ANSWER KEY

SECTION A	
Q.NO.	ANSWERS
1	a
2	c
3	c
4	b
5	c
6	a
7	b
8	a
9	c
10	a
11	a
12	c
13	a
14	c
	SECTION B
15	X=1,4
16	Y= $\frac{log(sinx)}{\log(x^2)}$ Y'= $\frac{\log(x^2)cotx-\log(sinx)(\frac{2}{x})}{[\log(x^2)]^2}$ = $\frac{x\log(x^2)cotx-2\log(sinx)}{x[\log(x^2)]^2}$
17	We have f (x) = $\cos(2x + \frac{\pi}{4})$ $\frac{dy}{dx} = -2\sin(2x + \frac{\pi}{4})$ Since, $\frac{3\pi}{8} < x < \frac{7\pi}{8}$ $\Rightarrow \frac{3\pi}{4} < 2x < \frac{7\pi}{4}$ $\Rightarrow \pi < 2x + \frac{\pi}{4} < 2\pi$ i.e 3^{rd} and 4^{th} quadrant So, $\sin(2x + \frac{\pi}{4})$ is negative $\Rightarrow \frac{dy}{dx}$ is positive Hence, f(x) is increasing in the given interval **Or**

Let the radius be r and volume be V of the ball at any instant, then.

$$V = \frac{4}{3}\pi r^2$$

Differentiating w.r.t. r, $\frac{dV}{dr} = \frac{4}{3}\pi\frac{d}{dr}r^3$

$$\Rightarrow \frac{dV}{dr} = \frac{4}{3}\pi \times 3r^2$$

$$\Rightarrow \frac{dV}{dr} = 4\pi r^2$$

when r = 5 cm, then $\frac{dV}{dr} = 4\pi(5)^2$

$$\frac{dV}{dr} = 100\pi \text{ cm}^3.$$

SECTION C

18

Let $\sin^{-1}\frac{3}{4} = \theta$

$$\frac{3}{4} = \sin\theta$$

$$\frac{3}{4} = \frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$$

$$3 + 3\tan^2\frac{\theta}{2} = 8\tan\frac{\theta}{2}$$

$$3\tan^2\frac{\theta}{2} - 8\tan\frac{\theta}{2} + 3 = 0$$

$$\tan\frac{\theta}{2} = \frac{8 \pm \sqrt{64-36}}{6}$$

$$= \frac{8 \pm \sqrt{28}}{6}$$

$$\tan\frac{\theta}{2} = \frac{4 \pm \sqrt{7}}{3}$$

So, $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)$

$$= \tan\frac{\theta}{2}$$

$$= \frac{4-\sqrt{7}}{3}$$

19	*Let A be a square matrix,* *We can write, A = A/2 + A/2* $= \frac{1}{2}(A + A^t) + \frac{1}{2}(A - A^t)$ *Let, A = P + Q* *Where,* $P = \frac{1}{2}(A + A^t), \quad Q = \frac{1}{2}(A - A^t)$ *Now, find* $\boldsymbol{P^t}$ *and* $\boldsymbol{Q^t}$ $P^t = (\frac{1}{2}(A + A^t))^t$ $= \frac{1}{2}(A + A^t)^t$ $= \frac{1}{2}(A^t + (A^t)^t)$ $= \frac{1}{2}(A + A^t)$ $= P$ *and,* $Q^t = (\frac{1}{2}(A - A^t))^t$ $= \frac{1}{2}(A - A^t)^t$ $= \frac{1}{2}(A^t - (A^t)^t)$ $= \frac{1}{2}(A^t - A)$ $= -\frac{1}{2}(A - A^t)$ $= -Q$ *So, here* ***P is symmetric and Q is skew-symmetric matrices and A is the sum of P and Q.***
20	Given $\quad x = \tan\left(\frac{1}{a}\log y\right)$ $\Rightarrow \quad \tan^{-1} x = \frac{1}{a}\log y$ $\Rightarrow \quad a\tan^{-1} x = \log y$ Differentiating w.r.t. x, we get $\Rightarrow \quad \frac{a}{1+x^2} = \frac{1}{y}.\frac{dy}{dx}$ $\Rightarrow \quad \frac{dy}{dx} = \frac{ay}{1+x^2}$ $\Rightarrow \quad (1+x^2)\frac{dy}{dx} = ay$ Differentiating w.r.t. x, we get $(1+x^2)\frac{d^2y}{dx^2} + 2x.\frac{dy}{dx} = a.\frac{dy}{dx}$ $\Rightarrow \quad (1+x^2)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$

