7 SAMPLE PAPER

[Average Concept]

Time Allowed: 3 Hours] [*Maximum Marks:* 80

General Instructions:

Read the following instructions very carefully and strictly follow them:

(*i*) *This Question paper contains **38** questions. **All** questions are **compulsory**.*

(*ii*) *This Question paper is divided into **five** Sections – **A**, **B**, **C**, **D** and **E**.*

(*iii*) *In **Section A**, Questions no. **1** to **18** are **multiple choice questions** (MCQs) and Questions no. **19** and **20** are **Assertion-Reason based** questions of **1 mark each**.*

(*iv*) *In **Section B**, Questions no. **21** to **25** are **Very Short Answer (VSA)-type** questions, carrying **2 marks each**.*

(*v*) *In **Section C**, Questions no. **26** to **31** are **Short Answer (SA)-type** questions, carrying **3 marks each**.*

(*vi*) *In **Section D**, Questions no. **32** to **35** are **Long Answer (LA)-type** questions, carrying **5 marks each**.*

(*vii*) *In **Section E**, Questions no. **36** to **38** are **Case study-based questions**, carrying **4 marks each**.*

(*viii*) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(*ix*) *Use of calculators is **not** allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. If area of a triangle with vertices $(k, 0)$, $(1, 1)$ and $(0, 3)$ is 5 sq units, then the value(s) of k is **[NCERT Part-I, Page 82]**

(*a*) $-\frac{7}{2}$ (*b*) $\frac{13}{2}$ (*c*) $\frac{7}{2}, \frac{-13}{2}$ (*d*) $-\frac{7}{2}, \frac{13}{2}$

2. How many matrices are possible having 24 elements? **[NCERT Part-I, Page 36]**

(*a*) 4 (*b*) 6 (*c*) 8 (*d*) 2

3. Let $f : R \rightarrow R$ be defined by $f(x) = \frac{1}{x}\ \forall x \in R$. Then f is **[NCERT Part-I, Page 7]**

(*a*) one-one (*b*) onto (*c*) bijective (*d*) f is not defined

4. Let Z be the set of integers and R be a relation defined in Z such that aRb if $(a - b)$ is divisible by 5. Then number of equivalence classes are **[NCERT Part-I, Page 4]**

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 5

5. The principal value of $\sec^{-1}(-2)$ is **[NCERT Part-I, Page 23]**

(*a*) $\frac{\pi}{3}$ (*b*) $\frac{\pi}{2}$ (*c*) $\frac{2\pi}{3}$ (*d*) not defined

6. Derivative of $\frac{x}{x-1}$ with respect to x, is **[NCERT Part-I, Page 119]**

(*a*) 2 (*b*) $\frac{1}{(x-1)^2}$ (*c*) $\frac{2x-1}{(x-1)^2}$ (*d*) $\frac{-1}{(x-1)^2}$

7. Given function $f(x) = x^2e^{-x}$, then 'f' increases in the interval **[NCERT Part-I, Page 153]**

(*a*) $(-\infty, \infty)$ (*b*) $(-2, 0)$ (*c*) $(2, \infty)$ (*d*) $(0, 2)$

8. State which of the following is continuous as well as differentiable for $x \in R$. **[Conceptual Application]**

(*a*) $|x|$ (*b*) $[x]$

(*c*) polynomial function (*d*) sgn (x)

9. Which of the following function is decreasing on $\left(0, \frac{\pi}{2}\right)$? **[NCERT Part-I, Page 153]**

(*a*) $\cos x$ (*b*) $-\cos 2x$ (*c*) $\cos 3x$ (*d*) $\tan x$

10. The general solution of the differential equation $\frac{dy}{dx} = 2xe^{x^2-y}$ is **[NCERT Part-II, Page 306-307]**

(*a*) $e^{x^2-y} = C$ (*b*) $e^{-y} + e^{x^2} = C$ (*c*) $e^y = e^{x^2} + C$ (*d*) $e^{x^2+y} = C$

11. The integrating factor for the differential equation $\frac{dy}{dx} + y\tan x - \sec x = 0$ is **[NCERT Part-II, Page 322-323]**

(*a*) $\tan x$ (*b*) $\sec^2 x$ (*c*) $\sec x$ (*d*) $\frac{\tan^2 x}{2}$

12. The sum of order and degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^3 = e^x$ is **[NCERT Part-II, Page 301-302]**

(*a*) 2 (*b*) 3 (*c*) 5 (*d*) 4

13. Area bounded between the parabola $y^2 = 4ax$ and latus rectum is **[Conceptual Application]**

(*a*) $\frac{4}{3}a^2$ sq units (*b*) $\frac{7}{3}a^2$ sq units (*c*) $\frac{8}{3}a^2$ sq units (*d*) $\frac{8}{3}a$ sq units

14. Area enclosed by the curve $3x^2 + 3y^2 = 15$ is **[Conceptual Application]**

(*a*) 5 sq units (*b*) 15 sq units (*c*) 3π sq units (*d*) 5π sq units

15. $\int_1^2 x^2 dx =$ **[NCERT Part-II, Page 268]**

(*a*) 1 (*b*) $\frac{7}{3}$ (*c*) $\frac{1}{3}$ (*d*) 0

16. The value of $\int_0^2 x[x]\, dx$ is **[Conceptual Application]**

(*a*) $\frac{7}{2}$ (*b*) $\frac{3}{2}$ (*c*) $\frac{5}{2}$ (*d*) None of these

17. If A and B are two independent events, such that $P(A) = 0.4$ and $P(B) = 0.3$, then $P(A \cup B) =$ **[Integrated Question]**

(*a*) 0.27 (*b*) 0.58 (*c*) 0.6 (*d*) 0.72

18. If A and B are two independent events, and $P(A) = 0.31$ and $P(B) = 0.41$, then $P(A \cap B) =$ **[Integrated Question]**

(*a*) 0.3141 (*b*) 0.2171 (*c*) 0.1271 (*d*) 0.123

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (*a*), (*b*), (*c*) and (*d*) as given below.)

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true but R is not the correct explanation of A.

(*c*) A is true but R is false.

(*d*) A is false but R is true.

19. **Assertion (*A*):** The vectors $\vec{a} = 2\hat{i} - \hat{j} + 5\hat{k}$ and $\vec{b} = 5\hat{i} + \hat{j} - 3\hat{k}$ are perpendicular to each other.

