CHAPTER 1
PATTERNS IN MATHEMATICS

Complete Notes | Concept Explanations | Solved Questions

1.1 What is Mathematics?

Core Idea: Mathematics is, in large part, the search for PATTERNS and the explanations as to WHY those patterns exist.

Patterns exist all around us — in nature, in our homes and schools, in the motion of the sun, moon, and stars. They appear in everything we do: shopping, cooking, throwing a ball, playing games, understanding weather, and using technology.

The search for patterns and their explanations can be a fun and creative endeavour. That is why mathematicians think of mathematics as both an ART and a SCIENCE.

Mathematics aims not just to find what patterns exist, but also the explanations for WHY they exist. Such explanations are then used in applications far beyond the context in which they were discovered — helping to propel humanity forward.

Examples of real-world impact:

- Understanding patterns in star/planet motion → Theory of Gravitation → Rockets to Moon & Mars
- Understanding patterns in genomes → Diagnosis and cure of diseases

FIGURE IT OUT — Section 1.1

Q1. Can you think of other examples where mathematics helps us in our everyday lives?

Ans: Examples include:

- Paying for groceries or vegetables — adding prices, calculating change
- Calculating speed of vehicles — time and distance problems
- Finding area of a house plot or room for tiling/painting
- Reading maps and measuring distances
- Cooking — measuring ingredients using fractions and ratios

Q2. How has mathematics helped propel humanity forward?

Ans: Mathematics has helped humanity in countless ways:

- Scientific experiments: Statistical tools help design and analyse experiments
- Economy & Democracy: Managing budgets, elections, and economic models
- Complex structures: Geometry and algebra are used to design bridges and buildings
- Technology: Computers, mobile phones, TVs use binary mathematics and algorithms

1.2 Patterns in Numbers

Number Theory: The branch of Mathematics that studies patterns in WHOLE NUMBERS (0, 1, 2, 3, 4, …) is called NUMBER THEORY.

Among the most basic patterns in mathematics are patterns of whole numbers.

Number Sequences: A NUMBER SEQUENCE is a list of numbers that follows a specific rule or pattern. They are the most basic and fascinating type of patterns in mathematics.

Table 1: Key Number Sequences

Sequence	Name	Rule
1, 1, 1, 1, 1, 1, 1, …	All 1's	Every term is 1
1, 2, 3, 4, 5, 6, 7, …	Counting Numbers	Add 1 each time
1, 3, 5, 7, 9, 11, 13, …	Odd Numbers	Add 2 each time (start at 1)
2, 4, 6, 8, 10, 12, 14, …	Even Numbers	Add 2 each time (start at 2)
1, 3, 6, 10, 15, 21, 28, …	Triangular Numbers	Add 1, 2, 3, 4, … successively
1, 4, 9, 16, 25, 36, 49, …	Square Numbers	n^2 ($1^2, 2^2, 3^2, \ldots$)
1, 8, 27, 64, 125, 216, …	Cube Numbers	n^3 ($1^3, 2^3, 3^3, \ldots$)
1, 2, 3, 5, 8, 13, 21, …	Virahānka Numbers	Each term = sum of previous two
1, 2, 4, 8, 16, 32, 64, …	Powers of 2	Multiply by 2 each time
1, 3, 9, 27, 81, 243, …	Powers of 3	Multiply by 3 each time

FIGURE IT OUT — Section 1.2

Q1. Can you recognise the pattern in each of the sequences in Table 1?

Ans: Yes! Here is the pattern/rule for each sequence:

All 1's: Every term is simply 1.

Counting Numbers: Each term increases by 1. → 1, 2, 3, 4, 5, …

Odd Numbers: Each term increases by 2, starting at 1.

Even Numbers: Each term increases by 2, starting at 2.