21	**(i) Reflexive:**
	
	If (a-b) is divisible by 2 then,
	
	⇒ (a-a)=0 is also divisible by 2
	
	⇒ (a,a) ∈ R
	
	Hence R is Reflexive ∀ (a,b) ∈ Z
	
	(ii) Symmetric:
	
	If (a-b) is divisible by 2 then,
	
	⇒ (b-a)=-(a-b) is also divisible by 2
	
	⇒ (a,b) ∈ R and (b,a) ∈ R
	
	Hence R is Symmetric ∀ (a,b) ∈ Z
	
	(iii)Transitive:
	
	If (a-b) and (b-c) are divisible by 2 then,
	
	⇒ a-c=(a-b)+(b-c) is also divisible by 2
	
	⇒ (a,b) ∈ R, (b,c) ∈ R and (a,c) ∈ R
	
	Hence R is Transitive ∀ (a,b) ∈ Z
	
	⇒ As Relation R is satisfying all the three parameters, hence R is an equivalence relation.
	
	Now equivalence class [0] is the set of all those elements in A which are related to 0 under relation.
	
	Now,
	
	(a,0) ∈ R
	
	⇒ a – 0 is divisible by 2 and a ∈ A.
	
	⇒ a ∈ A such that 2 divides a.
	
	⇒ a = 0, 2,4
	
	Thus [0] = {0,2,4}
	
	Or

Let f : W → W be defined as

$$f(n) = \begin{cases} n-1 & \text{if n is odd} \\ n+1 & \text{if n is even} \end{cases}$$

We need to prove that 'f' is invertible.

In order to prove that 'f' is invertible it is sufficient to prove that f is a bijection.

A function f: A→B is a one-one function or an injection, if

f(x)=f(y) ⇒ x=y for all x, y ∈ A

Case i:

If x and y are odd.

Let f(x) = f(y)

⇒x − 1 = y − 1

⇒x = y

Case ii:

If x and y are even,

Let f(x) = f(y)

⇒x + 1 = y + 1

⇒x = y

Thus, in both the cases, we have,

f(x) = f(y) ⇒ x = y for all x, y ∈ W.

Case iii:

If x is odd and y is even, then we will have x - 1 = y + 1.

⇒ x - y = 2 ,Not possible as difference of even and odd is always odd

Thus possibility of x being even and y being odd can also be ignored under a similar argument.

∴ Both x and y must be either odd or even.

Thus case iii would not be consider due to false result.

Hence f is injective.

Let n be an arbitrary element of W.

If n is an odd whole number, there exists an even whole number n − 1 ∈ W such that

f(n − 1) = n − 1 + 1 = n.

If n is an even whole number, then there exists an odd whole number n + 1 ∈ W such that f(n + 1) =

− 1 = n.

Also, f(1) = 0 and f(0) = 1

Thus, every element of W (co-domain) has its pre-image in W (domain).

So f is an onto function.

SECTION D

22	Q-22 (i) Sale 2021

$$\begin{array}{c|ccc} & \text{Hatchback} & \text{Sedan} & \text{SUV} \\ \hline A & 120 & 50 & 10 \\ B & 100 & 30 & 5 \\ C & 90 & 40 & 2 \end{array}$$

Sale in 2022

$$\begin{array}{c|ccc} & \text{Hatchback} & \text{Sedan} & \text{SUV} \\ \hline A & 300 & 150 & 20 \\ B & 200 & 50 & 6 \\ C & 100 & 60 & 5 \end{array}$$

(ii) Sale in 2023

$$= 2\left(\underbrace{\begin{bmatrix} 120 & 50 & 10 \\ 100 & 30 & 5 \\ 90 & 40 & 2 \end{bmatrix}}_{\text{Sale in 2021}} + \underbrace{\begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 60 & 5 \end{bmatrix}}_{\text{Sale in 2022}}\right)$$

$$= 2\begin{pmatrix} 420 & 200 & 30 \\ 300 & 80 & 11 \\ 190 & 100 & 7 \end{pmatrix}$$

$$= \begin{array}{c} \begin{bmatrix} 840 & 400 & 60 \\ 600 & 160 & 22 \\ 380 & 200 & 14 \end{bmatrix} \end{array} \begin{array}{c} A \\ B \\ C \end{array}$$

