Chapter 7
A TALE OF THREE INTERSECTING LINES

Complete Chapter Notes • Step-by-Step Solutions • All Figure It Out Questions

◇ Introduction to Triangles

A triangle is the most basic closed geometric shape. When three non-collinear points are joined by straight lines, we get a triangle.

What is a Triangle?
✓ A triangle has 3 vertices (corner points).
✓ A triangle has 3 sides (line segments joining the vertices).
✓ A triangle has 3 interior angles.
✓ The symbol used for a triangle is Δ (Delta).
✓ A triangle is named using its three vertices. For example: ΔABC, ΔXYZ, ΔPQR.
✓ The vertices can be written in any order — ΔABC = ΔBCA = ΔCAB.

In triangle ΔABC, the three angles are: **∠CAB (or ∠A)**, **∠ABC (or ∠B)**, and **∠BCA (or ∠C)**.

Key Question: What happens when three vertices lie on a straight line?
→ If all three vertices are collinear (on the same line), no triangle is formed.
→ For a triangle to exist, the three points must be NON-COLLINEAR.

◇ 7.1 Equilateral Triangles

Definition
An equilateral triangle is a triangle in which ALL THREE SIDES are of EQUAL length.
Because all sides are equal, all three angles are also equal (each = 60°).
It is the most symmetric of all triangles.

Construction: Equilateral Triangle with side 4 cm

We want to construct ΔABC where AB = BC = CA = 4 cm. Using only a ruler requires many trials — using a compass is far more efficient!

Step 1: Draw a straight line segment AB = 4 cm using a ruler. Label the endpoints A and B.

Step 2: Place the compass at point A. Set the radius to 4 cm. Draw a sufficiently long arc above the line AB. (Every point on this arc is exactly 4 cm from A.)

Step 3: Place the compass at point B. Keep the radius at 4 cm. Draw another arc so that it intersects the first arc. (Every point on this arc is exactly 4 cm from B.)

Step 4: Mark the intersection point of the two arcs as C.

Step 5: Join A to C and B to C using a ruler. ΔABC is the required equilateral triangle.

> ★ **Why does this work? Point C lies on BOTH arcs. So AC = 4 cm (radius of arc from A) AND BC = 4 cm (radius of arc from B). Together with AB = 4 cm, all three sides are equal!**

