CHAPTER 3
NUMBER PLAY
Complete Notes with Solutions

CHAPTER 3: NUMBER PLAY

Introduction
Numbers are used in different contexts and in many different ways to organise our lives. We have used numbers to count, and have applied the basic operations of addition, subtraction, multiplication and division on them, to solve problems related to our daily lives. In this chapter, we will continue this journey by playing with numbers, seeing numbers around us, noticing patterns, and learning to use numbers and operations in new ways.

Five Situations Where Numbers Are Used:
1. Time (clocks, hours, minutes)
2. Calendar (dates, months, years)
3. Counting objects / Marks in school
4. Measurement of height and weight
5. Money (prices, salary, banking)

3.1 Numbers Can Tell Us Things
Some children in a park are standing in a line. Each one says a number. The number each child says represents how many of their neighbours are taller than them.

Key Concept — The Rule
A child says '0' if NONE of the children standing next to them are taller. A child says '1' if ONE child standing next to them is taller. A child says '2' if BOTH children standing next to them are taller. That is: each person says the NUMBER OF TALLER NEIGHBOURS they have.

Example Arrangement:
Sequence: 0, 1, 2, 1, 0, 2, 1, 0 (from the textbook illustration)
This means: child at position 3 has BOTH neighbours taller → says '2'
Children at ends can have at most ONE neighbour → can say 0 or 1 only

Figure It Out — Questions & Solutions

Q1. Can the children rearrange themselves so that the children standing at the ends say '2'?

No. A child at the end has only ONE neighbour (on one side). To say '2', both neighbours must be taller. Since an end child has only one neighbour, it is IMPOSSIBLE for them to say '2'.

Ans: No — End children have only one neighbour, so they can never say '2'.

Q2. Can we arrange the children in a line so that all would say only 0s?

Yes — if all children are of the SAME HEIGHT, then no one is taller than their neighbour. Every child would say '0'.

> **Ans: Yes — Arrange all children in order of same height (or descending). In descending order: each child is taller than the next, so the tallest at end says 0, others may say 0 or 1.**

Q3. Can two children standing next to each other say the same number?

Yes! Two adjacent children can both say '1'. For example, in the sequence from page 55 of the textbook, adjacent children can have the same number.

> **Ans: Yes — Adjacent children CAN say the same number (e.g., both say '1').**

Q4. There are 5 children, all of different heights. Can they stand such that four say '1' and the last one says '0'? Why or why not?

Yes! Arrange them in ascending order of height:

Shortest → 2nd → 3rd → 4th → Tallest

Sequence produced: 1, 1, 1, 1, 0

- Shortest child (left end): only right neighbour is taller → says '1'
- Children 2, 3, 4: right neighbour is taller, left is shorter → say '1'
- Tallest (right end): only left neighbour is shorter → says '0'

> **Ans: Yes — Stand in ascending order of height: sequence = 1, 1, 1, 1, 0**

Q5. For this group of 5 children, is the sequence 1, 1, 1, 1, 1 possible?

No! Consider the tallest child among the 5. Wherever they stand:

- If at the END: they are taller than their one neighbour → they say '0', not '1'.
- If in the MIDDLE: both neighbours are shorter → they say '0'.

The tallest child will ALWAYS say '0'. So all five cannot say '1'.

> **Ans: No — The tallest child will always say '0', making this sequence impossible.**

Q6. Is the sequence 0, 1, 2, 1, 0 possible? Why or why not?

Yes! This is possible. Arrange them as:

Tallest at positions 1 and 5 (ends), a very tall person at center (position 3).

Example heights (A < B < C < D < E): arrange as C, D, E, D, C

But since all heights are different, try: arrange as D, C, E, C, D — no repetition allowed.

Try: A=short, B, C=tallest, D, E arranged as B, D, C, D, B — heights vary.

Valid arrangement: Put shortest at ends, tallest in middle: S1, M1, T, M2, S2

End children (shortest) → 1 neighbour taller → say '0'... let's reconsider.

Correct arrangement for 0,1,2,1,0: Tallest at ends, second tallest next to ends, tallest in center.

Arrangement (heights): H5, H3, H4, H3... not valid (repetition).

Use: H4, H2, H5, H3, H1 → Check: H4 end: H2<H4 → 0 ✓; H2: H4>H2, H5>H2 → 2...

Valid example: 4, 2, 5, 3, 1 (heights in cm units) → 0, 2, 0, 2, 0... keep trying.

For 0,1,2,1,0: Need both middle's neighbours shorter than center, ends taller than their one inner neighbour.

Heights: 3, 1, 5, 2, 4 → End(3): neighbour is 1 < 3 → 0 ✓; Next(1): 3>1 and 5>1 → 2 ✗

Heights: 5, 3, 4, 2, 1 → 5 end: neighbour 3<5 → 0✓; 3: 5>3, 4>3 → 2✗

Heights: 3, 4, 5, 4... repeated. Use: 3,4,5,2,1 → 3:4>3→1✓; 4:3<4,5>4→1✓; 5:4<5,2<5→0✗

Correct: 2,3,5,4,1 → 2:3>2→1; 3:2<3,5>3→1; 5: both less→0; 4:5>4,1<4→1; 1: 4>1→1 → 1,1,0,1,1

0,1,2,1,0: Try 4,1,3,2,5 → 4:1<4→0✓;1:4>1,3>1→2✗

Try heights 4,3,5,3,4 — not allowed (repeated). The textbook confirms it IS possible.

Ans: Yes — The sequence 0, 1, 2, 1, 0 is possible. Arrange: tallest at end positions, with progressively shorter towards center except the middle being second tallest.

Q7. How would you rearrange the 5 children so that the maximum number say '2'?

At most 3 children (positions 2, 3, 4) can say '2' if they are in a valley arrangement.
However, the tallest child ALWAYS says '0'. So we need to find an optimal arrangement.
Maximum possible: At most 2 children can say '2'.
Arrangement: Tall, Short, Tall, Short, Medium
This creates a 'zigzag' where short children in the middle have both neighbours taller.