(iii) Sale in 2022 Profit

$$\begin{array}{c} A \\ B \\ C \end{array}\begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 60 & 5 \end{bmatrix} \begin{bmatrix} 50,000 \\ 1,00,000 \\ 2,00,000 \end{bmatrix}$$

$$\begin{array}{c} A \\ B \\ C \end{array}\begin{bmatrix} 34,000,000 \\ 16,200,000 \\ 12,000,000 \end{bmatrix}$$ profit matrix by all 3 dealers

or

$$\begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 50 & 5 \end{bmatrix} \begin{bmatrix} 25000 \\ 10,000 \\ 75000 \end{bmatrix}$$ Loss

$$\begin{bmatrix} 10,500,000 \\ 5,950,000 \\ 3,375,000 \end{bmatrix}$$ Loss matrix by all 3 dealers

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23 (i) length $= 18 - 2x$
breadth $= 18 - 2x$
height $= x$

(ii) Volume $= l \times b \times h$
$= (18-2x)^2 x$

(iii) At $x = 3$, box has max volume.

or

Maximum Volume $= 432 \text{ cm}^3$ (for $x = 3$)

SECTION E

24

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

$$\text{Now } AB = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -2-9+12 & 0-2+2 & 1+3-4 \\ 0+18-18 & 0+4-3 & 0-6+6 \\ -6-18+24 & 0-4+4 & 3+6-8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore \quad AB = I_3 \qquad \therefore A^{-1} = B$

$$A^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}, X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ and } C = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$X = A^{-1}C$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -2+0+2 \\ 9+2-6 \\ 6+1-4 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$$

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$$y = (\log x)^x + x^{x\cos x}$$
$$\frac{dy}{dx} = \frac{d}{dx}((\log x)^x) + \frac{d}{dx}(x^{x\cos x})$$
$$y_1 = \log x^x$$
$$\log y_1 = x\log(\log x)$$
$$\frac{1}{y_1}\frac{dy_1}{dx} = x.\frac{1}{\log x} \times \frac{1}{x} + \log(\log x)$$
$$\frac{dy_1}{dx} = (\frac{1}{\log x}9 + \log\log x) \times (\log x)^2$$
$$y_2 = x^{x\cos x}$$
$$\log y_2 = x\cos x \log x$$
$$\frac{1}{y_2}\frac{dy_2}{dx} = x\cos x.\frac{1}{x} + x\log x(-\sin x) + \cos x\log x$$
$$\frac{dy_2}{dx} = x^{x\cos x}(\log x(\cos x - x\sin x) + \cos x)$$
$$\frac{dy}{dx} = (\log x)^x\left[\frac{1}{\log x} + \log\log x\right] + x^{x\cos x}\left[\log x(\cos x - x\sin x) + \cos x\right]$$

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Let a piece of length l be cut from the given wire to make a square.

Then, the other piece of wire to be made into a circle is of length $(28 - l)$ m.

Now, side of square $= \frac{l}{4}$.

Let r be the radius of the circle. Then, $2\pi r = 28 - l \Rightarrow r = \frac{1}{2\pi}(28 - l)$.

The combined areas of the square and the circle (A) is given by,

$$A = (\text{side of the square})^2 + \pi r^2$$
$$= \frac{l^2}{16} + \pi\left[\frac{1}{2\pi}(28 - l)\right]^2$$
$$= \frac{l^2}{16} + \frac{1}{4\pi}(28 - l)^2$$
$$\therefore \frac{dA}{dl} = \frac{2l}{16} + \frac{2}{4\pi}(28 - l)(-1) = \frac{l}{8} - \frac{1}{2\pi}(28 - l)$$
$$\frac{d^2A}{dl^2} = \frac{1}{8} + \frac{1}{2\pi} > 0$$
$$\text{Now, } \frac{dA}{dl} = 0 \Rightarrow \frac{l}{8} - \frac{1}{2\pi}(28 - l) = 0$$
$$\Rightarrow \frac{\pi l - 4(28 - l)}{8\pi} = 0$$
$$\Rightarrow (\pi + 4)l - 112 = 0$$

$\Rightarrow l = \frac{112}{\pi+4}$

Thus, when $l = \frac{112}{\pi+4}, \frac{d^2A}{dl^2} > 0.$

$\therefore$ By second derivative test, the area (A) is the minimum when $l = \frac{112}{\pi+4}$.

Hence, the combined area is the minimum when the length of the wire in making the square is $\frac{112}{\pi+4}$ cm while the length of the wire in making the circle is $28 - \frac{112}{\pi+4} = \frac{28\pi}{\pi+4}$ cm.