ASCII Diagram — Equilateral Triangle Construction

```
C            /\            /  \      4cm /     \ 4cm        /       \        A─────────B
4cm   [Arc from A] ∩ [Arc from B] = Point C
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7.2 Constructing a Triangle When Its Sides Are Given

We can construct any triangle using the compass-and-ruler method, as long as the three given side lengths actually form a valid triangle.

Construction: Triangle with sides 4 cm, 5 cm, 6 cm

Given: Three sides: 4 cm, 5 cm, 6 cm
To Construct: ΔABC with AB = 4 cm, AC = 5 cm, BC = 6 cm

Step 1: Draw base AB = 4 cm. Label A on left, B on right.

Step 2: From A, draw a sufficiently long arc of radius 5 cm (= AC).

Step 3: From B, draw an arc of radius 6 cm (= BC) so it intersects the arc from Step 2.

Step 4: The intersection point is vertex C.

Step 5: Join A to C and B to C. ΔABC is the required triangle.

> ★ **The point of intersection C satisfies AC = 5 cm AND BC = 6 cm simultaneously, because it lies on BOTH circles.**

Construct Triangles (Practice)

Construct triangles with the following side lengths (all in cm):

#	Sides (cm)	Type of Triangle
(a)	4, 4, 6	Isosceles triangle (two equal sides of 4 cm, base 6 cm)
(b)	3, 4, 5	Scalene triangle; this is a Pythagorean triple (right-angled!)
(c)	1, 5, 5	Isosceles triangle (two equal sides of 5 cm, base 1 cm)
(d)	4, 6, 8	Scalene triangle
(e)	3.5, 3.5, 3.5	Equilateral triangle with decimal sides

◇ Types of Triangles

Based on Sides

Type	Sides	Angles	Symmetry
Equilateral	All 3 sides equal	All 3 angles = 60°	Most symmetric
Isosceles	Any 2 sides equal	2 angles equal (base angles)	Has a line of symmetry
Scalene	All 3 sides different	All 3 angles different	No symmetry

Based on Angles

Type	Angle Condition	Example
Acute-Angled	All 3 angles < 90°	60°, 70°, 50°
Right-Angled	One angle = 90°	90°, 45°, 45°
Obtuse-Angled	One angle > 90°	120°, 35°, 25°

Figure It Out — Page 150–151 (Isosceles & Equilateral from Circles)

Q1. Use the points on the circle and/or the centre to form isosceles triangles.

Concept Used: All radii of a circle are equal.

Answer: Take any two points P and Q on the circle. Join the centre O to P and Q. Then OP = OQ = radius. So ΔOPQ is isosceles (two sides equal).

You can pick ANY two points on the circle and join them to the centre to always get an isosceles triangle. Many such triangles are possible.

Final Answer: ΔOPQ is isosceles since OP = OQ = radius. Infinitely many isosceles triangles can be formed this way.

Q2. Use the points on the circles and/or their centres to form isosceles and equilateral triangles. The circles are of the same size.

Concept Used: When two equal circles intersect, special symmetric triangles arise.

For two equal circles (centres A and B): Take intersection points C and D. Then AB = AC = BC = radius (all equal). So ΔABC is EQUILATERAL. Also, triangles like AC_1B, AC_2B (where C_1, C_2 are any points on circle B) are isosceles since $AC_1 = AC_2 = \ldots =$ radius of circle A.

For three equal circles (centres A, B, C): The centre-to-centre distances equal the radius, so ΔABC is equilateral. Various intersection points of pairs of circles form isosceles triangles.

Final Answer: Using equal circles: equilateral triangles form from the three centres; isosceles triangles from any two centres + a point on one circle.

Triangle Inequality — Are Triangles Possible for Any Lengths?

Real-Life Intuition: The Shortest Path
Imagine you are at a tent and want to reach a tree.
You can go DIRECTLY (straight red path) OR
You can go via a pole (yellow path = tent→pole→tree).
The DIRECT path is always SHORTER than going via a third point.
This is the KEY idea behind the triangle inequality!

The Triangle Inequality Rule
For three lengths a, b, c to form a triangle:
$a + b > c$ (sum of any two sides must be GREATER than the third side)
$b + c > a$
$a + c > b$
ALL THREE conditions must hold. If even ONE fails, no triangle exists.