Reason (*R*): $\vec{a} \times \vec{b}$ is a vector perpendicular to both $\vec{a}$ and $\vec{b}$. **[Conceptual Application]**

20. **Assertion (*A*):** $\begin{bmatrix} -7 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 0 & -7 \end{bmatrix}$ is a scalar matrix. **[NCERT Part-I, Page 40]**

Reason (*R*): All the elements of the principal diagonal are equal, it is called a scalar matrix.

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Find the general solution of the differential equation $\frac{dy}{dx} = \cos^3 x \sin^4 x$. **[NCERT Part-II, Page 306-307]**

22. The probabilities of *A*, *B*, *C*, solving a problem, are $\frac{1}{3}, \frac{2}{7}$ and $\frac{3}{8}$ respectively. If all the three try to solve the problem simultaneously, find the probability that exactly one of them can solve the problem.

[Conceptual Application]

OR

Bag *I* contains 3 red and 4 black balls and bag *II* contains 4 red and 5 black balls. One ball is transferred at random from bag *I* to bag *II* and then a ball is drawn from bag *II*. Find the probability that ball drawn is red. **[NCERT Part-II, Page 408]**

23. Suppose $\vec{a}, \vec{b}, \vec{c}$ are unit vectors, such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$. Prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$. **[Conceptual Application]**

24. Evaluate $\tan^{-1}\left(\tan \frac{9\pi}{8}\right)$ **[NCERT Part-I, Page 24]**

25. If $B = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $A = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then find the value of α such that $A = B^2$. **[NCERT Part-I, Page 41, 50]**

OR

If $[2x \quad 3]\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} x \\ 3 \end{bmatrix} = O$, find the value(s) of x. **[NCERT Part-I, Page 41 , 50]**

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. If $y = Ae^{mx} + Be^{nx}$, prove that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$ **[NCERT Part-I, Page 137]**

27. Find the interval(s) in which the function *f* given by $f(x) = \sin x + \cos x$, $0 \le x \le 2\pi$, is strictly increasing or strictly decreasing. **[NCERT Part-I, Page 153]**

28. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\tan x}{1 + m^2 \tan^2 x}\, dx$. **[Integrated Question]**

OR

If $y(x)$ is a solution of $\left(\frac{2 + \sin x}{1 + y}\right)\frac{dy}{dx} = -\cos x$ and $y(0) = 1$, then find the value of $y\left(\frac{\pi}{2}\right)$.

[NCERT Part-II, Page 306-307]

29. Show that the function $f : R - \{0\} \rightarrow R - \{0\}$ defined by $f(x) = \frac{1}{x}$ is one-one and onto. Is the result true, if the domain $R - \{0\}$ is replaced by N? **[NCERT Part-I, Page 7]**

30. Find the area of the region bounded by the curves $x = at^2$ and $y = 2at$ between the ordinate corresponding to $t = 1$ and $t = 2$. **[Conceptual Application]**

OR

Find the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the ordinates $x = ae$ and $x = 0$, where $b^2 = a^2(1 - e^2)$ and $e < 1$. **[Conceptual Application]**

31. Check the differentiability of the function **[NCERT Part-I, Page 118-119]**

$$f(x) = \begin{cases} x, & x < 1 \\ 2 - x, & 1 \le x \le 2 \\ -2 + 3x - x^2, & x > 2 \end{cases} \text{ at } x = 2.$$

OR

Differentiate, $\frac{8^x}{x^8}$ with respect to x. **[NCERT Part-I, Page 130]**

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32 If a variable line in two adjacent positions has direction cosines l, m, n and $l + \delta l, m + \delta m, n + \delta n$, show that the small angle $\delta\theta$ between two positions is given by $(\delta\theta)^2 = (\delta l)^2 + (\delta m)^2 + (\delta n)^2$.

[Conceptual Application]

OR

The vector $\vec{b} = 3\hat{i} + 4\hat{k}$ is to be written as the sum of two vectors $\vec{\alpha}$ and $\vec{\beta}$ where $\vec{\alpha}$ is parallel to $\vec{a} = \hat{i} + \hat{j}$ and $\vec{\beta}$ is perpendicular to $\vec{a}$. Find $\vec{\alpha}$ and $\vec{\beta}$. **[Conceptual Application]**

33. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. How we can use A^{-1} to solve the system of equations:

[NCERT Part-I, Page 94-95]

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3?$$

OR

For the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 4I = O$. Hence find A^{-1}.

[Conceptual Application]

34. Find the particular solution of the differential equation $dy = \cos x\,(2 - y \operatorname{cosec} x)dx$, given that $y = 2$ when $x = \frac{\pi}{2}$. **[NCERT Part-II, Page 322-323]**

35. Solve the following linear programming problem graphically. **[NCERT Part-II, Page 397-398]**

Minimise $Z = 3x + 4y + 370$

subject to the constraints,

$$\begin{aligned} y &\geq 0 \\ x + y &\leq 60 \\ x &\leq 40 \\ y &\leq 40 \\ x + y &\geq 10 \end{aligned}$$

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. In the office three employees Mehul, Janya and Charvi process incoming matter related to a particular project. Mehul processes 40% of the matter and Janya and Charvi process rest of the matter equally. It is found that 6% of matter processed by Mehul has an error whereas for Janya and Charvi error rate is 4% and 3% respectively. **[Conceptual Application]**

(*i*) Find the conditional probability that an error is committed in processing by Janya while processing the matter.

(*ii*) What is the probability that the matter processed by Janya has an error?

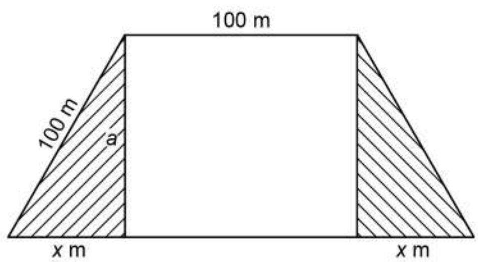
(*iii*) What is the probability of an error in processing the matter ?

OR

(*iii*) The processed matter is checked and the selected matter has an error, what is the probability that it was processed by Mehul?

Case Study - 2

37. A resort at the top of a hill decided to make a relaxation rectangular field with right triangular fields of equal shape and size, for planting flowers, attached to both sides as shown. They are also thinking of maximising the total area. The length of rectangle and hypotenuse of right triangular fields are 100 m each. **[Conceptual Application]**



(*i*) If base of triangular field is x m, then find width a of rectangular field.

(*ii*) Find the perimeter of total enclosed area.

(*iii*) Find the total covered area.

