# Chapter 6

# NUMBER PLAY

Complete Chapter Notes with Step-by-Step Solutions

## 📖 Chapter Overview

**Numbers are not just for counting!** In this chapter, we discover how numbers can tell us about arrangements, patterns, and hidden secrets. We will explore:

- ✦ How numbers describe the heights of people in a line
- ✦ Even and odd numbers (Parity) and their magical properties
- ✦ Grids and Magic Squares — a 2000-year-old puzzle!
- ✦ Virahanka–Fibonacci Numbers hidden in poetry and nature
- ✦ Cryptarithms — arithmetic with letters instead of digits

## 6.1 Numbers Tell Us Things

### 🎯 The Big Idea

**The Rule:** Each child calls out the NUMBER OF CHILDREN IN FRONT OF THEM who are TALLER than them.

### 📐 Understanding with a Stick Figure Example

**Example Arrangement 1:**

Arrangement: A B C D E F G (from left to right)

Heights (tall to short order for easy understanding):

| A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|
| **Tall** | Short | **Taller** | Med | **Tallest** | Short | V.Short |
| **0** | **1** | **0** | **2** | **0** | **2** | **1** |

**Explanation (Step-by-Step):**

Step 1: **Child A (Tallest) —** No one is in front. Says: 0

Step 2: **Child B —** Only A is in front. A is taller than B. Count = 1. Says: 1

Step 3: **Child C (Taller than B) —** A is in front and taller. Count = 1. Says: 0

Step 4: **Child D —** A and C are taller. Count = 2. Says: 2

Step 5: **Child E (Tallest of all!) —** No one in front is taller. Count = 0. Says: 0

**KEY OBSERVATION:** The FIRST person always says 0 (no one in front). The TALLEST person always says 0.

## 📌 Arrangement from Page 128 (Textbook)

**Given Arrangement (7 children from left to right):**

**Sequence given by children:** 0, 0, 1, 0, 3, 0, 3

Child 1: 0 taller in front → Says: **0**

Child 2: 0 taller in front → Says: **0**

Child 3: 1 taller in front → Says: **1**

Child 4: 0 taller in front → Says: **0**

Child 5: 3 taller in front → Says: **3**

Child 6: 0 taller in front → Says: **0**

Child 7: 3 taller in front → Says: **3**

**Final Answer: 0, 0, 1, 0, 3, 0, 3**

# Figure It Out — Section 6.1

**Q1. Arrange the stick figures such that the sequence reads: (a) 0,1,1,2,4,1,5 (b) 0,0,0,0,0,0,0 (c) 0,1,2,3,4,5,6 (d) 0,1,0,1,0,1,0 (e) 0,1,1,1,1,1,1 (f) 0,0,0,3,3,3,3**

**Given:** A sequence of numbers produced by children in a line.
**To Find:** The height arrangement that produces each sequence.
**Concept Used:** Each number = count of taller people in front.

**Solution (Step-by-Step):**

**(a) 0, 1, 1, 2, 4, 1, 5**

Step 1: First child says 0 → must be tallest so far.

Step 2: Second says 1 → 1 person taller in front = first child is taller.

Step 3: Third says 1 → only 1st is taller than 3rd (3rd is taller than 2nd).

Step 4: Fourth says 2 → 2 people in front are taller.

Step 5: Fifth says 4 → 4 people in front are taller → 5th is the shortest so far.

Step 6: Sixth says 1 → only 1 person in front is taller → 6th is quite tall.

Step 7: Seventh says 5 → 5 people in front are taller → 7th is the shortest.

**Arrangement (a):** Use cutouts F, C, B, G, A, D, E from textbook appendix. Heights arranged: 1st tallest, then shorter pattern.

**(b) 0, 0, 0, 0, 0, 0, 0**

Step 1: Every child says 0 → no one in front is taller.

Step 2: This means each next child is taller than all those before.

Step 3: Arrangement = strictly increasing heights from left to right.

**Arrangement (b):** Shortest to Tallest (heights keep increasing from left to right).

**(c) 0, 1, 2, 3, 4, 5, 6**

Step 1: 1st says 0 → must be tallest.

Step 2: 2nd says 1 → 1 person (1st) is taller.

Step 3: 3rd says 2 → 2 persons in front are taller → 3rd is shorter than both.

Step 4: Each next child is shorter than all previous → strictly decreasing heights.

**Arrangement (c):** Tallest to Shortest — heights strictly decrease from left to right.

**(d) 0, 1, 0, 1, 0, 1, 0**

Step 1: Alternating 0 and 1 → alternating tall and short children.

Step 2: Children at odd positions (1,3,5,7) are taller than the children just before them.

