8 SAMPLE PAPER

[Difficult Concept]

Time Allowed: 3 Hours] [*Maximum Marks:* 80

General Instructions:

Read the following instructions very carefully and strictly follow them:

(*i*) *This Question paper contains **38** questions. **All** questions are **compulsory**.*

(*ii*) *This Question paper is divided into **five** Sections – **A**, **B**, **C**, **D** and **E**.*

(*iii*) *In **Section A**, Questions no. **1** to **18** are **multiple choice questions** (MCQs) and Questions no. **19** and **20** are **Assertion-Reason based** questions of **1 mark each**.*

(*iv*) *In **Section B**, Questions no. **21** to **25** are **Very Short Answer (VSA)-type** questions, carrying **2 marks each**.*

(*v*) *In **Section C**, Questions no. **26** to **31** are **Short Answer (SA)-type** questions, carrying **3 marks each**.*

(*vi*) *In **Section D**, Questions no. **32** to **35** are **Long Answer (LA)-type** questions, carrying **5 marks each**.*

(*vii*) *In **Section E**, Questions no. **36** to **38** are **Case study-based questions**, carrying **4 marks each**.*

(*viii*) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(*ix*) *Use of calculators is **not** allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. Let R be a relation on the set N given by $R = \{(a, b) : a = b - 3, b > 5\}$. Then, **[NCERT Part-I, Page 2]**

(*a*) $(1, 2) \in R$ (*b*) $(5, 9) \in R$ (*c*) $(7, 10) \in R$ (*d*) $(10, 7) \in R$

2. Let 'f' : $R - \{2\} \to R - \{1\}$ be a function defined by $f(x) = \dfrac{x-1}{x-2}$, then 'f' is **[NCERT Part-I, Page 7]**

(*a*) into function (*b*) many one function

(*c*) bijective function (*d*) many one, into function.

3. If $AB = C$ then orders of matrices A, B, C is equal to **[NCERT Part-I, Page 51-52]**

(*a*) $A_{2 \times 3}, B_{3 \times 2}, C_{2 \times 3}$ (*b*) $A_{3 \times 2}, B_{2 \times 3}, C_{3 \times 3}$

(*c*) $A_{3 \times 3}, B_{2 \times 3}, C_{3 \times 3}$ (*d*) $A_{3 \times 2}, B_{2 \times 3}, C_{3 \times 2}$

4. If $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ -4 & 3 \end{bmatrix}, C = \begin{bmatrix} 2 & 12 \\ -7 & 9 \end{bmatrix}$ then $5A - 3B + C$ is equal to **[NCERT Part-I, Page 46]**

(*a*) $\begin{bmatrix} -1 & -1 \\ -20 & 20 \end{bmatrix}$ (*b*) $\begin{bmatrix} -1 & 1 \\ 20 & 20 \end{bmatrix}$ (*c*) $\begin{bmatrix} 1 & 1 \\ 20 & 20 \end{bmatrix}$ (*d*) $\begin{bmatrix} 1 & -1 \\ 20 & 20 \end{bmatrix}$

5. The value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$ is [NCERT Part-I, Page 21]

(a) $\frac{13\pi}{6}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{5}$

6. Solution of the differential equation $x\sqrt{1-y^2}\,dx + y\sqrt{1-x^2}\,dy = 0$ is [NCERT Part-II, Page 306-307]

(a) $\sqrt{1+x^2}+\sqrt{1-y^2}=C$ (b) $\sqrt{1-x^2}+\sqrt{1-y^2}=C$

(c) $\sqrt{1-x^2}-\sqrt{1-y^2}=C$ (d) $\left(\sqrt{1-x^2}\right)\left(\sqrt{1-y^2}\right)=C$

7. Integrating factor for the solution of differential equation $(x-y^3)dy + y\,dx = 0$ is [NCERT Part-II, Page 323]

(a) $\frac{1}{y}$ (b) $\log y$ (c) y (d) y^2

8. The area enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ is equal to [Conceptual Application]

(a) $\pi^2 ab$ sq units (b) πab sq units (c) $\pi a^2 b$ sq units (d) πab^2 sq units

9. The area enclosed by the circle $x^2 + y^2 = 2$ is equal to [Conceptual Application]

(a) 4π sq units (b) $2\sqrt{2}\,\pi$ sq units (c) $4\pi^2$ sq units (d) 2π sq units

10. $\int_0^{\frac{\pi}{4}}\sqrt{1-\sin 2x}\,dx =$ [NCERT Part-II, Page 241]

(a) $\sqrt{2}-1$ (b) $\sqrt{2}+1$ (c) $\sqrt{2}$ (d) $\sqrt{3}$

11. $\int_0^2 (x^2+3)dx =$ [NCERT Part-II, Page 268]

(a) $\frac{25}{3}$ (b) $\frac{26}{3}$ (c) 8 (d) $\frac{23}{3}$

12. The function $f(x) = \cos x - 2px$ is decreasing for [NCERT Part-I, Page 153]

(a) $p < \frac{1}{2}$ (b) $p > \frac{1}{2}$ (c) $p < 2$ (d) $p > 2$

13. For the function $y = x^3 + 21$, the value of x, when y increases 75 times as fast as x, is [NCERT Part-I, Page 147-148]

(a) ± 3 (b) $\pm 5\sqrt{3}$ (c) ± 5 (d) None of these

14. The function f, defined by $f(x) = \frac{x+1}{1+\sqrt{1+x}}$ is continuous at $x = 0$, when $f(0) =$ [NCERT Part-I, Page 105]

(a) 1 (b) –2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

15. The function $f(x) = \begin{cases} \frac{|x-4|}{2(x-4)}, & \text{if } x \neq 4 \\ 0, & \text{if } x = 4 \end{cases}$ is [NCERT Part-I, Page 105]

(a) continuous at all $x \in R$ (b) discontinuous at $x = 4$

(c) continuous only when $x = 4$ (d) None of these

16. The general point on the line $\vec{r} = (2\hat{i}+\hat{j}-4\hat{k})+\lambda(3\hat{i}+2\hat{j}-\hat{k})$ is [NCERT Part-II, Page 382]

(a) (2, 1, –4) (b) (3, 2, –1) (c) (–1, 1, 3) (d) $(2 + 3\lambda, 1 + 2\lambda, -4 - \lambda)$

17. A and B are two independent events, $P(A) = 0.2$ and $P(B) = 0.8$, then $P(A \cap \overline{B}) =$ [Conceptual Application]

(a) 0.03 (b) 0.04 (c) 0.4 (d) 0.3

18. A die is tossed twice. The probability of getting 1, 2, 3, 4 on first toss and 4, 5, 6 on the second toss is **[Conceptual Application]**

(*a*) $\frac{2}{5}$ (*b*) $\frac{1}{2}$ (*c*) $\frac{1}{3}$ (*d*) $\frac{1}{9}$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (*a*), (*b*), (*c*) and (*d*) as given below.)