OR

(*iii*) Find the value of x for which total area is maximum.

Case Study - 3

38. A class XII student appearing for a competitive examination was asked to attempt the following questions. **[Conceptual Application]**

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non zero vectors.

(*i*) If $\vec{a}$ and $\vec{b}$ are such that $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ then how $\vec{a}$ and $\vec{b}$ are related?

(*ii*) If $\vec{a} = \hat{i} - 2\hat{j}$, $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$ then find $(2\vec{a} + \vec{b}) \cdot [(\vec{a} + \vec{b}) \times (\vec{a} - 2\vec{b})]$.

SOLUTIONS

1. (*d*), as
$$\Delta = \frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 3 & 1 \end{vmatrix} = \frac{1}{2}[k(1-3) + 1(3-0)]$$
$$= \frac{1}{2}(-2k+3)$$
$$\therefore \quad \frac{1}{2}(-2k+3) = \pm 5 \Rightarrow -2k + 3 = \pm 10$$
$$\Rightarrow \quad -2k = 7 \text{ or } -13$$
$$\Rightarrow \quad k = -\frac{7}{2} \text{ or } \frac{13}{2}.$$

2. (*c*), as 24 can be written as product of two numbers:
$24 \to 1 \times 24, 2 \times 12, 3 \times 8, 4 \times 6, 6 \times 4, 8 \times 3, 12 \times 2, 24 \times 1$
$\therefore$ there are 8 possible orders.

3. (*d*),

4. (*d*), as remainder can be 0, 1, 2, 3, 4.

5. (*c*), Let $\theta = \sec^{-1}(-2) \Rightarrow \sec\theta = -2$
$$\Rightarrow \sec\theta = -\sec\frac{\pi}{3} = \sec\left(\pi - \frac{\pi}{3}\right) \Rightarrow \theta = \frac{2\pi}{3}$$

6. (*d*), $\frac{d}{dx}\left(\frac{x}{x-1}\right) = \frac{(x-1)\cdot 1 - x\cdot 1}{(x-1)^2} = \frac{-1}{(x-1)^2}$

7. (*d*)

8. (*c*), polynomial function

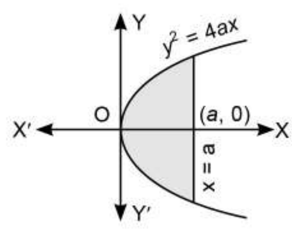
9. (*a*)

10. (*c*)
$$\frac{dy}{dx} = 2xe^{x^2-y}$$
$$\Rightarrow \quad e^y dy = 2xe^{x^2}dx$$
$$\Rightarrow \quad \int e^y dy = \int 2xe^{x^2}dx$$
$$\Rightarrow \quad e^y = \int e^t dt \qquad [\text{Put } x^2 = t \Rightarrow 2xdx = dt]$$
$$\Rightarrow \quad e^y = e^t + C \Rightarrow e^y = e^{x^2} + C$$

11. (*c*), as integrating factor $= e^{\int \tan x\, dx} = e^{\log|\sec x|} = \sec x$

12. (*b*), as given equation is $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^3 = e^x$
$$\text{Order} = 2, \text{degree} = 1$$
$$\therefore \quad \text{sum} = 2 + 1 = 3$$

13. (*c*)
$$\text{Required area} = 2\int_0^a \sqrt{4ax}\,dx$$
$$= 4\sqrt{a}\int_0^a x^{1/2}dx$$
$$= 4\sqrt{a} \times \frac{2}{3}[x^{3/2}]_0^a$$
$$= \frac{8}{3}\sqrt{a} \times a\sqrt{a} = \frac{8}{3}a^2 \text{ sq units}$$

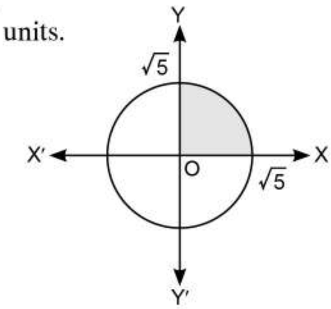


14. (*d*) The given curve is,

$$3x^2 + 3y^2 = 15$$

$$\Rightarrow \quad x^2 + y^2 = 5$$

It represents a circle having centre at origin (0, 0) and radius $\sqrt{5}$ units.



$$\text{Required area} = 4\int_0^{\sqrt{5}} \sqrt{5 - x^2}\,dx$$

$$= 4\left[\frac{x}{2}\sqrt{5 - x^2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_0^{\sqrt{5}}$$

$$= 4\left[0 + \frac{5}{2}\times\frac{\pi}{2} - 0\right]$$

$$= 5\pi \text{ sq units}$$

15. (*b*)
$$\int_1^2 x^2 dx = \frac{1}{3}[x^3]_1^2$$

$$= \frac{1}{3}[8 - 1] = \frac{7}{3}$$

16. (*b*)
$$\int_0^2 x[x]dx = \int_0^1 x[x]dx + \int_1^2 x[x]dx$$

$$= 0 + \int_1^2 x\,dx$$

$$= \left[\frac{x^2}{2}\right]_1^2 = 2 - \frac{1}{2} = \frac{3}{2}$$

17. (*b*), $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A)\,P(B)$ (as A and B are independent)

$= 0.4 + 0.3 - (0.4)\,(0.3)$

$= 0.4 + 0.3 - 0.12 = 0.7 - 0.12 = 0.58$

18. (*c*), A and B are independent events

$\therefore \quad P(A \cap B) = P(A)\,P(B) = 0.31 \times 0.41 = 0.1271$

19. (*d*), *A* is false *R* is true

as $\vec{a} \cdot \vec{b} = 10 - 1 - 15 = -6 \neq 0$

If two vectors are perpendicular then their dot product is 0. Hence $\vec{a}$ and $\vec{b}$ are not perpendicular.

$\vec{a} \times \vec{b}$ is vector given by,

$$\vec{a} \times \vec{b} = |\vec{a}\,||\vec{b}\,|\sin\theta\,\hat{n}$$

$\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$.

20. (*c*), Assertion is true but the reason is false.

