9 SAMPLE PAPER

[Difficult Concept]

Time Allowed: 3 Hours] [*Maximum Marks:* 80

General Instructions:

Read the following instructions very carefully and strictly follow them:

(*i*) *This Question paper contains **38** questions. **All** questions are **compulsory**.*

(*ii*) *This Question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(*iii*) *In **Section A**, Questions no. **1** to **18** are **multiple choice questions** (MCQs) and Questions no. **19** and **20** are **Assertion-Reason based** questions of **1 mark each**.*

(*iv*) *In **Section B**, Questions no. **21** to **25** are **Very Short Answer (VSA)-type** questions, carrying **2 marks each**.*

(*v*) *In **Section C**, Questions no. **26** to **31** are **Short Answer (SA)-type** questions, carrying **3 marks each**.*

(*vi*) *In **Section D**, Questions no. **32** to **35** are **Long Answer (LA)-type** questions, carrying **5 marks each**.*

(*vii*) *In **Section E**, Questions no. **36** to **38** are **Case study-based questions**, carrying **4 marks each**.*

(*viii*) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.*

(*ix*) *Use of calculators is **not** allowed.*

SECTION – A

(This section comprises of multiple choice questions (MCQs) of 1 mark each)

Select the correct option (Question 1 - Question 18):

1. If $\sin^{-1}x = y$, then **[NCERT Part-I, Page 19]**

(*a*) $0 \leq y < \pi$ (*b*) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

(*c*) $0 < y < \pi$ (*d*) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

2. Let R be a relation on the set L of lines defined by $l_1 R\, l_2$ if l_1 is perpendicular to l_2, then relation R is **[NCERT Part-I, Page 2]**

(*a*) reflexive and symmetric (*b*) symmetric and transitive

(*c*) equivalence relation (*d*) symmetric

3. Let R be a relation on the set N of natural numbers defined by nRm, if n divides m, then R is **[NCERT Part-I, Page 2]**

(*a*) reflexive and symmetric (*b*) transitive and symmetric

(*c*) equivalence (*d*) reflexive, transitive but not symmetric

4. $y = e^{-x} + ax + b$ is a solution of differential equation **[NCERT Part-II, Page 304-305]**

(*a*) $e^{-x}y'' = 1$ (*b*) $e^{x}y'' = 1$

(*c*) $e^{x}(y')^2 = 1$ (*d*) $e^{-x}(y')^2 = 1$

5. The area bounded by the curve $y = |x|$, the x-axis and between $x = -2$ to $x = 0$ is **[Conceptual Application]**

(a) 4 sq units (b) $\frac{3}{2}$ sq units

(c) 1 sq unit (d) 2 sq units

6. The degree of the differential equation $\left(1+\frac{dy}{dx}\right)^3 = \left(\frac{dy}{dx}\right)^2$ is **[NCERT Part-II, Page 302]**

(a) 1 (b) 2 (c) 3 (d) 4

7. The value of λ for which $\int \frac{4x^3 + \lambda 4^x}{4^x + x^4}dx = \log|4^x + x^4| + C$ is **[NCERT Part-II, Page 226-227]**

(a) 1 (b) $\log_e 4$ (c) $\log_4 e$ (d) 4

8. The point(s) on the curve $y = x^2$, at which y-coordinate is changing six times as fast as x-coordinate is/are **[NCERT Part-I, Page 147-148]**

(a) (2, 4) (b) (3, 9) (c) (3, 9), (9, 3) (d) (6, 2)

9. A function f is said to be continuous for $x \in R$, if **[Conceptual Application]**

(a) it is continuous at $x = 0$ (b) differentiable at $x = 0$

(c) continuous at two points (d) differentiable for $x \in R$

10. The rate of change of area of a circle with respect to its radius is **[NCERT Part-I, Page 147-148]**

(a) 2π (b) πr (c) $2\pi r$ (d) π

11. A function which is continuous at $x = 1$, but not differentiable at $x = 1$ is **[Conceptual Application]**

(a) $|x|$ (b) $[x]$ (c) $sgn(x)$ (d) $|x-1|$

12. Direction ratios of the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$ are **[NCERT Part-II, Page 382]**

(a) 2, 6, 3 (b) –2, 6, 3

(c) 2, – 6, 3 (d) None of these

13. Projection of a line segment joining the points (2, 0, 5) and (0, 3, 1) on the line whose direction ratios are 2, 3, 6 is **[Conceptual Application]**

(a) $-\frac{19}{7}$ (b) $\frac{19}{49}$ (c) $\frac{19}{7}$ (d) 19

14. If $\vec{a}, \vec{b}, \vec{c}$ be the position vectors of vertices A, B, C of a parallelogram $ABCD$, then the position vector of D is **[NCERT Part-II, Page 339, 344]**

(a) $\vec{a}+\vec{c}-\vec{b}$ (b) $\vec{a}-\vec{c}+\vec{b}$

(c) $\vec{a}-\vec{c}-\vec{b}$ (d) $\vec{c}-\vec{a}+\vec{b}$

15. Area of parallelogram, whose diagonals are along vectors $\hat{i}+2\hat{k}$ and $2\hat{j}-3\hat{k}$ is **[Conceptual Application]**

(a) $\sqrt{29}$ sq units (b) $(-4\hat{i}+3\hat{j}+2\hat{k})$ sq units

(c) $\frac{1}{2}\sqrt{29}$ sq units (d) None of these

16. If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ then **[Conceptual Application]**

(a) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$ (b) $\vec{a}+\vec{b}=\vec{b}+\vec{c}=\vec{c}+\vec{a}$

(c) $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=2(\vec{a}\times\vec{b})$ (d) None of these

17. If A and B are independent events then which of the following is not true **[NCERT Part-II, Page 417]**

(*a*) $P(A \cap B) = 0$ (*b*) $P(\overline{A} \cap B) = P(\overline{A})\, P(B)$

(*c*) $P(A \cap \overline{B}) = P(A)P(\overline{B})$ (*d*) $P(A \cup B) = 1 - P(\overline{A} \cap \overline{B})$

18. Bag A contains 3 red and 5 black balls and bag B contains 2 red and 4 black balls. A ball is drawn from one of the bags. The probability that ball drawn is red is **[NCERT Part-II, Page 424]**

(*a*) $\frac{17}{24}$ (*b*) $\frac{17}{48}$ (*c*) $\frac{3}{8}$ (*d*) $\frac{1}{3}$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (*a*), (*b*), (*c*) and (*d*) as given below.)