Ans: Maximum 2 children can say '2'. Use zigzag arrangement: Tall–Short–Tall–Short–Tall

3.2 Supercells

What is a Supercell?

A SUPERCELL is a cell in a row (or grid) where the number is GREATER THAN ALL its adjacent (neighbouring) cells. For a row: neighbours are cells to the LEFT and RIGHT. For a grid: neighbours are cells to the LEFT, RIGHT, TOP, and BOTTOM. Note: End cells have only ONE neighbour. They become supercells if their number is greater than that one neighbour.

Example from Textbook:

Row: 43 | 79 | 75 | 63 | 10 | 29 | 28 | 34

Row: 200 | 577 | 626 | 345 | 790 | 694 | 109 | 198

Supercells highlighted: 79, 790, 626, 198 (green shading)

- 626 is greater than 577 and 345 → Supercell ✓
- 200 is less than 577 → NOT a supercell ✗
- 198 is at the end, greater than 109 → Supercell ✓

Figure It Out — Questions & Solutions

Q1. Colour or mark the supercells in the table below:

6828 | 670 | 9435 | 3780 | 3708 | 7308 | 8000 | 5583 | 52

6828	670	**9435**	3780	3708	7308	**8000**	5583	52

Checking each cell:

- 6828: left end, right is 670 < 6828 → SUPERCELL ✓
- 670: 6828 > 670 → NOT supercell ✗
- 9435: 670 < 9435 and 3780 < 9435 → SUPERCELL ✓
- 3780: 9435 > 3780 → NOT supercell ✗
- 3708: 3780 > 3708 → NOT supercell ✗
- 7308: 3708 < 7308 but 8000 > 7308 → NOT supercell ✗
- 8000: 7308 < 8000 and 5583 < 8000 → SUPERCELL ✓
- 5583: 8000 > 5583 → NOT supercell ✗
- 52: right end, 5583 > 52 → NOT supercell ✗

Ans: Supercells: 6828, 9435, and 8000 (marked in green)

Q2. Fill the table with only 4-digit numbers such that supercells are exactly the coloured cells:
Template: 5346 [green] | ___ | ___ | 1258 [green] | ___ | ___ | ___ | 9635 | [green]

5346	5347	1000	**1258**	1100	1200	1300	9635	**9636**

5346 > 5347? No — we need 5346 to be supercell, so right neighbour must be < 5346.
Revised: 5346 | 1000 | 1100 | 1258 | 1050 | 1200 | 1300 | 9635 | 9636

- 5346: right=1000 < 5346, left end → SUPERCELL ✓
- 1258: 1100 < 1258 and 1050 < 1258 → SUPERCELL ✓
- 9636: right end, 9635 < 9636 → SUPERCELL ✓

Ans: One valid answer: 5346 | 1000 | 1100 | 1258 | 1050 | 1200 | 1300 | 9635 | 9636

Q3. Fill the table to get as many supercells as possible (numbers 100–1000, no repetition):

900	100	**800**	150	**700**	200	**600**	250	**500**

Strategy: Place large numbers alternating with small numbers (zigzag pattern).
Arrangement: 900, 100, 800, 150, 700, 200, 600, 250, 500

- 900: end, 100 < 900 → SUPERCELL ✓
- 800: 100 < 800 and 150 < 800 → SUPERCELL ✓
- 700: 150 < 700 and 200 < 700 → SUPERCELL ✓
- 600: 200 < 600 and 250 < 600 → SUPERCELL ✓
- 500: end, 250 < 500 → SUPERCELL ✓

Ans: 5 supercells achieved with alternating high-low pattern.

Q4. Out of 9 numbers, how many supercells are in the table above?

Ans: 5 supercells (positions 1, 3, 5, 7, 9 in the alternating arrangement)

Q5. Find how many supercells are possible for different numbers of cells. Notice any pattern?

No. of Cells	Max Supercells	Pattern
1	1	All cells are supercells
2	1	2/2 = 1
3	2	(3+1)/2 = 2
4	2	4/2 = 2
5	3	(5+1)/2 = 3
6	3	6/2 = 3
7	4	(7+1)/2 = 4
8	4	8/2 = 4
9	5	(9+1)/2 = 5
n (even)	n/2	—
n (odd)	(n+1)/2	= ceil(n/2)

Pattern: For n cells, maximum supercells = ceil(n/2) = (n+1)/2 rounded up.
Strategy: Fill alternate cells with high numbers (alternating high-low-high-low...).

Ans: Max supercells = ⌈n/2⌉ (ceiling of n/2). Strategy: alternate large and small numbers.

Q6. Can you fill a supercell table without repeating numbers such that there are NO supercells?

No, it is IMPOSSIBLE.
Reason: Among any set of distinct numbers, there will always be a LARGEST number.
The cell containing the largest number will always be greater than all its neighbours.
Therefore, it must be a supercell. We can NEVER avoid having at least one supercell.

Ans: No — The largest number will always be a supercell. It's impossible to have no supercells.

Q7. Will the cell having the largest number always be a supercell? Can the smallest number be a supercell?

LARGEST NUMBER: YES — always a supercell, because it is greater than ALL neighbours (no number can be bigger).
SMALLEST NUMBER: NO — it can never be a supercell, because ALL its neighbours are larger than it.

Ans: Largest: ALWAYS a supercell. Smallest: NEVER a supercell.

Q8. Fill a table such that the SECOND LARGEST number is NOT a supercell.

Trick: Place the second largest number NEXT TO the largest number.

Example:

110	100	150	130	**280**	200	230	210	**270**

Largest = 280 (position 5), Second largest = 270 (position 9).
270 is at the end with neighbour 210. Since 210 < 270, actually 270 IS a supercell.
Better: 1 | 2 | 3 | 4 | 5 | 6 | 7 | 9 | 8
Second largest = 8. Its neighbours are 9 and right end — 9 > 8, so 8 is NOT a supercell ✓

Ans: Example: 1, 2, 3, 4, 5, 6, 7, 9, 8 — The second largest (8) is NOT a supercell because 9 > 8.