(*a*) Both *A* and *R* are true and *R* is the correct explanation of *A*.

(*b*) Both *A* and *R* are true but *R* is not the correct explanation of *A*.

(*c*) *A* is true but *R* is false.

(*d*) *A* is false but *R* is true.

19. Assertion (*A*): Degree of the differential equation $\frac{dy}{dx} + \cos\left(\frac{dy}{dx}\right) = 0$ is not defined. **[NCERT Part-II, Page 302]**

Reason (*R*): The degree of those differential equations are not defined which can not be written as polynomial of derivatives.

20. Assertion (*A*): If *k* is a scalar and *A* is a 3×3 square matrix, then $|kA|$ is equal to $k^3 |A|$. **[NCERT Part-II, Page 80]**

Reason (*R*): If every element of a third order determinant of value Δ is multiplied by *n*, then the value of new determinant is $n^3\Delta$.

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Solve the differential equation $\frac{dy}{dx} = y^3 \operatorname{cosec} x$, given that $y\left(\frac{\pi}{4}\right) = 2$. **[NCERT Part-II, Page 306-307]**

22. *A* and *B* throw a pair of dice alternatively. A wins if he throws 6 before *B* and *B* wins if he throws 7 before *A* throws 6. If *A* begins, show that the odds in favour of *A* are 30 : 31. **[Conceptual Application]**

OR

E and F are events of an experiment such that $P(E) = 0.8$, $P(F) = 0.7$, $P(E \cap F) = 0.6$. Find $P\left(\frac{\overline{E}}{\overline{F}}\right)$. **[NCERT Part-II, Page 408]**

23. Find the area of the parallelogram whose one side and a diagonal are represented by coinitial vector $3\hat{i} - \hat{k}$ and $2\hat{i} + \hat{j} - 4\hat{k}$. **[Conceptual Application]**

24. If $\cos^{-1} x + \cos^{-1} y = 2\pi$, then find the value of $x^{100} + y^{200} + \frac{1}{x^{100}\, y^{200}}$. **[Conceptual Application]**

25. Given matrix $A = \begin{bmatrix} 4 & 3 \\ -1 & 2 \end{bmatrix}$, show that $A + A'$ is symmetric matrix and $A - A'$ is a skew symmetric matrix. **[NCERT Part-I, Page 63-64]**

OR

If *A* is a square matrix such that $A^2 = A$, show that $(I + A)^3 = 7A + I$. **[Conceptual Application]**

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. If $x = a \sin 2t(1 + \cos 2t)$ and $y = b \cos 2t(1 - \cos 2t)$, show that $\left(\frac{dy}{dx}\right)_{\text{at } t=\frac{\pi}{4}} = \frac{b}{a}$.

[NCERT Part-I, Page 134-135]

27. Show that $y = \log(1 + x) - \frac{2x}{2+x}, x > -1$ is an increasing function of x throughout its domain.

[NCERT Part-I, Page 153]

28. Evaluate $\int_2^3 \frac{x-1}{(x+1)^3} e^x dx$. **[NCERT Part-II, Page 262-263]**

OR

Solve the differential equation: $\frac{dy}{dx} = \sin(x + y) + \cos(x + y)$. **[Integrated Question]**

29. Let $f : W \to W$ be defined as $f(n) = n - 1$, if n is odd and $f(n) = n + 1$, if n is even. Show that f is injective as well as surjective. Here, W represents the set of whole numbers. **[NCERT Part-I, Page 7]**

30. Using integration, find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$. **[Conceptual Application]**

OR

Using the method of integration, find the area of the ΔABC, coordinates of whose vertices are $A(2, 0)$, $B(4, 5)$ and $C(6, 3)$. **[Conceptual Application]**

31. If $y = \frac{\log x}{x}$, show that $\frac{d^2y}{dx^2} = \frac{2\log x - 3}{x^3}$. **[NCERT Part-I, Page 137]**

OR

Examine the differentiability of the function $f(x) = \begin{cases} x[x], & \text{if } 0 \le x < 2 \\ (x-1)x, & \text{if } 2 \le x < 3 \end{cases}$ at $x = 2$.

[NCERT Part-I, Page 105]

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. Show that lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ intersect. Also find their point of intersection. **[Conceptual Application]**

OR

Find the value of λ, so that the lines $\frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{5z-10}{11}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other. **[NCERT Part-II, Page 383-384]**

33. If $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$, find A^{-1}. How we can use A^{-1} to solve the system of equations: $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2$; $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5$; $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = -4$? **[NCERT Part-I, Page 94-95]**

OR

Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and state how we can use it to solve the system of equations $x - y + z = 4; x - 2y - 2z = 9; 2x + y + 3z = 1.$ **[NCERT Part-I, Page 94-95]**

34. Find the particular solution of the differential equation $\tan x. \dfrac{dy}{dx} = 2x \tan x + x^2 - y; (\tan x \neq 0)$ given that $y = 0$ when $x = \dfrac{\pi}{2}$. **[NCERT Part-II, Page 322-323]**

35. Solve the following linear programming problem graphically.

Minimise $Z = 3x + y + 39500$ **[NCERT Part-II, Page 397-398]**

subject to the constraints,

$x \geq 0, y \geq 0, \; x + y \leq 7000, x \leq 4500, y \leq 3000, x + y \geq 3500$

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. A person has to reach a company for an interview. He has four options to reach the company i.e. by metro, by bus, by scooter or by other means of transport. The probabilities of using these means are $\dfrac{3}{10}, \dfrac{1}{5}, \dfrac{1}{10}$ and $\dfrac{2}{5}$ respectively. The probabilities that he will be late if he comes by metro, bus or scooter are $\dfrac{1}{4}, \dfrac{1}{3}$ and $\dfrac{1}{12}$ respectively, but if he comes by other means he will not be late. Using the above information answer the following questions. **[Conceptual Application]**

(*i*) What is the conditional probability of reaching late by other means of transport?

(*ii*) What is the probability that he travelled by bus and was late?