Triangular Numbers: Add 1, then 2, then 3, … → 1, 1+2=3, 3+3=6, 6+4=10, …

Squares: $n^2 \rightarrow 1^2=1, 2^2=4, 3^2=9, 4^2=16, 5^2=25, \ldots$

Cubes: $n^3 \rightarrow 1^3=1, 2^3=8, 3^3=27, 4^3=64, 5^3=125, \ldots$

Virahānka (Fibonacci): Each term = sum of previous two → 1, 2, 1+2=3, 2+3=5, 3+5=8, …

Powers of 2: Multiply by 2 → 1, 2, 4, 8, 16, … (i.e., $2^0, 2^1, 2^2, 2^3, \ldots$)

Powers of 3: Multiply by 3 → 1, 3, 9, 27, 81, … (i.e., $3^0, 3^1, 3^2, 3^3, \ldots$)

Q2. Rewrite each sequence of Table 1 in your notebook with the next three numbers, and explain the rule.

Ans: The next three terms for each sequence are:

Sequence	Next Three Terms	Rule
All 1's	1, 1, 1	Always 1
Counting Numbers	8, 9, 10	Add 1
Odd Numbers	15, 17, 19	Add 2
Even Numbers	16, 18, 20	Add 2
Triangular Numbers	36, 45, 55	Add 7, then 8, then 9, …
Squares	64, 81, 100	$8^2=64, 9^2=81, 10^2=100$
Cubes	343, 512, 729	$7^3, 8^3, 9^3$

Virahānka Numbers	34, 55, 89	Sum of previous two
Powers of 2	128, 256, 512	Multiply by 2
Powers of 3	729, 2187, 6561	Multiply by 3

1.3 Visualising Number Sequences

💡 **Why Visualise?:** Visualising mathematical objects through PICTURES or DIAGRAMS is a very powerful way to understand patterns and concepts!

Many number sequences can be represented using pictures. Let us look at pictorial representations of key sequences from Table 1.

Dot Pattern Representations

All 1's — Each group has exactly 1 dot:

●	●	●	●	●
1	1	1	1	1

Counting Numbers — Dots in a row:

●	● ●	● ● ●	● ● ● ●	● ● ● ● ●
1	2	3	4	5

Odd Numbers — L-shaped groups:

●	● ● ●	● ● ● ● ● ●	● ● ● ● ● ● ● ● ● ●	● ● ● ● ● ● ● ● ● ● ● ● ● ● ●
1	3	5	7	9

Even Numbers — Two-row grids:

● ●	● ● ● ●	● ● ● ● ● ●	● ● ● ● ● ● ● ●	● ● ● ● ● ● ● ● ● ●
2	4	6	8	10

Triangular Numbers — Triangle-shaped dot arrays:

●	● ● ●	● ● ● ● ● ●	● ● ● ● ● ● ● ● ● ●	● ● ● ● ● ● ● ● ● ● ● ● ● ● ●
1	3	6	10	15

💡 **Why 'Triangular'?:** These numbers form a triangular shape when dots are arranged in rows of 1, 2, 3, 4, 5, … — each row adds one more dot than the previous.

Square Numbers — Perfect square grids:

●	● ● ● ●	● ● ● ● ● ● ● ● ●	● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ●	● ● ● ● ● ● ● ● ● ● ● ● ● ●

				● ● ● ● ● ● ● ● ● ● ●
$1 = 1×1$	$4 = 2×2$	$9 = 3×3$	$16 = 4×4$	$25 = 5×5$

💡 **Why 'Square Numbers'?:** They form a perfect square when dots are arranged in a grid. 'n^2' means n rows × n columns.

Cube Numbers — 3D cube shapes:

▣	▣ ▣ ▣ ▣ ▣ ▣ ▣ ▣	▣▣▣ ▣▣▣ ▣▣▣ ▣▣▣ ▣▣▣ ▣▣▣ ▣▣▣ ▣▣▣ ▣▣▣	$64 = 4^3$ (4×4×4 cube)	$125 = 5^3$ (5×5×5 cube)
$1 = 1^3$	$8 = 2^3$	$27 = 3^3$	$64 = 4^3$	$125 = 5^3$

💡 **Why 'Cube Numbers'?:** They represent the number of unit cubes that fill a 3D cube of side n. A cube of side n has n^3 small cubes.