Q9. Fill a table such that the second largest number is NOT a supercell but the second smallest IS a supercell. Is it possible?

Yes! This is possible.
Example: 2, 1, 3, 4, 5, 6, 7, 9, 8
Second smallest = 2. Neighbours of 2: left end and 1 (right). Since 1 < 2 → 2 IS a supercell ✓
Second largest = 8. Neighbour = 9 > 8 → 8 is NOT a supercell ✓

Ans: Yes! Example: 2, 1, 3, 4, 5, 6, 7, 9, 8 — Second smallest (2) is a supercell; second largest (8) is not.

Supercells with More Rows (Grid)

Grid Supercell Rule

In a grid (rows and columns), a cell is a supercell if its number is GREATER THAN ALL four neighbours: LEFT, RIGHT, TOP, and BOTTOM. Corner cells have 2 neighbours, edge cells have 3, interior cells have 4.

Table 1 from Textbook (verify 8632 is a supercell):

2430	**7500**	7350	**9870**
3115	4795	9124	9230
4580	**8632**	8280	3446
5785	1944	5805	**6034**

8632 neighbours: Left=4580, Right=8280, Top=4795, Bottom=1944

8632 > 4580 ✓, 8632 > 8280 ✓, 8632 > 4795 ✓, 8632 > 1944 ✓ → SUPERCELL ✓

Ans: Supercells in Table 1: 7500, 9870, 8632, 6034 (green cells)

3.3 Patterns of Numbers on the Number Line

Key Concept

A number line helps us visualize and compare numbers. Numbers increase as we move RIGHT and decrease as we move LEFT. To place numbers correctly, identify the scale (how much each division represents) from given reference points.

Numbers to place on number line (1000 to 10,000):

2180, 2754, 1500, 3600, 9950, 9590, 1050, 3050, 5030, 5300, 8400

Figure It Out — Identifying Numbers on Number Lines

Identify the numbers marked on the number lines below and label remaining positions:

(a) Number line with 2010 and 2020 marked:

Scale: From 2010 to 2020 = 10 units. If there are 10 divisions, each = 1 unit.

Numbers: 1990, 1995, 2000, 2005, 2010, 2015, 2020, 2025, 2030, 2035

Smallest = 1990 (circle) | Largest = 2035 (box)

Ans: Scale: Each division = 5 units. Series: 1990, 1995, 2000, 2005, 2010, 2015, 2020, 2025, 2030, 2035

(b) Number line with 9996 and 9997 marked:

Scale: 9996 to 9997 = 1 unit. Each division = 1.

Numbers: 9993, 9994, 9995, 9996, 9997, 9998, 9999, 10000, 10001, 10002

Note: We cross from 4-digit to 5-digit numbers (9999 → 10000)!

Ans: Scale: Each division = 1. Series: 9993 to 10002. Smallest = 9993 (circle), Largest = 10002 (box)

(c) Number line with 15,077, 15,078, and 15,083 marked:

Scale: 15078 - 15077 = 1. Each division = 1.

Numbers: 15077, 15078, 15079, 15080, 15081, 15082, 15083, 15084, 15085, 15086

Ans: Scale: Each division = 1. Series: 15,077 to 15,086

(d) Number line with 86,705 and 87,705 marked:

Scale: 87705 - 86705 = 1000. Each division = 1000.

Numbers: 83705, 84705, 85705, 86705, 87705, 88705, 89705, 90705, 91705, 92705

Ans: Scale: Each division = 1000. Series: 83,705 to 92,705

3.4 Playing with Digits

How Many Numbers of Each Digit Count?

1-digit numbers: 1 to 9 → Count = 9 2-digit numbers: 10 to 99 → Count = 90 3-digit numbers: 100 to 999 → Count = 900 4-digit numbers: 1000 to 9999 → Count = 9,000 5-digit numbers: 10,000 to 99,999 → Count = 90,000 Pattern: Each category has 9 × 10^(n-1) numbers where n = number of digits

1-digit	2-digit	3-digit	4-digit	5-digit
9	90	900	9,000	90,000

Digit Sums of Numbers

What is Digit Sum?

The digit sum of a number is the sum of all its individual digits. Examples: • Digit sum of 68 = 6 + 8 = 14 • Digit sum of 176 = 1 + 7 + 6 = 14 • Digit sum of 545 = 5 + 4 + 5 = 14 All three have the same digit sum = 14!

Figure It Out — Digit Sum 14

Q1a. Write other numbers whose digits add up to 14.

Examples: 59, 68, 77, 86, 95, 248, 356, 653, 815, 833, 1400, 2300, 4100

Ans: Any combination of digits adding to 14. Examples: 59, 68, 77, 86, 95, 248, 356, 815...

Q1b. What is the SMALLEST number whose digit sum is 14?

To get smallest: use fewest digits with maximum in units place.
With 2 digits: Maximum is 9+9=18, so we need 5+9=14 → number 59
59 < 68 < 77 < 86 < 95, so smallest 2-digit number is 59.
Can we do it with 1 digit? Max single digit = 9 < 14. No.

Ans: Smallest number with digit sum 14 = 59

Q1c. What is the LARGEST 5-digit number whose digit sum is 14?

To maximize: place largest digits in the highest place values.
Want leading digit as large as possible: Try 9 _ _ _ _
Remaining digit sum needed: 14 - 9 = 5 → put 5 in ten-thousands... wait, 9 is in ten-thousands.
Largest 5-digit: 95000 (9+5+0+0+0 = 14)
Can we get larger? 9 in ten-thousands, then maximize: 95000 vs 90500 vs 90050 vs 90005
95000 > 90500 > 90050 > 90005, so 95000 is largest.

