ANSWER KEY

SECTION A	
Q.NO.	**ANSWERS**
1	**a**
2	**c**
3	**d**
4	**c**
5	**C**
6	**C**
7	**C**
8	**C**
	SECTION B
9	We have , $x = a\left(\cos t + \log\tan\frac{t}{2}\right)$ and y $= a\sin t$ On differentiating with respect to t, we get $\frac{dx}{dt} = \frac{d}{dt}\left[a\left(\cos t+\log\tan\frac{t}{2}\right)\right] = a\left(-\sin t + \frac{1}{\tan\frac{t}{2}}\times\sec^2\frac{t}{2}\times\frac{1}{2}\right)$ $= a\left(-\sin t + \frac{1}{2\sin\frac{t}{2}\cos\frac{t}{2}}\right) = a\left(-\sin t+\frac{1}{\sin t}\right)$ $= a\left(\frac{-\sin^2 t+1}{\sin t}\right) = a\left(\frac{\cos^2 t}{\sin t}\right)$ and $\frac{dy}{dt} = \frac{d}{dt}(a\sin t) = a\cos t$ Now, $\left(\frac{dy}{dx}\right) = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a\cos t}{a\frac{\cos^2 t}{\sin t}} = \tan t$ Therefore, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(\tan(t))$ $= \frac{d}{dt}(\tan(t))\times\frac{dt}{dx} = \sec^2 t\times\frac{\sin t}{a\cos^2 t}$ $= \left(\frac{\sin t}{a\cos^4 t}\right)$ $\left(\frac{d^2y}{dx^2}\right)_{t=\frac{\pi}{3}} = \left(\frac{\sin\left(\frac{\pi}{3}\right)}{a\cos^4\left(\frac{\pi}{3}\right)}\right) = \frac{\frac{\sqrt{3}}{2}}{a\left(\frac{1}{16}\right)} = \frac{8\sqrt{3}}{a}$ Hence, at t $= \frac{\pi}{3}, \frac{d^2y}{dx^2} = \frac{8\sqrt{3}}{a}$.
10	Put x = $\tan\theta$ $\theta = \tan^{-1}x$ $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$ $=\tan^{-1}\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}$ $=\tan^{-1}(\frac{\sec\theta-1}{\tan\theta})$ $=\tan^{-1}(\frac{1-\cos\theta}{\sin\theta})$ $=\tan^{-1}\tan(\frac{\theta}{2}) = \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$

11

$A = -A^T$

$\det A = \det(-A^T)$

$\det A = \det(-A)$

$\det A = (-1)^n \det A$, where n is the order of the matrix.

if n is odd , let n=1

then $\det A = -1 \det A$

$2\det A = 0$

$\det A = 0$

Or

Let $A = [a_{ij}]$ be a skew−symmetric matrix.

So,

$a_{ij} = -a_{ji}$ for all i , j

$\Rightarrow a_{ii} = -a_{ii}$ for all value of i

$\Rightarrow 2a_{ii} = 0 \Rightarrow a_{ii} = 0$ for all value of i

$\Rightarrow a_{11} = a_{22} = a_{33} = \ldots = a_{nn} = 0$

SECTION C

12

$$A^{-1} - \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

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Reflexive: xRx because x-x=0 which is in rational.

Symmetric: xRy implies x-y is in rational, which means y-x=-(x-y) is in rational so yRx.

Transitive: xRy and yRz implies x-y and y-z are both in rational. So if we add them the result will also be in rational: x-y+y-z=x-z, so xRz.

Hence R is an equivalance relation

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For differentiability at x = 0.

