

Chapter 4:

Expressions Using Letter-Numbers

Complete Notes with Step-by-Step Solutions | CBSE | Algebra Basics

★ Topics: Letter-Numbers • Algebraic Expressions • Simplification • Patterns • Calendar Algebra

4.1 THE NOTION OF LETTER-NUMBERS

► What are Letter-Numbers?

In mathematics, we often use LETTERS to represent unknown or changing numbers. These letters are called letter-numbers (also called variables). Using letter-numbers makes it easy to write general rules and formulas.

Letter-Number: A letter (like a, b, x, n) used to represent a number in a mathematical expression.

Algebraic Expression: A mathematical expression that uses letter-numbers along with numbers and operations (+, −, ×, ÷).

► Example 1: Aftab and Shabnam — Age Problem

Shabnam is always 3 years older than Aftab. We can write this as a formula:

$$\text{Shabnam's age} = \text{Aftab's age} + 3$$

Let 'a' = Aftab's age, 's' = Shabnam's age. Then:

$$s = a + 3$$

Aftab's Age (a)	Expression for Shabnam's Age	Shabnam's Age (s)
4	$4 + 3$	7
10	$10 + 3$	13
18	$18 + 3$	21
23	$23 + 3$	26
a	$a + 3$	$s = a + 3$

● If Aftab's age (a) is 23 years, what is Shabnam's age?

Given: $a = 23$, Expression: $s = a + 3$

To Find: Shabnam's age (s)

Concept Used: Substitution in algebraic expression

Step 1: Substitute $a = 23$ in the expression $s = a + 3$

Step 2: $s = 23 + 3 = 26$

✓ **Final Answer:** Shabnam's age = 26 years

● Given Shabnam's age (s), write an expression to find Aftab's age.

Since Aftab is 3 years YOUNGER than Shabnam:

$$a = s - 3$$

Step 1: If $s = 20$ (Shabnam's age), substitute in $a = s - 3$

Step 2: $a = 20 - 3 = 17$

☑ **Final Answer:** Aftab's age when Shabnam is 20 = 17 years

► Example 2: Matchstick Pattern (L-shapes)

Parthiv makes L-shapes using matchsticks. Each L needs EXACTLY 2 matchsticks.

$\begin{array}{c} | \\ | \\ | \end{array}$ $\begin{array}{c} | \\ | \\ | \end{array}$ $\begin{array}{c} | \\ | \\ | \end{array}$ ← Vertical sticks
 $\begin{array}{c} | \\ | \\ | \end{array}$ $\begin{array}{c} | \\ | \\ | \end{array}$ $\begin{array}{c} | \\ | \\ | \end{array}$ ← Horizontal sticks
 1 L 2 Ls 3 Ls Pattern: 2 sticks per L

Number of Ls	Number of Matchsticks	Pattern
1	2	1×2
5	10	5×2
7	14	7×2
45	90	45×2
n	$2n$	$n \times 2 = 2n$

$$\text{Number of matchsticks} = 2 \times n = 2n \quad (\text{where } n = \text{number of Ls})$$

► Example 3: Ketaki's Laddu Business — Cost Expression

Price of 1 coconut = ₹35 | Price of 1 kg jaggery = ₹60

Let c = number of coconuts, j = number of kg of jaggery

Item	Relationship	Algebraic Expression
Cost of coconuts	No. of coconuts $\times$ ₹35	$c \times 35 = 35c$
Cost of jaggery	No. of kg $\times$ ₹60	$j \times 60 = 60j$
Total Cost	Cost of coconuts + Cost of jaggery	$35c + 60j$

$$\text{Total Cost} = 35c + 60j$$

● **How much should she pay if she buys 10 coconuts and 5 kg jaggery?**

Step 1: Cost of 10 coconuts = $10 \times 35 = ₹350$

Step 2: Cost of 5 kg jaggery = $5 \times 60 = ₹300$

Step 3: Total = $350 + 300 = ₹650$

☑ **Final Answer:** Total cost = ₹650

● **How much should she pay if she buys 8 coconuts and 9 kg jaggery?**

Step 1: Cost = $8 \times 35 + 9 \times 60 = 280 + 540 = ₹820$

☑ **Final Answer:** Total cost = ₹820

● **Use the expression to find cost for 7 coconuts and 4 kg jaggery.**

Step 1: $c = 7, j = 4 \rightarrow 35(7) + 60(4) = 245 + 240 = ₹485$

☒ **Final Answer:** Total cost = ₹485

► Example 4: Perimeter of a Square

Perimeter of a square = $4 \times \text{side length}$

$$\text{Perimeter} = 4 \times q = 4q \quad (\text{where } q = \text{side length})$$

● **What is the perimeter of a square with side length 7 cm?**

Step 1: Substitute $q = 7$ in the formula: Perimeter = $4q = 4 \times 7 = 28$ cm

☒ **Final Answer:** Perimeter of square = 28 cm

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● **Q1. Write formulas for the perimeter of: (a) equilateral triangle, (b) regular pentagon, (c) regular hexagon**

Given: Regular polygons with all sides equal. Let side = a units.