21. Consider equation
$$\frac{dy}{dx} = \cos^3 x \sin^4 x$$

$$\Rightarrow \quad dy = (\cos^3 x \sin^4 x)dx$$

Integrating both sides, we get

$$\int dy = \int(\cos^3 x \sin^4 x)dx$$

$$\Rightarrow \quad y = \int \cos^3 x \sin^4 x\,dx \qquad ...(i)$$

Consider $\int \cos^3 x \sin^4 x\, dx = \int (1-t^2)t^4\, dt$ | Let $\sin x = t$ $\Rightarrow \cos x\, dx = dt$

$$= \int (t^4 - t^6)\, dt = \frac{t^5}{5} - \frac{t^7}{7} + C = \frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + C \quad ...(ii)$$

Substituting from (*ii*) in (*i*), we get

$y = \frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + C$ as the required solution where C is constant.

22. Given $P(A) = \frac{1}{3}, P(B) = \frac{2}{7}, P(C) = \frac{3}{8}$

$\Rightarrow \quad P(\overline{A}) = \frac{2}{3}, P(\overline{B}) = \frac{5}{7}, P(\overline{C}) = \frac{5}{8}$

All the three solve the problem simultaneously independently.

We can have case I: A can solve, B and C can not solve

Case II : B can solve, A and C can not solve

Case III: C can solve, A and B can not solve.

$\therefore \quad P(\text{only one solves}) = P(A\overline{B}\overline{C} \text{ or } \overline{A}B\overline{C} \text{ or } \overline{A}\,\overline{B}\,C)$.

$= \frac{1}{3} \times \frac{5}{7} \times \frac{5}{8} + \frac{2}{3} \times \frac{2}{7} \times \frac{5}{8} + \frac{2}{3} \times \frac{5}{7} \times \frac{3}{8}$

$= \frac{25 + 20 + 30}{168} = \frac{75}{168} = \frac{25}{56}$

OR

Bag *I*: 3 red + 4 black; bag *II*: 4 red + 5 black

Case *A*: when transferred ball is black.

Probability of drawing a black ball from bag $I = \frac{4}{7}$

Number of balls in bag *II*: 4 red + 6 black

Probability of red ball from bag $II = P(R/II) = \frac{4}{10} = \frac{2}{5}$

$\therefore$ probability of drawing a red ball from bag *II* when a black ball is transferred from bag *I* to bag *II*

$= \frac{4}{7} \times \frac{2}{5} = \frac{8}{35} \quad ... (i)$

Case *B*: when transferred ball is red

Probability of drawing a red ball from bag $I = \frac{3}{7}$

Number of balls in bag *II*: 5 red + 5 black

Probability of drawing a red ball from bag $II = P(R/II) = \frac{5}{10} = \frac{1}{2}$

Probability of drawing red ball from bag *II* when red ball is transferred from bag *I* to bag *II*

$= \frac{3}{7} \times \frac{1}{2} = \frac{3}{14} \quad ...(ii)$

$\therefore$ Probability of drawing a red ball from bag *II* when one ball is transferred from bag *I* to bag *II*.

$= \frac{8}{35} + \frac{3}{14}$ [from (*i*) and (*iii*)]

$= \frac{31}{70}$.

23. We have, $\vec{a} \perp \vec{b}$ and $\vec{a} \perp \vec{c}$.

Also we know that $(\vec{b} \times \vec{c})$ is perpendicular to both $\vec{b}$ and $\vec{c}$.

$\Rightarrow \quad \vec{a} \parallel \vec{b} \times \vec{c} \Rightarrow \vec{a} = \lambda(\vec{b} \times \vec{c})$ ...(*i*)

$\Rightarrow \quad |\vec{a}| = |\lambda|\,|\vec{b} \times \vec{c}| = |\lambda|\,|\vec{b}|\,|\vec{c}| \sin\frac{\pi}{6},$

$\therefore \quad \lambda = \pm 2,$ $[\vec{a}, \vec{b}, \vec{c}$ are unit vectors]

Substituting in (*i*), we get $\vec{a} = \pm 2(\vec{b} \times \vec{c})$.

24. The range of principle value branch of $\tan^{-1}$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

Now, $$\tan^{-1}\left(\tan\frac{9\pi}{8}\right) = \tan^{-1}\left[\tan\left(\pi + \frac{\pi}{8}\right)\right]$$

$$= \tan^{-1}\left(\tan\frac{\pi}{8}\right) = \frac{\pi}{8}.$$

25. $$A = B^2$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 + 0 & 0 + 0 \\ \alpha + 1 & 0 + 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha + 1 & 1 \end{bmatrix}$$

$\Rightarrow \alpha^2 = 1$ and $5 = \alpha + 1$

$\Rightarrow \alpha = \pm 1$ and $\alpha = 4$

There is no common value. Hence for no value of $\alpha, A = B^2$.

OR

$$[2x \quad 3]\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} x \\ 3 \end{bmatrix} = O \Rightarrow [2x \quad 3]\begin{bmatrix} x + 6 \\ -3x + 0 \end{bmatrix} = O$$

$$\Rightarrow [2x(x + 6) + 3(-3x)] = O \Rightarrow [2x^2 + 3x] = O$$

$$\Rightarrow 2x^2 + 3x = 0 \Rightarrow x(2x + 3) = 0$$

$$\Rightarrow x = 0 \text{ or } -\frac{3}{2}.$$

26. We have, $y = Ae^{mx} + Be^{nx}$ ...(*i*)

$\Rightarrow \quad \frac{dy}{dx} = Ame^{mx} + Bne^{nx}$ ...(*ii*)

and $\quad \frac{d^2y}{dx^2} = Am^2e^{mx} + Bn^2e^{nx}$ ...(*iii*)

LHS $= \frac{d^2y}{dx^2} - (m + n)\frac{dy}{dx} + mny = Am^2e^{mx} + Bn^2e^{nx} - (m + n)(Ame^{mx} + Bne^{nx}) + mn(Ae^{mx} + Be^{nx})$

[from (*i*), (*ii*) and (*iii*)]

$= Am^2e^{mx} + Bn^2e^{nx} - Am^2e^{mx} - Bmne^{nx} - Amne^{mx} - Bn^2e^{nx} + Amne^{mx} + Bmne^{nx} = 0.$

Alternatively:

Consider $y = Ae^{mx} + Be^{nx}$ ...(*i*)

$$\frac{dy}{dx} = Ame^{mx} + Bne^{nx} \quad ...(ii)$$

$$\frac{dy}{dx} = Ame^{mx} + n[y - Ae^{mx}] \text{[from } (i)]$$

$$\frac{dy}{dx} = A(m-n)e^{mx} + ny \quad ...(iii)$$

Again differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = Am\,(m-n)\,e^{mx} + n\frac{dy}{dx}$$

$$= m\left[\frac{dy}{dx} - ny\right] + n\frac{dy}{dx} \{\text{from } (iii)\}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = m\frac{dy}{dx} - mny + n\frac{dy}{dx}$$

$$= (m+n)\frac{dy}{dx} - mny$$

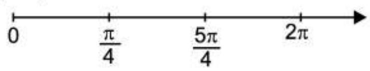
$$\Rightarrow \quad \frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$$