FIGURE IT OUT — Section 1.3

Q1. Copy the pictorial representations in Table 2 and draw the next picture for each sequence.

Ans: For each sequence, add one more row/layer:

- All 1's → Draw one more dot (1)
- Counting → Draw 6 dots in a row
- Odd → Draw L-shape with 11 dots
- Even → Draw 2 rows of 6 dots each (=12)
- Triangular → Add row of 6 to triangle (=21 dots)
- Squares → Draw 6×6 grid (=36 dots)
- Cubes → Represent $6^3 = 216$

Q2. Why are 1, 3, 6, 10, 15,… called triangular numbers? Why are 1, 4, 9, 16, 25,… called square numbers? Why are 1, 8, 27, 64, 125,… called cubes?

Ans — Triangular Numbers: When you arrange 1, 3, 6, 10, 15, … dots in rows of 1, 2, 3, 4, 5, …, they form a TRIANGLE shape. Hence the name 'Triangular Numbers'.

Ans — Square Numbers: When 1, 4, 9, 16, 25, … dots are arranged in equal rows and columns (1×1, 2×2, 3×3, …), they form a perfect SQUARE. Hence 'Square Numbers'.

Ans — Cube Numbers: 1, 8, 27, 64, 125, … represent the count of unit cubes in a 3D cube of side n (1×1×1, 2×2×2, 3×3×3, …). Hence 'Cube Numbers'.

✓ **Answer: Names come from the GEOMETRIC SHAPES the dot arrangements form.**

Q3. 36 is both a triangular number and a square number. Illustrate this!

Step 1: 36 as a Triangular Number: Arrange dots in rows 1+2+3+4+5+6+7+8 = 36 → Triangle of 8 rows.

Step 2: 36 as a Square Number: Arrange 36 dots in a 6×6 grid → Perfect square.

✓ **Answer: 36 = Triangular (T_8) AND Square (6^2). This shows the same number can have multiple geometric representations!**

Q4. What would you call the sequence 1, 7, 19, 37, …? What is the next number?

Ans: These are HEXAGONAL NUMBERS — each can be arranged in a hexagonal (6-sided) shape.

Pattern: Differences: 7−1=6, 19−7=12, 37−19=18 → differences increase by 6 each time.

Next: 37 + 24 = 61

✓ **Answer:** **These are called Hexagonal Numbers. The next number is 61.**

Q5. Can you think of pictorial ways to visualise Powers of 2 and Powers of 3?

Powers of 2: Think of dimensions growing: 1 point → 2 points in a line → 4 corners of a square → 8 corners of a cube → 16 corners of a hypercube. Pattern: 1, 2, 4, 8, 16, …

Powers of 3: Think of a point → triangle (3 vertices) → shape with 9 points → shape with 27 points. Pattern: 1, 3, 9, 27, …

✓ **Answer:** **Powers of 2: visualised as growing cubes/squares. Powers of 3: visualised as growing triangular structures.**

1.4 Relations Among Number Sequences

Key Idea: Sometimes, number sequences are RELATED to each other in beautiful and surprising ways. Discovering these relations is one of the great joys of mathematics!

Example 1: Adding Odd Numbers gives Square Numbers

Let us see what happens when we add consecutive odd numbers starting from 1:

Sum of Odd Numbers	Result	Square Number
1	1	$1^2 = 1$
1 + 3	4	$2^2 = 4$
1 + 3 + 5	9	$3^2 = 9$
1 + 3 + 5 + 7	16	$4^2 = 16$
1 + 3 + 5 + 7 + 9	25	$5^2 = 25$
1 + 3 + 5 + 7 + 9 + 11	36	$6^2 = 36$

Beautiful Pattern!: The sum of the first n odd numbers = n^2. This pattern holds FOREVER for any value of n!