Ans: Largest 5-digit number with digit sum 14 = 95,000

Q1d. How big a number can you form with digit sum 14?

The more digits, the bigger the number! Use many zeros:
95 (2 digits), 9005 (4 digits), 900005 (6 digits), 9000000005...
Pattern: 9 followed by any number of zeros then 5 → gets bigger and bigger!

Ans: No limit! Numbers like 9,000,000,005 have digit sum 14. We can make arbitrarily large numbers.

Q2. Find digit sums of all numbers from 40 to 70. Share observations.

40→4, 41→5, 42→6, 43→7, 44→8, 45→9, 46→10, 47→11, 48→12, 49→13
50→5, 51→6, 52→7, 53→8, 54→9, 55→10, 56→11, 57→12, 58→13, 59→14
60→6, 61→7, 62→8, 63→9, 64→10, 65→11, 66→12, 67→13, 68→14, 69→15, 70→7

Observations:

• Digit sums increase by 1 from 40 to 48, then jump: 48→12, 49→13, 50→5 (drops!)
• When units digit resets from 9 to 0 (e.g., 49→50), digit sum drops significantly.
• Digit sums range from 4 to 15 in this range.

Ans: Digit sums pattern: Increase by 1 until units digit hits 9, then drops when tens digit increases.

Q3. Calculate digit sums of 3-digit numbers with consecutive digits. See pattern?

123 → 1+2+3 = 6
234 → 2+3+4 = 9
345 → 3+4+5 = 12
456 → 4+5+6 = 15
567 → 5+6+7 = 18
678 → 6+7+8 = 21
789 → 7+8+9 = 24

Pattern: Each digit sum increases by 3. All are multiples of 3!

Reason: Three consecutive numbers a, a+1, a+2 → sum = 3a+3 = 3(a+1). Always multiple of 3.

Ans: Pattern: Digit sums = 6, 9, 12, 15, 18, 21, 24 — increase by 3. All are multiples of 3. The pattern continues!

Digit Detectives — How Many Times Does '7' Appear?

Among numbers 1–100, how many times does the digit '7' occur?

Units place: 7, 17, 27, 37, 47, 57, 67, 77, 87, 97 → 10 times
Tens place: 70, 71, 72, 73, 74, 75, 76, 77, 78, 79 → 10 times
Total = 10 + 10 = 20 times

Ans: Digit '7' occurs 20 times among numbers 1 to 100.

Among numbers 1–1000, how many times does the digit '7' occur?

Units place: 7, 17, 27, ... 997 → 100 times
Tens place: 70–79, 170–179, ... 970–979 → 100 times
Hundreds place: 700–799 → 100 times
Total = 100 + 100 + 100 = 300 times

Ans: Digit '7' occurs 300 times among numbers 1 to 1000.

3.5 Pretty Palindromic Patterns

What is a Palindrome?

A PALINDROME is a number that reads the same from LEFT TO RIGHT and from RIGHT TO LEFT. Examples: • 66 → reads as 66 from both sides ✓ • 848 → 848 reversed = 848 ✓ • 575, 797, 1111, 12321 — all palindromes! Non-examples: 123 reversed = 321 ✗

All 3-Digit Palindromes Using Digits 1, 2, 3:

Format: ABA where A and B are digits from {1, 2, 3}

• 111, 121, 131 (A=1, B=1/2/3)

• 212, 222, 232 (A=2, B=1/2/3)
• 313, 323, 333 (A=3, B=1/2/3)

Ans: All 3-digit palindromes using 1,2,3: 111, 121, 131, 212, 222, 232, 313, 323, 333 (9 total)

Reverse-and-Add Palindromes

The Reverse-and-Add Process

Step 1: Take any 2-digit number. Step 2: Reverse its digits. Step 3: Add the original and the reversed number. Step 4: If the result is a palindrome, STOP. Otherwise, repeat.

Examples:

34 + 43 = 77 ✓ (palindrome in 1 step)

29 + 92 = 121 ✓ (palindrome in 1 step)

48 + 84 = 132 → 132 + 231 = 363 ✓ (palindrome in 2 steps)

76 + 67 = 143 → 143 + 341 = 484 ✓ (palindrome in 2 steps)

Ans: For 2-digit numbers starting, reverse-and-add ALWAYS reaches a palindrome (eventually).

Puzzle Time: I am a 5-digit palindrome. I am odd. My 't' digit is double my 'u' digit. My 'h' digit is double my 't' digit. Who am I?

Let the 5-digit palindrome be: [tth][th][h][t][u]

Format: A B C B A (palindrome, so first = last, second = fourth)

Step 1: Let units digit (u) = 1 (must be odd)

Step 2: Tens digit (t) = 2 × u = 2 × 1 = 2

Step 3: Hundreds digit (h) = 2 × t = 2 × 2 = 4

Step 4: Palindrome: 1 2 4 2 1

Step 5: Check: 12421 reversed = 12421 ✓, it's odd ✓

Ans: The 5-digit palindrome is 12,421 (Twelve thousand four hundred twenty-one)

3.6 The Magic Number of Kaprekar

Who was D.R. Kaprekar?

D.R. Kaprekar was an Indian mathematician and mathematics teacher in a government school in Devlali, Maharashtra. He loved playing with numbers and discovered many beautiful patterns previously unknown to mathematics. In 1949, he discovered a MAGICAL phenomenon with 4-digit numbers now called the KAPREKAR CONSTANT.

The Kaprekar Process (4-digit numbers)

Step 1: Choose any 4-digit number with AT LEAST 2 different digits. Step 2: Arrange digits to form the LARGEST number (A). Step 3: Arrange digits to form the SMALLEST number (B). Step 4: Calculate C = A - B. Step 5: Repeat with C as the new number. Result: You will ALWAYS reach 6174 — the Kaprekar Constant!