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true but R is not the correct explanation of A.

(*c*) A is true but R is false.

(*d*) A is false but R is true.

19. Assertion (*A*): The area bounded by the curve $|x| + |y| = 1$ is 2 sq units. **[Conceptual Application]**

Reason (*R*): Curve is symmetrical to both axes and area bounded in each quadrant is 1 unit.

20. Assertion (*A*): The function $y = [x(x-2)]^2$ is increasing in $(0, 1) \cup (2, \infty)$. **[Conceptual Application]**

Reason (*R*): $\frac{dy}{dx} = 0$, when $x = 0, 1, 2$

SECTION – B

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

21. Solve the differential equation $xy\, dy = (y + 5)\, dx$, given that $y(5) = 0$. **[NCERT Part-II, Page 306-307]**

22. If A and B are events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$, find $P(\overline{A} \cap \overline{B})$. **[Conceptual Application]**

OR

Two dice are thrown simultaneously. If A be the event "getting 6 on the first die" and B be the event "getting 2 on the second die". Are the events independent? **[NCERT Part-II, Page 418]**

23. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors of equal magnitude, then show that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to the vectors $\vec{a}, \vec{b}$ and $\vec{c}$. **[Conceptual Application]**

24. Find the value of $\sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right]$. **[Conceptual Application]**

25. If $A = \begin{bmatrix} 0 & 3 \\ 2 & -5 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 4a \\ -8 & 5b \end{bmatrix}$, then find the values of k, a and b. **[NCERT Part-I, Page 41]**

OR

If $A = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$, find a matrix C such that $5A + 3B + 2C$ is a null matrix.

[NCERT Part-I, Page 43-44]

SECTION – C

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

26. If $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$. **[Integrated Question]**

27. Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ is always increasing on R.

[NCERT Part-I, Page 153]

28. Evaluate $\int \sin^{-1}\sqrt{\frac{x}{2+x}}\,dx$. **[Integrated Question]**

OR

Show that the differential equation, **[NCERT Part-II, Page 313-314]**

$xe^{y/x} - y + x\frac{dy}{dx} = 0$ is homogeneous and find the particular solution, given that $y = 0$ when $x = e$.

29. Let the relation S in the set of all real numbers R, be defined as $aSb \Leftrightarrow 1 + ab > 0$ for all $a, b \in R$. Is relation S an equivalence relation? **[NCERT Part-I, Page 2]**

30. Sketch the graph $y = |x - 1|$. Evaluate $\int_{-2}^{4}|x-1|dx$. What does the value of this integral represent on the graph? **[NCERT Part-II, Page 267]**

OR

Using method of integration find the area of the triangle ABC, coordinates of whose vertices are $A(1, -2)$, $B(3, 5)$ and $C(5, 2)$. **[Conceptual Application]**

31. If $\tan^{-1}\left(\frac{y}{x}\right) = \log\sqrt{x^2 + y^2}$, prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$. **[NCERT Part-I, Page 121]**

OR

If function f is differentiable at $x = a$, find $\lim_{x \to a}\frac{xf(a) - af(x)}{x - a}$. **[Integrated Question]**

SECTION – D

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

32. If $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}, \vec{b} = 4\hat{i} - 7\hat{j} + \hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 6$.

[Conceptual Application]

OR

Find the vector and Cartesian equations of a line passing through (1, 2, – 4) and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$. **[Conceptual Application]**

33. For the matrices A and B, verify that $(AB)' = B'A'$, where $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$, $B = [-1 \ \ 2 \ \ 1]$.

[NCERT Part-I, Page 61]

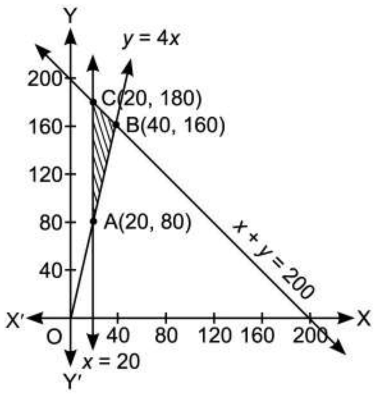
OR

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then show that $A^2 - 5A + 7I = O$. Hence, find A^{-1}. **[Conceptual Application]**

34. Solve the differential equation: **[NCERT Part-II, Page 322-323]**

$x\frac{dy}{dx} + y = x\cos x + \sin x$, given that $y = 1$ when $x = \frac{\pi}{2}$.

35. The feasible region of the system of linear constraints is shown below: **[Conceptual Application]**



Answer each of the following:

(*i*) If $Z = ax + by$ and $Z_{(20, 180)}$ is equal to $Z_{(40, 160)}$, what is relation between a and b?

(*ii*) If $Z = 400x + 300y$ be objective function, find value of maximum Z and the point of maximum.

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (*i*), (*ii*), (*iii*) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study - 1

36. Three persons A, B and C apply for the job of a manager in a private company the chances of their selection is given by the relation $4A = 2B = C$. The probability that if selected A, B and C can bring changes to improve profitability of the company are 0.8, 0.5 and 0.3 respectively.

[Conceptual Application]

(*i*) What is the probability that C is selected as a manager?

(*ii*) What is conditional probability that if change has taken place it is due to B?

(*iii*) What is the conditional probability that change does not take place due to selection of A?

OR

(*iii*) Find the probability that change does not take place.

Case Study - 2

37. An advertisement firm is supplied with decorative wire pieces of 34 m each and are asked to cut the wire into two pieces. From one piece a circular sign board is to be made and from other a square one and the idea is to keep the sum of the areas enclosed by a circle and square to be minimum for writing slogans. **[Conceptual Application]**

(*i*) If wire is cut at x m from one end and made into a circle of radius r, then find r.