Why Does This Happen? (Pictorial Explanation)

Think of square numbers as dots arranged in a square grid. We can partition those dots into L-shaped groups, where each L-shape contains an odd number of dots:

Partition of 6×6 square (36 dots) into L-shapes:

L-shape	Dots	Cumulative Sum
Innermost corner	1	1
2nd L-shape	3	1 + 3 = 4
3rd L-shape	5	1 + 3 + 5 = 9
4th L-shape	7	1 + 3 + 5 + 7 = 16
5th L-shape	9	1 + 3 + 5 + 7 + 9 = 25
6th L-shape	11	1 + 3 + 5 + 7 + 9 + 11 = 36

Since such an L-shaped picture can be drawn for a square of ANY size, this explains why adding odd numbers always gives square numbers!

Example 2: Adding Up and Down gives Square Numbers

Another beautiful pattern using counting numbers added up, then down:

Pattern	Result	Square
1	1	$1^2 = 1$
1 + 2 + 1	4	$2^2 = 4$
1 + 2 + 3 + 2 + 1	9	$3^2 = 9$
1 + 2 + 3 + 4 + 3 + 2 + 1	16	$4^2 = 16$
1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1	25	$5^2 = 25$
1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 + 3 + 2 + 1	36	$6^2 = 36$

💡 **Pattern:** Adding counting numbers up to n and then back down gives n^2. This is yet another way to get square numbers!

📋 FIGURE IT OUT — Section 1.4

Q1. Can you find a pictorial explanation for why adding counting numbers up and down gives square numbers?

Ans: Imagine a square grid rotated 45°. Each diagonal of the rotated square has a counting number of dots going up (1,2,3,…,n) and then back down (…,2,1). Together they form the complete square grid of n^2.

Visual: For n=3: Diagonals have 1, 2, 3, 2, 1 dots → total = 9 = 3^2

✓ **Answer: The diamond/rhombus arrangement of dots shows that 1+2+3+…+n+…+2+1 = n^2**

Q2. What is the value of 1 + 2 + 3 + … + 99 + 100 + 99 + … + 3 + 2 + 1?

Step 1: This is the 'up and down' sum pattern with n = 100.

Step 2: Using the pattern: 1+2+…+n+…+2+1 = n^2

Step 3: Here n = 100, so the answer = 100^2 = 10,000

✓ **Answer: 1 + 2 + 3 + … + 100 + … + 3 + 2 + 1 = 100^2 = 10,000**

Q3. What sequence do you get when you add the All 1's sequence up? And when you add it up and down?

Adding up: 1, 1+1, 1+1+1, 1+1+1+1, … = 1, 2, 3, 4, 5, … → Counting Numbers!

Adding up & down: 1, 1+(1+1)+1, 1+(1+1)+(1+1+1)+(1+1)+1, … = 1, 4, 9, 16, … → Square Numbers!

✓ **Answer: Adding All 1's up → Counting Numbers. Adding All 1's up and down → Square Numbers.**

Q4. What sequence do you get when you start to add counting numbers up?

Ans: 1, 1+2=3, 1+2+3=6, 1+2+3+4=10, 1+2+3+4+5=15, …

Result: This gives the TRIANGULAR number sequence: 1, 3, 6, 10, 15, …

Pictorial: Each partial sum forms a triangle. Adding one more row each time creates the next triangular number.

✓ **Answer: Adding counting numbers cumulatively gives the TRIANGULAR NUMBER sequence.**

Q5. What happens when you add pairs of consecutive triangular numbers: 1+3, 3+6, 6+10, 10+15, …?

Calculate: 1+3=4, 3+6=9, 6+10=16, 10+15=25, 15+21=36

Result: We get 4, 9, 16, 25, 36, … = SQUARE numbers!

Why?: Adjacent triangles (T_n and T_{n+1}) fit together to form a square. A triangle of n rows fits with one of n+1 rows to make an $(n+1)\times(n+1)$ square.