Example: Starting with 6382

Round	Largest (A)	Smallest (B)	C = A - B

1	8632	2368	8632 - 2368 = 6264
2	6642	2466	6642 - 2466 = 4176
3	7641	1467	7641 - 1467 = 6174
4	7641	1467	7641 - 1467 = 6174 (repeats!)

Ans: No matter what 4-digit number you start with, you will always reach 6174 = Kaprekar Constant!

Q. Carry out Kaprekar steps with 3-digit numbers. What number repeats?

Example: Starting with 321

Round	A	B	C = A - B
1	321	123	321 - 123 = 198
2	981	189	981 - 189 = 792
3	972	279	972 - 279 = 693
4	963	369	963 - 369 = 594
5	954	459	954 - 459 = 495
6	954	459	954 - 459 = 495 (repeats!)

Ans: For 3-digit numbers, the Kaprekar constant is 495.

Figure It Out

Q3 (Section 3.7). How many rounds does 5683 take to reach 6174?

Starting with 5683:

Round	A	B	C = A - B
1	8653	3568	8653 - 3568 = 5085
2	8550	5058	8550 - 5058 = 3492
3	9432	2349	9432 - 2349 = 7083
4	8730	3078	8730 - 3078 = 5652
5	6552	2556	6552 - 2556 = 3996
6	9963	3699	9963 - 3699 = 6264
7	6642	2466	6642 - 2466 = 4176
8	7641	1467	7641 - 1467 = 6174 ✓

Ans: 5683 takes 8 rounds to reach the Kaprekar constant 6174.

3.7 Clock and Calendar Numbers

Palindromic Times on a 12-Hour Clock

Types of Palindromic Times

Type 1 (Repeating digit in minutes): 1:11, 2:22, 3:33, 4:44, 5:55 Type 2 (Matching H:MM): 1:01, 2:02, 3:03...12:21 Type 3 (Palindromic): 10:01, 11:11, 12:21 A time H:MM is palindromic if reading the digits left-to-right equals right-to-left.

All palindromic times on 12-hour clock:
1:01, 1:11, 2:02, 2:12, 2:22, 3:03, 3:13, 3:23, 3:33, 4:04, 4:14, 4:24, 4:34, 4:44
5:05, 5:15, 5:25, 5:35, 5:45, 5:55, 10:01, 11:11, 12:21

Calendar Palindrome Dates

Meghana's birthday: 11/02/2011 — digits read same left-to-right and right-to-left.

Format DD/MM/YYYY as digits: 1 1 0 2 2 0 1 1 → 11022011 reversed = 11022011 ✓

Other palindrome dates: 01/02/2001 (01022010 — not palindrome)

Valid palindrome date format examples: 02/02/2020, 12/02/2021...

Figure It Out

Q1. Pratibha uses digits 4,7,3,2. Largest=7432, Smallest=2347, Difference=5085, Sum=9779.

(a) Choose 4 digits to make difference > 5085:

Use digits 7,4,3,1: Largest=7431, Smallest=1347

Difference = 7431 - 1347 = 6084 > 5085 ✓

Ans: Example: Digits 7,4,3,1 → Difference = 7431 - 1347 = 6084 > 5085

(b) Choose 4 digits to make difference < 5085:

Use digits 7,4,3,3: Largest=7433, Smallest=3347

Difference = 7433 - 3347 = 4086 < 5085 ✓

Ans: Example: Digits 7,4,3,3 → Difference = 7433 - 3347 = 4086 < 5085

(c) Choose 4 digits to make sum > 9779:

Use digits 7,4,3,3: Largest=7433, Smallest=3347

Sum = 7433 + 3347 = 10,780 > 9779 ✓

Ans: Example: Digits 7,4,3,3 → Sum = 7433 + 3347 = 10,780 > 9779

(d) Choose 4 digits to make sum < 9779:

Use digits 7,4,3,1: Largest=7431, Smallest=1347

Sum = 7431 + 1347 = 8778 < 9779 ✓

Ans: Example: Digits 7,4,3,1 → Sum = 7431 + 1347 = 8778 < 9779

Q2. What is the sum and difference of the smallest and largest 5-digit palindromes?

Smallest 5-digit palindrome: 10001 (1_0_0_0_1)
Largest 5-digit palindrome: 99999 (9_9_9_9_9)
Sum = 10001 + 99999 = 1,10,000
Difference = 99999 - 10001 = 89,998

Ans: Sum = 1,10,000 | Difference = 89,998

Q3. Time is 10:01. How many minutes until next palindromic time? And the one after?

Current time: 10:01 (which is itself a palindrome: 1001 reversed = 1001 ✓)
Next palindromic time: 11:11
Time from 10:01 to 11:11 = 1 hour 10 minutes = 70 minutes
One after that: 12:21
Time from 10:01 to 12:21 = 2 hours 20 minutes = 140 minutes

Ans: **Next palindrome: 11:11 → 70 minutes later. One after: 12:21 → 140 minutes from 10:01**

3.8 Mental Math

Mental Math — Adding Numbers from a Set

Given a set of 'middle' numbers, we can combine them (addition only, or addition and subtraction) to reach 'target' numbers on the sides. Middle numbers: 25,000 | 400 | 13,000 | 1,500 | 60,000 Targets on left: 38,800 | 28,000 | 61,600 | 31,000 Targets on right: 3,400 | 63,000 | 19,500 | 20,900 Numbers can be reused as many times as needed!

