STD 9 Maths [Manjari]
Exploring Algebraic Identities

*** Choose the right answer from the given options. [1 Marks Each]** **[25]**

1. What is the value of $(a+b)^2$?

(A) $\left(a^2+b^2\right)$ (B) $\left(a^2+2ab+b^2\right)$ (C) $\left(a^2-2ab+b^2\right)$ (D) $\left(a^2-ab+b^2\right)$

Ans. : (B) $\left(a^2+2ab+b^2\right)$

2. Which identity represents $(a-b)^2$?

(A) $\left(a^2-2ab+b^2\right)$ (B) $\left(a^2+b^2\right)$ (C) $\left(a^2+2ab+b^2\right)$ (D) $\left(a^2-ab-b^2\right)$

Ans.: (A) $\left(a^2-2ab+b^2\right)$

3. The product of (a + b) (a - b) is

(A) $\left(a^2+b^2\right)$ (B) $\left(a^2-b^2\right)$ (C) $\left(a^2+2ab+b^2\right)$ (D) $\left(a^2-2ab+b^2\right)$

Ans. : (B) $\left(a^2-b^2\right)$

4. Expand $(x+5)^2$.

(A) $x^2+10x+25$ (B) x^2+25 (C) $x^2-10x+25$ (D) $x^2+5x+25$

Ans.: (A) $x^2+10x+25$

5. What is the expansion of $(y-3)^2$?

(A) $\left(y^2+6y+9\right)$ (B) $\left(y^2-9\right)$ (C) $\left(y^2-6y+9\right)$ (D) $\left(y^2+3y+9\right)$

Ans. : (C) $\left(y^2-6y+9\right)$

6. Which identity is used for $\left.(a+b+c)^2\right)$?

(A) $\left(a^2+b^2+c^2\right)$

(B) $\left(a^2+b^2+c^2+2ab+2bc+2ca\right)$

(C) $\left(a^2-b^2-c^2\right)$

(D) $\left(a^2+b^2-c^2\right)$

Ans. : (B) $\left(a^2+b^2+c^2+2ab+2bc+2ca\right)$

7. Find the product of (2x + 3)(2x - 3).

(A) $\left(4x^2-9\right)$ (B) $\left(4x^2+9\right)$ (C) $\left(2x^2-9\right)$ (D) $\left(4x^2+6x-9\right)$

Ans.: (A) $\left(4x^2-9\right)$

8. What is $(m+n)^2-(m-n)^2$?

(A) (2mn) (B) (4mn) (C) $\left(m^2+n^2\right)$ (D) $\left(m^2-n^2\right)$

Ans. : (B) (4mn)

9. Expand $(2a+b)^2$.

(A) $\left(4a^2+b^2\right)$ (B) $\left(4a^2+4ab+b^2\right)$ (C) $\left(2a^2+b^2\right)$ (D) $\left(4a^2+2ab+b^2\right)$

Ans. : (B) $\left(4a^2+4ab+b^2\right)$

10. Which expression is equal to $\left(a^2-b^2\right)$?

(A) $(a+b)^2$ (B) $(a-b)^2$ (C) (a + b)(a - b) (D) (a - b)(a - b)

Ans. : (C) (a + b)(a - b)

11. simplify

(A) $\left(x^2+16\right)$ (B) $\left(x^2-16\right)$ (C) $\left(x^2-8x+16\right)$ (D) $\left(x^2+8x+16\right)$

Ans. : (B) $\left(x^2-16\right)$

12. Expand $(3p-q)^2$.

(A) $\left(9p^2-q^2\right)$ (B) $\left(9p^2-6pq+q^2\right)$ (C) $\left(3p^2-q^2\right)$ (D) $\left(9p^2+6pq+q^2\right)$

Ans. : (B) $\left(9p^2-6pq+q^2\right)$

13. Which of the following is a standard algebraic identity?

(A) $\left(a+b=a^2+b^2\right)$

(B) $\left((a+b)^2=a^2+2ab+b^2\right)$

(C) $\left(a-b=a^2-b^2\right)$

(D) $(ab=a+b)$

Ans. : (B) $\left((a+b)^2=a^2+2ab+b^2\right)$

14. $\text{Expand}(x+y+z)^2$.