**Arrangement (d):** Tall, Short, Tall, Short, Tall, Short, Tall — alternating heights.

**(e) 0, 1, 1, 1, 1, 1, 1**

Step 1: First child says 0 → tallest.

Step 2: All others say 1 → exactly 1 person (the first) is always taller than them.

Step 3: All children from 2nd to 7th have the same height (shorter than 1st but equal to each other).

**Arrangement (e):** 1st is tallest; all others are equal in height (and shorter than 1st).

**(f) 0, 0, 0, 3, 3, 3, 3**

Step 1: First 3 say 0 → among the first 3, each is taller than those before (or equal tallest group).

Step 2: Children 4-7 each say 3 → 3 people in front are taller → children 4-7 are shorter than 1, 2, and 3.

**Arrangement (f):** First 3 children are (strictly decreasing in height), last 4 children are all shorter than all 3 of the first group.

## Q2. For each statement, identify: Always True / Only Sometimes True / Never True

**Given:** Statements about the number-calling game.

**To Find:** Whether each statement is always, sometimes, or never true.

**Solution:**

| # | Statement | Answer |
|---|---|---|
| a | If a person says '0', they are the tallest. | **Only Sometimes True — First person always says 0 but may not be tallest overall.** |
| b | If a person is the tallest, their number is '0'. | **Always True — Tallest person has no one taller in front.** |
| c | The first person's number is '0'. | **Always True — No one is in front of the first person.** |
| d | If not first/last, they cannot say '0'. | **Only Sometimes True — Middle person can say 0 if taller than all in front.** |

| | | |
|---|---|---|
| **e** | Person with the largest number is the shortest. | **Only Sometimes True — Largest number means most people are taller, but not necessarily the shortest.** |
| **f** | Largest possible number in a group of 8? | **Answer: 7 (last person could be shortest, with 7 taller people in front)** |

# 6.2 Picking Parity

## 🎯 What is Parity?

**PARITY** means whether a number is **EVEN** or **ODD**. This simple property helps us solve many puzzles without calculating!

## Visual Understanding — Dot Model

**Even Numbers:** Can be arranged in PAIRS with NO leftover

2 = ●● 4 = ●●●● 6 = ●●●●●● 8 = ●●●●●●●●

**Odd Numbers:** ONE dot is always LEFT OVER after pairing

1 = ● 3 = ●●● 5 = ●●●●● 7 = ●●●●●●●

(extra) (extra) (extra) (extra)

## ⚡ Parity Rules for Addition & Subtraction

| Operation | Result | Example |
|---|---|---|
| Even + Even | **Even** | 4 + 6 = 10 (Even) |
| Odd + Odd | **Even** | 3 + 5 = 8 (Even) |
| Even + Odd | **Odd** | 4 + 3 = 7 (Odd) |
| Even − Even | **Even** | 8 − 4 = 4 (Even) |
| Odd − Odd | **Even** | 7 − 3 = 4 (Even) |
| Even − Odd | **Odd** | 8 − 3 = 5 (Odd) |
| Odd − Even | **Odd** | 7 − 4 = 3 (Odd) |

## 🧩 Kishor's Puzzle (The Sum = 30 Problem)

**Puzzle:** Cards available: 1, 3, 5, 7, 9, 11, 13 (all ODD). Choose exactly 5 cards that add to 30.

**Solution Using Parity:**

Step 1: All the cards (1, 3, 5, 7, 9, 11, 13) are ODD numbers.

Step 2: We need to add 5 cards = adding 5 ODD numbers.

Step 3: Sum of 2 odd = Even. Sum of 3 odd = Odd. Sum of 4 odd = Even. Sum of 5 odd = ODD.

Step 4: Any sum of 5 ODD numbers is always ODD.

Step 5: But 30 is EVEN!

**Conclusion: It is IMPOSSIBLE!** 5 odd numbers can NEVER add to an even number like 30.

## 👫 Martin and Maria's Age Problem

**Problem:** Two siblings born exactly 1 year apart. Today their ages sum to 112. Is this possible?

**Solution:**

Step 1: Ages are CONSECUTIVE numbers (e.g., n and n+1).

Step 2: In consecutive numbers, one is EVEN and one is ODD.

Step 3: Even + Odd = ODD (always!)

Step 4: But 112 is EVEN.

Step 5: Check: 55 + 57 = 112? No! 55 + 57 = 112 — wait, are these consecutive? NO! 56 + 57 = 113 (odd), 55 + 56 = 111 (odd).

**Conclusion: NOT POSSIBLE!** Sum of consecutive numbers is always ODD. 112 is EVEN.

## 🔢 Why 5 Odd Numbers Cannot Give an Even Sum — Proof

**Visual Proof with Dots:**

• Each odd number = pairs + 1 extra dot

• When we add 2 odd numbers, the 2 extra dots form a pair → Even!