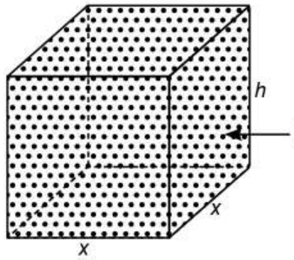
(*iii*) What is the probability of reaching late?

OR

(*iii*) What is the probability that person comes by Metro, given that when he arrives the company, he is late?

Case Study - 2

37. A village panchayat wants to dug out a square base tank for preparing fertilizers and wants capacity to be 250 cu metres. On calculations, it was found that cost of the land is ₹ 50 per square meter and cost of digging increases with depth and for the whole tank is ₹ 400 (depth)^2. Tank is shown as below. **[Conceptual Application]**



(*i*) If the side of the square base is x m and the height of the tank is h m, then establish relation between x and h.

(*ii*) Find the cost C for digging the tank in terms of x and h.

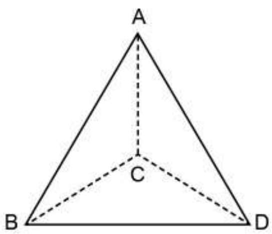
(*iii*) Find the cost C in terms of h only .

OR

(*iii*) Find the value of h for which cost C is minimum.

Case Study - 3

38. A building is to be constructed in the form of a triangular pyramid, $ABCD$ as shown in the figure. Let its angular points are $A(0, 1, 2)$, $B(3, 0, 1)$, $C(4, 3, 6)$ and $D(2, 3, 2)$ and G be the point of intersection of the medians of ΔBCD. **[Conceptual Application]**



(*i*) Find the coordinates of point G.

(*ii*) Find the length of vector $\overrightarrow{AG}$.

SOLUTIONS

1. (*c*), as $7 = 10 - 3$, so $(7, 10) \in R$.

2. (*c*)

3. (*b*)

4. (*d*)

$$5A - 3B + C = 5\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} - 3\begin{bmatrix} 2 & 1 \\ -4 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 12 \\ -7 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -10 \\ 15 & 20 \end{bmatrix} - \begin{bmatrix} 6 & 3 \\ -12 & 9 \end{bmatrix} + \begin{bmatrix} 2 & 12 \\ -7 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 5-6+2 & -10-3+12 \\ 15+12-7 & 20-9+9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ 20 & 20 \end{bmatrix}$$

5. (*b*), We know that the range of the principal value branch of $\cos^{-1}$ is $[0, \pi]$.

But $\frac{13\pi}{6} \notin [0, \pi]$

$$\therefore \quad \cos^{-1}\left(\cos\frac{13\pi}{6}\right) = \cos^{-1}\left[\cos\left(2\pi + \frac{\pi}{6}\right)\right]$$

$$= \cos^{-1}\left[\cos\left(\frac{\pi}{6}\right)\right]$$

$$= \frac{\pi}{6} \in [0, \pi]$$

$$\therefore \quad \cos^{-1}\left(\cos\frac{13\pi}{6}\right) = \frac{\pi}{6}.$$

6. (*b*),

$$x\sqrt{1-y^2}\,dx = -y\sqrt{1-x^2}\,dy$$

$$\Rightarrow \quad -\frac{y}{\sqrt{1-y^2}}dy = \frac{x}{\sqrt{1-x^2}}dx$$

$$\Rightarrow \quad \frac{1}{2}\int\frac{-2y}{\sqrt{1-y^2}}dy = -\frac{1}{2}\int\frac{-2x}{\sqrt{1-x^2}}dx$$

$$\Rightarrow \quad \frac{1}{2} \times 2\sqrt{1-y^2} = -\frac{1}{2} \times 2\sqrt{1-x^2} + C$$

$$\Rightarrow \quad \sqrt{1-y^2} + \sqrt{1-x^2} = C$$

7. (*c*) Equation is $(x - y^3)\,dy + y\,dx = 0$

$$\Rightarrow \quad \frac{dx}{dy} = -\frac{(x-y^3)}{y} \Rightarrow \frac{dx}{dy} + \frac{1}{y}\cdot x = y^2$$

Comparing with $\frac{dx}{dy} + P(y).x = Q(y)$, we get

$$P(y) = \frac{1}{y};\ Q(y) = y^2$$

Integrating factor $= e^{\int\frac{1}{y}dy} = e^{\log y} = y$

8. (*b*), since area $= 4\int_0^a \frac{b}{a}\sqrt{a^2 - x^2}\,dx$

$$= \frac{4b}{a}\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a = \pi ab \text{ sq units}$$

9. (*d*), since area $= 4\int_0^{\sqrt{2}} \sqrt{2 - x^2}\,dx$

$$= 4\left[\frac{x}{2}\sqrt{2 - x^2} + \sin^{-1}\frac{x}{\sqrt{2}}\right]_0^{\sqrt{2}} = 2\pi \text{ sq units.}$$

10. (*a*), $\int_0^{\frac{\pi}{4}} \sqrt{1 - \sin 2x}\,dx = \int_0^{\frac{\pi}{4}} \sqrt{\cos^2 x + \sin^2 x - 2\cos x \cdot \sin x}\,dx$

$$= \int_0^{\frac{\pi}{4}} |\cos x - \sin x|\,dx$$

$$= \int_0^{\frac{\pi}{4}} (\cos x - \sin x)\,dx$$

$$= \left[\sin x + \cos x\right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 0 - 1$$

$$= \sqrt{2} - 1$$

11. (*b*), $\int_0^2 (x^2 + 3)\,dx = \left[\frac{x^3}{3} + 3x\right]_0^2$

$$= \left(\frac{8}{3} + 6\right) - 0$$

$$= \frac{26}{3}$$

12. (*b*), As, $f(x) = \cos x - 2px$

$\Rightarrow$ $f'(x) = -\sin x - 2p = -(\sin x + 2p)$

For ↓ ing, $f'(x) < 0$

$\Rightarrow$ $-(\sin x + 2p) < 0$

$\Rightarrow$ $\sin x + 2p > 0$

$\Rightarrow$ $p > \frac{1}{2}$ $[\because -1 \le \sin x \le 1]$

13. (*c*), As, $y = x^3 + 21$

$\Rightarrow$ $\frac{dy}{dt} = 3x^2\frac{dx}{dt}$ ...(*i*)

Now, $\frac{dy}{dt} = 75\frac{dx}{dt}$ (given)