(A) $\left(x^2+y^2+z^2\right)$

(B) $\left(x^2+y^2+z^2+2xy+2yz+2zx\right)$

(C) $\left(x^2-y^2-z^2\right)$

(D) (x + y + z)

Ans. : (B) $\left(x^2+y^2+z^2+2xy+2yz+2zx\right)$

15. What is the square of $(5a)$?

(A) (10a) (B) (25a) (C) $\left(25a^2\right)$ (D) $\left(5a^2\right)$

Ans. : (C) $\left(25a^2\right)$

16. Expand $(a+2b)^2$.

(A) $\left(a^2+2ab+4b^2\right)$ (B) $\left(a^2+4ab+4b^2\right)$ (C) $\left(a^2-4ab+4b^2\right)$ (D) $\left(a^2+4b^2\right)$

Ans. : (B) $\left(a^2+4ab+4b^2\right)$

17. The expression (p – q) (p + q) simplifies to:

(A) $\left(p^2+q^2\right)$ (B) $\left(p^2-q^2\right)$ (C) $\left(p^2+2pq+q^2\right)$ (D) $\left(p^2-2pq+q^2\right)$

Ans. : (B) $\left(p^2 - q^2\right)$

18. Expand $(4x+1)^2$.

(A) $\left(16x^2 + 8x + 1\right)$ (B) $\left(16x^2 + 1\right)$ (C) $\left(8x^2 + 1\right)$ (D) $\left(16x^2 - 8x + 1\right)$

Ans.: (A) $\left(16x^2 + 8x + 1\right)$

19. Which identity is used to find the square of a binomial?

(A) $(a+b)^2$ (B) (a - b) (C) (a + b) (D) (ab)

Ans.: (A) $(a+b)^2$

20. Simplify $(7m-2n)^2$.

(A) $49m^2 - 28mn + 4n^2$ (B) $49m^2 + 4n^2$

(C) $14m^2 - 4n^2$ (D) $49m^2 + 28mn + 4n^2$

Ans.: (A) $49m^2 - 28mn + 4n^2$

21. What is the result of (x + 2)(x + 2)?

(A) $\left(x^2 + 2x + 4\right)$ (B) $\left(x^2 + 4x + 4\right)$ (C) $\left(x^2 - 4\right)$ (D) $\left(x^2 + 4\right)$

Ans. : (B) $\left(x^2 + 4x + 4\right)$

22. Which expression gives the difference of squares?

(A) $(a+b)^2$ (B) $(a-b)^2$ (C) (a + b)(a - b) (D) (ab)

Ans. : (C) (a + b)(a - b)

23. Expand $(2x-5)^2$.

(A) $\left(4x^2 - 20x + 25\right)$ (B) $\left(4x^2 + 25\right)$ (C) $\left(2x^2 - 25\right)$ (D) $\left(4x^2 + 20x + 25\right)$

Ans.: (A) $\left(4x^2 - 20x + 25\right)$

24. Which of the following is the correct expansion of $(a-b+c)^2$?

(A) $\left(a^2 + b^2 + c^2 - 2ab + 2ac - 2bc\right)$

(B) $\left(a^2 - b^2 + c^2\right)$

(C) (a + b + c)

(D) $\left(a^2 + 2ab + b^2\right)$

Ans.: (A) $\left(a^2 + b^2 + c^2 - 2ab + 2ac - 2bc\right)$

25. What is the value of $(3x+2y)^2$?

(A) $\left(9x^2 + 12xy + 4y^2\right)$

(B) $\left(9x^2 + 4y^2\right)$

(C) $\left(3x^2 + 2y^2\right)$

(D) $\left(9x^2 - 12xy + 4y^2\right)$

Ans.: (A) $\left(9x^2 + 12xy + 4y^2\right)$

* **Answer the following short questions. [2 Marks Each]** **[64]**

26. Factor using suitable identities:

$16y^2 - 24y + 9$

Ans. : $16y^2 = (4y)^2$

$9 = 3^2$

-24y = -2(4y)(3)

Therefore, $16y^2 - 24y + 9$

$= (4y)^2 - 2(4y)(3) + 3^2$

$= (4y - 3)^2$ [Using $a^2 - 2ab + b^2 = (a - b)^2$]

27. Factor using suitable identities:

$\frac{9}{4}s^2 + 6st + 4t^2$

Ans. : $(9/4)s^2 = [(3s)/2]^2$

$4t^2 = (2t)^2$

6st = 2 × (3s/2) × (2t)