(*ii*) Find the area enclosed by the circular ring.

(*iii*) Find the area enclosed by square frame.

OR

(*iii*) Find the value of x when combined area is minimum.

Case Study - 3

38. Three shopkeepers A, B and C go to a store to buy stationary. A purchase 12 dozen notebooks, 5 dozen pens and 6 dozen pencils. B purchase 10 dozen notebooks, 6 dozen pens and 7 dozen pencils. C purchase 11 dozen notebooks, 13 dozen pens and 8 dozen pencils. A notebook costs ₹ 40, a pen costs ₹ 12 and a pencil costs ₹ 3. **[Conceptual Application]**

(*i*) Represent the number of items purchased by shopkeepers A, B and C in matrix form.

(*ii*) If X represents the matrix formed in above part (*i*) and Y represents the matrix formed by the cost of each item, then find XY.

SOLUTIONS

1. (*b*)

2. (*d*), Not reflexive, as $l_1 R\, l_1 \Rightarrow l_1 \perp l_1$, not true

Symmetric, true as $l_1 R\, l_2 \Rightarrow l_1 \perp l_2 \Rightarrow l_2 \perp l_1 \Rightarrow l_2 R\, l_1$

Not Transitive, as $l_1 R\, l_2$ and $l_2 R\, l_3 \Rightarrow l_1 \perp l_2$ and $l_2 \perp l_3$

$$\Rightarrow l_1 \parallel l_3 \Rightarrow (l_1, l_3) \notin R$$

3. (*d*), Since n divides n, $\forall\, n \in N$, R is reflexive. R is not symmetric since for $3, 6 \in N$,

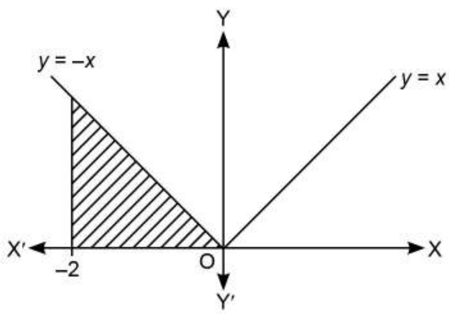
$3\, R\, 6 \not\Rightarrow 6\, R\, 3$.

R is transitive since for n, m, r whenever $n \mid m$ and $m \mid r \Rightarrow n \mid$r, i.e., if n divides m and m divides r, then n will divide r.

4. (*b*), as $y' = -e^{-x} + a$ and $y'' = e^{-x}$

$\Rightarrow e^x y'' = 1$

5. (*d*),



as $$\text{area} = \int_{-2}^{0} y\, dx = \int_{-2}^{0} |x|\, dx$$

$$= -\int_{-2}^{0} x\, dx$$

$$= -\left[\frac{x^2}{2}\right]_{-2}^{0} = -0 + \frac{4}{2} = 2 \text{ sq units}$$

6. (*c*), as differential equation is

$$1 + 3\frac{dy}{dx} + 3\left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dx}\right)^3 = \left(\frac{dy}{dx}\right)^2.$$

Highest exponent of the highest order derivative is 3.

7. (*b*), as $$\frac{d}{dx}[\log|4^x + x^4| + C] = \frac{1}{(4^x + x^4)} \cdot (4^x \cdot \log_e 4 + 4x^3)$$

$$= \frac{4x^3 + \log_e 4 \cdot 4^x}{4^x + x^4}$$

$\Rightarrow$ $$\lambda = \log_e 4$$

8. (*b*), as $$\frac{dy}{dt} = 2x \cdot \frac{dx}{dt}$$

$\Rightarrow$ $$6 \cdot \frac{dx}{dt} = 2x \cdot \frac{dx}{dt}$$

$\Rightarrow$ $$x = 3$$

From curve, $y = 9$, when $x = 3$ Point is (3, 9).

9. (*d*), as differentiable functions is continuous also.

10. (*c*), as $A = \pi r^2 \Rightarrow \dfrac{dA}{dr} = 2\pi r$

11. (*d*), as absolute value function $f(x) = |x - 1|$ is continuous at $x = 1$ but not differentiable at $x = 1$.

12. (*c*) as, $$\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$$

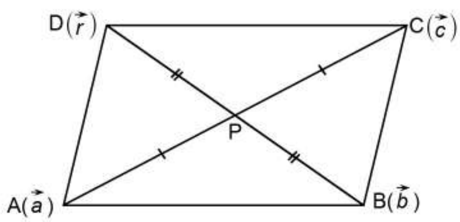
$\Rightarrow$ $$\frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3}$$

$\Rightarrow$ $$\frac{x-4}{2} = \frac{y}{-6} = \frac{z-1}{3}$$

$\therefore$ dr's are 2, –6, 3

13. (*c*)

14. (*a*),



We know, in a parallelogram, diagonals bisect each other.

Using this concept, we get $\dfrac{\vec{a} + \vec{c}}{2} = \dfrac{\vec{b} + \vec{r}}{2}$

$\Rightarrow$ $$\vec{r} = \vec{a} + \vec{c} - \vec{b}$$

15. (*c*), **Hint:** as area $= \dfrac{1}{2} \,|\, (\hat{i} + 2\hat{k}) \times (2\hat{j} - 3\hat{k}) \,|$ sq units.

16. (*a*), as $$\vec{a} + \vec{b} + \vec{c} = \vec{0}$$

$\Rightarrow$ $$\vec{a} \times (\vec{a} + \vec{b} + \vec{c}) = \vec{a} \times \vec{0}$$

$\Rightarrow$ $$\vec{a} \times \vec{a} + \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = \vec{0}$$

$\Rightarrow$ $$\vec{a} \times \vec{b} = \vec{c} \times \vec{a}$$

Similarly, we get $$\vec{b} \times \vec{c} = \vec{a} \times \vec{b}$$

Hence, $$\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$$

17. (*a*), As $P(A \cap B) = P(A)\,P(B)$ for independent events A and B.