$\therefore$ from (*i*), we get

$$75\frac{dx}{dt} = 3x^2\frac{dx}{dt} \Rightarrow x = \pm 5$$

14. (*c*), The function $f(x)$ is continuous at $x = 0$

$$\therefore \quad \underset{(x=0)}{\text{LHL}} = \underset{(x=0)}{\text{RHL}} = f(0)$$

$$\Rightarrow \quad \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$$

$$\Rightarrow \quad \lim_{h \to 0} \frac{0-h+1}{1+\sqrt{1+0-h}} = \lim_{h \to 0} \frac{0+h+1}{1+\sqrt{1+0+h}} = f(0)$$

$$\Rightarrow \quad \frac{1}{1+\sqrt{1}} = \frac{1}{1+\sqrt{1}} = f(0)$$

$$\Rightarrow \quad \frac{1}{2} = \frac{1}{2} = f(0) \Rightarrow f(0) = \frac{1}{2}$$

15. (*b*), The function is defined when $f(4) = 0$ (given)

$\therefore f(x)$ exists.

Consider LHL

$$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} \frac{|x-4|}{2(x-4)}$$

$$= \lim_{h \to 0} \frac{|4-h-4|}{2(4-h-4)}$$

$$= \lim_{h \to 0} \frac{|-h|}{2(-h)} = \lim_{h \to 0} \frac{-h}{2h} = \frac{-1}{2}$$

Consider RHL

$$\lim_{x \to 4^+} f(x) = \lim_{x \to 4^+} \frac{|x-4|}{2(x-4)}$$

$$= \lim_{h \to 0} \frac{|4+h-4|}{2(4+h-4)} = \lim_{h \to 0} \frac{|h|}{2h} = \frac{1}{2}$$

and $f(4) = 0$

As, $\underset{(x=4)}{\text{LHL}} \neq \underset{(x=4)}{RHL}$, so f is discontinuous at $x = 4$.

$\Rightarrow f(x)$ is discontinuous at $x = 4$

16. (*d*), As given line is $\vec{r} = (2\hat{i} + \hat{j} - 4\hat{k}) + \lambda\,(3\hat{i} + 2j - \hat{k})$

$\therefore$ position vector of a general point through which line passes, is given by

$$\vec{r} = (2+3\lambda)\hat{i} + (1+2\lambda)\hat{j} + (-4-\lambda)\hat{k}$$

$\therefore$ General point is $(2 + 3\lambda, 1 + 2\lambda, -4 - \lambda)$

17. (*b*), A and B are independent events.

$\therefore\ P(A \cap B) = P(\text{A}) \cdot P(B)$

Now $\quad P(A \cap \overline{B}) = P(A) \cdot P(\overline{B})$

$$= P(A)\,[1 - P(B)]$$

$$= 0.2\,(1 - 0.8)$$

$$= 0.2 \times 0.2 = 0.04$$

18. (*c*), A die is tossed twice.

$\therefore$ Total possible outcomes = 36

Let E : 1,2, 3, 4 appear on first toss.

$$\therefore \quad P(E) = \frac{4}{6} = \frac{2}{3}$$

Let F : 4, 5, 6 appear on second toss.

$$P(F) = \frac{3}{6} = \frac{1}{2}$$

Here E and F are independent events.

$$\therefore P(E \cap F) = P(E)\,P(F) = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$$

19. (a), Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (R).

20. (a), Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (R).

21. Consider $\frac{dy}{dx} = y^3 \operatorname{cosec} x \Rightarrow \frac{dy}{y^3} = \operatorname{cosec} x\, dx$

$$\Rightarrow \int \frac{dy}{y^3} = \int \operatorname{cosec} x\, dx$$

$$\Rightarrow \frac{y^{-2}}{-2} = \log|\operatorname{cosec} x - \cot x| + C$$

$$\Rightarrow -\frac{1}{2y^2} = \log|\operatorname{cosec} x - \cot x| + C \qquad ...(i)$$

given $y\left(\frac{\pi}{4}\right) = 2$, then from ($i$),

$$-\frac{1}{2 \times 4} = \log\left|\operatorname{cosec}\frac{\pi}{4} - \cot\frac{\pi}{4}\right| + C$$

$$\Rightarrow -\frac{1}{8} = \log|\sqrt{2} - 1| + C$$

$$\Rightarrow C = -\frac{1}{8} - \log|\sqrt{2} - 1|$$

Substituting in (i), we get

$$-\frac{1}{2y^2} = \log|\operatorname{cosec} x - \cot x| - \frac{1}{8} - \log|\sqrt{2} - 1|$$

22. Let $P(A) = \frac{5}{36}$ = Probability of throwing a total of 6

Now, $P(\overline{A}) = \frac{31}{36}$ $\qquad [\because P(X) + P(\overline{X}) = 1]$

Let $P(B) = \frac{6}{36} = \frac{1}{6}$ = Probability of throwing a total of 7

Now, $P(\overline{B}) = \frac{5}{6}$

$$\text{Probability of } A\text{'s win} = P(A) + P(\overline{A})P(\overline{B})P(A) + ...$$

$$= \frac{5}{36} + \frac{31}{36} \times \frac{5}{6} \times \frac{5}{36} + ...$$

$$= \frac{\frac{5}{36}}{1 - \frac{155}{216}} = \frac{5}{36} \times \frac{216}{61} = \frac{30}{61} \qquad \left[\because a + ar + ar^2 + ... = \frac{a}{1-r}\right]$$

Odds in favour of A are $P(A) : P(\overline{A})$

i.e., $\frac{30}{61} : \left(1 - \frac{30}{61}\right) = \frac{30}{61} : \frac{31}{61} = 30 : 31$

OR

$$P\left(\frac{\overline{E}}{\overline{F}}\right) = \frac{P(\overline{E} \cap \overline{F})}{P(\overline{F})}$$
$$= \frac{P(\overline{E \cup F})}{P(\overline{F})}$$
$$= \frac{1 - P(E \cup F)}{P(\overline{F})}$$
$$= \frac{1 - [P(E) + P(F) - P(E \cap F)]}{1 - P(F)}$$
$$= \frac{1 - [0.8 + 0.7 - 0.6]}{1 - 0.7}$$
$$= \frac{1 - 0.9}{0.3} = \frac{0.1}{0.3} = \frac{1}{3}.$$

23. Area of parallelogram $= |(3\hat{i} - \hat{k}) \times (2\hat{i} + \hat{j} - 4\hat{k})|$

$$= \left\| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & -1 \\ 2 & 1 & -4 \end{matrix} \right\| = |\hat{i} + 10\hat{j} + 3\hat{k}|$$
$$= \sqrt{1 + 100 + 9} = \sqrt{110} \text{ sq units}$$

24. Consider $\cos^{-1} x + \cos^{-1} y = 2\pi = \pi + \pi$

Also, maximum value of inverse cosine function in the principal value branch is π.