Examples:

38,800 = 25,000 + 400 × 2 + 13,000 = 25,000 + 800 + 13,000 = 38,800 ✓

3,400 = 1,500 + 1,500 + 400 = 3,400 ✓

28,000 = 13,000 + 13,000 + 1,500 + 400 + 400 + 400 - 700 → Hmm, try: 25,000 + 1,500 + 1,500 = 28,000 ✓

61,600 = 60,000 + 1,500 + 400 - 300 = ...try: 60,000 + 1,500 + 100 = no 100 available

61,600 = 60,000 + 400 × 4 = 60,000 + 1,600 = 61,600 ✓

63,000 = 60,000 + 1,500 + 1,500 = 63,000 ✓

Adding and Subtracting Section

Middle numbers: 40,000 | 7,000 | 300 | 1,500 | 12,000 | 800

Find combinations for: 39,800 | 45,000 | 5,900 | 17,500 | 21,400

39,800 = 40,000 - 800 + 300 + 300 = 40,000 - 200 = 39,800 ✓

45,000 = 40,000 + 7,000 - 1,500 - 300 - 300 = 40,000 + 5,000 = 45,000 ✓

OR: 40,000 + 7,000 - 800 - 800 - 800 - 800 + 1,200 = tricky

Simple: 40,000 + 7,000 - 1,500 - 300 - 300 = 44,900...

45,000 = 40,000 + 7,000 - 1,500 - 300 - 300 + 100 — not available

45,000 = 40,000 + 7,000 - 1,500 - 300 - 300 + ... try 40,000 + 7,000 - 800 - 800 - 800 + 400...

Simplest: 40,000 + 7,000 - 800 - 800 - 800 + 400? No 400 available.

45,000 = 40,000 + 12,000 - 7,000 = 45,000 ✓

5,900 = 7,000 - 1,500 + 300 + 300 - 300 + 100 ...

5,900 = 7,000 - 800 - 300 = 5,900 ✓

17,500 = 12,000 + 7,000 - 1,500 = 17,500 ✓

21,400 = 12,000 + 7,000 + 1,500 + 800 + 300 - 300 - 300 - 300 - 300 = ...

21,400 = 12,000 + 7,000 + 1,500 + 800 + 300 - 300 + 100 → no 100

21,400 = 12,000 + 7,000 + 1,500 + 800 + 300 = 21,600... off by 200

21,400 = 12,000 + 7,000 + 1,500 + 800 + 300 - 200 → need 200, not available

21,400 = 12,000 + 7,000 + 1,500 + 300 + 300 + 300 = 21,400 ✓

Figure It Out — Digits and Operations

Q1. Write examples for each scenario:

Scenario	Example / Reason	Result
5-digit + 5-digit > 90,250	45,000 + 45,400 = 90,400 > 90,250 ✓	POSSIBLE
5-digit + 3-digit = 6-digit	99,999 + 999 = 1,00,998 ✓	POSSIBLE

4-digit + 4-digit = 6-digit	NOT POSSIBLE (max: 9999+9999=19,998)	IMPOSSIBLE
5-digit + 5-digit = 6-digit	60,000 + 40,000 = 1,00,000 ✓	POSSIBLE
5-digit + 5-digit = 18,500	NOT POSSIBLE (min: 10,000+10,000=20,000)	IMPOSSIBLE
5-digit - 5-digit < 56,503	80,000 - 50,000 = 30,000 < 56,503 ✓	POSSIBLE
5-digit - 3-digit = 4-digit	10,000 - 999 = 9,001 ✓	POSSIBLE
5-digit - 4-digit = 4-digit	12,000 - 2,500 = 9,500 ✓	POSSIBLE
5-digit - 5-digit = 3-digit	50,999 - 50,000 = 999 ✓	POSSIBLE
5-digit - 5-digit = 91,500	NOT POSSIBLE (max difference: 99,999-10,000=89,999)	IMPOSSIBLE

Q2. Always, Sometimes, or Never True?

Statement	Answer	Reason
a. 5-digit + 5-digit = 5-digit	Sometimes	20000+80000=100000 (6-digit); 10000+10000=20000 (5-digit)
b. 4-digit + 2-digit = 4-digit	Sometimes	9999+99=10098 (5-digit); 1000+10=1010 (4-digit)
c. 4-digit + 2-digit = 6-digit	Never	Max: 9999+99=10098 — only 5 digits possible
d. 5-digit - 5-digit = 5-digit	Sometimes	12000-10000=2000 (4-digit); 99999-10000=89999 (5-digit)
e. 5-digit - 2-digit = 3-digit	Never	Min: 10000-99=9901 — always 4 digits

3.9 Playing with Number Patterns

Finding Sums Quickly Using Patterns

Instead of adding numbers one by one, look for patterns: • Count how many times each number appears • Multiply instead of repeatedly adding • Group identical numbers together This is much faster than adding one by one!

Pattern (a): Numbers 40 and 50

Count 40s: 12 times | Count 50s: 10 times

Sum = 40 × 12 + 50 × 10 = 480 + 500 = 980

Ans: Sum = 40 × 12 + 50 × 10 = 480 + 500 = 980

Pattern (b): Dot pattern

Count the pattern: rows × columns with repeated elements

Look at the grid structure and count systematically.

Pattern (c): Grid of 32s and 64s

32s: 4 rows × 8 columns = 32 cells → 32 × 32 = 1024

64s: 4 rows × 4 columns (partial) = 12 cells → 64 × 12 = 768

Total = 1024 + 768 = 1792

Ans: Sum = 32 × 32 + 64 × 12 = 1024 + 768 = 1792

Pattern (f): Circle pattern with 125, 250, 500, 1000

Count: 1000 appears 1 time, 500 appears 4 times, 250 appears 8 times, 125 appears many times

Sum = 1×1000 + 4×500 + 8×250 + 16×125 = 1000 + 2000 + 2000 + 2000 = 7000

Ans: Sum = 1000 + 2000 + 2000 + 2000 = 7000 (count each ring)

3.10 An Unsolved Mystery — The Collatz Conjecture!

The Collatz Rule

Start with ANY whole number. • If the number is EVEN: divide by 2 • If the number is ODD: multiply by 3 and add 1 Repeat until you reach 1. Collatz's Conjecture (1937): No matter what number you start with, you will ALWAYS eventually reach 1! This has been verified for incredibly large numbers, but NO ONE has proven it is true for ALL numbers. It remains one of mathematics' most famous UNSOLVED PROBLEMS!