✓ **Answer:** **$1+3=4=2^2$, $3+6=9=3^2$, $6+10=16=4^2$, … → We get SQUARE NUMBERS.**

Q6. What happens when you add powers of 2: 1, 1+2, 1+2+4, 1+2+4+8,…? Add 1 to each — what do you get?

Partial sums: 1, 1+2=3, 1+2+4=7, 1+2+4+8=15, 1+2+4+8+16=31, …

Result: We get 1, 3, 7, 15, 31, … (one less than powers of 2)

Add 1: 1+1=2, 3+1=4, 7+1=8, 15+1=16, 31+1=32, … → Powers of 2!

Why?: Sum of powers of 2 from 2^0 to 2^(n−1) = $2^n - 1$. Adding 1 gives 2^n.

✓ **Answer:** **Cumulative sums: 1, 3, 7, 15, 31, … Adding 1: 2, 4, 8, 16, 32, … (Powers of 2)**

Q7. What happens when you multiply triangular numbers by 6 and add 1?

Calculate: (1×6)+1=7, (3×6)+1=19, (6×6)+1=37, (10×6)+1=61, (15×6)+1=91, …

Result: We get 7, 19, 37, 61, 91, … = HEXAGONAL NUMBERS!

✓ **Answer:** **6 × (Triangular Number) + 1 = Hexagonal Number**

Q8. What happens when you add hexagonal numbers: 1, 1+7, 1+7+19, 1+7+19+37, …?

Calculate: 1, 1+7=8, 1+7+19=27, 1+7+19+37=64, …

Result: We get 1, 8, 27, 64, … = CUBE NUMBERS!

Pictorial: A cube can be 'built up' by adding hexagonal layers. Each hexagonal ring represents a layer of a growing cube.

✓ **Answer:** **Sum of hexagonal numbers = Cube numbers (1, 8, 27, 64, 125, …)**

Q9. Find your own patterns or relations in and among the sequences in Table 1.

Suggestion 1: Every even number = 2 × (counting number) → 2, 4, 6, 8, …

Suggestion 2: Every odd number can be written as difference of two consecutive squares: $n^2 - (n-1)^2 = 2n-1$

Suggestion 3: Every triangular number T_n = n(n+1)/2

Suggestion 4: Powers of 2 double each time; powers of 3 triple each time

✓ **Answer:** **There are many beautiful patterns! Mathematics is about discovering them and explaining WHY they work.**

1.5 Patterns in Shapes

💡 **Geometry:** The branch of Mathematics that studies patterns in SHAPES is called GEOMETRY. Shapes may be 1D, 2D, 3D or even higher-dimensional!

Just like number sequences, there are also SHAPE SEQUENCES — ordered lists of shapes that follow a specific pattern or rule.

Table 3: Key Shape Sequences

Shape Sequence	Description	What Changes
Regular Polygons	Triangle → Square → Pentagon → Hexagon → Heptagon → Octagon → Nonagon → Decagon → …	Number of sides increases by 1 each time
Complete Graphs (K2, K3, K4, …)	Connect every point to every other point: 2 points, 3 points, 4 points, …	Number of points (and therefore lines) increases
Stacked Squares	1×1 → 2×2 → 3×3 → 4×4 → 5×5 → …	Side length increases by 1; total small squares = n^2

Stacked Triangles	Small triangle → 4-triangle → 9-triangle → 16-triangle → …	Each row adds one more; total small triangles = n^2
Koch Snowflake	Start with triangle; replace each side with a 'speed-bump' repeatedly	Each step multiplies line segments by 4

Regular Polygons — Explained

A polygon is called REGULAR if all its sides are equal in length AND all its angles (corners) are equal.

Sides	Name	Example
3	Triangle (Regular)	Equilateral Triangle
4	Quadrilateral / Square	Square
5	Pentagon	Regular Pentagon
6	Hexagon	Regular Hexagon
7	Heptagon	Regular Heptagon
8	Octagon	Regular Octagon
9	Nonagon	Regular Nonagon
10	Decagon	Regular Decagon

Complete Graphs — Explained

A COMPLETE GRAPH K_n is a figure where n points are drawn and EVERY point is connected to every other point with a line.

