ANSWER KEY

SECTION A	
Q.NO.	**ANSWERS**
1	**d**
2	**b**
3	**a**
4	**d**
5	**a**
6	**a**
7	**a**
8	**a**
	SECTION B
9	$\frac{3+4\sqrt{3}}{10}$.
10	R = {(a, b); $a \leq b$} Clearly (a, a) ∈ R as $a = a$. ∴R is reflexive. Now, (2, 4) ∈ R (as 2 < 4) But, (4, 2) ∉ R as 4 is greater than 2. ∴ R is not symmetric. Now, let (a, b), (b, c) ∈ R. Then, $a \leq b$ and $b \leq c$ ⇒ $a \leq c$ ⇒ (a, c) ∈ R ∴R is transitive. Hence,R is reflexive and transitive but not symmetric. **Or**

$R = \{(a, b) : a \le b^2\}$

It can be observed that $\left(\frac{1}{2}, \frac{1}{2}\right) \notin R$, since $\frac{1}{2} > \left(\frac{1}{2}\right)^2 = \frac{1}{4}$.

$\therefore$ R is not reflexive.

Now, $(1, 4) \in R$ as $1 < 4^2$

But, 4 is not less than 1^2.

$\therefore (4, 1) \notin R$

$\therefore$ R is not symmetric.

Now, $(3, 2), (2, 1.5) \in R$

$(as 3 < 2^2 = 4 and 2 < (1.5)^2 = 2.25)$

But, $3 > (1.5)^2 = 2.25$

$\therefore (3, 1.5) \notin R$

$\therefore$ R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

11

$A^2 = 2A$

$\Rightarrow |AA| = |2A|$

$\Rightarrow |A||A| = 8|A|$

$(\because |AB| = |A|\,|B| \text{ and } |2A| = 2^3|A|)$

$\Rightarrow |A|\,(|A| - 8) = 0$

$|A| = 0$ or 8

SECTION C

12

$A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$

$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix}$

$5A = \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix}$

$A^2 - 5A - 14I = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

As $AB = I = BA$

$\Rightarrow B = A^{-1}$ & $A^{-1} = \frac{1}{14}[A - 5I] = \frac{1}{14}\begin{bmatrix} -2 & -5 \\ -4 & -3 \end{bmatrix} = \frac{-1}{14}\begin{bmatrix} 2 & 5 \\ 4 & 3 \end{bmatrix}$

13	**Reflexivity:** Let a be an arbitrary element of A. Then, a ∈ R ⇒ a = a [Since, every element is equal to itself] ⇒ (a, a) ∈ R for all a ∈ A Clearly, R is reflexive on A. **Symmetry:** Let (a, b) ∈ R ⇒ ab ⇒ b = a ⇒ (b, a) ∈ R for all a, b ∈ A Therefore, R is symmetric on A. **Transitivity:** Let (a, b) and (b, c) ∈ R ⇒ a = b and b = c ⇒ a = bc ⇒ a = c ⇒ (a, c) ∈ R Clearly, R is transitive on A. Hence, R is an equivalence relation on A.
14	$(adj\ A)\cdot(A) = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $= \begin{pmatrix} \cos^2\alpha + \sin^2\alpha & 0 & 0 \\ 0 & \sin^2\alpha + \cos^2\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad ...(ii)$

and $\quad |A| = \begin{vmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$= 1\cdot(\cos^2\alpha + \sin^2\alpha) = 1$ ...(iii)

(expanding along R_3)

From Eqs. (i), (ii) and (iii), we get

$$A\,(adj\,A) = (adj\,A)\cdot A = |A|\,I_3$$

Or

$\Rightarrow AA' = I$

$$\Rightarrow \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4y^2+z^2 & 2y^2-z^2 & -2y^2+z^2 \\ 2y^2-z^2 & x^2+y^2+z^2 & x^2-y^2-z^2 \\ -2y^2+z^2 & x^2-y^2+z^2 & x^2+y^2+z^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow 4y^2 + z^2 = 1$

$2y^2 - z^2 = 0$

$x^2 + y^2 + z^2 = 1$

$x^2 - y^2 - z^2 = 0$

$\Rightarrow y^2 = 1/6,\ z^2 = 1/3,\ x^2 = 1/2$

$\Rightarrow x = \pm\dfrac{1}{\sqrt{2}}$

$\Rightarrow y = \pm\dfrac{1}{\sqrt{6}}$

and $z = \pm\dfrac{1}{\sqrt{3}}$

15

$f : R^+ \rightarrow [-5, \infty]$ given by $f(x) = 9x^2 + 6x - 5$

To show: f is one-one & onto

Let us assume that f is not one-one.