Examples from Textbook:

Starting Number	Collatz Sequence
12	12→6→3→10→5→16→8→4→2→1
17	17→52→26→13→40→20→10→5→16→8→4→2→1
21	21→64→32→16→8→4→2→1
22	22→11→34→17→52→26→13→40→20→10→5→16→8→4→2→1

Make Collatz sequences for your favourite numbers. Do you always reach 1?

Starting with 28:

28→14→7→22→11→34→17→52→26→13→40→20→10→5→16→8→4→2→1 ✓

Starting with 19:

19→58→29→88→44→22→11→34→17→52→26→13→40→20→10→5→16→8→4→2→1 ✓

Figure It Out

Q8. Why is Collatz conjecture correct for all powers of 2?

Powers of 2: 1, 2, 4, 8, 16, 32, 64, 128, ...

Each power of 2 is EVEN. So we keep dividing by 2:

64→32→16→8→4→2→1 ✓

2^n → 2^(n-1) → 2^(n-2) → ... → 2 → 1

Since all powers of 2 are even, we just halve repeatedly until reaching 1.

Ans: Powers of 2 are always even, so we keep halving: 2^n → 2^(n-1) → ... → 1. Always reaches 1!

Q9. Check Collatz Conjecture for starting number 100:

100→50→25→76→38→19→58→29→88→44→22→11→34→17→52→26→13→40→20→10→5→16→8→4→2→1 ✓

Ans: Yes! Collatz Conjecture holds for 100. It takes 25 steps to reach 1.

3.11 Simple Estimation

What is Estimation?

Estimation means finding an approximate answer that is close enough to be useful — without knowing or needing the EXACT value. Good estimation skills help in: • Quick mental calculations • Checking if answers are reasonable • Real-life planning and decision making Key: Your estimate doesn't need to be exact. It needs to be REASONABLE.

Figure It Out — Estimation Questions

Q1. Estimate steps to walk:

a. Seat to classroom door: about 20-30 steps
b. Across school ground: about 200-500 steps
c. Classroom door to school gate: about 100-300 steps
d. School to home: depends on distance (1000 to 10,000+ steps)

Q2. Times you blink eyes / take breaths:

Action	Per Minute	Per Hour	Per Day
Eye blinks	~15-20 times	~900-1200	~15,000-20,000
Breaths	~12-16 times	~720-960	~17,000-23,000

Q3. Objects around you:

a. A few thousand: grains of rice in a cup, pages in a book collection, tiles in a floor
b. More than ten thousand: hair on your head (~100,000), words in a novel, grains of sand in a handful

Estimate the Answer

Q1. Number of words in your maths textbook: More or Less than 5000?

Estimate: A typical page has ~250 words. If textbook has ~100 pages:
Total ≈ 250 × 100 = 25,000 words >> 5000

Ans: More than 5000 words in the maths textbook.

Q3. Roshan estimates fruit custard for 5 people costs ₹100. Agree?

Milk (1 litre) ≈ ₹50-60
3 types of fruit (small quantities) ≈ ₹30-50
Sugar, custard powder ≈ ₹20
Total ≈ ₹100-130
For 5 people, ₹100 might be tight but possible with careful buying.

Ans: Partly agree — ₹100 is possible but tight. Actual cost might be slightly higher.

Q5. Sheetal (Grade 6) says she has spent ~13,000 hours in school. Agree?

Grade 6 = approximately 7 years in school (Nursery, KG, 1-6)
School hours per day = 6 hours
School days per year ≈ 200 days
Total hours = 6 × 200 × 7 = 8,400 hours
13,000 hours ÷ (6 × 200) = 10.8 years — that's too high for Grade 6!

Ans: No — A Grade 6 student would have about 8,400 school hours, not 13,000. Sheetal's estimate is too high.

Q4. Estimate distance from Gandhinagar (Gujarat) to Kohima (Nagaland):

These cities are at opposite ends of India (west to northeast).

India's width ≈ 3,000 km, India's length ≈ 3,200 km

Straight line distance ≈ 2,500 km (diagonal across India)

Ans: Estimated distance ≈ 2,500 km (by road would be more — about 3,500 km)

3.12 Games and Winning Strategies

Game Theory in Mathematics

Numbers can be used to play games and develop WINNING STRATEGIES. By analyzing patterns, you can figure out which positions are 'winning positions' and plan your moves backward from the goal.

Game #1 — The 21 Game

Rules: Player 1 says 1, 2, or 3. Players alternate adding 1, 2, or 3. First to reach 21 WINS!

Finding the Winning Strategy:

Work backwards from 21:

- To WIN, you want to say 21. You need the previous number to be 18, 19, or 20.
- If you say 21, you need to get to 17 — then your opponent says 18, 19, or 20, and you say 21.
- If you say 17, opponent must say 18, 19, or 20. Then you add to reach 21.
- Key positions: 21, 17, 13, 9, 5, 1 (each 4 apart)
- Pattern: Say numbers that are of the form 4k+1: 1, 5, 9, 13, 17, 21
- Player 1 starts by saying 1, then always says: previous opponent's number + (4 - what opponent just added)

Ans: Winning strategy for Game 1: Player 1 wins. Start with 1, then always reach 5→9→13→17→21. Maintain multiples of 4 plus 1.

Game #2 — The 99 Game

Rules: Player 1 says 1-10. Players alternate adding 1-10. First to reach 99 WINS!

Finding the Winning Strategy:

Work backwards from 99:

- Key winning positions: 99, 88, 77, 66, 55, 44, 33, 22, 11 (each 11 apart)
- Pattern: Multiples of 11: 11, 22, 33, 44, 55, 66, 77, 88, 99
- Player 2 wins! Player 1 cannot say 0, so whatever Player 1 says (1-10), Player 2 adds to reach 11.
- Then Player 2 always responds to keep total at next multiple of 11.
- Example: P1 says 7 → P2 says 4 (total=11). P1 says 3 → P2 says 8 (total=22)... continues to 99.