OR

We have, $f(x) = (x-1)(x-2)^2$

Now, $f'(x) = (x-2)^2(1) + 2(x-1)(x-2)$

$= x^2 + 4 - 4x + 2(x^2 - 3x + 2)$

$= x^2 + 4 - 4x + 2x^2 - 6x + 4$

$= 3x^2 - 10x + 8$

i) For critical point,

$f'(x) = 0$

$3x^2 - 10x + 8 = 0$

$\Rightarrow x = 2, \frac{4}{3}$

ii) For strictly increasing $3x^2 - 10x + 8 > 0$

$\Rightarrow x < \frac{4}{3} \cup x > 2$

For strictly decreasing $3x^2 - 10x + 8 < 0$

$\Rightarrow \frac{4}{3} < x < 2$

(iii) $f''(x) = 6x - 10$

At $x = \frac{4}{3}$

$f''(x) = -2 < 0$

So, it is point of maxima.

At $x = 2$

$f''(x) = 2 > 0$

So, it is point of minima.

Local minima value is 0.

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Solution

Let:

Radius of the base = r,

Height = h,

Slant height = l,

Volume = V,

Curved surface area = C

As, Volume , $V = \frac{1}{3}\pi r^2 h$

$\Rightarrow h = \frac{3V}{\pi r^2}$

Also, the slant height, $l = \sqrt{h^2 + r^2}$

$= \sqrt{\left(\frac{3V}{\pi r^2}\right)^2 + r^2}$

$= \sqrt{\frac{9V^2}{\pi^2 r^4} + r^2}$

$= \sqrt{\frac{9V^2 + \pi^2 r^6}{\pi^2 r^4}}$

$\Rightarrow l = \frac{\sqrt{9V^2 + \pi^2 r^6}}{\pi r^2}$

Now,

CSA, $C = \pi r l$

$\Rightarrow C(r) = \pi r \frac{\sqrt{9V^2 + \pi^2 r^6}}{\pi r^2}$

$\Rightarrow C(r) = \frac{\sqrt{9V^2 + \pi^2 r^6}}{r}$

$$\Rightarrow C\prime(r) = \frac{r \times \frac{6\pi^2 r^5}{2\sqrt{9V^2+\pi^2 r^6}} - \sqrt{9V^2+\pi^2 r^6}}{r^2}$$

$$= \frac{\left[\frac{3\pi^2 r^6 - \left(9V^2+\pi^2 r^6\right)}{\sqrt{9V^2+\pi^2 r^6}}\right]}{r^2}$$

$$= \frac{3\pi^2 r^6 - 9V^2 - \pi^2 r^6}{r^2\sqrt{9V^2+\pi^2 r^6}}$$

$$= \frac{2\pi^2 r^6 - 9V^2}{r^2\sqrt{9V^2+\pi^2 r^6}}$$

For maxima or minima, $C\prime(r) = 0$

$$\Rightarrow \frac{2\pi^2 r^6 - 9V^2}{r^2\sqrt{9V^2+\pi^2 r^6}} = 0$$

$$\Rightarrow 2\pi^2 r^6 - 9V^2 = 0$$

$$\Rightarrow 2\pi^2 r^6 = 9V^2$$

$$\Rightarrow V^2 = \frac{2\pi^2 r^6}{9}$$

$$\Rightarrow V = \sqrt{\frac{2\pi^2 r^6}{9}}$$

$$\Rightarrow V = \frac{\pi r^3\sqrt{2}}{3} \text{ or } r = \left(\frac{3V}{\pi\sqrt{2}}\right)^{\frac{1}{3}}$$

$$\text{So, } h = \frac{3}{\pi r^2} \times \frac{\pi r^3\sqrt{2}}{3}$$

$$\Rightarrow h = r\sqrt{2}$$

$$\Rightarrow \frac{h}{r} = \sqrt{2}$$

$$\Rightarrow \cot\theta = \sqrt{2}$$

$$\therefore \theta = \cot^{-1}\left(\sqrt{2}\right)$$

Also,

Since, for $r < \left(\frac{3V}{\pi\sqrt{2}}\right)^{\frac{1}{3}}, C\prime(r) < 0$ and for $r > \left(\frac{3V}{\pi\sqrt{2}}\right)^{\frac{1}{3}}, C\prime(r) > 0$

So, the curved surface for $r = \left(\frac{3V}{\pi\sqrt{2}}\right)^{\frac{1}{3}}$ or $V = \frac{\pi r^3\sqrt{2}}{3}$ is the least.