$\therefore$ there exists 2 as more numbers for which images are same

For $x_1, x_2 \in R+$ & $x_1 \pm x_2$

Let $f(x_1) = f(x_2)$

$\Rightarrow 9x_1^2 + 6x_1 - 5 = 9x_1^2 + 6x_1 - 5$

$9x_1^2 + 6x_2 = 9x_1^2 + 6x_2$

$9(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0$

$(x_1 - x_2)[9(x_1 - x_2) + 6] = 0.$

Since x_1 & x_2 are positive

$9(x_1 - x_2) + 6 > 0$

$\therefore x_1 - x_2 = 0 \Rightarrow x_1 = x_2$

Therefore, it contradicts are assumption.

Hence the function is one-one

FOR ONTO ,Let f(x)=y

$$9x^2 + 6x - 5 = y$$

$$9x^2 + 6x - 5 - y = 0$$

$$9x^2 + 6x - (5 + y) = 0 \quad \Rightarrow \quad x = \frac{-6 \pm \sqrt{36 + 4(9)(5 + y)}}{2 \times 9}$$

$$x = \frac{-6 \pm 6\sqrt{1 + 5 + y}}{18} \quad = \quad \frac{-1 \pm \sqrt{y + 6}}{3}$$

$$x = \frac{-1 + \sqrt{y + 6}}{3}, \quad \frac{-1 - \sqrt{y + 6}}{3}$$

$$\frac{-1 + \sqrt{(6 + y)}}{3} \geq 0$$

$$-1 + \sqrt{(6 + y)} \geq 0$$

$$\sqrt{(6 + y)} \geq 0 + 1$$

$$\sqrt{(6 + y)} \geq 1$$

$$6 + y \geq 1$$

$$y \geq 1 - 6$$

$$y \geq -5$$

SECTION D

16

(i)

$$\begin{array}{c} \text{fertilizers} \\ \begin{array}{ccc} \text{I} & \text{II} & \text{III} \end{array} \\ \begin{bmatrix} 200 & 300 & 500 \end{bmatrix} \end{array} \begin{array}{c} \text{Constituents} \\ \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0.5 & 0 & 0.5 & 0 \\ 0.2 & 0.3 & 0 & 0.5 \\ 0.2 & 0.2 & 0.1 & 0.5 \end{bmatrix} \end{array} \text{(In kg)}$$

$$= \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 260 & 190 & 150 & 400 \end{bmatrix} \end{array} \text{(In kg)}$$

Each constituent used given in above matrix.

(ii)

$$\begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \text{(In kg)} \end{array} \\ \begin{bmatrix} 0.5 & 0 & 0.5 & 0 \\ 0.2 & 0.3 & 0 & 0.5 \\ 0.2 & 0.2 & 0.1 & 0.5 \end{bmatrix} \end{array} \begin{array}{c} \text{cost} \\ \begin{bmatrix} 5 \\ 6 \\ 7.5 \\ 10 \end{bmatrix} \end{array} \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \text{(per 100 gm)}$$

$$\begin{array}{c} \text{I} \\ \text{II} \\ \text{III} \end{array} \begin{bmatrix} 500 & 0 & 500 & 0 \\ 200 & 300 & 0 & 500 \\ 200 & 200 & 100 & 500 \end{bmatrix} \text{(In gram)} \begin{bmatrix} 5/100 \\ 6/100 \\ 75/1000 \\ 10/100 \end{bmatrix} \text{per gram}$$

$$= \begin{bmatrix} 25+0+37.5+0 \\ 10+18+0+50 \\ 10+12+7.5+50 \end{bmatrix} = \begin{bmatrix} 62.5 \\ 78 \\ 79.5 \end{bmatrix}$$

(iii)