Therefore, $(9/4)s^2 + 6st + 4t^2$

$= [(3s)/2]^2 + 2 \times (3s/2) \times (2t) + (2t)^2$

$= [(3s)/2 + 2t]^2 \left[U \operatorname{sing} a^2 + 2ab + b^2 = (a + b)^2\right]$

28. Factor using suitable identities:

$\frac{p^2}{16} - 2 + \frac{16}{p^2}$

Ans. : Writing -2 as:

-2 = -2 × (p/4) × (4/p)

Now, $p^2/16 = (p/4)^2$

$16/p^2 = (4/p)^2$

So, $p^2/16 - 2 + 16/p^2$

$= (p/4)^2 - 2(p/4)(4/p) + (4/p)^2$

$= (p/4 - 4/p)^2$ [Using $a^2 - 2ab + b^2 = (a - b)^2$]

29. Factor the following:

$9a^2 + b^2 + 4c^2 - 6ab + 12ac - 4bc$

Ans. : $9a^2 + b^2 + 4c^2 - 6ab + 12ac - 4bc$

$= (3a)^2 + (b)^2 + (2c)^2 + 2(3a)(-b) + 2(-b)(2c) + 2(3a)(2c)$

$= (3a - b + 2c)^2 \left[U \operatorname{sing} x^2 + y^2 + z^2 + 2xy + 2xz + 2yz = (x + y + z)^2\right]$

30. Factor the following:

$16s^2 + 25t^2 - 40st$

Ans. : Now, $16s^2 + 25t^2 - 40st$

$= 16s^2 - 40st + 25t^2$ [Rearranging the terms]

$= (4s)^2 - 2(4s)(5t) + (5t)^2$
$= (4s-5t)^2$ [Using $a^2 - 2ab + b^2 = (a-b)^2$]

31. Factor the following:

$r^2 - r - 42$

Ans. : We need two numbers whose product = −42 and Sum = −1.
These numbers are −7 and 6.
So, $r^2 - r - 42$
$= r^2 - 7r + 6r - 42$
= r(r − 7) + 6(r − 7)
= (r − 7)(r + 6)

32. Factor the following:

$49g^2 + 14gh + h^2$

Ans. : $49g^2 + 14gh + h^2$
$= (7g)^2 + 2(7g)(h) + (h)^2$
$= (7g+h)^2 \left[U \operatorname{sing} a^2 + 2ab + b^2 = (a+b)^2\right]$

33. Factor the following:

$64u^2 + 121v^2 + 4w^2 - 176uv - 32uw + 44vw$

Ans. : $64u^2 + 121v^2 + 4w^2 - 176uv - 32uw + 44vw$
$= 64u^2 + 121v^2 + 4w^2 - 176uv + 44vw - 32uw$ [Rearranging the terms]
$= (8u)^2 + (11v)^2 + (2w)^2 + 2(8u)(-11v) + 2(-11v)(-2w) + 2(8u)(-2w)$
$= (8u - 11v$
$-2w)^2 \left[U \operatorname{sing} x^2 + y^2 + z^2 + 2xy + 2xz + 2yz == (x+y+z)^2\right.$

34. Use suitable identity to find the following product:

$(-3x+4)^2$

Ans. : $(-3x+4)^2$
$= (-3x)^2 + 2(-3x)(4) + 4^2$ [Using the identity $(a+b)^2 = a^2 + 2ab + b^2$]
$= 9x^2 - 24x + 16$

35. Use suitable identity to find the following product:

$(2s+7)(2s-7)$

Ans. : (2s + 7)(2s − 7)
$= (2s)^2 - 7^2$ [Using the identity $(a+b)(a-b) = a^2 - b^2$]
$= 4s^2 - 49$

36. Use suitable identity to find the following product:

$\left(p^2 + \frac{1}{2}\right)\left(p^2 - \frac{1}{2}\right)$

Ans. : $\left(p^2+1/2\right)\left(p^2-1/2\right)$

$=\left(p^2\right)^2-(1/2)^2$ [Using the identity $(a+b)(a-b)=a^2-b^2$]

$=p^4-1/4$

37. Use suitable identity to find the following product:

$(2n+7)(2n-7)$

Ans. : (2n + 7)(2n – 7)

$=(2n)^2-7^2$ [Using the identity $(a+b)(a-b)=a^2-b^2$]