SHORTCUT: Only check if the largest side < sum of the other two sides.

TRIANGLE INEQUALITY: Each side < Sum of the other two sides → Triangle EXISTS

Worked Example 1: Sides 10 cm, 15 cm, 30 cm

Given: Three sides: 10 cm, 15 cm, 30 cm

To Check: Does a triangle exist?

Step 1: Check: $10 + 15 = 25 < 30$ ✗ The sum of two smaller sides is LESS than the largest side.

Step 2: Since even one condition fails, no triangle is possible.

Step 3: Explanation: If we try to construct this triangle, the two arcs from A and B (radii 10 and 15) would NOT reach each other — they fall short of intersecting.

Final Answer:
Triangle with sides 10 cm, 15 cm, 30 cm does NOT exist. (10 + 15 = 25 < 30)

Worked Example 2: Sides 3 cm, 4 cm, 5 cm

Step 1: Check: 3 + 4 = 7 > 5 ✓

Step 2: Check: 3 + 5 = 8 > 4 ✓

Step 3: Check: 4 + 5 = 9 > 3 ✓

Step 4: All three conditions are satisfied.

Final Answer: Triangle with sides 3 cm, 4 cm, 5 cm EXISTS. (It is also a right-angled triangle!)

Figure It Out — Page 154 (Triangle Inequality)

Q1. We checked by construction that there are no triangles having sidelengths 3 cm, 4 cm and 8 cm; and 2 cm, 3 cm and 6 cm. Check if you could have found this without trying to construct the triangle.

Answer: Yes! For 3, 4, 8: check 3 + 4 = 7 < 8. The two smaller sides don't reach the largest side.

For 2, 3, 6: check 2 + 3 = 5 < 6. Again, the two smaller sides are insufficient.

In both cases, the triangle inequality is violated — we can determine IMPOSSIBILITY without construction.

Final Answer: Both sets violate the triangle inequality. No construction needed — algebraic check is sufficient.

Q2. Can we say anything about the existence of a triangle for each of the following?

Lengths	Check	Conclusion
(a) 10 km, 10 km, 25 km	10 + 10 = 20 < 25	**Triangle does NOT exist**
(b) 5 mm, 10 mm, 20 mm	5 + 10 = 15 < 20	**Triangle does NOT exist**
(c) 12 cm, 20 cm, 40 cm	12 + 20 = 32 < 40	**Triangle does NOT exist**

Final Answer: All three sets violate the triangle inequality — no triangles exist for any of them.

Q3. For any set of lengths, will there always be at least two comparisons where the direct length is less than the sum of the other two?

Answer: Yes, this always happens! If we arrange the three lengths in increasing order as $a \le b \le c$, then:

- $a < b + c$ always holds (since both b and c are ≥ a, so their sum > a)
- $b < a + c$ always holds (since $c \ge b$, so $a + c > b$)
- $c < a + b$ is the one that MAY or MAY NOT hold — this is the critical check!

So at minimum two comparisons ALWAYS pass. Only the LARGEST side vs. sum of the other two is the decisive check.

Final Answer: At least two comparisons always pass. Only check: Largest side < Sum of the two smaller sides.

Existence of Triangle Using Circles

When constructing a triangle given side lengths, we draw two circles centred at the base vertices. The existence of a triangle depends on whether these circles intersect.

Case	Circles	Triangle?
Case 1	Circles TOUCH at one point	**NO — only one point of contact, no triangle**
Case 2	Circles do NOT intersect (too far apart)	**NO — arcs never meet, no third vertex**
Case 3	Circles INTERSECT at two points	**YES — either intersection point gives a valid triangle**

CONCLUSION: Triangle inequality satisfied → circles intersect (Case 3) → Triangle EXISTS. If inequality fails → Case 1 or 2 → Triangle does NOT exist.

Figure It Out — Page 156 (Can these be side lengths?)

Q1. Which of the following lengths can be the sidelengths of a triangle? Explain your answers.

Lengths	Check	Verdict
(a) 2, 2, 5	$2 + 2 = 4 < 5$	**NO — triangle does NOT exist**
(b) 3, 4, 6	$3+4=7>6$, $3+6=9>4$, $4+6=10>3$	**YES — triangle exists**
(c) 2, 4, 8	$2 + 4 = 6 < 8$	**NO — triangle does NOT exist**
(d) 5, 5, 8	$5+5=10>8$, $5+8=13>5$	**YES — isosceles triangle exists**

(e) 10, 20, 25	10+20=30>25, 20+25=45>10, 10+25=35>20	**YES — triangle exists**
(f) 10, 20, 35	10 + 20 = 30 < 35	**NO — triangle does NOT exist**