Concept Used: Perimeter = number of sides $\times$ side length

Shape	No. of Sides	Perimeter Formula
Equilateral Triangle (all sides equal)	3	Perimeter = $3 \times a = 3a$ units
Regular Pentagon	5	Perimeter = $5 \times a = 5a$ units
Regular Hexagon	6	Perimeter = $6 \times a = 6a$ units

☒ **Final Answer:** (a) $3a$ (b) $5a$ (c) $6a$ (where a = side length)

● **Q2. Munirathna has a 20 m long pipe. He joins another pipe of length ' k ' metres. Give expression for combined length.**

Step 1: Combined length = existing pipe + new pipe = $20 + k$

☒ **Final Answer:** Combined length = $(20 + k)$ metres

● **Q3. What is the total amount Krithika has with different notes of ₹100, ₹20, and ₹5? Complete the table.**

₹100 notes	₹20 notes	₹5 notes	Expression and Total
3	5	6	$3 \times 100 + 5 \times 20 + 6 \times 5 = 300 + 100 + 30 = ₹430$
6	4	3	$6 \times 100 + 4 \times 20 + 3 \times 5 = 600 + 80 + 15 = ₹695$
8	4	z	$8 \times 100 + 4 \times 20 + z \times 5 = 880 + 5z$
x	y	z	$100x + 20y + 5z$

● **Q4. Venkatalakshmi's flour mill: 10 sec to start, 8 sec/kg to grind. Which expression gives time for ' y ' kg?**

Concept: Time = Start-up time + (grinding time per kg $\times$ number of kg)

Step 1: Start-up time = 10 seconds

Step 2: Grinding time = $8 \times y$ seconds

Step 3: Total = $10 + 8y$

☒ **Final Answer: (d) $10 + 8 \times y$ is correct!**

Why others are wrong: (a) $10+8+y$ adds y directly — wrong. (b) $(10+8) \times y$ multiplies startup+grind by y — wrong. (c) $10 \times 8 \times y$ — wrong. (e) $10y+8$ multiplies startup by y — wrong.

Q5. Write algebraic expressions: (a) 5 more than a number, (b) 4 less than a number, (c) 2 less than 13 times a number, (d) 13 less than 2 times a number

Let the number be 'd'.

Description	Expression
5 more than a number	$d + 5$
4 less than a number	$d - 4$
2 less than 13 times a number	$13d - 2$
13 less than 2 times a number	$2d - 13$

Q6. Describe situations for: (a) $8 \times x + 3 \times y$, (b) $15 \times j - 2 \times k$

(a) $8 \times x + 3 \times y$:

A shopkeeper sells pens at ₹8 each and notebooks at ₹3 each. Abha buys x pens and y notebooks. Total cost = $8x + 3y$.

(b) $15 \times j - 2 \times k$:

A factory makes 15 chairs per day for j days = $15j$ chairs. But 2 chairs break per day for k days = $2k$ chairs broken. Good chairs = $15j - 2k$.

Q7. In a 2×3 calendar grid, if the bottom middle cell is date 'w', write expressions for all blank cells.

In a calendar, dates in the SAME row differ by 1. Dates in the NEXT row differ by 7.

Bottom Row →	Left Cell	Middle Cell	Right Cell
Bottom row	$w - 1$	w	$w + 1$
Top row (7 days back)	$w - 8$	$w - 7$	$w - 6$

4.2 REVISITING ARITHMETIC EXPRESSIONS

► Key Rules for Evaluating Expressions

Rule	Description	Example
BODMAS/Order	Multiplication/Division BEFORE Addition/Subtraction	$23 - 10 \times 2 = 23 - 20 = 3$
Swapping	Terms can be added in any order (commutative)	$83+28-13+32 = (83-13)+(28+32) = 70+60 = 130$
Grouping	Group convenient terms to simplify calculation	Group positives and negatives separately

Brackets +ve	Positive sign outside: remove brackets as is	$+(18+13) = +18+13$
Brackets -ve	Negative sign outside: flip ALL signs inside	$-(18+13) = -18-13$

● Evaluate: 1. $23 - 10 \times 2$

Concept: Multiplication first (BODMAS)

Step 1: Write as sum of terms: $23 + (-10 \times 2)$

Step 2: Calculate $-10 \times 2 = -20$

Step 3: $23 + (-20) = 23 - 20 = 3$

☑ **Final Answer:** $23 - 10 \times 2 = 3$

● Evaluate: 2. $83 + 28 - 13 + 32$

Concept: Swapping and grouping like terms

Step 1: Rewrite: $83 + 28 + (-13) + 32$

Step 2: Group conveniently: $(83 + (-13)) + (28 + 32)$

Step 3: $= 70 + 60 = 130$

☑ **Final Answer:** $83 + 28 - 13 + 32 = 130$

● Evaluate: 3. $34 - 14 + 20$

Step 1: $= 34 + (-14) + 20 = (34+20) + (-14) = 54 - 14 = 40$

☑ **Final Answer:** $34 - 14 + 20 = 40$

● Evaluate: 4. $42 + 15 - (8 - 7)$

Step 1: Solve bracket first: $8 - 7 = 1$

Step 2: $= 42 + 15 - 1 = 56$

☑ **Final Answer:** $42 + 15 - (8 - 7) = 56$

● Evaluate: 5. $68 - (18 + 13)$

Method 1 (Solve bracket first):

Step 1: $18 + 13 = 31$

Step 2: $68 - 31 = 37$

Method 2 (Remove bracket with negative sign):

Step 1: $68 - (18 + 13) = 68 - 18 - 13$

Step 2: $= (68-18) - 13 = 50 - 13 = 37$

☑ **Final Answer:** $68 - (18 + 13) = 37$

● Evaluate: 6. $7 \times 4 + 9 \times 6$

Step 1: $7 \times 4 = 28$ and $9 \times 6 = 54$

Step 2: $28 + 54 = 82$

☑ **Final Answer:** $7 \times 4 + 9 \times 6 = 82$

● Evaluate: 7. $20 + 8 \times (16 - 6)$

Step 1: Bracket: $16 - 6 = 10$

Step 2: $8 \times 10 = 80$

Step 3: $20 + 80 = 100$

☑ **Final Answer:** $20 + 8 \times (16 - 6) = 100$

4.3 OMISSION OF THE MULTIPLICATION SYMBOL

► The Shorthand Notation

In algebra, when a NUMBER is multiplied by a LETTER-NUMBER, we DROP the multiplication sign.