$=4n^2-49$

38. Use suitable identity to find the following product:

$(s-2t)\left(s^2+2st+4t^2\right)$

Ans. : $(s-2t)\left(s^2+2st+4t^2\right)$

$=s^3-(2t)^3$ [Using the identity $(a-b)\left(a^2+ab+b^2\right)=a^3-b^3$]

$=s^3-8t^3$

39. Use suitable identity to find the following product:

$\left(\frac{1}{2r}-4r\right)^2$

Ans. : $[1/(2r)-4r]^2$

$=(1/(2r))^2-2(1/(2r))(4r)+(4r)^2$ [Using the identity $(a-b)^2=a^2-2ab+b^2$]

$=1/\left(4r^2\right)-4+16r^2$

40. Use suitable identity to find the following product:

$(-3m+4k-l)^2$

Ans. : So, $(-3m+4k-l)^2$

$=(-3m)^2+(4k)^2+(-l)^2+2(-3m)(4k)+2(4k)(-l)+2(-3m)(-l)$

[Using $(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$]

$=9m^2+16k^2+l^2-24mk-8kl+6ml$

41. Use suitable identity to find the following product:

$\left(x-\frac{1}{3}y\right)^3$

Ans. : $(x-1/3y)^3$

$=x^3-3x^2(y/3)+3x(y/3)^2-(y/3)^3$

[Using the identity $(a-b)^3=a^3-3a^2b+3ab^2-b^3$]

$=x^3-x^2y+3x\left(y^2/9\right)-y^3/27$

$=x^3-x^2y+(1/3)xy^2-y^3/27$

42. Find the value using suitable identity:

17 x 21

Ans. : Using identity: $(a-b)(a+b)=a^2-b^2$

We have 17 × 21 = (19 − 2)(19 + 2)

$= 19^2 - 2^2$
= 361 − 4
= 357

43. Find the value using suitable identity:
 104 x 96

Ans. : Using identity: $(a+b)(a-b) = a^2 - b^2$
We have 104 × 96 = (100 + 4)(100 − 4)
$= 100^2 - 4^2$
= 10000 − 16
= 9984

44. Find the value using suitable identity:
 24 x 16

Ans. : Using identity: $(a+b)(a-b) = a^2 - b^2$
We have 24 × 16 = (20 + 4)(20 − 4)
$= 20^2 - 4^2$
= 400 − 16
= 384

45. Find the value using suitable identity:
 147^3

Ans. : We know that 147 = 150 − 3
So, $147^3 = (150-3)^3$
$= 150^3 - 3 \times 150^2 \times 3 + 3 \times 150 \times 3^2 - 3^3$ [Using identity $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$]
= 3375000 − 202500 + 4050 − 27
= 3176523

46. Find the value using suitable identity:
 199^3

Ans. : 199 = 200 − 1
So, $199^3 = (200-1)^3$
$= 200^3 - 3 \times 200^2 \times 1 + 3 \times 200 \times 1^2 - 1$ [Using identity: $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$]
= 8000000 − 120000 + 600 − 1
= 7880599

47. Find the value using suitable identity:
 127^3

Ans. : Here, 127 = 130 − 3
So, $127^3 = (130-3)^3$
$= 130^3 - 3 \times 130^2 \times 3 + 3 \times 130 \times 3^2 - 3^3$ [Using identity: $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
]

= 2197000 − 152100 + 3510 − 27
= 2048383

48. Factor the following algebraic expression:
$4y^2+1+\frac{1}{16y^2}$

Ans. : Here, we have $4y^2=(2y)^2$
$1/\left(16y^2\right)=(1/4y)^2$
and $2\times 2y\times 1/(4y)=1$
So, this is of the form: $a^2+2ab+b^2=(a+b)^2$
Therefore, $4y^2+1+1/\left(16y^2\right)$
$=(2y+1/(4y))^2$

49. Factor the following algebraic expression:
$9m^2-\frac{1}{25n^2}$

Ans. : $9m^2-1/\left(25n^2\right)$
$=(3m)^2-(1/5n)^2$
$=(3m+1/5n)(3m-1/5n)\left[\text{ Using } a^2-b^2=(a+b)(a-b)\right]$

50. Factor the following algebraic expression:
$27b^3-\frac{1}{64b^3}$

Ans. : $27b^3-1/\left(64b^3\right)$
$=[3b-1$
$/(4b)]\left[(3b)^2+(3b)(1/4b)+(1/4b)^2\right]\left[U\operatorname{sing} a^3-b^3\right.$
$\left.=(a-b)\left(a^2+ab+b^2\right)\right]$