• When we add 4 odd numbers, 4 extras form 2 pairs → Even!

• When we add 5 odd numbers, 5 extras = 2 pairs + 1 leftover → ODD!

• Pattern: Sum of EVEN NUMBER of odds = Even; Sum of ODD NUMBER of odds = Odd

**GOLDEN RULE:** Sum of ODD COUNT of odd numbers = ODD. Sum of EVEN COUNT of odd numbers = EVEN.

# Figure It Out — Section 6.2

**Q1. Find the parity of: (a) 2 even + 2 odd (b) 2 odd + 3 even (c) 5 even (d) 8 odd**

**Concept Used:** Parity rules for addition.

**Solution:**

**(a) 2 Even + 2 Odd:**

Step 1: 2 even numbers → sum is Even.

Step 2: 2 odd numbers → sum is Even (even count of odds).

Step 3: Even + Even = Even.

**(a) Answer: EVEN**

**(b) 2 Odd + 3 Even:**

Step 1: 2 odd numbers → sum is Even.

Step 2: 3 even numbers → sum is Even.

Step 3: Even + Even = Even.

**(b) Answer: EVEN**

**(c) 5 Even numbers:**

Any number of even numbers → sum is always Even.

**(c) Answer: EVEN**

**(d) 8 Odd numbers:**

8 is EVEN count of odd numbers → sum is EVEN.

**(d) Answer: EVEN**

**Q2. Lakpa has odd number of ₹1 coins, odd number of ₹5 coins, even number of ₹10 coins. Total = ₹205. Did he make a mistake?**

**Given:** Odd ₹1 coins + Odd ₹5 coins + Even ₹10 coins = ₹205

**Concept Used:** Parity of sums.

**Solution:**

Step 1: Value from ₹1 coins (odd count × 1) = Odd number.

Step 2: Value from ₹5 coins (odd count × 5) = Odd × Odd = Odd number.

Step 3: Value from ₹10 coins (even count × 10) = Even number.

Step 4: Total = Odd + Odd + Even = Even + Even = Even.

Step 5: But 205 is ODD! Contradiction!

**YES, Lakpa made a mistake!** The total must be even with those coin counts, but 205 is odd.

**Q3. Find parity of: (d) even−even (e) odd−odd (f) even−odd (g) odd−even**

**Solution:**

(d) even − even = Even (e) odd − odd = Even (f) even − odd = Odd (g) odd − even = Odd

**(d) Even (e) Even (f) Odd (g) Odd**

# Parity of Expressions

## Expression: 3n + 4

The parity of an algebraic expression depends on the value of the variable.

| n | Value of 3n + 4 | Parity |
|---|---|---|
| 3 | 13 | Odd |
| 8 | 28 | Even |
| 10 | 34 | Even |
| 5 | 19 | Odd |

## 📐 nth Even and nth Odd Numbers

**Key Formulas:**

- **nth Even Number = 2n**

n=1: 2×1=2, n=2: 2×2=4, n=3: 2×3=6, ..., n=100: 200

- **nth Odd Number = 2n − 1**

n=1: 2×1−1=1, n=2: 2×2−1=3, n=3: 2×3−1=5, ..., n=100: 199

**DERIVATION:** Even numbers: 2,4,6,8... → nth term = 2n. Odd numbers are 1 less than even: 1,3,5,7... → nth term = 2n−1.

## ☐ Small Squares in Grids — Parity Rule

**Rule:** For a grid with m rows and n columns, total squares = m × n.

- **If BOTH m and n are ODD → Product is ODD**
- **If at least ONE is EVEN → Product is EVEN**

**Find parity of: (a) 27×13 (b) 42×78 (c) 135×654**

**(a) 27×13:** Both ODD × ODD = ODD

**(b) 42×78:** Even × Even = EVEN

**(c) 135×654:** Odd × Even = EVEN (654 is even)

**Answers: (a) Odd (b) Even (c) Even**

**Give examples of expressions with: (i) Always Even (ii) Always Odd (iii) Both**

**(i) Always Even:** 2n, 4k, 100p, 48w − 2, 6m + 2

Reason: Multiples of 2 are always even.

**(ii) Always Odd:** 2n+1, 2n−1, 4m+3, 8a+3

Reason: 2n is even; adding 1 makes it odd.

**(iii) Both Possible:** 3n+4, 5n+2, n+1

Reason: Parity changes depending on whether n is even or odd.

# 6.3 Some Explorations in Grids

## ☐ The Grid Activity

**Rule:** Fill a 3×3 grid using numbers 1–9 (each used exactly once). The circled numbers outside the grid show the ROW and COLUMN SUMS.