18. (*b*), as $P(\text{red}) = P(A) \cdot P(R/A) + P(B) \cdot P(R/B)$

$$= \frac{1}{2} \cdot \frac{3}{8} + \frac{1}{2} \cdot \frac{2}{6}$$

$$= \frac{3}{16} + \frac{1}{6} = \frac{17}{48}.$$

19. (*c*), curve is $|x| + |y| = 1$

$\Rightarrow \quad x + y = 1, \quad \text{if } x > 0, y > 0$

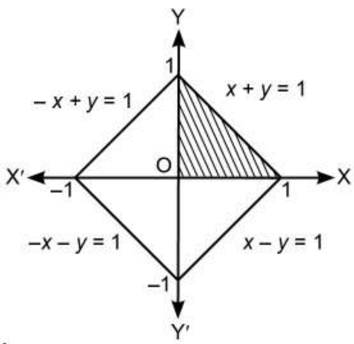
$\Rightarrow \quad x - y = 1, \quad \text{if } x > 0, y < 0$

$\Rightarrow \quad -x + y = 1, \quad \text{if } x < 0, y > 0$

$\quad -x - y = 1, \quad \text{if } x < 0, y < 0$

$$\text{Area} = 4\int_0^1 y\,dx = 4\int_0^1 (1-x)\,dx$$

$$= 4\left[x - \frac{x^2}{2}\right]_0^1 = 2 \text{ sq units}$$



A is true.

$$\text{Area bounded in each quadrant} = \frac{1}{2} \times 1 \times 1$$

$$= \frac{1}{2} \text{ sq unit}$$

So, R is false.

20. (*b*), Both A and R are true but R is not the correct explanation of A.

21. We have, $xy\,dy = (y + 5)dx$

$$\Rightarrow \quad \frac{y}{y+5}dy = \frac{dx}{x}$$

Integrating both sides, we get

$$\int \frac{y}{y+5}dy = \int \frac{dx}{x}$$

$$\Rightarrow \quad \int \frac{(y+5)-5}{y+5}dy = \int \frac{dx}{x}$$

$$\Rightarrow \quad \int \left\{1 - \frac{5}{y+5}\right\}dy = \int \frac{dx}{x}$$

$$\Rightarrow \quad y - 5\log|y + 5| = \log|x| + C \quad ...(i)$$

Given when $x = 5, y = 0$, then from (*i*), we get

$$0 - 5\log 5 = \log 5 + C \Rightarrow C = -6\log 5$$

Substituting in (*i*), we get

$$y - 5\log|y + 5| = \log|x| - 6\log 5, \text{ is the required solution.}$$

22. $P(\overline{A} \cap \overline{B}) = 1 - P(A \cup B)$

$$= 1 - [P(A) + P(B) - P(A \cap B)]$$

$$= 1 - \left[\frac{1}{4} + \frac{1}{2} - \frac{1}{8}\right] = 1 - \frac{5}{8} = \frac{3}{8}$$

Alternatively:

$P(\overline{A} \cap \overline{B}) = P(\overline{A})\, P(\overline{B})$

As events A and B are independent.

OR

Let S be the sample space.

$$\therefore \quad n(S) = 36$$

$$A = \{(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$$

$$B = \{(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2)\}$$

$$A \cap B = \{(6, 2)\}$$

$$P(A) = \frac{6}{36} = \frac{1}{6}, P(B) = \frac{6}{36} = \frac{1}{6}, P(A \cap B) = \frac{1}{36}$$

As $P(A)\,P(B) = P(A \cap B)$

Hence, events A and B are independent.

23. Given $\quad |\vec{a}| = |\vec{b}| = |\vec{c}|$

and $\vec{a} \cdot \vec{b} = 0,\ \vec{b} \cdot \vec{c} = 0,\ \vec{c} \cdot \vec{a} = 0 \qquad ...(i)$

Let α is angle between $\vec{a}$ and $\vec{a} + \vec{b} + \vec{c}$

then
$$\cos\alpha = \frac{\vec{a} \cdot (\vec{a} + \vec{b} + \vec{c})}{|\vec{a}||\vec{a} + \vec{b} + \vec{c}|} = \frac{\vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}}{|\vec{a}||\vec{a} + \vec{b} + \vec{c}|}$$

$$= \frac{|\vec{a}|^2 + 0 + 0}{|\vec{a}||\vec{a} + \vec{b} + \vec{c}|} = \frac{|\vec{a}|}{|\vec{a} + \vec{b} + \vec{c}|} \qquad ...(ii)\ [\text{from } (i)]$$

Similarly, we can show if β and γ are the angles which $\vec{b}$ and $\vec{c}$ make with $\vec{a} + \vec{b} + \vec{c}$ respectively, then

$$\cos\beta = \frac{|\vec{b}|}{|\vec{a} + \vec{b} + \vec{c}|} \text{ and } \cos\gamma = \frac{|\vec{c}|}{|\vec{a} + \vec{b} + \vec{c}|} \qquad ...(iii)$$

From (i), (ii) and (iii), we get $\cos\alpha = \cos\beta = \cos\gamma$

Hence, $\alpha = \beta = \gamma$,

24. As
$$\sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right] = \sin^{-1}\left[\cos\left(6\pi + \frac{3\pi}{5}\right)\right]$$

$$= \sin^{-1}\left[\cos\left(\frac{3\pi}{5}\right)\right]$$

$$= \sin^{-1}\left[\cos\left(\neq - \frac{2\neq}{5}\right)\right]$$

$$= \sin^{-1}\left[-\cos\left(\frac{2\pi}{5}\right)\right]$$

$$= \sin^{-1}\left[-\sin\left(\frac{\neq}{2} - \frac{2\neq}{5}\right)\right]$$

$$= \sin^{-1}\left[-\sin\left(\frac{\pi}{10}\right)\right] = \sin^{-1}\left[\sin\left(\frac{-\pi}{10}\right)\right] = \frac{-\pi}{10}$$

25. Given
$$A = \begin{bmatrix} 0 & 3 \\ 2 & -5 \end{bmatrix} \text{ and } kA = \begin{bmatrix} 0 & 4a \\ -8 & 5b \end{bmatrix}$$

$$\Rightarrow \quad k\begin{bmatrix} 0 & 3 \\ 2 & -5 \end{bmatrix} = \begin{bmatrix} 0 & 4a \\ -8 & 5b \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 0 & 3k \\ 2k & -5k \end{bmatrix} = \begin{bmatrix} 0 & 4a \\ -8 & 5b \end{bmatrix}$$

$$\Rightarrow \quad 3k = 4a,\ 2k = -8,\ -5k = 5b$$

$$\Rightarrow \quad k = -4,\ a = -3,\ b = 4$$

OR

As matrices A and B are of order 2×2. Therefore, matrix C is also of order 2×2 for sum to define.