$=(3b-1/4b)\left[9b^2+3/4+1/16b^2\right]$

51. Factor the following algebraic expression:
$x^2+\frac{5x}{6}+\frac{1}{6}$

Ans. : We need two numbers whose sum is 5/6 and product is 1/6.
These numbers are 1/2 and 1/3.
So, $x^2+5x/6+1/6$
$=x^2+x/2+x/3+1/6$ [Splitting the middle term]
= x(x + 1/2) + 1/3(x + 1/2)
= (x + 1/3)(x + 1/2)

52. Factor the following algebraic expression:
$27u^3-\frac{1}{125}-\frac{27u^2}{5}+\frac{9u}{25}$

Ans. : Given expression:
$27u^3-1/125-27u^2/5+9u/25$
$=27u^3-27u^2/5+9u/25-1/125$ [Rearranging the terms]

$= (3u)^3 - 3(3u)^2(1/5) + 3(3u)(1/5)^2 - (1/5)^3$

$= (3u - 1/5)^3$ [Using $a^3 - 3a^2b + 3ab^2 - b^3 = (a-b)^3$]

53. Factor the following algebraic expression:

$64y^3 + \frac{1}{125}z^3$

Ans. : $64y^3 + 1/125z^3$

$= (4y)^3 + (z/5)^3$

$= (4y + z/5)\left[(4y)^2 - (4y)(z/5) + (z/5)^2\right]$ [Using $a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$]

$= (4y + z/5)\left(16y^2 - 4yz/5 + z^2/25\right)$

54. Factor the following algebraic expression:

$p^3 + 27q^3 + r^3 - 9pqr$

Ans. : $p^3 + 27q^3 + r^3 - 9pqr$

$= p^3 + (3q)^3 + r^3 - 3(p)(3q)(r)$

$= (p + 3q + r)\left[p^2 + (3q)^2 + r^2 - p(3q) - (3q)r - pr)\right]$

[Using $a^3 + b^3 + c^3 - 3abc = (a+b+c)\left(a^2 + b^2 + c^2 - ab - bc - ca\right)$]

$= (p + 3q + r)\left(p^2 + 9q^2 + r^2 - 3pq - 3qr - pr\right)$

55. Factor the following algebraic expression:

$9m^2 - 12m + 4$

Ans. : $9m^2 - 12m + 4$

$= (3m)^2 - 2(3m)(2) + (2)^2$

$= (3m - 2)^2$ [Using $a^2 - 2ab + b^2 = (a-b)^2$]

56. Factor the following algebraic expression:

$4x^2 + 9y^2 + 36z^2 + 12xz + 36yz + 24xy$

Ans. : $4x^2 + 9y^2 + 36z^2 + 12xz + 36yz + 24xy$

$= 4x^2 + 9y^2 + 36z^2 + 24xy + 36yz + 12xz$

$= (2x)^2 + (3y)^2 + (6z)^2 + 2(2x)(3y) + 2(3y)(6z) + 2(2x)(6z)$

$= (2x + 3y + 6z)^2 \left[U\sin g\, a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a+b+c)^2\right]$

57. Factor the following algebraic expression:

$27u^3 - \frac{1}{216} - \frac{9u^2}{2} + \frac{u}{4}$

Ans. : $27u^3 - 1/216 - 9u^2/2 + u/4$

$= 27u^3 - 9u^2/2 + u/4 - 1/216$ [Rearranging the terms]

$= (3u)^3 - 3(3u)^2(1/6) + 3(3u)(1/6)^2 - (1/6)^3$

$= (3u - 1/6)^3$ [Using $a^3 - 3a^2b + 3ab^2 - b^3 = (a-b)^3$] .