(g) 24, 26, 28	24+26=50>28, 26+28=54>24, 24+28=52>26	**YES — triangle exists**

Figure It Out — Page 159 (Does a triangle exist?)

Q1. Check if a triangle exists for each of the following set of lengths:

(a) 1, 100, 100 1 + 100 = 101 > 100 ✓ 100 + 100 = 200 > 1 ✓

Ans: Triangle EXISTS ☑

(b) 3, 6, 9 3 + 6 = 9 = 9 (NOT strictly greater) ✗

Ans: Triangle CANNOT exist ✖

(c) 1, 1, 5 1 + 1 = 2 < 5 ✗

Ans: Triangle CANNOT be constructed ✖

(d) 5, 10, 12 5+10=15>12✓, 5+12=17>10✓, 10+12=22>5✓

Ans: Triangle EXISTS ☑

Q2. Does there exist an equilateral triangle with sides 50, 50, 50? In general, does there exist an equilateral triangle of any sidelength? Justify your answer.

Answer: Yes, an equilateral triangle with sides 50, 50, 50 EXISTS. Check: 50 + 50 = 100 > 50 ✓ (and all three conditions hold by symmetry).

In general, for ANY positive length s, we have s + s = 2s > s. So equilateral triangles exist for ALL possible side lengths.

☑ **Final Answer: Equilateral triangles exist for every positive sidelength s, since s + s = 2s > s always.**

Q3. For each of the following, give at least 5 possible values for the third length so there exists a triangle:

(a) 1, 100: Third side must be between |100−1| and 100+1, i.e., between 99 and 101 (exclusive).

Possible values: 99.5, 100, 100.5, 100.7, 99.4

(b) 5, 5: Third side must satisfy: 5 + 5 > c → c < 10 AND 5 + c > 5 → c > 0.

So: 0 < c < 10. Possible values: 1, 3, 4.9, 5, 7

(c) 3, 7: Third side must satisfy: |7−3| < c < 7+3 → 4 < c < 10.

Possible values: 5, 6, 6.4, 7, 8

◇ 7.3 Construction of Triangles When Some Sides and Angles Are Given

Case A: Two Sides and the Included Angle (SAS)

What is the Included Angle?
The included angle is the angle BETWEEN the two given sides.
Example: If sides are AB and AC, then ∠A is the included angle.
IMPORTANT: A triangle can ALWAYS be constructed with two sides and any included angle
EXCEPT when the included angle ≥ 180° (the rays would point away from each other).

Construction: ΔABC with AB = 5 cm, AC = 4 cm, ∠A = 45°

Given: AB = 5 cm, AC = 4 cm, ∠A = 45°
To Construct: Triangle ABC

Step 1: Draw base AB = 5 cm.
Step 2: At point A, construct an angle of 45° using a protractor. Draw the ray forming this angle.
Step 3: On the ray from Step 2, mark point C such that AC = 4 cm.
Step 4: Join B to C. ΔABC is the required triangle with AB = 5 cm, AC = 4 cm, ∠A = 45°.

Final Answer: Triangle ABC constructed with AB = 5 cm, AC = 4 cm, ∠BAC = 45°. BC can be measured.

Figure It Out — Page 161 (SAS Constructions)

Q1. Construct triangles for the following measurements where the angle is included between the sides:

(a) 3 cm, 75°, 7 cm: Steps: Draw base 3 cm → angle 75° → mark 7 cm on ray → join to get triangle.

(b) 6 cm, 25°, 3 cm: Steps: Draw base 6 cm → angle 25° → mark 3 cm on ray → join to get triangle.

(c) 3 cm, 120°, 8 cm: Steps: Draw base 3 cm → angle 120° (obtuse) → mark 8 cm on ray → join.

When is a triangle NOT possible (SAS)? When the included angle is ≥ 180°. For example: 4 cm, 210°, 6 cm — the two sides point in the same direction or backward, so they cannot enclose a triangle.

Final Answer: SAS always gives a unique triangle UNLESS the included angle ≥180°

Case B: Two Angles and the Included Side (ASA)

What is the Included Side?
The included side is the side BETWEEN the two given angles.
Example: If angles are ∠A and ∠B, the included side is AB.
Condition for existence: The sum of the two given angles must be LESS than 180°.

Construction: ΔABC with AB = 5 cm, ∠A = 45°, ∠B = 80°

Given: AB = 5 cm, ∠A = 45°, ∠B = 80°

Check: 45° + 80° = 125° < 180° ✓ Triangle is possible.

Step 1: Draw base AB = 5 cm.

Step 2: At A, construct ∠A = 45°. Draw ray from A.

Step 3: At B, construct ∠B = 80°. Draw ray from B.

Step 4: The point where the two rays from Steps 2 and 3 intersect is vertex C.

Step 5: ΔABC is the required triangle.

Final Answer: Triangle ABC with AB = 5 cm, ∠A = 45°, ∠B = 80° is constructed. ∠C = 180° − 45° − 80° = 55°.