Total cost per week

Cost of each fertilizer per week $= 1000 \times$ cost of 1kg tin of each fertilizer

$$= 1000 \times \begin{bmatrix} 62.5 \\ 78 \\ 79.5 \end{bmatrix}$$

$$= \begin{bmatrix} 62500 \\ 78000 \\ 79500 \end{bmatrix}$$

$\therefore$ Total cost

$62500 + 78000 + 79500$

$= 220000$

SECTION E

17

$$AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$\Rightarrow AB = 6I$

$\Rightarrow AB = 6I$

$\Rightarrow A\left(\frac{1}{6}B\right) = I \;\; \Rightarrow A^{-1} = \frac{1}{6}(B)$

The given equations can be written as

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$AX = D, \;\; where\; D = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$\Rightarrow \;\; X = A^{-1}D$$

$$\Rightarrow \;\; \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix}$$

$$\Rightarrow \;\; \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$x = 2, \qquad y = -1, \qquad z = 4$

18

Let $\quad (a, b) \in N \times N$

then,

$\because \quad a^2 + b^2 = a^2 + b^2$

$\therefore \quad (a, b)\, R\, (a, b)$

Hence R is reflexive.

Let $(a, b), (c, d) \in N \times N$ be such that

$(a, b)\, R\, (c, d)$

$\Rightarrow \quad a^2 + d^2 = b^2 + c^2$

$\Rightarrow \quad c^2 + b^2 = d^2 + a^2$

$\Rightarrow \quad (c, d)\, R\, (a, b)$

Hence, R is symmetric.

Let $(a, b), (c, d), (e, f) \in N \times N$ be such that

$(a, b)\, R\, (c, d), (c, d)\, R\, (e, f)$.

$\Rightarrow \quad a^2 + d^2 = b^2 + c^2$ ...(i)

and $\quad c^2 + f^2 = d^2 + e^2$ ...(ii)

Adding eqn. (i) and (ii),

$\Rightarrow a^2 + d^2 + c^2 + f^2 = b^2 + c^2 + d^2 + e^2$

$\Rightarrow \quad a^2 + f^2 = b^2 + e^2$

$\Rightarrow \quad (a, b)\, R\, (e, f)$

Since, R is reflexive, symmetric and transitive. Therefore, R is an equivalence relation.

Or

It is given that

$f(x) \in (-1, 1)$

$f(x) = \dfrac{x}{1+|x|}$

we know that : $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

$$\Rightarrow f(x) = \begin{cases} \dfrac{x}{1+x} & \text{if } x \geq 0 \\ \dfrac{x}{1-x} & \text{if } x < 0 \end{cases}$$

Case (i) : If $x_1, x_2 < 0$ then

$$\frac{x_1}{1-x_1} = \frac{x_2}{1-x_2}$$

or $\quad x_1 - x_1x_2 = x_2 - x_1x_2$

or $\quad x_1 = x_2$

Case (ii) : If $x_1, x_2 > 0$ then

$$\frac{x_1}{1+x_1} = \frac{x_2}{1+x_2}$$

$x_1 + x_1x_2 = x_2 + x_1x_2$

or $\quad x_1 = x_2$

Case (iii) : If $x_1 > 0, x_2 < 0$ (Similarly for $x_1 < 0, x_2 > 0$)

or $\quad \dfrac{x_1}{1+|x_1|} \neq \dfrac{x_2}{1-|x_2|}$

$x_1 \neq x_2$

or $\quad f(x_1) \neq f(x_2)$

$\therefore$ From (i), (ii) & (iii) f is a one-one function.

	Onto Case (i) $x \geq 0$ $f(x) = \frac{x}{1+x}$ Let $y = f(x)$ $y = \frac{x}{1+x}$ $y + yx = x$ $y = (1-y)x$ $\frac{y}{1-y} = x$ as $x > 0 \Rightarrow \frac{y}{1-y} \geq 0$ $-$ 0 $+$ 1 $-$ $\therefore y \in [0, 1)$ Case (ii) $x < 0$ $f(x) = \frac{x}{1-x}$ $y = \frac{x}{1-x}$ $x = \frac{y}{1+y} < 0$ $+$ -1 $-$ 0 $+$ $y \in (-1, 0)$ $\therefore y \in (-1, 0) \cup [0, 1] \Rightarrow y \in (-1, 1)$

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