*** Answer the following questions. [3 Marks Each]** **[96]**

58. Using the identity $(a+b)^2=a^2+2ab+b^2$, expand the following:

(i) $(7x+4y)^2$

(ii) $\left(\frac{7}{5}x+\frac{3}{2}y\right)^2$

(iii) $(2.5p+1.5q)^2$

Ans. : (i) Using $(a+b)^2=a^2+2ab+b^2$

Here, a = 7x and b = 4y

$=(7x)^2+2(7x)(4y)+(4y)^2$

$=49x^2+56xy+16y^2$

(ii) Here, a = (7/5)x and b = (3/2)y

$=[(7/5)x]^2+2\times(7/5)x\times(3/2)y+[(3/2)y]^2$

$=(49/25)x^2+2\times(21/10)xy+(9/4)y^2$

$=(49/25)x^2+(42/10)xy+(9/4)y^2$

Simplify:

$=(49/25)x^2+(21/5)xy+(9/4)y^2$

(iii) Here, a = 2.5p and b = 1.5q

$=(2.5p)^2+2(2.5p)(1.5q)+(1.5q)^2$

$=6.25p^2+7.5pq+2.25q^2$

59. Using the identity $(a+b)^2=a^2+2ab+b^2$, expand the following:

(i) $\left(\frac{3}{4}s+8t\right)^2$

(ii) $\left(x+\frac{1}{2y}\right)^2$

(iii) $\left(\frac{1}{x}+\frac{1}{y}\right)^2$

Ans. : (i) Here, a = (3/4)s and b = 8t

$=[(3/4)s]^2+2\times(3/4)s\times 8t+(8t)^2$

$=(9/16)s^2+2\times(24/4)st+64t^2$

$=(9/16)s^2+12st+64t^2$

(ii) Here, a = x and b = 1/(2y)

$=x^2+2\times x\times[1/(2y)]+[1/(2y)]^2$

$=x^2+(2x)/(2y)+1/\left(4y^2\right)$

$=x^2+x/y+1/\left(4y^2\right)$

(iii) Here, a = 1/x and b = 1/y

$=(1/x)^2+2(1/x)(1/y)+(1/y)^2$

$=1/x^2+2/(xy)+1/y^2$

60. Using the same identity, find the values of the following:

(i) $(64)^2$

(ii) $(105)^2$

(iii) $(205)^2$

Ans. : (i) Write 64 = 60 + 4

Using $(a+b)^2 = a^2 + 2ab + b^2$:

$(60+4)^2 = 60^2 + 2 \times 60 \times 4 + 4^2$

= 3600 + 480 + 16

= 4096

(ii) Writing 105 = 100 + 5

Now $(105)^2 = (100+5)^2$

$= 100^2 + 2 \times 100 \times 5 + 5^2 \left[U \operatorname{sing}(a+b)^2 = a^2 + 2ab + b^2\right]$

= 10000 + 1000 + 25

= 11025

(iii) Write 205 = 200 + 5

So, $(205)^2 = (200+5)^2$

$= 200^2 + 2 \times 200 \times 5 + 5^2 \left[U \operatorname{sing}(a+b)^2 = a^2 + 2ab + b^2\right]$

= 40000 + 2000 + 25

= 42025

61. Find the values of the following using the identity $(a-b)^2 = a^2 - 2ab + b^2$.

(i) $(79)^2$

(ii) $(193)^2$

(iii) $(299)^2$

Ans. : (i) 79 = 80 − 1

$= (80-1)^2$

$= 80^2 - 2 \times 80 \times 1 + 1^2 \left[\text{Using}(a-b)^2 = a^2 - 2ab + b^2\right]$

= 6400 − 160 + 1

= 6241

(ii) 193 = 200 − 7

$= (200-7)^2$

$= 200^2 - 2 \times 200 \times 7 + 7^2 \left[U \operatorname{sing}(a-b)^2 = a^2 - 2ab + b^2\right]$

= 40000 − 2800 + 49

= 37249

(iii) 299 = 300 − 1

$= (300-1)^2$

$= 300^2 - 2 \times 300 \times 1 + 1^2 \left[\text{Using}(a-b)^2 = a^2 - 2ab + b^2\right]$

= 90000 − 600 + 1

= 89401

62. Find the following squares using one of the above identities. Determine which of these identities will make these calculations easier.

(i) 117^2

(ii) 78^2

Ans. : (i) 117 = 110 + 7

So, $117^2 = (110+7)^2$

$= 110^2 + +2(110)(7) + 7^2 \left[U \operatorname{sing}(a+b)^2 = a^2 + 2ab + b^2\right]$

= 12100 + 1540 + 49

= 13689

(ii) 78 = 80 – 2

So, $78^2 = (80-2)^2$

$= 80^2 - 2(80)(2) + 2^2 \left[\text{ Using } (a-b)^2 = a^2 - 2ab + b^2\right]$

= 6400 – 320 + 4

= 6084

63. Find the following squares using one of the above identities. Determine which of these identities will make these calculations easier.