## 📊 Sample Grid (from textbook)

**Filled Grid:**

| | | |
|---|---|---|
| **4** | **7** | **5** |
| **6** | **1** | **2** |
| **3** | **9** | **8** |

Row sums: 4+7+5=16 ✓, 6+1+2=9 ✓, 3+9+8=20 ✓

Column sums: 4+6+3=13 ✓, 7+1+9=17 ✓, 5+2+8=15 ✓

Sum of all row sums = 16+9+20 = 45 = 1+2+3+4+5+6+7+8+9 ✓

## 📌 Important Observation — Why Sum is Always 45

The sum of ALL row sums = sum of ALL numbers used = 1+2+3+4+5+6+7+8+9 = 45.

Similarly, sum of ALL column sums = 45.

## ✏ Filling the Grid Problem 1 (from textbook)

**Fill Grid: Row sums = 13, 14, 18. Column sums = 24, 9, 12.**

**Given:** Grid row sums: 13, 14, 18; Column sums: 24, 9, 12. One cell has 9, one has 5.

**Solution:**

Step 1: Total = 13+14+18 = 45. Check: 24+9+12 = 45 ✓

Step 2: 9 is in Row 1. 5 is in Row 3, Col 3.

Step 3: Row 1 contains 9. Sum of Row 1 = 13 → remaining two cells sum to 4 = 1+3.

Step 4: Try combinations systematically.

| | | |
|---|---|---|
| **9** | **1** | **3** |
| **8** | **2** | **4** |
| **7** | **6** | **5** |

**Answer: Grid filled as shown above.**

## ✕ Impossible Grid — Why?

**Grid with row sums 5, 21, 19 and column sums 9, 11, 26. Is this possible?**

**Solution:**

Step 1: Smallest possible row sum = 1+2+3 = 6.

Step 2: But given row sum of 5 < 6. IMPOSSIBLE!

Step 3: Largest possible column sum = 9+8+7 = 24.

Step 4: But given column sum of 26 > 24. IMPOSSIBLE!

**This grid is IMPOSSIBLE** because sums are out of valid range (6 to 24).

# Magic Squares

## ✨ Definition

**Magic Square:** A square grid of numbers where EVERY row, EVERY column, and BOTH diagonals add to the SAME number. This number is called the **Magic Sum.**

## 🔑 Key Results for 3×3 Magic Square (using numbers 1–9)

**Result 1:** Sum of numbers 1 to 9 = 45

**Result 2:** Magic Sum = 45 ÷ 3 = 15  (since 3 equal rows each sum to 15)

**Result 3:** The CENTER number MUST be 5

**Result 4:** Numbers 1 and 9 CANNOT be in corners — they must be in EDGE positions

## 🔍 Proving the Centre Must Be 5

### Why can't 9 be at centre?

Step 1: If 9 is at centre, the next number 8 shares a row or column with 9.

Step 2: We need 8 + 9 + X = 15 → X = −2. IMPOSSIBLE (only positive integers 1–9)!

### Why can't 1 be at centre?

Step 1: If 1 is at centre, 2 shares a row/column with 1.

Step 2: We need 2 + 1 + X = 15 → X = 12. IMPOSSIBLE (max is 9)!

Step 3: Similarly, check 2, 3, 4, 6, 7, 8 at centre — all fail.

Step 4: Only 5 works: 5 in centre means row = 15 → other two sum to 10 (possible: 1+9, 2+8, 3+7, 4+6).

**CONCLUSION: Centre number MUST be 5.**

## 🔍 Why 1 and 9 Cannot Be at Corners

If 1 is at a corner, it lies on 3 lines (1 row + 1 col + 1 diagonal = 3 lines through that corner).

We need 3 different pairs of numbers that each sum to 14 (= 15 − 1):

14 can be made as: 5+9, 6+8. That's only 2 pairs → NOT ENOUGH!

A corner needs 3 lines through it → we need 3 pairs. Only 2 exist → IMPOSSIBLE.

**1 and 9 must be at edge (middle of sides) positions, NOT corners.**

## ⌛ The Standard Magic Square

| | | |
|---|---|---|
| **2** | **7** | **6** |
| **9** | **5** | **1** |
| **4** | **3** | **8** |

Check Row 1: 2+7+6 = 15 ✓   Row 2: 9+5+1 = 15 ✓   Row 3: 4+3+8 = 15 ✓

Check Col 1: 2+9+4 = 15 ✓   Col 2: 7+5+3 = 15 ✓   Col 3: 6+1+8 = 15 ✓

Diagonal 1: 2+5+8 = 15 ✓   Diagonal 2: 6+5+4 = 15 ✓

## Figure It Out — Magic Squares

**Q1. How many different magic squares can be made using numbers 1–9?**

**Solution:**

Step 1: Once we fix the centre as 5 and the positions of 1 and 9, the rest is determined.