Graph	Points	Lines (connections)
K2	2	1
K3	3	3
K4	4	6
K5	5	10
K6	6	15

💡 **Notice:** The number of lines in complete graphs gives: 1, 3, 6, 10, 15, … = TRIANGULAR NUMBERS!

Koch Snowflake — Explained

Start with a triangle. In each step, replace every straight line segment '—' with a 'speed bump' shape '^'. As this is done more and more times, a beautiful snowflake-like fractal emerges!

Step	Line Segments	Pattern
Step 0 (Triangle)	3	$3 = 3 \times 4^0$
Step 1	12	$12 = 3 \times 4^1$
Step 2	48	$48 = 3 \times 4^2$
Step 3	192	$192 = 3 \times 4^3$
Step 4	768	$768 = 3 \times 4^4$

💡 **Koch Snowflake Sequence:** Number of line segments = 3, 12, 48, 192, … = 3 × Powers of 4

FIGURE IT OUT — Section 1.5

Q1. Can you recognise the pattern in each of the sequences in Table 3?

Regular Polygons: Sides go 3, 4, 5, 6, 7, … (counting numbers starting from 3). Each new shape adds one side.

Complete Graphs: Lines go 1, 3, 6, 10, 15, … (Triangular numbers). Each new point adds as many new lines as there are existing points.

Stacked Squares: Little squares: 1, 4, 9, 16, 25, … (Square numbers). Each $n \times n$ grid has n^2 small squares.

Stacked Triangles: Little triangles: 1, 4, 9, 16, 25, … (Square numbers again!). Row sums go as 1, 1+3, 1+3+5, …

Koch Snowflake: Line segments: 3, 12, 48, 192, … (3 × Powers of 4). Each line becomes 4 lines in next step.

✓ **Answer:** **Each shape sequence has a clear mathematical pattern connected to number sequences.**

Q2. Try to redraw each sequence in Table 3 and draw the next shape in each sequence.

Regular Polygons: Draw next: 11-sided polygon (Hendecagon)

Complete Graphs: Draw K7: 7 points, all connected → 21 lines (next triangular number)

Stacked Squares: Draw 6×6 grid = 36 small squares

Stacked Triangles: Draw triangle with 6 rows = 36 small triangles

Koch Snowflake: Cannot practically draw Step 4+ by hand (segments are very tiny), but the pattern is 3×4^n

✓ **Answer:** **Most sequences can be extended by following their pattern rule. Koch Snowflake becomes too complex after a few steps.**

1.6 Relation to Number Sequences

Big Idea: Shape sequences are often related to number sequences in SURPRISING ways. These relationships help us understand both the shapes and the numbers better!

FIGURE IT OUT — Section 1.6

Q1. Count sides in Regular Polygons. Which number sequence? Same for corners?

Sides: Triangle=3, Square=4, Pentagon=5, Hexagon=6, Heptagon=7, Octagon=8, Nonagon=9, Decagon=10, …

Sequence: 3, 4, 5, 6, 7, 8, 9, 10, … = Counting numbers starting from 3

Corners: Triangle=3, Square=4, Pentagon=5, … Same as sides!

Why same?: In ANY closed polygon, number of sides = number of corners (vertices). Each side connects two corners!

✓ **Answer:** **Sides AND corners of Regular Polygons: 3, 4, 5, 6, 7, 8, 9, 10, … (Counting numbers from 3). Both are equal because each polygon has one corner per side.**

Q2. Count lines in Complete Graphs. Which number sequence?

Count: K2: 1 line, K3: 3 lines, K4: 6 lines, K5: 10 lines, K6: 15 lines, …

Sequence: 1, 3, 6, 10, 15, … = TRIANGULAR numbers!