(i) 198^2

(ii) 214^2

Ans. : (i) 198 = 200 – 2

So, $198^2 = (200-2)^2$

$= 200^2 - 2(200)(2) + 2^2 \left[\text{ Using } (a-b)^2 = a^2 - 2ab + b^2\right]$

= 40000 – 800 + 4

= 39204

(ii) 214 = 200 + 14

So, $214^2 = (200+14)^2$

$= 200^2 + 2(200)(14) + 14^2 \left[\text{ Using } (a-b)^2 = a^2 - 2ab + b^2\right]$

= 40000 + 5600 + 196

= 45796

64. Find the following squares using one of the above identities. Determine which of these identities will make these calculations easier.

(i) 1104^2

(ii) 1120^2

Ans. : (i) 1104 = 1100 + 4

So, $1104^2 = (1100+4)^2$

$= 1100^2 + 2(1100)(4) + 4^2 \left[\text{ Using } (a+b)^2 = a^2 + 2ab + b^2\right]$

= 1210000 + 8800 + 16

= 1218816

(ii) 1120 = 1100 + 20

So, $1120^2 = (1100+20)^2$

$= 1100^2 + 2(1100)(20) + 20^2 \left[\text{ Using } (a+b)^2 = a^2 + 2ab + b^2\right]$

= 1210000 + 44000 + 400

= 1254400

65. Factor using suitable identities:

$9a^2+4b^2+c^2-12ab+6ac-4bc$

Ans. : We have:

$9a^2=(3a)^2$

$4b^2=(-2b)^2$ [Here $4b^2=(-2b)^2$ as in two negative terms $-12ab$ and $-4bc, b$ is common]

$c^2=c^2$

Now, $9a^2+4b^2+c^2-12ab+6ac-4bc$

$=(3a)^2+(-2b)^2+c^2+2(3a)(-2b)+2(3a)(c)+2(-2b)(c)$

$=(3a-2b+c)^2\left[U\operatorname{sing} x^2+y^2+z^2+2xy+2yz+2zx=(x+y+z)^2\right]$

66. Fill in the blanks to complete the following identities:

(i) $10x^2-11x-6=$ (2x - __) (__ + 2)

(ii) $6x^2+7x+2=$ (__________) (__________)

Ans. : (i) $LHS=10x^2-11x-6$

$=10x^2-15x+4x-6$

= 5x(2x − 3) + 2(2x − 3)

= (5x + 2)(2x − 3)

(2x − 3)(5x + 2)

So, $10x^2-11x-6=(2x-3)(5x+2)$

(ii) $=6x^2+3x+4x+2$

= 3x(2x + 1) + 2(2x + 1)

= (3x + 2)(2x + 1)

So, $6x^2+7x+2==(3x+2)(2x+1)$

67. Select and use the identity that will help you to find the following products without multiplying directly:

(i) $(41)^2$

(ii) $(27)^2$

Ans. : (i) 41 = 40 + 1

$=(40+1)^2$

$=40^2+2\times 40\times 1+1^2\left[\text{ Using } (a+b)^2=a^2+2ab+b^2\right]$

= 1600 + 80 + 1

= 1681

(ii) 27 = 30 − 3

So, $(27)^2=(30-3)^2$

$=(30)^2-2\times 30\times 3+(3)^2\left[U\operatorname{sing}(a-b)^2=a^2-2ab+b^2\right]$

= 900 − 180 + 9

= 729

68. Select and use the identity that will help you to find the following products without multiplying directly:

(i) (23×17)

(ii) $(135)^2$

Ans. : (i) Use identity:

23 × 17 = (20 + 3)(20 − 3)

$= 20^2 - 3^2 \left[\text{Using}(a+b)(a-b) = a^2 - b^2\right]$

= 400 − 9

= 391

(ii) 135 = 100 + 35

So, $(135)^2$

$= (100+35)^2$

$= (100)^2 + 2 \times 100 \times 35 + (35)^2$ [Using $(a+b)^2 = a^2 + 2ab + b^2$]

= 10000 + 7000 + 1225

= 18225

69. Select and use the identity that will help you to find the following products without multiplying directly:

(i) $(97)^2$

(ii) (18×29)

Ans. : (i) 97 = 100 − 3

So, $(97)^2$

$= (100-3)^2$

$= (100)^2 - 2 \times 100 \times 3 + (3)^2 \left[\text{Using}(a-b)^2 = a^2 - 2ab + b^2\right]$