Full Form	Shorthand (Algebraic)	Read As
$4 \times n$	$4n$	Four times n
$5 \times m + 3$	$5m + 3$	Five m plus three
$7 \times k$	$7k$	Seven k
$2 \times a \times b$	$2ab$	Two a b

★ **RULE:** Write the NUMBER first, then the LETTER. Never write $n4$ — always write $4n$.

★ **RULE:** We CANNOT omit the sign between two numbers: $3 \times 4 \neq 34$. Only number $\times$ letter allows omission.

► Number Sequence: Multiples of 4

Sequence: 4, 8, 12, 16, 20, 24, 28, ... (multiples of 4)

Position (n)	Term	Expression
1	4	$4 \times 1 = 4n$ when $n=1$
3	12	4×3
29	116	4×29
nth	?	$4 \times n = 4n$

nth term of 4, 8, 12, 16, ... = $4n$

● Find the value of $7k$ when $k = 4$.

Step 1: $7k = 7 \times k = 7 \times 4 = 28$

☑ Final Answer: $7k = 28$ when $k = 4$

● Find the value of $5m + 3$ when $m = 2$.

Step 1: $5m = 5 \times m = 5 \times 2 = 10$

Step 2: $5m + 3 = 10 + 3 = 13$

☑ Final Answer: $5m + 3 = 13$ when $m = 2$

⚠ MIND THE MISTAKE, MEND THE MISTAKE (Section 4.3)

Find and correct the errors in these substitutions:

#	Expression	Given (Wrong)	Error	Correct Answer
1	If $a=-4$, then $10-a=?$	6 ✗	$10-(-4)=10+4=14$, not $10-4=6$	14
2	If $d=6$, then $3d=?$	36 ✗	$3d=3 \times 6=18$, not 36	18
3	If $s=7$, then $3s-2=?$	15 ✓	$3 \times 7 - 2 = 21 - 2 = 19$... actually check: $3 \times 7 = 21$, $21 - 2 = 19$	19

4	If $r=8$, then $2r+1=?$	29 ✗	$2 \times 8 + 1 = 16 + 1 = 17$, not 29	17
5	If $j=5$, then $2j=?$	10 ✓	$2 \times 5 = 10$ ✓ Correct!	10
6	If $m=-6$, then $3(m+1)=?$	19 ✗	$3(-6+1) = 3(-5) = -15$, not 19	-15
7	If $f=3, g=1$, then $2f-2g=?$	2 ✗	$2 \times 3 - 2 \times 1 = 6 - 2 = 4$, not 2	4
8	If $t=4, b=3$, then $2t+b=?$	24 ✗	$2 \times 4 + 3 = 8 + 3 = 11$, not 24	11
9	If $h=5, n=6$, then $h-(3-n)=?$	4 ✗	$5-(3-6) = 5-(-3) = 5+3 = 8$, not 4	8

4.4 SIMPLIFICATION OF ALGEBRAIC EXPRESSIONS

► Like Terms vs Unlike Terms

Type	Definition	Examples
Like Terms	Same letter-number(s), possibly different coefficients	$5c, 3c, 10c \rightarrow$ all have 'c'
Unlike Terms	Different letter-numbers	$18c$ and $11d \rightarrow$ different letters
Can Simplify?	ONLY like terms can be combined	$5c+3c+10c = 18c$ BUT $18c+11d$ CANNOT simplify

★ Like terms can be **ADDED** or **SUBTRACTED**. Unlike terms cannot be combined further.

★ When adding like terms: add the **COEFFICIENTS** (numbers) and keep the letter-number same.

► Perimeter of a Rectangle — Simplification

Perimeter = length + breadth + length + breadth = $l + b + l + b$

Step	Expression	Explanation
Initial	$l + b + l + b$	Sum of all four sides
Rearrange (swap)	$l + l + b + b$	Group like terms together
Simplify	$2l + 2b$	$l+l = 2l$ and $b+b = 2b$
Verify: $l=3, b=4$	$2(3)+2(4) = 6+8 = 14$	Also: $3+4+3+4 = 14$ ✓

Perimeter of Rectangle = $2l + 2b$

► Example 5: Pencil and Eraser Shop

Price per pencil = c , Price per eraser = d

Item	Day 1	Day 2	Day 3	Total Sold
Pencils (price c)	5	3	10	18
Erasers (price d)	4	6	1	11

● Find total money earned from pencils.

Step 1: Money from pencils = $5c + 3c + 10c$

Step 2: Use distributive property: $(5+3+10) \times c = 18 \times c = 18c$

✓ Final Answer: Total from pencils = $18c$

● If $c = ₹50$, find the total amount earned by the sale of pencils.

Step 1: Total = $18c = 18 \times 50 = ₹900$

✓ Final Answer: Total earned from pencils = ₹900

● Find total money earned from erasers.

Step 1: Money from erasers = $4d + 6d + 1d = (4+6+1)d = 11d$

✓ Final Answer: Total from erasers = $11d$

● Can $18c + 11d$ be simplified further?

NO! $18c$ and $11d$ are UNLIKE TERMS (different letter-numbers c and d). They cannot be combined.