27. Given $f(x) = \sin x + \cos x$ in $[0, 2\pi]$

$$f'(x) = \cos x - \sin x \quad ...(i)$$

For critical points, $f'(x) = 0$

$$\Rightarrow \quad \tan x = 1 \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$$



From (*i*), $f'(x) = \cos x - \sin x$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right)$$

$$= \sqrt{2}\cos\left(x + \frac{\pi}{4}\right) \quad ...(ii)$$

(*a*) $0 < x < \frac{\pi}{4} \Rightarrow \frac{\pi}{4} < x + \frac{\pi}{4} < \frac{\pi}{2}$

$\Rightarrow x + \frac{\pi}{4} \in$ Ist quadrant

$\therefore \cos\left(x + \frac{\pi}{4}\right) > 0 \Rightarrow f'(x) > 0$

$\Rightarrow f$ is increasing

(*b*) $\frac{\pi}{4} < x < \frac{5\pi}{4} \Rightarrow \frac{\pi}{2} < x + \frac{\pi}{4} < \frac{3\pi}{2}$

$\Rightarrow x + \frac{\pi}{4} \in$ II, III quadrant

$\therefore \cos\left(x + \frac{\pi}{4}\right) < 0 \Rightarrow f'(x) < 0$

$\Rightarrow f$ is decreasing

(c) $\frac{5\pi}{4} < x < 2\pi$

$\Rightarrow \quad \frac{3\pi}{2} < x + \frac{\pi}{4} < 2\pi + \frac{\pi}{4}$

$\Rightarrow x + \frac{\pi}{4} \in$ IV, I quadrant

$\therefore \cos\left(x + \frac{\pi}{4}\right) > 0$

$\Rightarrow f'(x) > 0 \Rightarrow f$ is increasing

Strictly increasing in $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right)$;

Strictly decreasing in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

28. $\int_0^{\frac{\pi}{2}} \frac{\tan x}{1 + m^2 \tan^2 x} dx$

Consider $I = \int \frac{\tan x}{1 + m^2 \tan^2 x} dx$

Let $\tan^2 x = t \Rightarrow 2\tan x \cdot \sec^2 x \, dx = dt$

$$\Rightarrow \quad \tan x \, dx = \frac{1}{2\sec^2 x} dt = \frac{dt}{2(1+t)}$$

$$\therefore \quad I = \frac{1}{2}\int \frac{1}{(1+t)(1+m^2 t)} dt$$

Let $\frac{1}{(1+t)(1+m^2 t)} = \frac{A}{1+t} + \frac{B}{1+m^2 t}$

$$\Rightarrow \quad 1 = A(1 + m^2 t) + B(1 + t)$$

$$\Rightarrow \quad 1 = t(Am^2 + B) + (A + B)$$

Comparing coefficients, we get

$$A m^2 + B = 0, A + B = 1$$

$$\Rightarrow \quad A = \frac{-1}{m^2 - 1}, B = \frac{m^2}{m^2 - 1}$$

$$\therefore \quad \frac{1}{(1+t)(1+m^2 t)} = \frac{m^2}{m^2 - 1} \cdot \frac{1}{1 + m^2 t} - \frac{1}{(m^2 - 1)(1 + t)}$$

Integrating both sides w.r.t., t we get

$$\int \frac{1}{(1+t)(1+m^2 t)} \cdot dt = \frac{m^2}{m^2 - 1} \cdot \int \frac{1}{1+m^2 t} dt - \frac{1}{(m^2 - 1)} \cdot \int \frac{dt}{1+t}$$

$$= \frac{m^2}{(m^2 - 1)} \cdot \frac{\log|1 + m^2 t|}{m^2} - \frac{1}{(m^2 - 1)} \cdot \log|1 + t|$$

$$\therefore \quad I = \frac{1}{2(m^2 - 1)} \log\left|\frac{1 + m^2 t}{1 + t}\right|$$

$$= \frac{1}{2(m^2 - 1)} \log\left|\frac{1 + m^2 \tan^2 x}{1 + \tan^2 x}\right|$$

$$\therefore \quad \int_0^{\pi/2} \frac{\tan x}{1+m^2\tan^2 x}dx = \frac{1}{2(m^2-1)}\left[\log\left|\frac{1+m^2\tan^2 x}{1+\tan^2 x}\right|\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2(m^2-1)}\left[\log\left|\frac{\cot^2\frac{\pi}{2}+m^2}{\cot^2\frac{\pi}{2}+1}\right| - \log\left|\frac{1+m^2\tan^2 0}{1+\tan^2 0}\right|\right]$$

$$= \frac{1}{2(m^2-1)}[\log m^2 - 0] = \frac{2\log m}{2(m^2-1)} = \frac{\log m}{m^2-1}$$

OR

Consider $\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx} = -\cos x$

$\Rightarrow \quad \int \frac{1}{1+y}dy = -\int \frac{\cos x}{2+\sin x}dx$

$\Rightarrow \quad \log|1+y| = -\log|2+\sin x| + \log C$ $\quad \left[\text{using} \int \frac{f'(x)}{f(x)}dx = \log|f(x)| + C\right]$

$\Rightarrow \quad \log|1+y| = \log\left|\frac{C}{2+\sin x}\right|$

$\Rightarrow (1+y)(2+\sin x) = C$, where C is constant of integration. ...(*i*)

Given when $x = 0, y = 1$

$\Rightarrow (1+1)(2+\sin 0) = C \Rightarrow C = 4$

Substituting in (*i*), we get

$(1+y)(2+\sin x) = 4$

When $x = \frac{\pi}{2}$,

$(1+y)(2+\sin\frac{\pi}{2}) = 4$

$\Rightarrow (1+y)(3) = 4$

$\Rightarrow y = \frac{4}{3} - 1 = \frac{1}{3}$.

29. Given $f : R - \{0\} \to R - \{0\}$, defined by $f(x) = \frac{1}{x}$.

For one-one: Let for $x, y \in R - \{0\}$, (domain), $f(x) = f(y)$.

Now, $f(x) = f(y) \Rightarrow \frac{1}{x} = \frac{1}{y} \Rightarrow x = y$. Hence, one-one.