Step 2: If we consider rotations and reflections as the same, there is exactly ONE unique 3×3 magic square.

Step 3: But if rotations/reflections count as different, there are 8 arrangements.

**Answer: 1 unique magic square (ignoring rotations and reflections); 8 if all counted.**

**Q2. Create a magic square using numbers 2–10.**

**Solution:**

Strategy: Take the 1–9 magic square and ADD 1 to every number.

Original (1–9): Centre = 5, Magic Sum = 15

New (2–10): Centre = 6, Magic Sum = 15 + 3 = 18

| | | |
|---|---|---|
| **3** | **8** | **7** |
| **10** | **6** | **2** |
| **5** | **4** | **9** |

Check: All rows, columns, diagonals = 18 ✓

**Magic Square 2–10 shown above. Magic Sum = 18.**

**Q3. Take a magic square: (a) Increase each number by 1 (b) Double each number. Is result still magic?**

**Solution:**

**(a) Add 1 to each:**

Original magic sum = 15. New sum = 15 + 3 = 18. YES, still a magic square!

| | | |
|---|---|---|
| **9** | **2** | **7** |
| **4** | **6** | **8** |
| **5** | **10** | **3** |

**(b) Double each:**

Original magic sum = 15. New sum = 15 × 2 = 30. YES, still a magic square!

| | | |
|---|---|---|
| 16 | 2 | 12 |
| 6 | 10 | 14 |
| 8 | 18 | 4 |

**YES, both operations preserve magic square property.**

**Q4. What other operations can be performed on a magic square to yield another magic square?**

Adding, subtracting, multiplying, or dividing every element by the same constant preserves the magic square.

**Answer: Addition, Subtraction, Multiplication, Division by a constant all work.**

**Q5. Discuss ways of creating magic squares using 9 consecutive numbers like 3–11, 9–17, etc.**

**Solution:**

Method: Take the 1–9 magic square. To get numbers starting from k, add (k−1) to every element.

For 3–11: Add 2 to each element of 1–9 magic square.

For 9–17: Add 8 to each element of 1–9 magic square.

**Formula: Take 1–9 magic square, add (starting_number − 1) to each element.**

# Generalising a 3×3 Magic Square

## Using variable 'm' at centre

If the centre number is m, then all other numbers can be expressed as:

| | | |
|---|---|---|
| m+3 | m−4 | m+1 |
| m−2 | m | m+2 |
| m−1 | m+4 | m−3 |

Verify Row 1: (m+3) + (m−4) + (m+1) = 3m + 0 = 3m ✓

Magic Sum = 3m (always 3 times the centre number)

## Figure It Out — Generalising

**Q1. Using generalised form, find magic square with centre number 25.**

**Solution:** Substitute m = 25 in the generalised form.

| | | |
|---|---|---|
| 28 | 21 | 26 |
| 23 | 25 | 27 |
| 24 | 29 | 22 |

Magic Sum = 3 × 25 = 75. Check Row 1: 28+21+26 = 75 ✓

**Magic Square with centre 25, magic sum = 75.**

**Q2. Expression for adding 3 terms of any row, column or diagonal?**

Step 1: Take any row, e.g., Row 1: (m+3) + (m−4) + (m+1)

Step 2: = m+3+m−4+m+1 = 3m + 0 = 3m

**Answer: 3m (where m is the centre number)**

**Q3(a). Add 1 to every term in generalised form.**

New grid: Replace each cell (m+k) with (m+k+1) = (m+1) + k

| | | |
|---|---|---|
| **m+4** | **m−3** | **m+2** |
| **m−1** | **m+1** | **m+3** |
| **m** | **m+5** | **m−2** |

**New magic sum = 3m+3 = 3(m+1)**

**Q3(b). Double every term in generalised form.**

| | | |
|---|---|---|
| **2m+6** | **2m−8** | **2m+2** |
| **2m−4** | **2m** | **2m+4** |
| **2m−2** | **2m+8** | **2m−6** |

**New magic sum = 6m = 2×3m**

**Q4. Create a magic square with magic sum = 60.**

**Solution:**

Step 1: Magic sum = 3m. So 3m = 60 → m = 20.

Step 2: Use m=20 in generalised form.

| | | |
|---|---|---|
| **23** | **16** | **21** |
| **18** | **20** | **22** |
| **19** | **24** | **17** |

Check: 23+16+21 = 60 ✓, 18+20+22 = 60 ✓, 19+24+17 = 60 ✓

**Magic square with magic sum = 60, centre = 20.**

**Q5. Is it possible to make a magic square with nine non-consecutive numbers?**

**Solution:** YES! It is possible. Example using multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27.

| | | |
|---|---|---|
| **24** | **3** | **18** |
| | **15** | **21** |
| **12** | **27** | **6** |

YES, magic squares can be made with non-consecutive numbers.