Left hand derivative

$$f(0-0) = \lim_{h \to 0} \frac{f(0-h) - f(0)}{-h}$$

$$= \lim_{h \to 0} \frac{|0-h| - |0|}{-h}$$

$$= \lim_{h \to 0} \frac{|-h| - 0}{-h}$$

$$= \lim_{h \to 0} \frac{h}{-h}$$

$$= \lim_{h \to 0} (-1) = -1$$

Right hand derivative

$$f'(0+0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{|0+h| - |0|}{h}$$

$$= \lim_{h \to 0} \frac{|h| - 0}{h}$$

$$= \lim_{h \to 0} \frac{h}{h}$$

$$= \lim_{h \to 0} 1$$

$$= 1$$

$\therefore \quad f'(0-0) \neq f'(0+0)$

Hence, f(x) is not differentiable at x = 0.

Or

Proof : Let a function $f(x)$ be differentiable at x = c . Then,

$$\lim_{x \to c} \frac{f(x)-f(c)}{x-c} \text{ exists finitely .}$$

$$\text{Let } \lim_{x \to c} \frac{f(x)-f(c)}{x-c} = f'(c)$$

In order to prove that f(x) is continous at x = c , it is sufficient to show that $\lim_{x \to c} f(x) = f(c)$

$$\lim_{x \to c} f(x) = \lim_{x \to c} \left\{ \left(\frac{f(x)-f(c)}{x-c} \right)(x-c) + f(c) \right\}$$

$$\Rightarrow \lim_{x \to c} f(x) = \lim_{x \to c} \left[\left\{ \frac{f(x)-f(c)}{x-c} \right\} (x-c) \right] + f(c)$$

$$\Rightarrow \lim_{x \to c} f(x) = \lim_{x \to c} \left\{ \frac{f(x)-f(c)}{x-c} \right\} . \lim_{x \to c} (x-c) + f(c)$$

$$\Rightarrow \lim_{x \to c} f(x) = f'(c) \times 0 + f(c)$$

$$\Rightarrow \lim_{x \to c} f(x) = f(c)$$

Hence, f(x) is continuous at x $= c$.

Converse is not true as modulus function is continuous everywhere but not differentiable at change points

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$$y = \sin^{-1} \frac{\sqrt{1+x}+\sqrt{1-x}}{2}$$

Put $x = \cos\theta$

$$\therefore y = \sin^{-1} \frac{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}{2}$$

$$= \sin^{-1} \frac{\sqrt{2\cos^2(\frac{\theta}{2})}+\sqrt{2\sin^2(\frac{\theta}{2})}}{2}$$

$$= \sin^{-1} \frac{\sqrt{2}(\cos\frac{\theta}{2}+\sin\frac{\theta}{2})}{2}$$

$$= \sin^{-1} (\frac{1}{\sqrt{2}}\cos\frac{\theta}{2} + \frac{1}{\sqrt{2}}\sin\frac{\theta}{2})$$

$$= \sin^{-1} (\sin\frac{\pi}{4}\cos\frac{\theta}{2} + \cos\frac{\pi}{4}\sin\frac{\theta}{2})$$

$$= \sin^{-1}\sin (\frac{\pi}{4} + \frac{\theta}{2})$$

$$= \frac{\pi}{4} + \frac{\theta}{2}$$

$$\therefore y = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x$$

$$\therefore \frac{dy}{dx} = \frac{1}{2} \frac{-1}{\sqrt{1-x^2}}$$

$$= \frac{-1}{2\sqrt{1-x^2}}$$

SECTION D

16

i) **-1**
ii) **-1**
iii) **No or f’(2)=-2**

	SECTION E
17	**Proving**
18	Let the first , second & third number be x, y, z respectively Given, $\therefore x + y + z = 6$ $y + 3z = 11$ $x + z = 2y$ or $x - 2y + z = 0$ Step 1 Write equation as AX = B $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$ Hence $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ & $B = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$ Solution of given system of equations is $X = A^{-1}B$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 42 & -33 & +0 \\ 18 & +0 & +0 \\ -6 & +33 & +0 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 9 \\ 18 \\ 27 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\therefore x = 1, y = 2, z = 3$