✓ Final Answer: Simplest form = $18c + 11d$

► Example 6: Rectangle Area — Distributive Property

A big rectangle has a vertical side 'v' and is split horizontally into widths 4 and 3.

$\begin{array}{|c|c|c|} \hline \leftarrow 4 \rightarrow & \leftarrow 3 \rightarrow & \\ \hline \end{array}$
| Area= $4v$ | Area= $3v$ | Height = v

Step 1: Area of big rectangle = $v \times (4+3) = 7v$

Step 2: OR: Sum of smaller areas = $4v + 3v = (4+3)v = 7v$

$4v + 3v = 7v$ ← This demonstrates the Distributive Property!

► Example 7: Chair and Table Rental — Complex Simplification

Item	Rent (₹)	Refund (₹)
Chair	40	6
Table	75	10

● Write and simplify the expression for total amount paid for x chairs and y tables.

Step 1: Amount paid at start = $40x + 75y$

Step 2: Amount returned = $6x + 10y$

Step 3: Net amount paid = $(40x + 75y) - (6x + 10y)$

Step 4: Open bracket with negative sign: = $40x + 75y - 6x - 10y$

Step 5: Group like terms: = $(40x - 6x) + (75y - 10y)$

Step 6: = $(40-6)x + (75-10)y = 34x + 65y$

✓ Final Answer: Net amount paid = $34x + 65y$

► Example 8: Charu's Quiz Scores — Adding Expressions

Round 1 score: $7p - 3q$ | Round 2: $8p - 4q$ | Round 3: $6p - 2q$

(p = score for correct answer, q = penalty for wrong answer)

● If $p=4$, $q=1$, find Charu's score in Round 1.

Step 1: Score = $7p - 3q = 7 \times 4 - 3 \times 1 = 28 - 3 = 25$

☑ Final Answer: Round 1 score = 25

● Find Charu's TOTAL score after all 3 rounds.

Step 1: Total = $(7p-3q) + (8p-4q) + (6p-2q)$

Step 2: Remove brackets: $= 7p - 3q + 8p - 4q + 6p - 2q$

Step 3: Group like terms: $= (7+8+6)p + (-3-4-2)q$

Step 4: $= 21p + (-9)q = 21p - 9q$

☑ Final Answer: Charu's total score = $21p - 9q$

● Krishita's score is $23p - 7q$. How much more did Krishita score than Charu?

Step 1: Difference = $(23p - 7q) - (21p - 9q)$

Step 2: $= 23p - 7q - 21p + 9q$ [note: $-(-9q) = +9q$]

Step 3: $= (23-21)p + (-7+9)q = 2p + 2q$

☑ Final Answer: Krishita scored $2p + 2q$ more than Charu

► Example 9: Distributive Property in Simplification

● Simplify: $4(x + y) - y$

Step 1: Apply distributive property: $4(x+y) = 4x + 4y$

Step 2: Expression becomes: $4x + 4y - y$

Step 3: Group like terms (y terms): $4x + (4-1)y$

Step 4: $= 4x + 3y$

☑ Final Answer: $4(x + y) - y = 4x + 3y$

► Example 10: Are $5u$ and $5+u$ Equal?

$5u$ means 5 TIMES u . $5+u$ means 5 MORE THAN u . These are DIFFERENT operations!

u	$5u (= 5 \times u)$	$5+u$	Equal?
2	10	7	NO
5	25	10	NO
8	40	13	NO
11	55	16	NO

☑ Final Answer: $5u$ and $5+u$ are NOT equal (they give different values for almost all values of u)

● Are the expressions $10y - 3$ and $10(y - 3)$ equal?

$10y - 3$ means: 3 less than 10 times y

$10(y - 3)$ means: 10 times (3 less than y)

y	$10y - 3$	$10(y - 3)$	Equal?
0	-3	-30	NO
2	17	-10	NO
7	67	40	NO
10	97	70	NO

☑ Final Answer: $10y - 3$ and $10(y - 3)$ are NOT equal!

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Q1. Add numbers in the picture grids and write simplified expressions.

► **Grid 1: $5y$, -6 , x , x , 2 , $5y$**

Step 1: Total = $5y + (-6) + x + x + 2 + 5y$

Step 2: Group: $(5y + 5y) + (x + x) + (-6 + 2)$

Step 3: = $10y + 2x + (-4) = 10y + 2x - 4$

☒ **Final Answer: Sum = $10y + 2x - 4$**

► **Grid 2: $2p$, $3q$, -2 , 3 , $3q$, $2p$, 3 , -2 , $2p$, $3q$, $3q$, $2p$**

Step 1: Count: $2p$ appears 4 times = $8p$; $3q$ appears 4 times = $12q$

Step 2: Numbers: $(-2) + 3 + 3 + (-2) = 2$

Step 3: Total = $8p + 12q + 2$

☒ **Final Answer: Sum = $8p + 12q + 2$**

► **Grid 3: $-5g$, $5k$, $5k$, $-5g$ (corners); $5k$ in all middle spots**

Step 1: $-5g$ appears 4 times: $-20g$; $5k$ appears 12 times: $60k$

Step 2: Total = $-20g + 60k$

☒ **Final Answer: Sum = $-20g + 60k$**

Q2. Simplify each expression:

Expression	Simplified Form	Working
$p+p+p+p$	$4p$	4 times p
$p+p+p+q$	$3p + q$	3 p and one q
$p+q+p-q$	$2p$	$p+p=2p$, $q-q=0$
$p-q+p-q$	$2p - 2q$	2 p and $-2q$
$p+q-p+q$	$2q$	$p-p=0$, $q+q=2q$
$p+q-(p+q)$	0	$= p+q-p-q = 0$
$p-q-p-q$	$-2q$	$p-p=0$, $-q-q=-2q$
$2d-d-d-d$	$-d$	$2d-2d=0$, then $-d$
$2d-d-d-c$	$-c$	$2d-2d=0$, leaves $-c$
$2d-d-(d-c)$	c	$=2d-d-d+c=0+c=c$
$2d-(d-d)-c$	$2d-c$	$d-d=0$, so $2d-0-c=2d-c$
$2d-d-c-c$	$d-2c$	$2d-d=d$, then $-c-c=-2c$

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#	Expression	Given (Wrong)	Error Explanation	Correct Form
1	$3a + 2b$	5	Cannot add unlike terms $3a$ and $2b$	$3a + 2b$
2	$3b - 2b - b$	0	$3b-2b=b$, then $b-b=0$ ✓ Correct!	0

3	$6(p + 2)$	$6p + 8$	$6 \times p = 6p$ ✓, but $6 \times 2 = 12$ not 8	$6p + 12$
4	$(4x+3y)-(3x+4y)$	$x + y$	$4x-3x=x$ ✓, but $3y-4y=-y$ not $+y$	$x - y$
5	$5-(2-6z)$	$3 - 6z$	$-(-6z)=+6z$ not $-6z$	$3 + 6z$
6	$2+(x+3)$	$2x - 6$	No multiplication here! Just add. $2+x+3=x+5$	$x + 5$
7	$2y+(3y-6)$	$-y + 6$	$2y+3y=5y$ not $-y$. Brackets have + outside.	$5y - 6$
8	$7p-p+5q-2q$	$7p + 3q$	$7p-p=6p$ not $7p$. $5q-2q=3q$ ✓	$6p + 3q$
9	$5(2w+3x+4w)$	$10w+15x+20w$	$5(6w+3x)=30w+15x$ (combine $2w+4w=6w$ first)	$30w + 15x$
10	$3j+6k+9h+12$	$3(j+2k+3h+4)$	$3 \times j = 3j$ ✓ $3 \times 2k = 6k$ ✓ $3 \times 3h = 9h$ ✓ $3 \times 4 = 12$ ✓	$3j+6k+9h+12$
11	$4(2r+3s+5)-20$	$-8r-12s$	$4 \times 2r = 8r$, $4 \times 3s = 12s$, $4 \times 5 = 20$; $20-20=0$; so $8r+12s$	$8r + 12s + 20$

4.5 PICK PATTERNS AND REVEAL RELATIONSHIPS

► Formula Detective — Number Machines

A number machine takes 2 inputs (top of Y), performs an operation, and outputs a result (bottom).

● **Machine 1 (Black): Inputs $5, 2 \rightarrow 8$; $8, 1 \rightarrow 15$; $9, 11 \rightarrow 7$; $10, 10 \rightarrow 10$; $a, b \rightarrow ?$**

Step 1: Try $2 \times 5 - 2 = 8$ ✓ | $2 \times 8 - 1 = 15$ ✓ | $2 \times 9 - 11 = 7$ ✓ | $2 \times 10 - 10 = 10$ ✓

Formula: $2a - b$ (2 times first input minus second input)

☑ **Final Answer: Last expression: $2a - b$**

● **Machine 2 (Blue): Inputs $5, 2 \rightarrow 5$; $8, 1 \rightarrow 7$; $9, 11 \rightarrow 18$; $10, 10 \rightarrow 18$; $a, b \rightarrow ?$**

Step 1: Try $a+b-2$: $5+2-2=5$ ✓ | $8+1-2=7$ ✓ | $9+11-2=18$ ✓ | $10+10-2=18$ ✓

Formula: $a + b - 2$

☑ **Final Answer: Last expression: $a + b - 2$**

● **Machine 3 (Purple): Inputs $4, 1 \rightarrow 5$; $6, 0 \rightarrow 1$; $3, 2 \rightarrow 7$; $10, 3 \rightarrow ?$; $a, b \rightarrow ?$**

Step 1: Try $a \times b + 1$: $4 \times 1 + 1 = 5$ ✓ | $6 \times 0 + 1 = 1$ ✓ | $3 \times 2 + 1 = 7$ ✓ | $10 \times 3 + 1 = 31$ ✓

Formula: $a \times b + 1 = ab + 1$

☑ **Final Answer: 4th expression: 31 | General: $ab + 1$**

► Example 12: Saree Design Pattern — Algebra in Art

Pattern A, B, C, A, B, C, A, B, C,... repeats with period 3.

Design	Positions	Formula for nth occurrence
A (position 1 in group)	1, 4, 7, 10, 13, ...	$3n - 2$
B (position 2 in group)	2, 5, 8, 11, 14, ...	$3n - 1$

C (position 3 in group)	3, 6, 9, 12, 15, ...	$3n$
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● Which design appears at Position 122?

Step 1: Divide 122 by 3: $122 = 3 \times 40 + 2$ (remainder = 2)

Step 2: Remainder 2 means position is '2 mod 3' → DESIGN B

☑ Final Answer: Design B appears at position 122

● What design appears at positions 99, 122, and 148?

Step 1: $99 \div 3 = 33$ remainder 0 → Remainder 0 means multiple of 3 → Design C

Step 2: $122 \div 3 = 40$ remainder 2 → Design B

Step 3: $148 \div 3 = 49$ remainder 1 → Design A

☑ Final Answer: Position 99: Design C | Position 122: Design B | Position 148: Design A

► Patterns in a Calendar — Diagonal Sums

In any 2×2 square of calendar dates, the two diagonal sums are ALWAYS EQUAL!