Figure It Out — Page 162–163 (ASA and Angle Conditions)

Q1. Construct triangles for the following measurements (two angles and the included side):

(a) 75°, 5 cm, 75°: 75 + 75 = 150° < 180° ✓ — Triangle exists. Third angle = 30°.

(b) 25°, 3 cm, 60°: 25 + 60 = 85° < 180° ✓ — Triangle exists. Third angle = 95°.

(c) 120°, 6 cm, 30°: 120 + 30 = 150° < 180° ✓ — Triangle exists. Third angle = 30°.

Q1 (Page 163). For each of the following angles, find another angle for which a triangle is (a) possible, (b) not possible.

RULE: Triangle with angles A and B is possible if 0° < A + B < 180°. It is NOT possible if A + B ≥ 180°.

Angle	Triangle POSSIBLE (second angle)	Triangle NOT POSSIBLE (second angle)
30°	Less than 150°; e.g., 50° or 90°	≥ 150°; e.g., 150° or 170°

70°	Less than 110°; e.g., 60° or 80°	≥ 110°; e.g., 110° or 140°
54°	Less than 126°; e.g., 64° or 90°	≥ 126°; e.g., 126° or 154°
144°	Less than 36°; e.g., 20° or 35°	≥ 36°; e.g., 36° or 90°

Q2. Determine which of the following pairs can be the angles of a triangle and which cannot:

(a) 35°, 150°: 35 + 150 = 185° > 180° → **CANNOT form a triangle** ❌

(b) 70°, 30°: 70 + 30 = 100° < 180° → **CAN form a triangle** ☑ **(Third angle = 80°)**

(c) 90°, 85°: 90 + 85 = 175° < 180° → **CAN form a triangle** ☑ **(Third angle = 5°)**

(d) 50°, 150°: 50 + 150 = 200° > 180° → **CANNOT form a triangle** ❌

> ★ **RULE: Two angles A and B can be angles of a triangle if and only if 0° < A + B < 180°.**

◇ Finding the Third Angle — Using Parallel Lines

Given two angles of a triangle, we can find the third angle without constructing the triangle, by using the parallel line method.

Method: Finding Third Angle when ∠B = 50°, ∠C = 70°

Step 1: Draw triangle ABC with ∠B = 50°, ∠C = 70°.

Step 2: Through vertex A, draw line XY parallel to base BC.

Step 3: Since XY ∥ BC and AB is a transversal: ∠XAB = ∠B = 50° (alternate interior angles)

Step 4: Since XY ∥ BC and AC is a transversal: ∠YAC = ∠C = 70° (alternate interior angles)

Step 5: ∠XAB + ∠BAC + ∠YAC = 180° (angles on a straight line)

Step 6: 50° + ∠BAC + 70° = 180° → ∠BAC = 180° − 120° = 60°

> ☑ **Final Answer:** **Third angle ∠A = 60°. Verification: 50° + 70° + 60° = 180° ✓**

Figure It Out — Page 165 (Third Angle)

Q1. Find the third angle of a triangle when two of the angles are:

(a) 36°, 72°: 36 + 72 = 108° → **Third angle = 180° − 108° = 72°**

(b) 150°, 15°: 150 + 15 = 165° → **Third angle = 180° − 165° = 15°**

(c) 90°, 30°: 90 + 30 = 120° → **Third angle = 180° − 120° = 60°**

(d) 75°, 45°: 75 + 45 = 120° → **Third angle = 180° − 120° = 60°**

Q2. Can you construct a triangle all of whose angles are equal to 70°? If two of the angles are 70°, what would the third angle be? If all angles are equal, what must the measure be?

- **All angles = 70°:** $70 + 70 + 70 = 210° \neq 180°$. NOT possible.
- **If two angles = 70°:** Third angle $= 180° - 70° - 70° = 40°$.
- **For all angles equal:** $3 \times \text{angle} = 180° \rightarrow \text{angle} = 60°$. An EQUILATERAL triangle has all angles = 60°.

Final Answer: **No triangle with three 70° angles (sum > 180°). Two 70° angles → third angle = 40°. Equal angles must each be 60°.**

Q3. In a triangle ∠B = ∠C and ∠A = 50°. Find ∠B and ∠C.

Given: $\angle A = 50°,\ \angle B = \angle C$

Using: $\angle A + \angle B + \angle C = 180° \rightarrow 50° + 2\angle B = 180° \rightarrow \angle B = 65°$

Final Answer: $\angle B = \angle C = 65°$

Angle Sum Property of Triangles

Theorem: Angle Sum Property

The sum of all three interior angles of ANY triangle = 180°.

This is called the ANGLE SUM PROPERTY of triangles.

$\angle A + \angle B + \angle C = 180°$ for any triangle ABC.

Proof (Using Parallel Lines)

Step 1: Take any triangle ABC.

Step 2: Through vertex A, draw line XY parallel to base BC.

Step 3: XY ∥ BC with transversal AB: $\angle XAB = \angle B$ (alternate angles)