Let $$C = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$\therefore$ $$5A + 3B + 2C = O$$

$\Rightarrow$ $$5\begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix} + 3\begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix} + 2\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow$ $$\begin{bmatrix} 45 & 5 \\ 35 & 40 \end{bmatrix} + \begin{bmatrix} 3 & 15 \\ 21 & 36 \end{bmatrix} + \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow$ $$\begin{bmatrix} 45 + 3 + 2a & 5 + 15 + 2b \\ 35 + 21 + 2c & 40 + 36 + 2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow$ $$\begin{bmatrix} 48 + 2a & 20 + 2b \\ 56 + 2c & 76 + 2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow$ $$48 + 2a = 0, \quad 20 + 2b = 0$$
$$56 + 2c = 0, \quad 76 + 2d = 0$$

$\Rightarrow$ $$a = -24, b = -10$$
$$c = -28, d = -38$$

$\therefore$ Matrix $$C = \begin{bmatrix} -24 & -10 \\ -28 & -38 \end{bmatrix}$$

26. Given: $$x = a(\theta - \sin\theta) \text{ and } y = a(1 + \cos\theta)$$

$$\frac{dx}{d\theta} = a(1 - \cos\theta) \text{ and } \frac{dy}{d\theta} = -a\sin\theta \qquad ...(i)$$

$\therefore$ $$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta} = \frac{-a\sin\theta}{a(1 - \cos\theta)}$$

$$= \frac{-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} = -\cot\frac{\theta}{2}$$

$\therefore$ $$\frac{d^2y}{dx^2} = -\frac{d}{dx}\left(\cot\frac{\theta}{2}\right) = \text{cosec}^2\frac{\theta}{2} \cdot \left(\frac{1}{2} \cdot \frac{d\theta}{dx}\right)$$

$$= \frac{1}{2}\text{cosec}^2\frac{\theta}{2} \cdot \frac{1}{a(1 - \cos\theta)} \qquad \text{[From } (i)\text{]}$$

$$= \frac{1}{2a} \cdot \text{cosec}^2\frac{\theta}{2} \cdot \frac{1}{2\sin^2\frac{\theta}{2}} = \frac{1}{4a}\text{cosec}^4\frac{\theta}{2}$$

$\therefore$ $$\left.\frac{d^2y}{dx^2}\right]_{\theta = \frac{\pi}{2}} = \frac{1}{4a}\text{cosec}^4\frac{\pi}{4} = \frac{1}{4a}(\sqrt{2})^4 = \frac{1}{a}$$

27. Consider the function, $$f(x) = 4x^3 - 18x^2 + 27x - 7$$

$$f'(x) = 12x^2 - 36x + 27$$
$$= 12\left(x^2 - 3x + \frac{9}{4}\right)$$
$$= 12\left(x - \frac{3}{2}\right)^2$$

$\therefore$ $f'(x) \geq 0$, for $x \in R$. Hence, function always increasing on R.

28. Consider $I = \int \sin^{-1}\sqrt{\frac{x}{2+x}}\,dx$

Let $$x = 2\tan^2\theta$$

$\Rightarrow$ $$dx = 4\tan\theta\sec^2\theta\, d\theta$$

Now, $$I = \int \sin^{-1}\sqrt{\frac{2\tan^2\theta}{2+2\tan^2\theta}} \times 4\tan\theta\sec^2\theta\, d\theta$$

$$= 4\int \sin^{-1}(\sin\theta)\cdot(\tan\theta\sec^2\theta)d\theta$$

$$= 4\int \theta\cdot(\tan\theta\sec^2\theta)d\theta$$

$$= 4\left[\theta\cdot\frac{\tan^2\theta}{2} - \int 1\cdot\frac{\tan^2\theta}{2}d\theta\right] \qquad \left[\because \int \tan\theta\cdot\sec^2\theta\, d\theta = \frac{\tan^2\theta}{2}\right]$$

$$= 2\left[\theta\tan^2\theta - \int(\sec^2\theta - 1)\right]d\theta$$

$$= 2[\theta\tan^2\theta - \tan\theta + \theta] + C$$

$$= 2\left[\tan^{-1}\sqrt{\frac{x}{2}}\cdot\frac{x}{2} - \sqrt{\frac{x}{2}} + \tan^{-1}\sqrt{\frac{x}{2}}\right] + C$$

$$= (x+2)\tan^{-1}\sqrt{\frac{x}{2}} - \sqrt{2x} + C.$$

OR

Consider equation, $xe^{y/x} - y + x\frac{dy}{dx} = 0$

$\Rightarrow$ $$x\frac{dy}{dx} = y - x\,e^{y/x}$$

$\Rightarrow$ $$\frac{dy}{dx} = \frac{y}{x} - e^{y/x} \qquad ...(i)$$

Let $$f(x, y) = \frac{y}{x} - e^{y/x}$$

$$f(\lambda x, \lambda y) = \frac{\lambda y}{\lambda x} - e^{\frac{\lambda y}{\lambda x}} = \frac{y}{x} - e^{y/x} = f(x, y)$$

Hence, function is homogeneous, so corresponding differential equation is homogeneous.