$\therefore$ $\cos^{-1} x = \pi, \cos^{-1} y = \pi$

$\Rightarrow$ $x = \cos \pi, y = \cos \pi$

$\Rightarrow$ $x = -1, y = -1$

$\therefore$ $x^{100} + y^{200} + \dfrac{1}{x^{100} y^{200}} = (-1)^{100} + (-1)^{200} + \dfrac{1}{(-1)^{100} (-1)^{200}}$

$$= 1 + 1 + 1 = 3$$

25. Given $A = \begin{bmatrix} 4 & 3 \\ -1 & 2 \end{bmatrix}$

$$P = A + A' = \begin{bmatrix} 4 & 3 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & -1 \\ 3 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 4+4 & 3-1 \\ -1+3 & 2+2 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 2 & 4 \end{bmatrix}$$

Now $P' = \begin{bmatrix} 8 & 2 \\ 2 & 4 \end{bmatrix} = P$

$\Rightarrow$ P is symmetric matrix

Again $Q = A - A' = \begin{bmatrix} 4 & 3 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 4 & -1 \\ 3 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 4-4 & 3+1 \\ -1-3 & 2-2 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$$
$$Q' = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix} = -Q$$

$\Rightarrow$ Q is skew symmetric matrix.

OR

$$\begin{aligned}(I + A)^3 &= (I + A)(I + A)(I + A)\\ &= (I^2 + IA + AI + A^2)(I + A)\\ &= (I + A + A + A)(I + A)\\ &= (I + 3A)(I + A) \qquad [\because A^2 = A]\\ &= I^2 + IA + 3AI + 3A^2\\ &= I + A + 3A + 3A = I + 7A.\end{aligned}$$

26. Given $x = a \sin 2t(1 + \cos 2t)$;

Differentiating both sides w.r.t. t, we get

$$\begin{aligned}\frac{dx}{dt} &= a[\sin 2t(-2 \sin 2t) + 2\cos 2t(1 + \cos 2t)]\\ &= 2a(-\sin^2 2t + \cos 2t + \cos^2 2t)\\ &= 2a(\cos 2t + \cos 4t) \qquad ...(i)\end{aligned}$$

and $y = b \cos 2t\,(1 - \cos 2t)$

Differentiating both sides w.r.t. t, we get

$$\begin{aligned}\frac{dy}{dt} &= b[\cos 2t(2 \sin 2t) - 2\sin 2t(1 - \cos 2t)]\\ &= 2b(\cos 2t \sin 2t - \sin 2t + \sin 2t \cos 2t)\\ &= 2b(2 \sin 2t \cos 2t - \sin 2t)\\ &= 2b(\sin 4t - \sin 2t) \qquad ...(ii)\end{aligned}$$

$$\therefore \quad \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{2b\,(\sin 4t - \sin 2t)}{2a\,(\cos 2t + \cos 4t)} \qquad \text{[from } (i) \text{ and } (ii)]$$

$$\left.\frac{dy}{dx}\right]_{\text{at } t=\frac{\pi}{4}} = \frac{b\left(\sin \pi - \sin\frac{\pi}{2}\right)}{a\left(\cos\frac{\pi}{2} + \cos \pi\right)} = \frac{b\,(0-1)}{a\,(0-1)} = \frac{b}{a}.$$

27. Given $y = \log(1 + x) - \dfrac{2x}{2+x}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{1+x} - \left[\frac{(2+x).2 - 2x.1}{(2+x)^2}\right]\\ &= \frac{1}{1+x} - \frac{4}{(2+x)^2} = \frac{(2+x)^2 - 4(1+x)}{(1+x)(2+x)^2}\\ &= \frac{4 + 4x + x^2 - 4 - 4x}{(1+x)(2+x)^2} = \frac{x^2}{(1+x)(2+x)^2} \qquad ...(i)\end{aligned}$$

Now x^2, $(2 + x)^2$ are always positive, also $1 + x > 0$ for $x > -1$.

$\therefore$ from (i), $\dfrac{dy}{dx} > 0$ for $x > -1$. Hence, function is increasing for $x > -1$

28. Consider

$$\begin{aligned}\int \frac{x-1}{(x+1)^3} e^x dx &= \int e^x \left\{\frac{(x+1) - 2}{(x+1)^3}\right\} dx\\ &= \int e^x \left\{\frac{1}{(x+1)^2} - \frac{2}{(x+1)^3}\right\} dx\end{aligned}$$

Let $$f(x) = \frac{1}{(x+1)^2}, \text{ then } f'(x) = \frac{-2}{(x+1)^3}$$

Using $$\int e^x \{f(x) + f'(x)\} dx = e^x . f(x) + C \text{ we get}$$

$$\int \frac{x-1}{(x+1)^3} e^x dx = e^x . \frac{1}{(x+1)^2} + C.$$

Now, $$\int_2^3 \frac{x-1}{(x+1)^3} e^x dx = \left[\frac{e^x}{(x+1)^2}\right]_2^3 = \frac{e^3}{16} - \frac{e^2}{9}.$$

OR

Consider equation $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$ ...(*i*)

Let $x + y = t \Rightarrow 1 + \frac{dy}{dx} = \frac{dt}{dx}$

From (*i*), $$\frac{dt}{dx} - 1 = \sin t + \cos t$$

$$\frac{dt}{dx} = 1 + \sin t + \cos t$$

$$\Rightarrow \frac{dt}{1 + \sin t + \cos t} = dx$$

Integrating both sides, we get

$$\int \frac{dt}{1 + \sin t + \cos t} = \int dx \quad ...(ii)$$

Let $$\tan \frac{t}{2} = z \Rightarrow t = 2 \tan^{-1} z$$

$$\Rightarrow dt = \frac{2}{1+z^2} . dz$$

$$\Rightarrow \sin t = \frac{2z}{1+z^2}, \cos t = \frac{1-z^2}{1+z^2}$$

$\therefore$ From (*ii*), we get

$$\int \frac{dz}{1 + \frac{2z}{1+z^2} + \frac{1-z^2}{1+z^2}} . \frac{2}{1+z^2} = \int dx$$

$$\Rightarrow 2\int \frac{dz}{1 + z^2 + 2z + 1 - z^2} = \int dx$$

$$\Rightarrow \int \frac{dz}{1+z} = \int dx$$

$$\Rightarrow \log |1 + z| = x + C$$

$$\Rightarrow \log \left|1 + \tan \frac{t}{2}\right| = x + C, \text{ where } C \text{ is constant of integration.}$$

$$\Rightarrow \log \left|1 + \tan \frac{x+y}{2}\right| = x + C \text{ is required solution.}$$

29. Given $$f(n) = \begin{cases} n-1, & n \text{ is odd} \\ n+1, & n \text{ is even} \end{cases}$$

For one-one: Let $n_1, n_2 \in W$ be even.

