{12}+\frac{11}{12}\times\frac{2}{3}\times\frac{7}{12}=\frac{10+55+154}{432}=\frac{219}{432}$$

38. (*i*) The dc's of line $\overleftrightarrow{AB}$ are $\left\langle\frac{-0.5}{\sqrt{0.51}},\frac{0.5}{\sqrt{0.51}},\frac{-0.1}{\sqrt{0.51}}\right\rangle$

$\Rightarrow \langle\cos\alpha,\cos\beta,\cos\gamma\rangle = \left\langle\frac{-0.5}{\sqrt{0.51}},\frac{0.5}{\sqrt{0.51}},\frac{-0.1}{\sqrt{0.51}}\right\rangle$

where α,β,γ, are the angles made by the line $\overleftrightarrow{AB}$ with positive direction of x-axis, y-axis and z-axis.

So, $\cos\gamma = \frac{-0.1}{\sqrt{0.51}} \Rightarrow \gamma = \cos^{-1}\left(\frac{-0.1}{\sqrt{0.51}}\right)$

(*ii*) The Cartesian equation of a line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

So, Cartesian equation of line AB is

$$\frac{x-10.5}{-0.5}=\frac{y-10}{0.5}=\frac{z-1}{-0.1}$$

The vector equation of a line passing through the points having position vectors as $\vec{a}$ and $\vec{b}$ is

$$\vec{r}=\vec{a}+\lambda(\vec{b}-\vec{a})$$

Now, $\vec{a}=10.5\hat{i}+10\hat{j}+\hat{k}$

$\vec{b}=10\hat{i}+10.5\hat{j}+0.9\hat{k}$

So, $\vec{r}=(10.5\hat{i}+10\hat{j}+\hat{k})+\lambda(-0.5\hat{i}+0.5\hat{j}-0.1\hat{k})$