Let $$y = vx \;\Rightarrow\; \frac{dy}{dx} = v + x\frac{dv}{dx}$$

From (i), we get

$$v + x\frac{dv}{dx} = v - e^v$$

$\Rightarrow$ $$x\frac{dv}{dx} = -e^v$$

$\Rightarrow$ $$\frac{1}{e^v}dv = -\frac{dx}{x}$$

$\Rightarrow$ $$\int e^{-v}dv = -\int\frac{dx}{x}$$

$\Rightarrow$ $$\frac{e^{-v}}{-1} = -\log|x| + C$$

$\Rightarrow$ $$-e^{-v} = -\log|x| + C$$

$\Rightarrow \qquad -e^{-y/x} = -\log|x| + C \qquad$ (*C* is constant of integration)

$\Rightarrow \qquad e^{-y/x} = \log|x| - C \qquad$ ...(*i*)

Given $\qquad y = 0$ when $x = e$

$\Rightarrow \qquad e^0 = \log|e| - C \Rightarrow 1 = 1 - C \Rightarrow C = 0$

Substituting in (*i*), we get

$$e^{-y/x} = \log|x| \Rightarrow -\frac{y}{x} = \log\{\log|x|\}$$

$\Rightarrow \qquad y = -x \log\{\log|x|\}$ is particular solution.

29. Relation S is defined as

$$S = \{(a, b) \in R \times R | 1 + ab > 0\}$$

For reflexive: Let $\qquad a \in R$

then $\qquad (a, a) \in S$

$\Rightarrow \qquad 1 + a \cdot a > 0$

$\Rightarrow \qquad 1 + a^2 > 0$, true for $a \in R$

Hence, reflexive.

For symmetric: Let $\qquad a, b \in R$

then $\qquad (a, b) \in S$

$\Rightarrow \qquad 1 + ab > 0$

$\Rightarrow \qquad 1 + ba > 0$

$\Rightarrow \qquad (b, a) \in S$

As $(a, b) \in S \Rightarrow (b, a) \in S$, for $a, b \in R$

Hence, symmetric.

For transitive: For $\qquad a, b, c \in R$

Let $\qquad (a, b) \in S$ and $(b, c) \in S$

$\Rightarrow \qquad 1 + ab > 0$ and $1 + bc > 0$ but this may not imply $1 + ac > 0$, i.e. $(a, c) \in S$.

For example:

Let $\qquad a = 1, b = \frac{1}{2}, c = -\frac{3}{2}$

Now, $\qquad (a, b) \in S$

$\Rightarrow \qquad \left(1, \frac{1}{2}\right) \in S$

$\Rightarrow \qquad 1 + 1 \times \frac{1}{2} > 0$

$\Rightarrow \qquad 1 + \frac{1}{2} > 0$, true

Now, $\qquad (b, c) \in S$

$\Rightarrow \qquad \left(\frac{1}{2}, -\frac{3}{2}\right) \in S$

$\Rightarrow \qquad 1 + \frac{1}{2} \times \frac{-3}{2} > 0$

$\Rightarrow \qquad 1 - \frac{3}{4} > 0 \Rightarrow \frac{1}{4} > 0$, true

Now, $(a, c) \in S$

$\Rightarrow \quad \left(1, \frac{-3}{2}\right) \in S \Rightarrow 1 + 1 \times \frac{-3}{2} > 0$

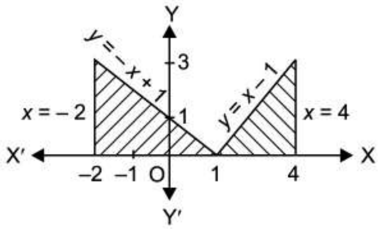
$\Rightarrow \quad 1 - \frac{3}{2} > 0 \Rightarrow -\frac{1}{2} > 0$, false

$\therefore (a, b) \in S, (b, c) \in S$ but $(a, c) \notin S$.

Not transitive

Hence, relation S is not an equivalence relation.

30. Graph of the curve $y = |x - 1|$ is shown as



$$\int_{-2}^{4} |x-1|\, dx = \int_{-2}^{1} -(x-1)\, dx + \int_{1}^{4} (x-1)dx$$

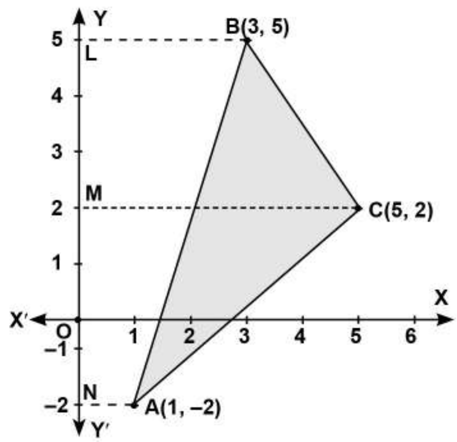
$$= \left[\frac{-x^2}{2} + x\right]_{-2}^{1} + \left[\frac{x^2}{2} - x\right]_{1}^{4}$$

$$= \left(-\frac{1}{2}+1\right) - \left(-\frac{4}{2}-2\right) + \left(\frac{16}{2}-4\right) - \left(\frac{1}{2}-1\right) = 9 \text{ sq units}$$

Value of this integral represents area bounded by the curve $y = |x - 1|$, the x-axis and between $x = -2$ to $x = 4$.

OR

Consider vertices as $A(1, -2)$, $B(3, 5)$ and $C(5, 2)$.