Now, $$f(n_1) = f(n_2)$$

$$\Rightarrow \quad n_1 + 1 = n_2 + 1 \quad \Rightarrow \quad n_1 = n_2$$

Let $n_1, n_2 \in W$ be odd.

Now, $$f(n_1) = f(n_2) \quad \Rightarrow \quad n_1 - 1 = n_2 - 1 \Rightarrow n_1 = n_2$$

Let $n_1, n_2 \in W$ such that n_1 is even and n_2 is odd.

Now, $n_1 \neq n_2$. Then $n_1 + 1$ is odd and $n_2 - 1$ is even

$$\Rightarrow \quad f(n_1) \neq f(n_2)$$

Let $n_1, n_2 \in W$ such that n_1 is odd and n_2 is even. So, $n_1 \neq n_2$.

Then, $n_1 - 1$ is even and $n_2 + 1$ is odd. So, $f(n_1) \neq f(n_2)$.

From above, we notice f is one-one function.

For onto: Let for m (even) $\in W$ (co-domain), there exists $n \in W$ (domain)

such that $$m = f(n)$$

$$m = n - 1$$

$$\Rightarrow \quad n = m + 1 \in W$$

$$f(m + 1) = m + 1 - 1 = m \qquad \text{...}(i)$$

For m (odd) $\in W$ (co-domain), there exists $n \in W$ (domain)

such that $$m = f(n)$$

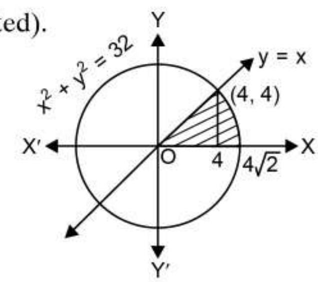
$$m = n + 1 \Rightarrow \quad n = m - 1$$

$$f(m - 1) = m - 1 + 1 = m \qquad \text{...}(ii)$$

$\therefore$ f is onto or surjective

So, f is injective as well as surjective.

30. Eliminating y from $x^2 + y^2 = 32$ and $x = y$, we get $x = 4$ or $x = -4$ (rejected).



$$\therefore \text{ Required area} = \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{32 - x^2}\,dx$$

$$= \left[\frac{x^2}{2}\right]_0^4 + \left[\frac{x}{2}\sqrt{32 - x^2} + \frac{32}{2}\sin^{-1}\frac{x}{\sqrt{32}}\right]_4^{4\sqrt{2}}$$

$$= \left(\frac{16}{2} - 0\right) + \left(\frac{4\sqrt{2}}{2} \times 0 + 16\sin^{-1}1\right) - \left(\frac{4}{2}\sqrt{32 - 16} + 16\sin^{-1}\frac{1}{\sqrt{2}}\right)$$

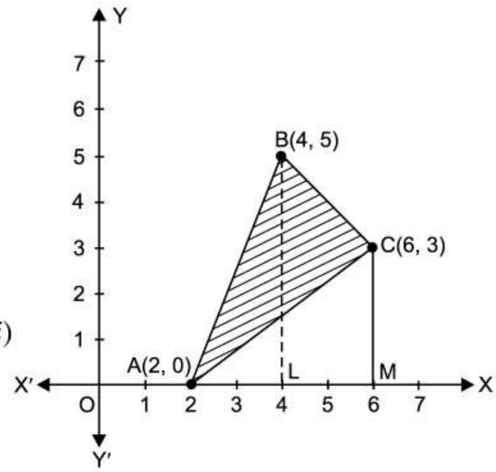
$$= 8 + 16 \times \frac{\pi}{2} - 2 \times 4 - 16 \times \frac{\pi}{4}$$

$$= 8\pi - 4\pi = 4\pi \text{ sq units}$$

OR

Given points are $A(2, 0)$, $B(4, 5)$ and $C(6, 3)$.

Plotting the points, we get triangle ABC.



$\text{ar}(\Delta ABC) = \text{ar}(ABL) + \text{ar}(LBCM) - \text{ar}(ACM)$

Equation of AB: $A(2, 0)$, $B(4, 5)$

$$y - 0 = \frac{5-0}{4-2}(x-2)$$

$$\Rightarrow \quad y = \frac{5}{2}(x-2) \quad ...(i)$$

Equation of BC: $B\,(4, 5)$, $C\,(6, 3)$

$$y - 5 = \frac{3-5}{6-4}(x-4)$$

$$\Rightarrow \quad y - 5 = -x + 4$$

$$\Rightarrow \quad y = -x + 9 \quad ...(ii)$$

Equation of CA: $C\,(6, 3)$, $A(2, 0)$

$$y - 0 = \frac{3-0}{6-2}(x-2)$$

$$\Rightarrow \quad y = \frac{3}{4}(x-2) \quad ...(iii)$$

Using (i), (ii) and (iii), we get

$$ar\,(\Delta ABC) = \int_2^4 \frac{5}{2}(x-2)\,dx + \int_4^6 (-x+9)\,dx - \int_2^6 \frac{3}{4}(x-2)\,dx$$

$$= \frac{5}{2}\left[\frac{x^2}{2} - 2x\right]_2^4 + \left[9x - \frac{x^2}{2}\right]_4^6 - \frac{3}{4}\left[\frac{x^2}{2} - 2x\right]_2^6$$

$$= 7 \text{ sq units}$$

31. Given $$y = \frac{\log x}{x} \Rightarrow y' = \frac{x \cdot \frac{1}{x} - \log x}{x^2} = \frac{1 - \log x}{x^2}$$

Again differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{x^2 \cdot \left(-\frac{1}{x}\right) - (1-\log x)(2x)}{x^4}$$