Example from November 2024: Square with 12, 13, 19, 20

Diagonal 1: $12 + 20 = 32$ | Diagonal 2: $13 + 19 = 32$ → Equal!

● PROVE that diagonal sums are always equal for any 2×2 calendar square.

Let top-left number = a . In a standard 7-day week calendar:

Position	Value	Reason
Top-left	a	Given
Top-right	$a + 1$	Next day (1 more)
Bottom-left	$a + 7$	Next week (7 more)
Bottom-right	$a + 8$	$7+1$ more

a | $a+1$
 $a+7$ | $a+8$

Step 1: Diagonal 1 sum: $a + (a+8) = a + a + 8 = 2a + 8$

Step 2: Diagonal 2 sum: $(a+1) + (a+7) = a + 1 + a + 7 = 2a + 8$

Step 3: Both diagonals give $2a + 8$ → They are always EQUAL!

Both diagonal sums = $2a + 8$ (8 more than twice the top-left number)

☑ Final Answer: Proved! Diagonal sums are ALWAYS equal for any 2×2 calendar square.

► Plus-Sign Pattern in Calendar

Consider the + shape: centre = 15, with 8 above, 22 below, 14 left, 16 right.

Step 1: Sum = $8 + 14 + 15 + 16 + 22 = 75$

Step 2: $75 = 5 \times 15$ (5 times the centre)

PROOF: Let centre = a . Then: above = $a-7$, below = $a+7$, left = $a-1$, right = $a+1$

Step 1: Sum = $(a-7) + (a-1) + a + (a+1) + (a+7)$

Step 2: = $a-7+a-1+a+a+1+a+7$

Step 3: = $5a + (-7-1+0+1+7) = 5a + 0 = 5a$

Sum of + shape = $5 \times$ (centre number) — ALWAYS!

► Matchstick Pattern — Triangles

Pattern: 1 triangle = 3 sticks, 2 triangles = 5 sticks, 3 triangles = 7 sticks...

Step (y)	Triangles	Matchsticks	Pattern
1	1	3	3
2	2	5	3+2
3	3	7	3+2+2
4	4	9	3+2+2+2
5	5	11	3+2+2+2+2
y	y	$2y + 1$	$3 + 2(y-1)$

Number of matchsticks in Step $y = 2y + 1$

Verification: $3 + 2(y-1) = 3 + 2y - 2 = 2y + 1 \checkmark$

● How many matchsticks in Step 33, Step 84, Step 108?

Step 1: Step 33: $2(33) + 1 = 66 + 1 = 67$ matchsticks

Step 2: Step 84: $2(84) + 1 = 168 + 1 = 169$ matchsticks

Step 3: Step 108: $2(108) + 1 = 216 + 1 = 217$ matchsticks

☒ **Final Answer:** Step 33: 67 | Step 84: 169 | Step 108: 217 matchsticks

FIGURE IT OUT — Pages 102–105

● Q1. One plate of Jowar roti costs ₹30 and Pulao costs ₹20. x plates roti and y plates pulao ordered. Which expression gives total?

Step 1: Total = (cost per roti × no. of roti plates) + (cost per pulao × no. of pulao plates)

Step 2: $= 30 \times x + 20 \times y = 30x + 20y$

☒ **Final Answer:** (a) $30x + 20y$ is correct!

Why others are wrong: (b) $(30+20) \times (x+y) = 50(x+y)$ adds both prices per item — wrong. (c) $20x+30y$ swaps prices. (d) and (e) are wrong operations.

● Q2. Pushpita gives flags: p customers bought champak only, q bought marigold only, r bought both. How many flags?

Step 1: Each of the $p+q+r$ customers gets exactly 1 flag

Step 2: Total customers = $p + q + r$ (r customers bought BOTH but are still COUNTED ONCE)

☒ **Final Answer:** (a) $p + q + r$ flags

● Q3. A snail climbs 'u' cm/day and slips 'd' cm/night for 10 days and 10 nights. (a) Expression for distance from start, (b) What if $d > u$?

Step 1: Net movement per day = climb up – slip down = $u - d$

Step 2: For 10 days: total net movement = $10(u - d)$

☒ **Final Answer:** (a) Distance from start = $10(u - d)$ cm

(b) If $d > u$, then $u - d$ is NEGATIVE, meaning the snail slides DOWN overall. The snail will NEVER reach the top.

● Q4. Radha cycles 5 km/day in Week 1. Each week she increases by 'z' km/day. How many km after 3 weeks?

Step 1: Week 1: 5 km/day for 7 days = $5 \times 7 = 35$ km

Step 2: Week 2: $(5+z)$ km/day for 7 days = $7(5+z) = 35+7z$ km

Step 3: Week 3: $(5+2z)$ km/day for 7 days = $7(5+2z) = 35+14z$ km

Step 4: Total = $35 + (35+7z) + (35+14z) = 105 + 21z$ km

☒ **Final Answer: Total distance after 3 weeks = $105 + 21z$ km**

Q5. Fill in the missing blanks on the expression transformer paths.