Equation of AB: $A(1, -2)$, $B(3, 5)$

$$y + 2 = \frac{5+2}{3-1}(x-1)$$

$\Rightarrow \quad y = \frac{7}{2}x - \frac{7}{2} - 2 \Rightarrow 2y = 7x - 11$

Equation of BC: $B(3, 5)$, $C(5, 2)$

$$y - 5 = \frac{2-5}{5-3}(x-3)$$

$$\Rightarrow \quad y - 5 = -\frac{3}{2}x + \frac{9}{2}$$

$$\Rightarrow \quad 2y - 10 = -3x + 9$$

$$\Rightarrow \quad 3x + 2y - 19 = 0$$

Equation of AC: $A(1, -2)$, $C(5, 2)$

$$y + 2 = \frac{2+2}{5-1}(x - 1)$$

$$\Rightarrow \quad y + 2 = x - 1 \quad \Rightarrow \quad x - y - 3 = 0$$

$$ar(ABC) = ar(CBLM) + ar(MCAN) - ar(BLNA)$$

$$= \int_2^5 x_{BC}\, dy + \int_{-2}^2 x_{AC}\, dy - \int_{-2}^5 x_{AB}\, dy$$

$$= \int_2^5 \frac{19 - 2y}{3}\, dy + \int_{-2}^2 (y + 3)\, dy - \int_{-2}^5 \left(\frac{2y + 11}{7}\right) dy$$

$$= \frac{1}{3}[19y - y^2]_2^5 + \left[\frac{y^2}{2} + 3y\right]_{-2}^2 - \frac{1}{7}[y^2 + 11y]_{-2}^5$$

$$= \frac{1}{3}[(95 - 25) - (38 - 4)] + \left[\left(\frac{4}{2} + 6\right) - \left(\frac{4}{2} - 6\right)\right] - \frac{1}{7}[(25 + 55) - (4 - 22)]$$

$$= \frac{1}{3}(70 - 34) + (8 + 4) - \frac{1}{7}(80 + 18)$$

$$= 12 + 12 - 14 = 10 \text{ sq units.}$$

31. Consider $\tan^{-1}\left(\frac{y}{x}\right) = \log\sqrt{x^2 + y^2}$

$$\Rightarrow \quad \tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log(x^2 + y^2)$$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{1 + \frac{y^2}{x^2}} \cdot \left\{\frac{xy' - y \cdot 1}{x^2}\right\} = \frac{1}{2} \cdot \frac{1}{x^2 + y^2}\{2x + 2yy'\}$$

$$\Rightarrow \quad \frac{x^2}{x^2 + y^2} \cdot \left\{\frac{xy' - y}{x^2}\right\} = \frac{1}{x^2 + y^2}\{x + yy'\}$$

$$\Rightarrow \quad xy' - y = x + yy'$$

$$\Rightarrow \quad (x - y)y' = x + y \Rightarrow y' = \frac{x + y}{x - y}$$

$$\therefore \quad \frac{dy}{dx} = \frac{x + y}{x - y}$$

OR

Consider $\lim_{x \to a} \frac{xf(a) - af(x)}{x - a} = \lim_{x \to a} \frac{xf(a) - af(a) + af(a) - af(x)}{(x - a)}$

$$= \lim_{x \to a} \frac{(x - a)f(a) - a[f(x) - f(a)]}{x - a}$$

$$= \lim_{x \to a}\left[f(a) - a \cdot \frac{f(x) - f(a)}{x - a}\right]$$

$$= f(a) - a \cdot \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= f(a) - af'(a).$$

32. Let $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$ ...(*i*)

$\therefore$ $$\vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ x & y & z \end{vmatrix}$$

$$= \hat{i}(z + y) - \hat{j}(2z + x) + \hat{k}(2y - x)$$

As $\vec{a} \times \vec{c} = \vec{b}$

$\Rightarrow$ $(y + z)\hat{i} - (2z + x)\hat{j} + (2y - x)\hat{k} = 4\hat{i} - 7\hat{j} + \hat{k}$

$\therefore$ $y + z = 4$ ...(*ii*), $2z + x = 7$ ...(*iii*),

and $2y - x = 1$ ...(*iv*)

Also $\vec{a} \cdot \vec{c} = 6 \Rightarrow (2\hat{i} + \hat{j} - \hat{k}).(x\hat{i} + y\hat{j} + z\hat{k}) = 6$

$\Rightarrow$ $2x + y - z = 6$ ...(*v*)

From (*ii*) and (*v*), we get

$$2x + 2y = 10 \Rightarrow x + y = 5 \quad ...(vi)$$

Also $2y - x = 1$ [from (*iv*)]

Solving (*vi*) and (*iv*), we get

$$3y = 6 \Rightarrow y = 2, x = 3 \text{ and } z = 2$$

$\therefore$ vector $\vec{c} = 3\hat{i} + 2\hat{j} + 2\hat{k}$

OR

Equations of lines in vector form are

$$\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$$

and $\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$

Let line through the point (1, 2, –4) be

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda'\vec{m}, \text{ where } \lambda' \text{ is a scalar} \quad ...(i)$$

Line (*i*) is perpendicular to lines

$$\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$$

and $\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$

$\therefore$ $(3\hat{i} - 16\hat{j} + 7\hat{k}) \cdot \vec{m} = 0$ and $(3\hat{i} + 8\hat{j} - 5\hat{k}) \cdot \vec{m} = 0$

$\Rightarrow$ $$\vec{m} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix} = 24\hat{i} + 36\hat{j} + 72\hat{k}$$

$\therefore$ From (*i*) line is

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda'(24\hat{i} + 36\hat{j} + 72\hat{k})$$

or $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda''(2\hat{i} + 3\hat{j} + 6\hat{k})$,

where $\lambda'' = 12\lambda'$, is a scalar

Vector form of line is $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda''(2\hat{i} + 3\hat{j} + 6\hat{k})$

$\therefore$ Point through which line passes is (1, 2, – 4) and DR's of line are 2, 3, 6.

$\therefore$ Cartesian form is $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z+4}{6}$.

33.
$$AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}\begin{bmatrix} -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$$

$\therefore$
$$(AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

Now,
$$A' = \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}, B' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

$\therefore$
$$B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & -4 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

Hence, $(AB)' = B'A'$.

OR

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$A^2 = A\,.\,A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence, $A^2 - 5A + 7I = O$.