# 6.4 The Virahanka–Fibonacci Numbers

## Historical Background

**Origin:** These numbers were first discovered by the great Prakrit scholar **Virahanka (around 700 CE)** in the context of Sanskrit/Prakrit POETRY — counting rhythms made of short (1 beat) and long (2 beats) syllables!

- Virahanka was inspired by the legendary scholar Pingala (around 300 BCE).
- Later: Gopala (c. 1135 CE) and Hemachandra (c. 1150 CE) also wrote about these numbers.
- In Europe, Fibonacci (1202 CE) — ~500 years AFTER Virahanka — wrote about them.

**The Virahanka Sequence:** **1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...**
**Rule:** Each number = sum of the two numbers before it.

## The Rhythm Problem

**Question:** In how many ways can you write the number n as a sum of 1s and 2s? (Short = 1 beat, Long = 2 beats)

| n | Ways to write as sum of 1s and 2s | Count |
|---|---|---|
| n=1 | 1 | **1** |
| n=2 | 1+1, 2 | **2** |
| n=3 | 1+1+1, 1+2, 2+1 | **3** |
| n=4 | 1+1+1+1, 1+1+2, 1+2+1, 2+1+1, 2+2 | **5** |
| n=5 | 8 ways (see text) | **8** |
| n=6 | 13 ways | **13** |
| n=7 | 21 ways | **21** |
| n=8 | 34 ways | **34** |

## The Key Insight

### Why does this give Fibonacci numbers?

Step 1: Every n-beat rhythm starts with either 1+ or 2+.

Step 2: If it starts with 1+, the remaining must be an (n−1)-beat rhythm.

Step 3: If it starts with 2+, the remaining must be an (n−2)-beat rhythm.

Step 4: So: ways(n) = ways(n−1) + ways(n−2) = Fibonacci rule!

**Number of 8-beat rhythms = 8th Virahanka number = 34**

## Parity Pattern in Virahanka Sequence

### Sequence with parity:

1(O), 2(E), 3(O), 5(O), 8(E), 13(O), 21(O), 34(E), 55(O), 89(O), 144(E), ...

**Pattern: ODD, EVEN, ODD, ODD, EVEN, ODD, ODD, EVEN, ... (repeats every 3 terms)**

**Pattern (repeat of 3): Odd, Even, Odd → Odd, Even, Odd → ...**

Position 1,4,7,10,... → ODD | Position 2,5,8,11,... → EVEN | Position 3,6,9,12,... → ODD

## Next Numbers in Sequence

1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...

89 + 55 = 144 (even)    144 + 89 = 233 (odd)    233 + 144 = 377 (odd)

**Next 3 numbers: 144, 233, 377**

## Virahanka Numbers in Nature

Daisies with 13 petals, 21 petals, 34 petals — all Virahanka numbers!

These numbers appear in spirals of sunflowers, pine cones, shells, and many natural patterns.

# 6.5 Digits in Disguise (Cryptarithms)

## What are Cryptarithms?

**Definition:** Arithmetic puzzles where digits are replaced by letters. Each letter stands for a unique digit (0–9). You must find which digit each letter represents.

## Example 1: T + T + T = UT

**Solve: T + T + T = UT  (where U and T are digits)**

**Given:** Three T's added together give a 2-digit number UT (units digit = T).

**To Find:** Values of U and T.

**Solution:**

Step 1: T + T + T = 3T. This is a 2-digit number.

Step 2: The units digit of 3T must be T (the number ends in T).

Step 3: Try T = 5: 3 × 5 = 15. Units digit = 5 = T ✓

Step 4: So U = 1, T = 5, UT = 15.

Check: T=2: 3×2=6 (single digit, not 2-digit). T=5: 3×5=15 (units digit 5=T) ✓

**T = 5,  U = 1,  UT = 15**

## Example 2: K2 + K2 = HMM

**Solve: K2 + K2 = HMM  (K2 is a 2-digit number, HMM is a 3-digit number)**

**Given:** K2 + K2 = HMM. K2 means tens digit K, units digit 2. HMM: H is hundreds, M repeated in tens and units.

**Solution:**

Step 1: K2 + K2 = 2 × K2.

Step 2: Units: 2 + 2 = 4. So M = 4.

Step 3: 2 × K2 = HMM = H44. Since tens digit is also M=4.

Step 4: Try K=6: 62+62=124. M=2, H=1, tens=2 ✓  HMM=122. Check: 62+62=124 ≠ 122. No.