Starting expression: $w + 2$

Path	Step 1	After Step 1	Step 2	Final Result
Top path (given)	$\times 3 \rightarrow$ $3(w+2)=3w+6$	$w+5$ [wrong, look again]	$+3$ then $\times 4$	$4w+20$
Top: $w+2 \rightarrow -5 \rightarrow w-3 \rightarrow$ $(\times 3) \rightarrow$	$w+2-5=w-3$	$\times 3 = 3w-9$	$\leftarrow$	$3w-9$
Bottom: $w+2 \rightarrow -4 \rightarrow w-2 \rightarrow$ $\times 3 \rightarrow$	$w+2-4=w-2$	$\times 3 = 3w-6$	$\leftarrow$	$3w-6$

☒ **Final Answer: Top path back: $3w - 9$ | Bottom path: $3w - 6$**

Q6. Train Yahapur to Vahapur: stops at 3 stations. Travel time between stations = t min. Stop time = 2 min each. (a) If $t=4$, total time? (b) Expression?

Step 1: Diagram: Yahapur $\rightarrow$ Station1 $\rightarrow$ Station2 $\rightarrow$ Station3 $\rightarrow$ Vahapur

Step 2: There are 4 intervals of travel, each t minutes

Step 3: There are 3 stops, each 2 minutes

Step 4: Total time = $4t + 3 \times 2 = 4t + 6$ minutes

Step 5: If $t = 4$: Total = $4(4) + 6 = 16 + 6 = 22$ minutes

☒ **Final Answer: (a) 22 minutes | (b) $4t + 6$ minutes**

Q7. Simplify the following expressions:

Expression	Simplified Step-by-Step	Final Answer
(a) $3a+9b-6+8a-4b-7a+16$	Group: $(3a+8a-7a)+(9b-4b)+(-6+16) = 4a+5b+10$	$4a + 5b + 10$
(b) $3(3a-3b)-8a-4b-16$	$9a-9b-8a-4b-16 = (9a-8a)+(-9b-4b)-16 = a-13b-16$	$a - 13b - 16$
(c) $2(2x-3)+8x+12$	$4x-6+8x+12 = (4x+8x)+(-6+12) = 12x+6$	$12x + 6$
(d) $8x-(2x-3)+12$	$8x-2x+3+12 = 6x+15$	$6x + 15$
(e) $8h-(5+7h)+9$	$8h-5-7h+9 = (8h-7h)+(-5+9) = h+4$	$h + 4$
(f) $23+4(6m-3n)-8n-3m-18$	$23+24m-12n-8n-3m-18 =$ $(24m-3m)+(-12n-8n)+(23-18) = 21m-20n+5$	$5 + 21m - 20n$

Q8. Add the expressions:

Pair	Expression 1 + Expression 2	Sum
(a)	$(4d-7c+9) + (8c-11+9d)$	$13d + c - 2$
(b)	$(-6f+19-8s) + (-23+13f+12s)$	$7f + 4s - 4$
(c)	$(8d-14c+9) + (16c-11-9d)$	$2c - d - 2$

(d)	$(6f-20+8s) + (23-13f-12s)$	$-7f - 4s + 3$
(e)	$(13m-12n) + (12n-13m)$	0
(f)	$(-26m+24n) + (26m-24n)$	0

● **Q9. Subtract the expressions:**

Problem	Calculation	Answer
(a) $9a-6b+14$ FROM $6a+9b-18$	$(6a+9b-18)-(9a-6b+14) = 6a+9b-18-9a+6b-14 = -3a+15b-32$	$-3a + 15b - 32$
(b) $-15x+13-9y$ FROM $7y-10+3x$	$(7y-10+3x)-(-15x+13-9y) = 7y-10+3x+15x-13+9y = 16y+18x-23$	$16y + 18x - 23$
(c) $17g+9-7h$ FROM $11-10g+3h$	$(11-10g+3h)-(17g+9-7h) = 11-10g+3h-17g-9+7h = 10h-27g+2$	$10h - 27g + 2$
(d) $9a-6b+14$ FROM $6a-(9b+18)$	$(6a-9b-18)-(9a-6b+14) = 6a-9b-18-9a+6b-14 = -3a-3b-32$	$-(3a+3b+32)$
(e) $10x+2+10y$ FROM $-3y+8-3x$	$(-3y+8-3x)-(10x+2+10y) = -3y+8-3x-10x-2-10y = -13y-13x+6$	$-13y - 13x + 6$
(f) $8g+4h-10$ FROM $7h-8g+20$	$(7h-8g+20)-(-8g+4h-10) = 7h-8g+20+8g-4h+10 = 3h-4h+30$	$3h - 4h + 30$

● **Q10. Describe situations for: (a) $8x + 3y$, (b) $15x - 2x$**

(a) $8x + 3y$:

A fruit seller sells mangoes at ₹8 each and bananas at ₹3 each. If a customer buys x mangoes and y bananas, total cost = $8x + 3y$.

(b) $15x - 2x$:

A shopkeeper has 15 pencils, each costing ₹ x . Two pencils are returned/unsold. Amount received = $15x - 2x = 13x$.

● **Q11. Rope folding: 0 folds→2 pieces, 1 fold→3 pieces, 2 folds→? Find pattern and expression for r folds.**

Folds (r)	Pieces	Pattern
0	2	2
1	3	2+1
2	4	2+2
3	5	2+3
10	12	2+10
r	$r+2$	$r+2$

Number of pieces when folded r times = $r + 2$

☑ **Final Answer: 10 folds → 12 pieces | r folds → $r + 2$ pieces**

● **Q12. Matchstick squares pattern (connected squares): How many matchsticks for 10 squares? For w squares?**

Step 1: 1 square = 4 matchsticks. Each new square adds 3 matchsticks (shares 1 side).