For onto: Let $y \in R - \{0\}$ (co-domain), then there must exists $x \in R - \{0\}$ (domain), such that $f(x) = y$

$\Rightarrow \frac{1}{x} = y \Rightarrow x = \frac{1}{y}$.

Clearly for each $y \in R - \{0\}$ (co-domain) there exists unique $x = \frac{1}{y} \in R - \{0\}$ domain *s.t.*

$f(x) = f\left(\frac{1}{y}\right) = y$

Hence, onto.

Consider the function $f : N \to R - \{0\}$ defined by $f(x) = \frac{1}{x}$.

Let $x, y, \in N$ such that $f(x) = f(y)$.

Now, $f(x) = f(y) \Rightarrow \frac{1}{x} = \frac{1}{y} \Rightarrow x = y$.

So, f is one-one.

Also, for each $y\ (< 0) \in R - \{0\}$ we must have $x \in N$ such that $y = f(x)$.

Now, $y = f(x) \Rightarrow y = \frac{1}{x}$.

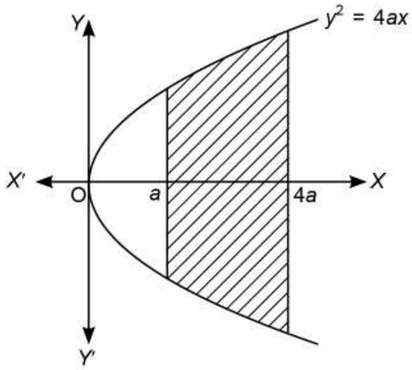
$\Rightarrow x = \frac{1}{y} < 0 \notin N$. So, f is not onto. So, result is not true if domain is replaced by N.

30. Curve is $x = at^2$ and $y = 2at$

$\Rightarrow x = a\left(\frac{y}{2a}\right)^2 \Rightarrow y^2 = 4ax$, which represents a parabola.

Also, for $t = 1, x = a$ and for $t = 2, x = 4a$.

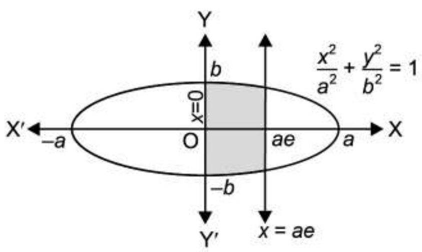
Plotting the curve, we have to find the shaded area.



Curve is symmetrical to the x-axis as function is even with respect to y.

$$\begin{aligned} \text{Area} &= 2 \times \text{area in the first quadrant.} \\ &= 2\int_a^{4a} \sqrt{4ax}\, dx = \left[4\sqrt{a} \cdot \frac{2}{3}x^{3/2}\right]_a^{4a} \\ &= \frac{8\sqrt{a}}{3}\left[(4a)^{3/2} - (a)^{3/2}\right] = \frac{8\sqrt{a}}{3}[8a^{3/2} - a^{3/2}] \\ &= \frac{8\sqrt{a}}{3} \times 7 \times a\sqrt{a} = \frac{56}{3}a^2 \text{ sq units.} \end{aligned}$$

OR



We have to find the shaded area

As curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is symmetrical about the x-axis.

$$\therefore \quad \text{area} = 2\int_0^{ae} y\, dx$$

$$= 2\frac{b}{a}\int_0^{ae} \sqrt{a^2 - x^2}\, dx$$

$$= \frac{2b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^{ae}$$

$$= \frac{2b}{a}\left[\left\{\frac{ae}{2}\sqrt{a^2-a^2e^2} + \frac{a^2}{2}\sin^{-1}e\right\} - \{0+0\}\right]$$

$$= \frac{2b}{a}\left[\frac{ae}{2}\sqrt{a^2(1-e^2)} + \frac{a^2}{2}\sin^{-1}e\right]$$

$$= \frac{2b}{a}\left[\frac{ae}{2}.b + \frac{a^2}{2}\sin^{-1}e\right] \quad [\because b^2 = a^2(1-e^2)]$$

$$= (b^2e + ab\sin^{-1}e) \text{ sq units.}$$

31. $$\text{LHD} = \text{L}f'(2) = \lim_{h\to 0}\frac{f(2-h)-f(2)}{-h}$$

$$= \lim_{h\to 0}\frac{(2-2+h)-0}{-h} = \lim_{h\to 0}(-1) = -1$$

$$\text{RHD} = \text{R}f'(2) = \lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$$

$$= \lim_{h\to 0}\frac{[-2+3(2+h)-(2+h)^2]-0}{h}$$

$$= \lim_{h\to 0}\frac{-2+6+3h-4-h^2-4h}{h} = \lim_{h\to 0}(-1-h) = -1$$

As $\text{L}f'(2) = \text{R}f'(2) \Rightarrow f$ is differentiable at $x = 2$.

OR

Let $y = \frac{8^x}{x^8}$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{x^8\cdot\frac{d}{dx}(8^x) - 8^x\cdot\frac{d}{dx}x^8}{(x^8)^2} = \frac{x^8\cdot 8^x\cdot\log_e 8 - 8^x\cdot 8x^7}{x^{16}}$$

$$= \frac{8^x\cdot x^7[x\log_e 8 - 8]}{x^{16}} = \frac{8^x[x\log_e 8 - 8]}{x^9}.$$

32. Since l, m, n and $l+\delta l, m+\delta m, n+\delta n$ are direction cosines of a variable line in two different positions, therefore, $l^2+m^2+n^2 = 1$ ...(*i*)

and $(l+\delta l)^2 + (m+\delta m)^2 + (n+\delta n)^2 = 1$ ...(*ii*)

Now, $(l+\delta l)^2 + (m+\delta m)^2 + (n+\delta n)^2 = 1$

$\Rightarrow (l^2+m^2+n^2) + 2(l\cdot\delta l + m\cdot\delta m + n\cdot\delta n) + (\delta l)^2 + (\delta m)^2 + (\delta n)^2 = 1$

$\Rightarrow 1 + 2(l\,\delta l + m\,\delta m + n\,\delta n) + (\delta l)^2 + (\delta m)^2 + (\delta n)^2 = 1$ [using (*i*)]

$\Rightarrow 2(l\,\delta l + m\,\delta m + n\,\delta n) = -[(\delta l)^2 + (\delta m)^2 + (\delta n)^2]$

$\Rightarrow (l\,\delta l + m\,\delta m + n\,\delta n) = -\frac{1}{2}[(\delta l)^2 + (\delta m)^2 + (\delta n)^2]$ ...(*iii*)