Step 5: Try K=7: 72+72=144. M=4, H=1. HMM=144. Check: 72+72=144 ✓

**K = 7,  H = 1,  M = 4. So: 72 + 72 = 144**

## More Cryptarithms

**YY + Z = ZOO:**

YY is two-digit: YY = 11×Y. 11Y + Z = ZOO = 100Z + 11×0 + 0 = No wait — Z00 means hundreds Z, tens 0, units 0.

11Y + Z = 100Z+0+0. 11Y = 99Z. Y = 9Z. If Z=1, Y=9: 99+1=100. Check: 99+1=100 ✓ ZOO=100!

**Y=9, Z=1. So: 99 + 1 = 100 (ZOO=100)**

# Final Figure It Out — Mixed Problems

**Q1. A light bulb is ON. Dorjee toggles its switch 77 times. Will it be on or off?**

**Given:** Bulb starts ON. Toggle count = 77.

**Concept Used:** Every toggle changes ON↔OFF. Even toggles → same state. Odd toggles → opposite state.

**Solution:**

Step 1: Bulb starts ON.

Step 2: 77 is ODD.

Step 3: Odd number of toggles → state is opposite of start → OFF.

**The bulb will be OFF. (77 is odd, so final state is opposite of starting state)**

**Q2. Liswini counted 50 loose sheets. Can the sum of all page numbers be 6000?**

**Given:** 50 sheets. Each sheet has 2 pages (consecutive page numbers: odd and even).

**Concept Used:** Parity of sums.

**Solution:**

Step 1: Each sheet has pages numbered as consecutive integers: one ODD, one EVEN.

Step 2: 50 sheets = 50 odd pages + 50 even pages.

Step 3: Sum of 50 even numbers = EVEN.

Step 4: Sum of 50 odd numbers (even count of odds) = EVEN.

Step 5: EVEN + EVEN = EVEN. And 6000 is EVEN. So it is possible!

**YES, the sum CAN be 6000.**

**Q3. Fill a 2×3 grid with 3 odd and 3 even numbers so: Row 1 sum = Odd, Row 2 sum = Even, Col sums = Even, Even, Odd.**

**Solution:**

Row 1 (odd sum): Use 2 even + 1 odd → even+even+odd = odd ✓

Row 2 (even sum): Use 1 even + 2 odd → even+odd+odd = even ✓

Check columns: Col1 = e+o=odd... adjust accordingly.

One valid filling:

| | | |
|---|---|---|
| o | e | e |
| o | e | o |

o=odd, e=even. Row 1 sum (o+e+e) = odd ✓. Row 2 sum (o+e+o) = even ✓

Col1 (o+o)=even ✓. Col2 (e+e)=even ✓. Col3 (e+o)=odd ✓

**Q4. Make a 3×3 magic square with magic sum = 0. (Not all zeros. Use negative numbers.)**

**Solution:**

Step 1: Magic sum = 0 → 3m = 0 → m (centre) = 0.

Step 2: Use m=0 in generalised form.

| | | |
|---|---|---|
| **3** | **−4** | **1** |
| **−2** | **0** | **2** |
| **−1** | **4** | **−3** |

Check Row 1: 3+(−4)+1 = 0 ✓  Diagonal: 3+0+(−3)=0 ✓

**Magic square with magic sum = 0 shown above.**

**Q5. Fill blanks with 'odd' or 'even': (a) Sum of odd number of even numbers (b) Sum of even number of odd numbers (c) Sum of even number of even numbers (d) Sum of odd number of odd numbers**

**Solution:**

(a) Any number of even numbers → sum is always EVEN.

(b) Even count of odd numbers → EVEN.

(c) Any number of even numbers → EVEN.

(d) Odd count of odd numbers → ODD.

**(a) Even  (b) Even  (c) Even  (d) Odd**

**Q6. What is the parity of the sum of numbers from 1 to 100?**

**Solution:**

Step 1: Sum from 1 to 100 = 100×101/2 = 5050.

Step 2: 5050 is EVEN.

Alternative: Pair them: (1+100)+(2+99)+...+(50+51) = 101×50 = 5050. Each pair sums to 101 (odd). 50 odd numbers → sum is even.

**Sum from 1 to 100 = 5050. Parity = EVEN.**

**Q7. Two consecutive Virahanka numbers are 987 and 1597. Find next 2 and previous 2.**

**Solution:**

Step 1: Next two: 987+1597=2584,  1597+2584=4181.

Step 2: Previous two: 1597−987=610,  987−610=377.

**Sequence: ..., 377, 610, 987, 1597, 2584, 4181, ...**

**Q8. Angaan climbs 8-step staircase taking 1 or 2 steps at a time. How many ways?**

**Solution:**

Step 1: This is exactly the Virahanka number problem! Number of ways to climb n steps = V(n).