Step 4: XY ∥ BC with transversal AC: $\angle YAC = \angle C$ (alternate angles)

Step 5: ∠XAB, ∠BAC, ∠YAC together form a straight angle (180°) along line XY.

Step 6: Therefore: $\angle B + \angle A + \angle C = 180°$

PROVEN: $\angle A + \angle B + \angle C = 180°$ for every triangle. (Angle Sum Property)

Historical Note: This elegant proof using a parallel line appears in **'The Elements'**, written by Greek mathematician **Euclid** (~300 BCE). It remains a beautiful example of creative geometric reasoning!

Exterior Angle Property of Triangles

Definition: Exterior Angle

When a side of a triangle is extended beyond a vertex, the angle formed between the extension and the adjacent side is called an EXTERIOR ANGLE.

In ΔABC, if side BC is extended to D, then $\angle ACD$ is an exterior angle.

PROPERTY: Exterior angle = Sum of the two non-adjacent interior angles (remote interior angles).

Worked Example: Find $\angle ACD$ if $\angle A = 50°$, $\angle B = 60°$

Given: $\angle A = 50°$, $\angle B = 60°$, BC extended to D

Find: Exterior angle $\angle ACD$

Step 1: Using angle sum property: $\angle A + \angle B + \angle ACB = 180°$

Step 2: $50° + 60° + \angle ACB = 180° \rightarrow \angle ACB = 70°$

Step 3: $\angle ACD + \angle ACB = 180°$ (linear pair / straight angle)

Step 4: $\angle ACD = 180° - 70° = 110°$

Key Observation: $\angle ACD = \angle A + \angle B = 50° + 60° = 110°$ ✓

EXTERIOR ANGLE PROPERTY: Exterior angle = Sum of the two opposite interior angles. $\angle ACD = \angle A + \angle B$

Proof of Exterior Angle Property

From angle sum: $\angle A + \angle B + \angle ACB = 180°$

From linear pair: $\angle ACD + \angle ACB = 180°$

Therefore: $\angle A + \angle B = \angle ACD$ (subtracting $\angle ACB$ from both sides)

7.4 Altitudes of Triangles

Definition: Altitude of a Triangle

The altitude from vertex A is the perpendicular drawn from A to the opposite side BC.

The foot of the perpendicular on BC is usually called D.

AD is the altitude from A. Its length = height of triangle when BC is taken as base.

Every triangle has THREE altitudes — one from each vertex.

In a right-angled triangle, TWO of the sides are themselves altitudes!

Construction of Altitude Using Set Square

Step 1: Draw triangle ABC. Fix BC as the base.

Step 2: Align the ruler along the base BC.

Step 3: Place the set square on the ruler such that one edge of the right angle touches the ruler.

Step 4: Slide the set square along the ruler until the vertical edge of the set square passes through vertex A.

Step 5: Draw the perpendicular from A to BC along the vertical edge of the set square. The foot is D.

Step 6: AD is the altitude from A to BC.

Special Case: Obtuse Triangle
In an obtuse-angled triangle, the foot of the altitude (from the vertex opposite the obtuse angle) falls OUTSIDE the triangle.
We extend the opposite side and then drop the perpendicular.

Altitudes by Paper Folding

Cut out a triangular piece of paper. Fix one side as the base. Fold the triangle so that the top vertex touches the base line and the fold is perpendicular to the base. The crease formed IS the altitude!

Why? Because folding brings the vertex directly 'above' the base, and the fold line forms the shortest (perpendicular) distance.

Figure It Out — Page 170 (Altitudes and Triangle Types)

Q1. Construct a triangle ABC with BC = 5 cm, AB = 6 cm, CA = 5 cm. Construct an altitude from A to BC.

Given: BC = 5 cm, AB = 6 cm, CA = 5 cm (This is an isosceles triangle: CA = BC = 5 cm)

Step 1: Construct ΔABC using compass: Draw BC = 5 cm. Arc from B (radius 6 cm) and arc from C (radius 5 cm) intersect at A.

Step 2: Place ruler along BC. Set square with one edge on ruler.

Step 3: Slide set square till vertical edge touches vertex A.

Step 4: Draw perpendicular from A to BC. Label foot D.

Step 5: AD is the required altitude.

Note: In isosceles triangles, the altitude from the vertex angle (A) bisects the base BC. So D is the midpoint of BC.