Also $\cos(\delta\theta) = l(l+\delta l) + m(m+\delta m) + n(n+\delta n)$

$= (l^2+m^2+n^2) + (l\,\delta l + m\,\delta m + n\,\delta n)$

$= 1 - \frac{1}{2}[(\delta l)^2 + (\delta m)^2 + (\delta n)^2]$ [using (*i*) and (*iii*)]

$\Rightarrow 2(1 - \cos \delta\theta) = (\delta\, l)^2 + (\delta\, m)^2 + (\delta\, n)^2$

$\Rightarrow 2 \times 2 \sin^2 \dfrac{\delta\theta}{2} = (\delta\, l)^2 + (\delta\, m)^2 + (\delta\, n)^2$ $\quad [\because\ 1 - \cos 2\theta = 2 \sin^2 \theta]$

$\Rightarrow 4\left[\dfrac{\delta\theta}{2}\right]^2 = (\delta l)^2 + (\delta m)^2 + (\delta n)^2$ $\quad$ [if θ is small, then $\sin \theta \to \theta$]

$\Rightarrow (\delta\theta)^2 = (\delta l)^2 + (\delta m)^2 + (\delta n)^2$

OR

Given $\quad \vec{b} = \vec{\alpha} + \vec{\beta}$ $\quad$ ...(*i*)

where $\vec{\alpha}$ is parallel to $\vec{a} = \hat{i} + \hat{j}$ and $\vec{\beta}$ is perpendicular to $\vec{a}$.

From (*i*), we get $3\hat{i} + 4\hat{k} = \lambda(\hat{i} + \hat{j}) + \vec{\beta}$

$\Rightarrow (3 - \lambda)\hat{i} - \lambda\hat{j} + 4\hat{k} = \vec{\beta}$ $\quad$ ...(*ii*)

As $\vec{\beta} \perp \vec{a} \Rightarrow [(3 - \lambda)\hat{i} - \lambda\hat{j} + 4\hat{k}] \cdot (\hat{i} + \hat{j}) = 0$

$\Rightarrow \quad 3 - \lambda - \lambda = 0$

$\Rightarrow \quad 3 - 2\lambda = 0$

$\Rightarrow \quad \lambda = \dfrac{3}{2}$

$\therefore\ \vec{\alpha} = \dfrac{3}{2}(\hat{i} + \hat{j})$ and $\vec{\beta} = \dfrac{3}{2}\hat{i} - \dfrac{3}{2}\hat{j} + 4\hat{k}$

$\therefore \quad 3\hat{i} + 4\hat{k} = \left(\dfrac{3}{2}\hat{i} + \dfrac{3}{2}\hat{j}\right) + \left(\dfrac{3}{2}\hat{i} - \dfrac{3}{2}\hat{j} + 4\hat{k}\right)$

33. Consider $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$$

$$= 2(0) + 3(-2) + 5(1) = -1 \neq 0$$

Let A_{ij} be cofactor of a_{ij} in $|A|$.

$A_{11} = 0, \quad A_{21} = -1, \quad A_{31} = 2$

$A_{12} = 2, \quad A_{22} = -9, \quad A_{32} = 23$

$A_{13} = 1, \quad A_{23} = -5, \quad A_{33} = 13$

$$\text{Adj}\, A = \begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\, \text{adj}\, A = -\frac{1}{1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \quad \text{...(i)}$$

Consider equations,

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3$

Corresponding matrix equation is

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

i.e. $AX = B$. Its solution is $X = A^{-1}B$ ...(*ii*)

We had already calculated A^{-1}, So we substitute in (*ii*) and find X and hence x, y, z.

$$X = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow x = 1; \quad y = 2; \quad z = 3.$$

OR

Given $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$$\therefore A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$5A = 5\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 10 & -5 & 5 \\ -5 & 10 & -5 \\ 5 & -5 & 10 \end{bmatrix}$$

$$4I = 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\therefore A^2 - 5A + 4I = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \begin{bmatrix} 10 & -5 & 5 \\ -5 & 10 & -5 \\ 5 & -5 & 10 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6-10+4 & -5+5+0 & 5-5+0 \\ -5+5+0 & 6-10+4 & -5+5+0 \\ 5-5+0 & -5+5+0 & 6-10+4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O; \ \therefore A^2 - 5A + 4I = O$$

Pre-multiply both sides by A^{-1}, we get

$\therefore \quad A^{-1}(AA) - 5A^{-1}A + 4A^{-1}I = A^{-1}O$

$\Rightarrow \quad (A^{-1}A)A - 5I + 4A^{-1} = O$

$\Rightarrow \quad IA - 5I + 4A^{-1} = O$

$$\Rightarrow \quad 4A^{-1} = 5I - A = 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$\Rightarrow \quad 4A^{-1} = \begin{bmatrix} 5-2 & 0+1 & 0-1 \\ 0+1 & 5-2 & 0+1 \\ 0-1 & 0+1 & 5-2 \end{bmatrix} = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$\Rightarrow \quad A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

34. Consider equation $dy = \cos x(2 - y \operatorname{cosec} x)dx$

$$\Rightarrow \frac{dy}{dx} = 2\cos x - y\cot x$$

$$\Rightarrow \quad \frac{dy}{dx} + \cot x \cdot y = 2\cos x$$

Here $P(x) = \cot x$, $Q(x) = 2\cos x$

Integrating factor (I.F.) $= e^{\int \cot x\, dx}$

$= e^{\log|\sin x|} = \sin x$

Its solution is (I.F.) $y = \int \{(\text{I.F}).Q(x)\}dx$

$$\Rightarrow \quad \sin x.y = \int \sin x . (2\cos x)dx = \int \sin 2x\, dx$$

$$\Rightarrow \quad \sin x.y = -\frac{\cos 2x}{2} + C \qquad ...(i)$$

Given $y = 2$, when $x = \frac{\pi}{2}$

$$\Rightarrow \quad \sin\frac{\pi}{2}\cdot 2 = -\frac{\cos\pi}{2} + C$$

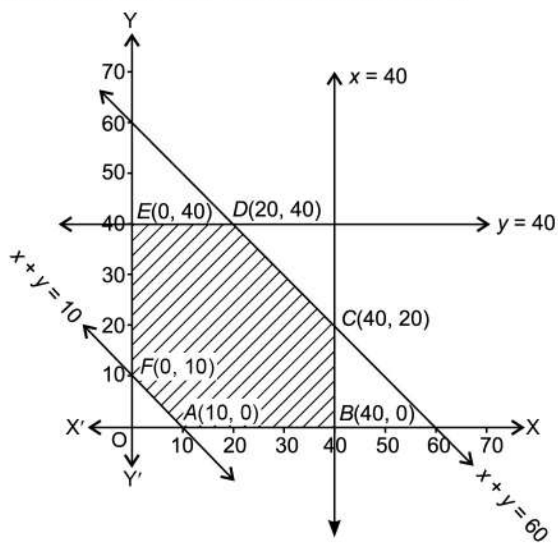
$$\Rightarrow \quad 2 = \frac{1}{2} + C$$

$$\Rightarrow \quad C = \frac{3}{2}$$

Substituting in (i), we get

$\sin x . y = -\frac{1}{2}\cos 2x + \frac{3}{2}$ as particular solution.