Step 2: V(1)=1, V(2)=2, V(3)=3, V(4)=5, V(5)=8, V(6)=13, V(7)=21, V(8)=34.

**Number of ways = 34 (8th Virahanka number).**

**Q9. What is the parity of the 20th term of the Virahanka sequence?**

**Solution:**

Step 1: Pattern repeats every 3: O, E, O, O, E, O, O, E, O, ...

Step 2: 20 ÷ 3 = 6 remainder 2. So 20th term has same parity as 2nd term.

Step 3: 2nd term = 2 = EVEN.

**The 20th term of the Virahanka sequence is EVEN.**

**Q10. Identify the true statements: (a) 4m−1 always odd (b) All even = 6j−4 (c) 2p+1 and 2q−1 describe ALL odds (d) 2f+3 gives both**

**Solution:**

**(a) 4m − 1:** 4m is always even (multiple of 4). Even−1 = ODD. ALWAYS TRUE ✓

**(b) 6j − 4:** For j=1: 2 (even). For j=2: 8. For j=3: 14. Misses 4, 6, 10, 12... NOT all even numbers. FALSE ✗

**(c) 2p+1 and 2q−1:** 2p+1 gives ALL odd numbers (p=0→1, p=1→3...). 2q−1 also gives ALL odds (q=1→1, q=2→3...). Both describe all odds. TRUE ✓ (but statement says BOTH describe ALL odds... partially true. Actually both are true individually.)

**(d) 2f+3:** 2f is always even; even+3 = ODD. ALWAYS ODD, never even. FALSE ✗

**TRUE: (a) and (c) only.**

**Q11. Solve the cryptarithm: UT + TA = TAT**

**Given:** UT is a 2-digit number. TA is a 2-digit number. TAT is a 3-digit number.

**To Find:** Values of U, T, A.

**Solution (Step-by-Step):**

Step 1: TAT is a 3-digit number. UT + TA gives 3-digit sum → there is a carry.

Step 2: UT = 10U + T. TA = 10T + A. TAT = 100T + 10A + T = 101T + 10A.

Step 3: (10U + T) + (10T + A) = 101T + 10A

Step 4: 10U + 11T + A = 101T + 10A

Step 5: 10U = 90T + 9A = 9(10T + A)

Step 6: 10U = 9(10T + A). So 10T + A must be a multiple of 10 that gives valid digits.

Step 7: Try T=1, A=0: 10U = 9(10+0) = 90 → U=9. Check: UT=91, TA=10, 91+10=101=TAT ✓

**U = 9, T = 1, A = 0. Check: 91 + 10 = 101 = TAT ✓**

## Chapter Summary

### KEY CONCEPTS MASTERED

**★ 6.1 Numbers Tell Us Things**
Each child says: count of TALLER people in front. First person ALWAYS says 0. Tallest person ALWAYS says 0.

**★ 6.2 Parity (Even/Odd)**
Even + Even = Even | Odd + Odd = Even | Even + Odd = Odd Sum of ODD count of odds = ODD. Sum of EVEN count of odds = EVEN.

**★ nth Formulas**
nth Even number = 2n | nth Odd number = 2n − 1

**★ Grid Parity**
Grid m×n: ODD only if BOTH m and n are odd. EVEN if at least one is even.

**★ 6.3 Grids & Magic Squares**
Sum of 1–9 = 45. Magic sum = 15. Centre = 5. Numbers 1 and 9 are NEVER in corners.

**★ Generalised Magic Square**

Centre = m → Magic Sum = 3m. Adding k to each element → new sum = 3(m+k). Multiplying by k → new sum = 3mk.

**★ 6.4 Virahanka-Fibonacci**

Sequence: 1,2,3,5,8,13,21,34,55,89... Rule: each term = sum of previous two. Parity pattern: O,E,O (repeating)

**★ 6.5 Cryptarithms**

Letters replace digits. Use logic + algebra to find unique digit values.

## 📐 Important Formulas at a Glance

| Formula | Explanation |
|---|---|
| **nth Even = 2n** | n=1→2, n=2→4, n=100→200 |
| **nth Odd = 2n−1** | n=1→1, n=2→3, n=100→199 |
| **Sum 1 to 9 = 45** | 1+2+3+4+5+6+7+8+9 |
| **Magic Sum = 15 (for 1-9)** | 45 ÷ 3 rows = 15 |
| **Magic Sum = 3m** | m = centre number |
| **V(n) = V(n-1)+V(n-2)** | Virahanka rule |
| **Sum 1 to n = n(n+1)/2** | e.g., 1 to 100 = 5050 |

**✦ Keep Practising, Keep Growing! ✦**