Q2. Construct a triangle TRY with RY = 4 cm, TR = 7 cm, ∠R = 140°. Construct an altitude from T to RY.

Given: RY = 4 cm, TR = 7 cm, ∠R = 140° (Obtuse-angled triangle)

Step 1: Draw RY = 4 cm.

Step 2: At R, construct ∠R = 140° (obtuse angle). Mark T on the ray such that TR = 7 cm.

Step 3: Join T to Y to complete ΔTRY.

Step 4: For the altitude from T: extend RY beyond Y (since the triangle is obtuse, the foot falls outside segment RY).

Step 5: Using set square, drop perpendicular from T to the extended line RY. Label foot H.

Step 6: TH is the altitude from T to line RY.

Q3. Construct a right-angled triangle ΔABC with ∠B = 90°, AC = 5 cm. How many different triangles exist with these measurements?

Answer: Infinitely many! With ∠B = 90° fixed, AC = 5 cm is the hypotenuse. Vertex B can be placed anywhere on the semicircle with AC as diameter (Thales' theorem). Since ∠A and ∠C can take any complementary values (∠A + ∠C = 90°), there are infinitely many right-angled triangles with this hypotenuse.

☑ **Final Answer: Infinitely many right-angled triangles exist with ∠B = 90° and AC = 5 cm.**

Q4. Through construction, explore if it is possible to construct an equilateral triangle that is (i) right-angled (ii) obtuse-angled. Also construct an isosceles triangle that is (i) right-angled (ii) obtuse-angled.

Equilateral right-angled? NOT POSSIBLE. An equilateral triangle has all angles = 60°. None of them is 90°.

Equilateral obtuse-angled? NOT POSSIBLE. All angles are 60° < 90°, so the triangle is actually acute-angled.

Isosceles right-angled? POSSIBLE! Example: angles 90°, 45°, 45°. The two equal sides are the legs; the hypotenuse is the unequal side.

Isosceles obtuse-angled? POSSIBLE! Example: angles 120°, 30°, 30°. The two equal sides meet at the obtuse angle.

CHAPTER SUMMARY

Key Formulas & Rules

1. ANGLE SUM PROPERTY: ∠A + ∠B + ∠C = 180° (for any triangle)
2. EXTERIOR ANGLE: Exterior angle = sum of two opposite interior angles
3. TRIANGLE INEQUALITY: Each side < sum of the other two sides
4. THIRD ANGLE: ∠C = 180° − ∠A − ∠B
5. CONDITION (ASA): Triangle exists if ∠A + ∠B < 180°
6. ALTITUDE: Perpendicular from vertex to opposite side

Types of Triangles (Summary)

By sides → Equilateral (all 3 equal) | Isosceles (2 equal) | Scalene (all different)
By angles → Acute (all < 90°) | Right (one = 90°) | Obtuse (one > 90°)

Equilateral triangle is also acute-angled (all angles = 60°).
A right-angled triangle can be isosceles (45°-45°-90°) but NOT equilateral.

Construction Methods
SSS (3 sides given): Draw base, arcs from both endpoints, intersection = third vertex.
SAS (2 sides + included angle): Draw base, construct angle, mark length on ray, join.
ASA (2 angles + included side): Draw base, construct both angles, rays intersect at third vertex.
Altitude: Use set square aligned to ruler; slide till it touches the opposite vertex.

Triangle Inequality: a + b > c | Angle Sum: ∠A + ∠B + ∠C = 180° | Exterior Angle = sum of remote interior angles

Quick Reference: Figure It Out Solutions at a Glance

Page	Question	Answer
p.150	Isosceles from circle	Centre + 2 rim points → isosceles (radii equal)
p.151	Equilateral from two equal circles	Both intersection points + any centre → equilateral
p.154 Q1	3,4,8 and 2,3,6 without construction	3+4=7<8 and 2+3=5<6 → triangle inequality fails
p.154 Q2a	10,10,25 km	10+10=20<25 → no triangle
p.154 Q2b	5,10,20 mm	5+10=15<20 → no triangle
p.154 Q2c	12,20,40 cm	12+20=32<40 → no triangle
p.156 Q1b	3,4,6	Triangle EXISTS (all conditions satisfied)
p.156 Q1d	5,5,8	Triangle EXISTS (isosceles)
p.156 Q1g	24,26,28	Triangle EXISTS
p.159 Q1a	1,100,100	Triangle EXISTS
p.159 Q1b	3,6,9	Triangle CANNOT exist (3+6=9, not strictly greater)
p.159 Q2	Equilateral of any size	Exists for any positive side length
p.163 Q2a	35°, 150°	Cannot form triangle (sum > 180°)
p.163 Q2b	70°, 30°	Can form triangle (third angle = 80°)
p.165 Q1a	36°, 72°	Third angle = 72°
p.165 Q3	∠B = ∠C, ∠A = 50°	∠B = ∠C = 65°