35. On plotting the inequations we notice shaded portion is feasible solution. Possible points for minimum Z are $A(10, 0)$, $B(40, 0)$, $C(40, 20)$, $D(20, 40)$, $E(0, 40)$, $F(0, 10)$



Points	$Z = 3x + 4y + 370$	Values	
$A(10, 0)$	$30 + 0 + 370$	400	← Minimum
$B(40, 0)$	$120 + 0 + 370$	490	
$C(40, 20)$	$120 + 80 + 370$	570	
$D(20, 40)$	$60 + 160 + 370$	590	
$E(0, 40)$	$0 + 160 + 370$	530	
$F(0, 10)$	$0 + 40 + 370$	410	

Z is minimum for $A(10, 0)$, i.e. $x = 10, y = 0$.

36. Considering the following events.

E_1: matter processed by Mehul

E_2: matter processed by Janya

E_3: matter processed by Charvi

E: matter has an error

(*i*) $P(E/E_2) = 0.04$

(*ii*) Probability $= P(E_2) \,.\, P(E/E_2)$

$= 0.3 \times 0.04 = 0.012.$

(*iii*) $P(E) = P(E_1)\,P(E/E_1) + P(E_2)\,P(E/E_2) + P(E_3)P(E/E_3)$

$= 0.4 \times 0.06 + 0.3 \times 0.04 + 0.3 \times 0.03$

$= 0.024 + 0.012 + 0.009 = 0.045.$

OR

(*iii*) $P(E_1/E) = \dfrac{P(E_1)\,P(E/E_1)}{0.045} = \dfrac{0.024}{0.045} = \dfrac{8}{15}.$

37. (*i*) In right angled triangle,

$100^2 = a^2 + x^2 \Rightarrow a^2 = 10000 - x^2$

$\Rightarrow a = \sqrt{10000 - x^2}$ m

(*ii*) Perimeter $= 100 + 100 + 100 + x + 100 + x$

$= (400 + 2x)$m

(*iii*) Total covered area is area of trapezium

$= \dfrac{1}{2}[100 + 100 + 2x] \,.\, a$

$= (100 + x)\sqrt{10000 - x^2}$ m^2

OR

(*iii*) $A = (100 + x)\sqrt{10000 - x^2}$

$$\frac{dA}{dx} = \frac{(100+x)(-2x)}{2\sqrt{10000-x^2}} + \sqrt{10000-x^2}$$

$$= \frac{-100x - x^2 + 10000 - x^2}{\sqrt{10000-x^2}}$$

$$\frac{dA}{dx} = \frac{-2x^2 - 100x + 10000}{\sqrt{10000-x^2}}$$

For maximum area

$$\frac{dA}{dx} = 0 \Rightarrow -2x^2 - 100x + 10000 = 0$$

$\Rightarrow x^2 + 50x - 5000 = 0$

$\Rightarrow (x + 100)(x - 50) = 0$

$\Rightarrow x = -100$(rejected) or $x = 50$ m

$$\frac{d^2A}{dx^2} = \frac{\sqrt{10000-x^2} \times (-4x - 100) - (-2x^2 - 100x + 10000) \times \left(\frac{-x}{\sqrt{10000-x^2}}\right)}{(10000 - x^2)}$$

$$\Rightarrow \frac{d^2A}{dx^2} = \frac{(-4)(x+25)\sqrt{10000-x^2} + \frac{2x}{\sqrt{10000-x^2}}(-x^2 - 50x + 5000)}{(10000 - x^2)}$$

Now, $\left[\frac{d^2A}{dx^2}\right]_{x=50} = \frac{-300 \times 50\sqrt{3} + 0}{7500} = -2\sqrt{3}$

$\frac{d^2A}{dx^2} < 0$ for $x = 50$

Hence, area is maximum for $x = 50$ m.

38. (*i*) We have, $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$

$\Rightarrow |\vec{a}+\vec{b}|^2 = |\vec{a} - b|^2 \Rightarrow (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b})$

$\Rightarrow |\vec{a}^2| + |\vec{b}^2| + 2\vec{a} \cdot \vec{b} = |\vec{a}^2| + |\vec{b}^2| - 2\vec{a} \cdot \vec{b}$

$\Rightarrow 4\vec{a} \cdot \vec{b} = 0 \Rightarrow \vec{a} \cdot \vec{b} = 0 \Rightarrow \vec{a} \perp \vec{b}$

(*ii*) $2\vec{a} + \vec{b} = 2\hat{i} - 4\hat{j} + 2\hat{i} + \hat{j} + 3\hat{k} = 4\hat{i} - 3\hat{j} + 3\hat{k}$

$\vec{a} + \vec{b} = \hat{i} - 2\hat{j} + 2\hat{i} + \hat{j} + 3\hat{k} = 3\hat{i} - \hat{j} + 3\hat{k}$

$\vec{a} - 2\hat{b} = \hat{i} - 2\hat{j} - 4\hat{i} - 2\hat{j} - 6\hat{k} = -3\hat{i} - 4\hat{j} - 6\hat{k}$

$$(\vec{a} + \vec{b}) \times (\vec{a} - 2\vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 3 \\ -3 & -4 & -6 \end{vmatrix}$$

$$= 18\hat{i} + 9\hat{j} - 15\hat{k}$$

Now, $(2\vec{a} + \vec{b}) \cdot \left[(\vec{a} + \vec{b}) \times (\vec{a} - 2\vec{b})\right]$

$= (4\hat{i} - 3\hat{j} + 3\hat{k}) \cdot (18\hat{i} + 9\hat{j} - 15\hat{k})$

$= 72 - 27 - 45 = 0$