Step 2: Pattern: 4, 7, 10, 13, ... (starts at 4, adds 3 each time)

Step 3: For 10 squares: $4 + 3 \times 9 = 4 + 27 = 31$ matchsticks

Step 4: For w squares: $4 + 3(w-1) = 4 + 3w - 3 = 3w + 1$

Number of matchsticks for w squares = $3w + 1$

☒ **Final Answer: 10 squares: 31 matchsticks | w squares: $3w + 1$**

Q13. Traffic signal colours: Red(1), Yellow(2), Green(3), Yellow(4), Red(5)... Pattern repeats every 4. Find colours at positions 90, 190, 343.

Position	$\div 4$ remainder	Colour
90	$90 \div 4 = 22 \text{ R } 2 \rightarrow \text{remainder } 2$	Yellow
190	$190 \div 4 = 47 \text{ R } 2 \rightarrow \text{remainder } 2$	Yellow
343	$343 \div 4 = 85 \text{ R } 3 \rightarrow \text{remainder } 3$	Green
Colour	Expression for positions	Examples
Red	$4n - 3$ ($n = 1, 2, 3, \dots$)	1, 5, 9, 13, ...
Yellow	$4n - 2$ and $4n$ (two yellow per cycle)	2, 4, 6, 8, ...even positions
Green	$4n - 1$ ($n = 1, 2, 3, \dots$)	3, 7, 11, 15, ...

☒ **Final Answer: Position 90: Yellow | Position 190: Yellow | Position 343: Green**

Q14. Square pattern: Step 1=5, Step 2=9, Step 3=13 squares. Find formula. How many squares in Step 4, 10, 50?

Step 1: Pattern: 5, 9, 13, 17, ... (starts at 5, adds 4 each time)

Step 2: General formula: $5 + (n-1) \times 4 = 5 + 4n - 4 = 4n + 1$

Step 3: Step 4: $4(4)+1 = 17$ | Step 10: $4(10)+1 = 41$ | Step 50: $4(50)+1 = 201$

Number of squares in Step $n = 4n + 1$

Step 4: Each square has 4 vertices, so vertices = $4 \times (4n+1) = 16n + 4$

☒ **Final Answer: Step 4: 17 | Step 10: 41 | Step 50: 201 | General: $4n+1$ | Vertices: $16n+4$**

Q15. Numbers in a 4-column endless grid: 1,2,3,4; 5,6,7,8; 9,10,11,12... Find expressions for each column, locate numbers.

Step 1: Pattern: Row 1 has 1-4, Row 2 has 5-8, Row r has $(4r-3)$ to $4r$

Step 2: General: Number in row r , column $c = 4(r-1) + c$

Column	Expression for r th row
1	$4(r-1)+1 = 4r-3$
2	$4(r-1)+2 = 4r-2$
3	$4(r-1)+3 = 4r-1$
4	$4(r-1)+4 = 4r$

(b) Finding row and column of 124, 147, 201:

Step 1: 124: $124 \div 4 = 31 \text{ R } 0 \rightarrow 4 \times 30 + 4 \rightarrow \text{row } 31, \text{ column } 4$

Step 2: 147: $147 \div 4 = 36 \text{ R } 3 \rightarrow \text{row } 37, \text{ column } 3$

Step 3: 201: $201 \div 4 = 50 \text{ R } 1 \rightarrow \text{row } 51, \text{ column } 1$

☒ **Final Answer: 124: Row 31, Col 4 | 147: Row 37, Col 3 | 201: Row 51, Col 1**

(c) Number in row r , column $c = 4(r-1) + c$

(d) Multiples of 3 pattern:

Multiples of 3 appear in columns in pattern: 3, 2, 1, 4, 3, 2, 1, 4, ... (repeating cycle of 4)

Also: Even numbers always in columns 2 and 4. Odd numbers always in columns 1 and 3.



CHAPTER SUMMARY

► Key Vocabulary

Term	Meaning	Example
Letter-Number (Variable)	A letter used to represent a number	a, b, x, n, p
Algebraic Expression	Expression using numbers and letter-numbers	$3a + 2b, 4x - 5$
Coefficient	The number multiplying a letter-number	In $5x$, coefficient = 5
Like Terms	Terms with same letter-numbers	$3x, 5x, -2x$ (all have x)
Unlike Terms	Terms with different letter-numbers	$3x$ and $5y$
Simplification	Combining like terms to reduce expression	$3x + 5x + 2x = 10x$
Formula	An algebraic expression for a general relationship	Perimeter = $2l + 2b$

► Key Formulas

Perimeter of Equilateral Triangle = $3a$

Perimeter of Square = $4q$

Perimeter of Rectangle = $2l + 2b$

Perimeter of Regular Pentagon = $5a$

Perimeter of Regular Hexagon = $6a$

n th term of 4, 8, 12, 16, ... = $4n$

Matchstick triangles at step $y = 2y + 1$

Matchstick squares for w squares = $3w + 1$

Rope pieces after r folds = $r + 2$

► Important Rules

★ **BODMAS:** Brackets → Orders → Division → Multiplication → Addition → Subtraction

★ Like terms can be combined; unlike terms **CANNOT**

★ Negative sign outside bracket: **FLIP** all signs inside: $-(a+b) = -a-b$

★ Positive sign outside bracket: keep signs same: $+(a+b) = a+b$

★ Distributive property: $a(b+c) = ab + ac$

★ Multiplication of number $\times$ letter: write number first → $4 \times n = 4n$

★ Minimum terms in simplified form $\geq$ Number of different letter-numbers



Keep Practicing, Keep Growing!