Why?: In K_n, each new point connects to all $n-1$ existing points. So lines added: 1, 2, 3, 4, 5, … Cumulative: 1, 3, 6, 10, 15, … (Triangular numbers)

✓ **Answer:** **Number of lines in Complete Graphs K2, K3, K4, K5, K6, … = 1, 3, 6, 10, 15, … = TRIANGULAR NUMBERS.**

Q3. Count little squares in Stacked Squares sequence. Which number sequence?

Count: 1×1=1, 2×2=4, 3×3=9, 4×4=16, 5×5=25, …

Sequence: 1, 4, 9, 16, 25, … = SQUARE numbers!

Why?: An n×n grid has n rows, each with n squares. So total = n×n = n^2.

✓ **Answer: Number of small squares in n×n Stacked Square = n^2 (Square Numbers: 1, 4, 9, 16, 25, …).**

Q4. Count little triangles in Stacked Triangles. Which number sequence?

Count: Row 1 has 1 (upward). Next: adds 3 (2 up + 1 down). Then 5, then 7, …

Totals: 1, 1+3=4, 4+5=9, 9+7=16, 16+9=25, … = SQUARE numbers!

Alternative view: Using 'up and down' pattern: rows have 1, 1+2+1, 1+2+3+2+1, … triangles = 1, 4, 9, … squares.

Hint explanation: In step n, row k (from top) has 2k−1 little triangles. Sum = 1+3+5+…+(2n−1) = n^2.

✓ **Answer: Number of little triangles in Stacked Triangles = 1, 4, 9, 16, 25, … = SQUARE NUMBERS.**

Q5. How many line segments in each Koch Snowflake shape? What is the number sequence?

Step 0: Start with equilateral triangle: 3 line segments

Step 1: Each segment splits into 4 → 3 × 4 = 12 segments

Step 2: Each of 12 splits into 4 → 12 × 4 = 48 segments

Step 3: 48 × 4 = 192 segments

Pattern: 3, 12, 48, 192, 768, … = 3 × 1, 3 × 4, 3 × 4^2, 3 × 4^3, … = 3 × Powers of 4

✓ **Answer: Koch Snowflake line segments: 3, 12, 48, 192, 768, … = 3 × Powers of 4.**

★ CHAPTER SUMMARY ★

Concept	Key Point
Mathematics	is the search for PATTERNS and for the explanations as to WHY those patterns exist.
Number Theory	is the branch of Math that studies patterns in whole numbers (0, 1, 2, 3, …).
Key Number Sequences	Counting, Odd, Even, Triangular, Square, Cube, Virahānka, Powers of 2 & 3.
Relations Among Sequences	Sum of first n odd numbers = n^2. Consecutive triangular numbers add to give squares. Adding up powers of 2 then adding 1 gives the next power of 2.
Visualising Sequences	Dot patterns, grids, triangles, cubes — pictures help us understand WHY patterns hold!
Shape Sequences	Regular Polygons, Complete Graphs, Stacked Squares/Triangles, Koch Snowflake.
Geometry	The branch of Math that studies patterns in shapes (1D, 2D, 3D, …).
Shapes ↔ Numbers	Sides of regular polygons: 3, 4, 5, … (counting). Lines in K_n: triangular numbers. Squares/triangles: square numbers. Koch Snowflake: 3 × powers of 4.

Quick Formula Card

Formula / Fact	Meaning
$1+3+5+\ldots+(2n-1) = n^2$	Sum of first n odd numbers = n^2
$1+2+3+\ldots+n+\ldots+2+1 = n^2$	Counting numbers up and down to n, sum = n^2
$T_n = n(n+1)/2$	n-th Triangular number
$T_n + T_{n+1} = (n+1)^2$	Sum of consecutive triangular numbers = Square
$1+2+4+8+\ldots+2^{(n-1)} = 2^n-1$	Sum of first n powers of 2
$6 \times T_n + 1 = H_n$	Hexagonal number from triangular number
$H_1+H_2+\ldots+H_n = n^3$	Sum of hexagonal numbers = Cube number
Koch Snowflake = 3×4^n	Line segments at step n