$\therefore$ $A\,.\,A - 5A = -7I$

$\Rightarrow$ $A \cdot A(A^{-1}) - 5AA^{-1} = -7IA^{-1}$ [post-multiplying by A^{-1}]

$\Rightarrow$ $A(AA^{-1}) - 5I = -7A^{-1}$

$\Rightarrow$ $AI - 5I = -7A^{-1}$ [$AA^{-1} = I$]

$\Rightarrow$ $A^{-1} = -\frac{1}{7}(A - 5I) = \frac{1}{7}(5I - A)$ [$AI = A$]

$$= \frac{1}{7}\left(\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\right)$$

$$= \frac{1}{7}\begin{bmatrix} 5-3 & 0-1 \\ 0+1 & 5-2 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

$\therefore$
$$A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

34. Consider equation $x\frac{dy}{dx} + y = x\cos x + \sin x$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{y}{x} = \cos x + \frac{\sin x}{x}$$

Here, $$P(x) = \frac{1}{x},\ Q(x) = \cos x + \frac{\sin x}{x}$$

$$\text{Integrating factor} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$$

$$\text{Solution is (I.F.)}\, y = \int \{(\text{I.F.})\, Q(x)\}\, dx$$

$$x.y = \int x \, . \left(\cos x + \frac{\sin x}{x}\right) dx$$

$$= \int x \cos x \, dx + \int \sin x \, dx$$

$$= x \sin x - \int 1 \, . \sin x \, dx + \int \sin x \, dx$$

$$\Rightarrow \quad xy = x \sin x + C \qquad \text{...}(i)$$

Given $y = 1$, when $x = \frac{\pi}{2}$, then from (i), we get

$$\frac{\pi}{2} \, . \, 1 = \frac{\pi}{2} \, . \sin\frac{\pi}{2} + C \ \Rightarrow \ C = 0$$

$\therefore$ From (i), solution is $xy = x \sin x \Rightarrow y = \sin x$.

35. (i) $$Z_{(20,\,180)} = 20a + 180b \qquad \text{...}(i)$$

$$Z_{(40,\,160)} = 40a + 160b \qquad \text{...}(ii)$$

Since, $Z_{(20,\,180)} = Z_{(40,\,160)}$, then

from (i) and (ii), we get

$$20a + 180b = 40a + 160b$$

$$20a = 20b \Rightarrow a = b$$

(ii) Possible points for maximum Z are $A(20, 80)$, $B(40, 160)$ and $C(20, 180)$.

Points	$Z = 400x + 300y$	**Values**	
$A(20, 80)$	8,000 + 24,000	32,000	
$B(40, 160)$	16,000 + 48,000	64,000	← Maximum
$C(20, 180)$	8,000 + 54,000	62,000	

Z is maximum at $B(40, 160)$, i.e. for $x = 40$ and $y = 160$, maximum value of $Z = 64,000$

36. (i) $$4A = 2B = C \Rightarrow \frac{A}{1} = \frac{B}{2} = \frac{C}{4}$$

$$A : B : C = 1 : 2 : 4$$

$$\therefore \quad P(C) = \frac{4}{7}$$

(ii) Let E be the event that the person selected bring the changes.

So, required probability = $P(E|B) = 0.5$

(iii) Required probabilities = $1 - P(E/A) = 1 - 0.8 = 0.2$

OR

(*iii*) $$P(E) = P(A)P(E/A) + P(B)P(E/B) + P(C)\,P(E/C)$$

$$= \frac{1}{7}\times 0.8 + \frac{2}{7}\times 0.5 + \frac{4}{7}\times 0.3 = \frac{0.8+1.0+1.2}{7} = \frac{3}{7}$$

$$P(\overline{E}) = 1 - P(E) = 1 - \frac{3}{7} = \frac{4}{7}$$

37. (*i*) $$2\pi r = x \Rightarrow r = \frac{x}{2\pi} \qquad ...(i)$$

(*ii*) $$\text{Area of circle} = \pi\left(\frac{x}{2\pi}\right)^2 = \frac{1}{4\pi}x^2\ \text{m}^2 \qquad \left[\text{Using } r = \frac{x}{2\pi} \text{ from } (i)\right]$$

(*iii*) Perimeter of square frame = $(34 - x)$ m

So, $$\text{side} = \left(\frac{34-x}{4}\right)\text{m}$$

$$\text{Area of square frame} = \frac{1}{16}(34-x)^2\ \text{m}^2$$

OR

(*iii*) Combined area,

$$A = \frac{x^2}{4\pi} + \frac{1}{16}(34-x)^2$$

Now, $$\frac{dA}{dx} = \frac{2x}{4\pi} + \frac{1}{16}\cdot 2(34-x)(-1)$$

For minimum area,

$$\frac{dA}{dx} = 0$$

$$\Rightarrow \quad \frac{x}{2\pi} - \frac{34-x}{8} = 0$$

$$\Rightarrow \quad x = \frac{34\pi}{4+\pi}$$

$$\frac{d^2A}{dx^2} = \frac{1}{2\pi} + \frac{1}{8}$$

Now, $$\frac{d^2A}{dx^2} > 0 \text{ for } x = \frac{34\pi}{4+\pi}$$

$\therefore$ Combined area is minimum for $x = \frac{34\pi}{4+\pi}$ m

38. (*i*) Number of items purchased by shopkeepers A, B and C can be written in matrix form as

$$X = \begin{array}{c} \text{Notebooks} \quad \text{pens} \quad \text{pencils} \\ \begin{bmatrix} 144 & 60 & 72 \\ 120 & 72 & 84 \\ 132 & 156 & 96 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix} \end{array}$$

Cost per item (in ₹)

(*ii*) Since, $$Y = \begin{bmatrix} 40 \\ 12 \\ 3 \end{bmatrix} \begin{matrix} \text{Notebook} \\ \text{Pen} \\ \text{Pencil} \end{matrix}$$

$\therefore$ $$XY = \begin{bmatrix} 144 & 60 & 72 \\ 120 & 72 & 84 \\ 132 & 156 & 96 \end{bmatrix}\begin{bmatrix} 40 \\ 12 \\ 3 \end{bmatrix} = \begin{bmatrix} 5760 + 720 + 216 \\ 4800 + 864 + 252 \\ 5280 + 1872 + 288 \end{bmatrix} = \begin{bmatrix} 6696 \\ 5916 \\ 7440 \end{bmatrix}$$