$$= \frac{-x - 2x + 2x\log x}{x^4} = \frac{2\log x - 3}{x^3}$$

OR

Given function $$f(x) = \begin{cases} x\,[x] \;, & \text{if } 0 \le x < 2 \\ (x-1)x, & \text{if } 2 \le x < 3 \end{cases}$$

$$\text{LHD} = Lf'(2) = \lim_{h \to 0} \frac{f(2-h) - f(2)}{-h}$$

$$= \lim_{h \to 0} \frac{(2-h)[2-h]-2}{-h} = \lim_{h \to 0} \frac{(2-h)(1)-2}{-h}$$

$$= \lim_{h \to 0} \frac{-h}{-h} = \lim_{h \to 0}(1) = 1.$$

and $$\text{RHD} = Rf'(2) \lim_{h \to 0} \frac{f(2+h)-f(2)}{h}$$

$$= \lim_{h \to 0} \frac{(2+h-1)(2+h)-2}{h}$$

$$= \lim_{h \to 0} \frac{(h+1)(h+2)-2}{h}$$

$$= \lim_{h \to 0} \frac{h^2+3h+2-2}{h}$$

$$= \lim_{h \to 0}(h+3) = 3$$

So, $$\underset{x=2}{\text{LHD}} \neq \underset{x=2}{\text{RHD}}$$

Since, LHD $\neq$ RHD at $x = 2$,

hence, the given function is not differentiable at $x = 2$.

32. Consider line $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ ...(*i*)

and the line $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ ...(*ii*)

Position vector of general point on line (*i*) is

$$\vec{r} = (1+3\lambda)\hat{i} + (1-\lambda)\hat{j} - \hat{k} \quad \text{...(iii)}$$

and position vector of general point on line (*ii*) is

$$\vec{r} = (4+2\mu)\hat{i} + (-1+3\mu)\hat{k} \quad \text{...(iv)}$$

If the lines (*i*) and (*ii*) intersect, then they have common point. So, for some values of λ and μ, we must have,

$$(1+3\lambda)\hat{i} + (1-\lambda)\hat{j} - \hat{k} = (4+2\mu)\hat{i} + (-1+3\mu)\hat{k}$$

So, $$1 + 3\lambda = 4 + 2\mu \quad \text{...(v)}$$

$$1 - \lambda = 0 \Rightarrow \lambda = 1 \quad \text{...(vi)}$$

$$-1 = -1 + 3\mu \Rightarrow \mu = 0 \quad \text{...(vii)}$$

Substituting in (*v*) for λ and μ, we get

$$1 + 3 = 4 + 0, \text{ true}$$

Hence, for $\lambda = 1$ and $\mu = 0$, the lines intersect.

Substituting for λ in (*iii*) or μ in (*iv*), we get the position vector of point of intersection as $\vec{r} = 4\hat{i} - \hat{k}$.

Point of intersection is (4, 0, –1).

OR

Consider, line $\frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{5z-10}{11}$

$$\Rightarrow \quad \frac{-(x-1)}{3} = \frac{7(y-2)}{2\lambda} = \frac{5(z-2)}{11}$$

$$\Rightarrow \quad \frac{x-1}{-105} = \frac{y-2}{10\lambda} = \frac{z-2}{77}$$

Direction ratios of line are –105, 10λ, 77. ... (*i*)

Now consider, line $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$

$$\Rightarrow \quad \frac{-7(x-1)}{3\lambda} = \frac{y-5}{1} = \frac{-(z-6)}{5}$$

$$\Rightarrow \quad \frac{x-1}{3\lambda} = \frac{y-5}{-7} = \frac{z-6}{35}$$

Direction ratios of line are 3λ, –7, 35 ... (*ii*)

If lines are perpendicular, then

$$(-105)(3\lambda) + (10\lambda)(-7) + 77 \times 35 = 0$$

$$\Rightarrow \quad -315\lambda - 70\lambda + 2695 = 0$$

$$\Rightarrow \quad 385\lambda = 2695 \Rightarrow \lambda = 7.$$

33. Consider $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$

We have $A^{-1} = \frac{1}{|A|}(\text{adj } A)$

$$|A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}$$

$$= 2(120-45) - 3(-80-30) + 10(36+36)$$

$$= 150 + 330 + 720 = 1200 \neq 0$$

Hence, A^{-1} exists.

Matrix formed by cofactors of each element in $|A|$ is given by,

$$\begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}$$

$$\therefore \quad \text{Adj } A = \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}'$$

$$= \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \quad \text{...}(i)$$

Consider equations

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = -4$$

Corresponding matrix equation is

$$\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix} \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$$

i.e., $AX = B$ is matrix equation.

Its solution is $X = A^{-1}B$ ...(*ii*)

We had already calculated A^{-1}. So we can substitute from (*i*) in (*ii*) and find X and hence x, y, z.

From (*ii*), we get

$$X = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{-1}{3} \\ \frac{1}{5} \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{-1}{3} \\ \frac{1}{5} \end{bmatrix} \Rightarrow x = 2, y = -3, z = 5$$

OR

Consider $$A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

and $$B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$AB = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -4+4+8 & 4-8+4 & -4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I \quad \text{...}(i)$$

Consider equations

$$x - y + z = 4$$
$$x - 2y - 2z = 9$$
$$2x + y + 3z = 1$$

Corresponding matrix equation is

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

We can note the matrix of coefficient is A or B and write accordingly.

$\Rightarrow$ $BX = C$ is matrix equation.

Its solution is $X = B^{-1}C$ ...(*ii*)

Now from (*i*), we can find B^{-1} as

$$AB = 8I \Rightarrow \left(\frac{1}{8}A\right)B = I$$

$$\Rightarrow \quad B^{-1} = \frac{1}{8}A$$

This value of B^{-1}, we can substitute in (*ii*) and after writing matrix A, we can find X and hence x, y, z.