Ans: Winning strategy for Game 2: PLAYER 2 wins. Always make total = next multiple of 11 (11, 22, 33...99).

General Pattern for These Games

Win Target	Max Add Per Turn	Key Positions	Winner
21	1, 2, or 3	1, 5, 9, 13, 17, 21 (diff=4)	Player 1
99	1 to 10	11, 22, 33...99 (diff=11)	Player 2

Any target T	1 to n	T, T-(n+1), T-2(n+1)...	Depends

General Rule: Key step = (max you can add) + 1. Winning positions are multiples of this step.

Q10. Starting with 0, players alternate adding 1-3. First to reach 22 wins. Strategy?

Key step = 3 + 1 = 4. Winning positions: 2, 6, 10, 14, 18, 22 (NOT multiples of 4)
Starting from 0, Player 1 first says a number. If P1 says 2, then key positions: 2, 6, 10, 14, 18, 22.
P1 should start with 2, then always add to reach next position in the series.
Whichever player says 22 first wins.

Ans: Player 1 wins. Start with 2, then maintain positions 2, 6, 10, 14, 18, 22.

Figure It Out — Final Questions

Q1. Grid supercell puzzle: Only one supercell. Swap two digits of one number to get 4 supercells.

16,200	39,344	29,765
23,609	**62,871**	45,306
19,381	50,319	38,408

Currently only 62,871 is a supercell (greater than all 4 neighbours).
If we swap digits 6 and 1 in 62,871 → we get 12,876

16,200	39,344	**29,765**
23,609	12,876	**45,306**
19,381	**50,319**	38,408

After swap (62,871 → 12,876):
• 16,200: left end, neighbour=39,344 > 16,200 → NOT supercell...
• Let's check all: 39,344 > 16,200, 12,876, 29,765? 39,344 > 12,876 ✓ and 39,344 > 16,200 ✓ → check if > neighbours above/below (no rows above row 0)
• 16,200 (row0,col0): right=39,344>16,200, below=23,609>16,200 → NOT supercell ✗
• 39,344 (row0,col1): left=16,200<39,344✓, right=29,765<39,344✓, below=12,876<39,344✓ → SUPERCELL ✓
• 29,765 (row0,col2): left=39,344>29,765 → NOT supercell ✗... wait, below=45,306>29,765 also
• 45,306 (row1,col2): right end, left=12,876<45,306✓, above=29,765<45,306✓, below=38,408<45,306✓ → SUPERCELL ✓
• 50,319 (row2,col1): left=19,381<50,319✓, right=38,408<50,319✓, above=12,876<50,319✓, no below → SUPERCELL ✓
• 29,765 could be a supercell if... above: none, below: 45,306>29,765 → NOT supercell.
• 39,344, 45,306, 50,319 are supercells. One more needed.
• 23,609: left=12,876<23,609✓ — wait, 12,876 replaced 62,871. Check: right=12,876<23,609✓, left=end... actually col0. Left end.
• 23,609 (row1,col0): right=12,876<23,609✓, above=16,200<23,609✓, below=19,381<23,609✓ → SUPERCELL ✓

Ans: Swap digits 6 and 1 in 62,871 to get 12,876. New supercells: 39,344 | 23,609 | 45,306 | 50,319 = 4 supercells!

Q3. 5-digit numbers between 35,000 and 75,000 with ALL ODD digits. Largest, Smallest, Closest to 50,000?

Odd digits: 1, 3, 5, 7, 9 (all must be odd)

Range: 35,000 to 75,000 (but digits must ALL be odd)

Smallest: Must start with 3 (odd, ≥35000). Smallest odd digits: 35,111

Check: 35,111 > 35,000 ✓, all digits odd: 3,5,1,1,1 ✓

Largest: Must start with 7 (≤75,000). Largest: 73,999

Check: 73,999 < 75,000 ✓, all digits odd: 7,3,9,9,9 ✓

Closest to 50,000: 51,111 (all odd digits, difference = 1,111) or 49,999 (all odd, diff=1)

49,999: 4 is EVEN — not valid!

51,111: all odd digits ✓, difference from 50,000 = 1,111

Can we get closer? 51,111 → try 51,113, 51,131... all farther. Try 51,111 vs 59,999: 51,111 closer.

Ans: Smallest: 35,111 | Largest: 73,999 | Closest to 50,000: 51,111 (difference = 1,111)

Q6. Write one 5-digit and two 3-digit numbers that sum to 18,670:

18,000 + 300 + 370 = 18,670 ✓

Or: 17,000 + 900 + 770 = 18,670 ✓

Or: 18,000 + 500 + 170 = 18,670 ✓

Ans: Example: 18,000 + 300 + 370 = 18,670 (18000 is 5-digit; 300 and 370 are 3-digit)

Chapter Summary

Key Learnings from Chapter 3 — Number Play

1. NUMBERS TELL US THINGS: Numbers can convey information in context (like height relationships).
2. SUPERCELLS: A cell is a supercell if its value exceeds all adjacent cells. Maximum supercells = ⌈n/2⌉.
3. NUMBER LINE PATTERNS: Numbers increase rightward; scale helps identify positions.
4. DIGIT SUMS: Sum of all digits of a number; numbers with same digit sum share properties.
5. PALINDROMES: Numbers that read the same forwards and backwards. Reverse-and-add always gives palindromes for 2-digit numbers.
6. KAPREKAR CONSTANT: Any 4-digit number (with ≥2 different digits) always reaches 6174 through the Kaprekar process. For 3-digit: 495.
7. COLLATZ CONJECTURE: Start any number → if even halve, if odd triple+1. Always reaches 1? STILL UNSOLVED!
8. ESTIMATION: Approximate answers that are close enough to be useful.
9. GAME STRATEGIES: Use backward reasoning to find winning positions.
10. COMPUTATIONAL THINKING: Formulating step-by-step procedures to solve number problems.