From (*ii*), we get

$$X = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

$\therefore x = 3; y = -2; z = -1$

34. Consider equation $\tan x . \frac{dy}{dx} = 2x \tan x + x^2 - y$

$$\Rightarrow \quad \tan x . \frac{dy}{dx} + y = 2x \tan x + x^2$$

$$\Rightarrow \quad \frac{dy}{dx} + \cot x \cdot y = (2x \tan x + x^2) \cot x$$

Here $P(x) = \cot x$, $Q(x) = (2x \tan x + x^2) \cot x$

$$\text{Integrating factor (I.F.)} = e^{\int \cot x \, dx}$$

$$= e^{\log|\sin x|} = \sin x$$

$\therefore$ Solution is $(\text{I.F.})y = \int \{\text{I.F.} \times Q(x)\} dx$

$$\Rightarrow \quad \sin x . y = \int \sin x (2x \tan x + x^2) . \cot x \, dx$$

$$= \int (2x \sin x + x^2 \cos x) dx$$

$$= \int 2x \sin x \, dx + \int \underset{①}{x^2} \underset{②}{\cos x} \, dx$$

$$= \int 2x \sin x \, dx + x^2 \cdot \sin x - \int 2x \cdot \sin x \, dx$$

$$\sin x . y = x^2 \sin x + C \quad \text{...(i)}$$

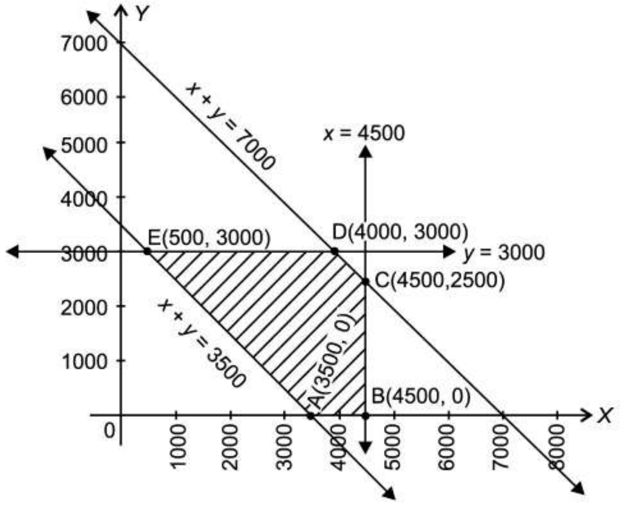
Put $x = \frac{\pi}{2}, y = 0$ in (*i*), we get $\quad 0 = \frac{\pi^2}{4} \cdot \sin\frac{\pi}{2} + C$

$$\Rightarrow \quad C = -\frac{\pi^2}{4}$$

Substituting in (*i*), we get

$\sin x . y = x^2 \sin x - \frac{\pi^2}{4}$ is required solution.

35. Plotting the graph of above inequations, we notice shaded portion is optimum solution. Possible points for minimum Z are $A(3500, 0)$, $B(4500, 0)$, $C(4500, 2500)$, $D(4000, 3000)$, $E(500, 3000)$.



Points	$Z = 3x + y + 39500$	Values
$A(3500, 0)$	$10500 + 0 + 39500$	50,000
$B(4500, 0)$	$13500 + 0 + 39500$	53000
$C(4500, 2500)$	$13500 + 2500 + 39500$	55,500
$D(4000, 3000)$	$12000 + 3000 + 39500$	54,500
$E(500, 3000)$	$1500 + 3000 + 39500$	44,000 ← Minimum

Z is minimum for $E(500, 3000)$, i.e. $x = 500, y = 3000$

36. Consider the following events.

A : Person comes by Metro

B : Person comes by Bus

C : Person comes by Scooter

D : Person comes by other means.

E : Person reaches late

(*i*) $P(E/D) = 0.$

(*ii*) Required probability $= P(B)\,P(E/B)$

$$= \frac{1}{5} \times \frac{1}{3} = \frac{1}{15}$$

(*iii*) $$P(E) = P(A)\,P(E/A) + P(B)\,P(E/B) + P(C)\,P(E/C) + P(\text{D})\,P(E/D)$$

$$= \frac{3}{10}\cdot\frac{1}{4} + \frac{1}{5}\cdot\frac{1}{3} + \frac{1}{10}\cdot\frac{1}{12} + \frac{2}{5}\cdot 0$$

$$= \frac{9+8+1}{120} = \frac{18}{120} = \frac{3}{20}$$

OR

(*iii*)
$$P(A/E) = \frac{P(A)\times P(E/A)}{P(A)\cdot P(E/A)+P(B)\cdot P(E/B)+P(C)\cdot P(E/C)+P(D)\cdot P(E/D)}$$

$$= \frac{\frac{3}{10}\times\frac{1}{4}}{\frac{3}{10}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{3}+\frac{1}{10}\times\frac{1}{12}+\frac{2}{5}\times 0}$$

$$= \frac{\frac{3}{40}}{\frac{3}{40}+\frac{1}{15}+\frac{1}{120}}$$

$$= \frac{3}{40}\times\frac{20}{3} = \frac{1}{2}$$

37. (*i*) Volume $= x.\,x.\,h = 250$

$\Rightarrow \quad x^2h = 250$

(*ii*) Cost $(C) = 50\times x^2 + 400\,(h)^2$

$= ₹\,(50\,x^2 + 400\,h^2)$

(*iii*) $C = 50.\dfrac{250}{h} + 400\,h^2$ [from (*i*)]

$= ₹\left(\dfrac{12500}{h} + 400\,h^2\right)$

OR

(*iii*) We have, $C = \dfrac{12500}{h} + 400h^2$

$\Rightarrow \quad \dfrac{dC}{dh} = \dfrac{-12500}{h^2} + 800h$

For minimum C, $\dfrac{dC}{dh} = 0$

$\Rightarrow \quad \dfrac{-12500}{h^2} + 800\,h = 0$

$\Rightarrow \quad h^3 = \dfrac{125}{8}$

$\Rightarrow \quad h = \dfrac{5}{2} = 2.5$ m

Also, $\dfrac{d^2C}{dh^2} = \dfrac{25000}{h^3} + 800$

Now, $\left.\dfrac{d^2C}{dh^2}\right]_{h\,=\,2.5} = 2400 > 0$

So, C is minimum at $h = 2.5$ m.

38. (*i*) Clearly, G be the centroid of ΔBCD, therefore coordinates of G are

$$\left(\frac{3+4+2}{3}, \frac{0+3+3}{3}, \frac{1+6+2}{3}\right) = (3, 2, 3)$$

(*ii*) Since, $A \equiv (0, 1, 2)$ and $G \equiv (3, 2, 3)$

$$\therefore \overrightarrow{AG} = (3-0)\hat{i} + (2-1)\hat{j} + (3-2)\hat{k} = 3\hat{i} + \hat{j} + \hat{k}$$

$$\Rightarrow \quad |\overrightarrow{AG}| = \sqrt{3^2+1^2+1^2}$$

$$= \sqrt{9+1+1}$$

$$= \sqrt{11}$$