= 10000 − 600 + 9

= 9409

(ii) 18 × 29 =

= (20 − 2)(20 + 9)

$= 20^2 + (-2+9) \times 20 - 18 \left[U\,\text{sing}(x+a)(x+b) = x^2 + (a+b)x + a \times b\right]$

= 400 + 140 − 18

= 522

70. Select and use the identity that will help you to find the following products without multiplying directly:

(i) (34×43)

(ii) $(205)^2$

Ans. : (i) = (38 − 4)(38 + 5)

$= 38^2 + (-4+5) \times 38 - 20 \left[\text{Using}(x+a)(x+b) = x^2 + (a+b)x + a \times b\right]$

$= 38^2 + 38 - 20$

= 1444 + 38 − 20

= 1462

(ii) 205 = 200 + 5

$=(200+5)^2$

$=(200)^2+2\times 200\times 5+(5)^2$ [Using $(a+b)^2=a^2+2ab+b^2$]

= 40000 + 2000 + 25

= 42025

71. Simplify the following rational expression assuming that the expression in the denominator are not equal to zero:

$\frac{3p^2-3pq-18q^2}{p^2+3pq-10q^2}$

Ans. : First factorising the numerator:

$3p^2-3pq-18q^2$

$=3\left(p^2-pq-6q^2\right)$

$=3\left[p^2-3pq+2pq-6q^2\right]$

= 3[p(p − 3q) + 2q(p − 3q)

= 3(p − 3q)(p + 2q)

Now factorising the denominator:

$p^2+3pq-10q^2$

$=p^2+5pq-2pq-10q^2$

= p(p + 5q) − 2q(p + 5q)

= (p + 5q)(p − 2q)

So, $\left(3p^2-3pq-18q^2\right)/\left(p^2+3pq-10q^2\right)$

= 3(p − 3q)(p + 2q)/[(p + 5q)(p − 2q)]

No common factor cancels.

72. Simplify the following rational expression assuming that the expression in the denominator are not equal to zero:

$\frac{n^3-3n^2m+3nm^2-m^3}{5m^2-10mn+5n^2}$

Ans. : Factorising the numerator using identity:

$n^3-3n^2m+3nm^2-m^3$

$=(n-m)^3$ [Using $a^3-3a^2b+3ab^2-b^3=(a-b)^3$]

Factorising the denominator:

$5m^2-10mn+5n^2$

$=5\left(m^2-2mn+n^2\right)$

$=5(m-n)^2\left[U\operatorname{sing} a^2-2ab+b^2=(a-b)^2\right]$

Now, $(n-m)^3/\left[5(m-n)^2\right]$

$=-(m-n)^3/\left[5(m-n)^2\right]$ [Since, $(n-m)=-(m-n)$, so, $(n-m)^3=-(m-n)^3$]

$=-(m-n)/5$

73. Simplify the following rational expression assuming that the expression in the denominator are not equal to zero:

$\frac{4y^2-20yz+25z^2}{(25z^2-4y^2)}$

Ans. : Factor numerator:

$4y^2-20yz+25z^2$

$=(2y)^2-2(2y)(5z)+(5z)^2$

$=(2y-5z)^2$ [Using $a^2-2ab+b^2=(a-b)^2$]

Factorising the denominator:

$25z^2-4y^2$

$=(5z-2y)(5z+2y)\left[U\operatorname{sing} a^2-b^2=(a-b)(a+b)\right]$

So, $\left(4y^2-20yz+25z^2\right)/\left(25z^2-4y^2\right)$

$=(5z-2y)^2/[(5z-2y)(5z+2y)]$ [Since $(2y-5z)^2=(-1)^2(5z-2y)^2=(5z-2y)^2$]

= (5z – 2y)/(5z + 2y)

74. Simplify the following rational expression assuming that the expression in the denominator are not equal to zero:

$\frac{\left(x^2+x-6\right)\left(x^2-7x+12\right)}{(x^2-6x+8)(x^2-9)}$

Ans. : Factorising each polynomial:

$x^2+x-6=x^2+3x-2x-6=x(x+3)-2(x+3)=(x+3)(x-2)$

$x^2-7x+12=x^2-3x-4x+12=x(x-3)-4(x-3)=(x-3)(x-4)$

$x^2-6x+8=x^2-4x-2x+8=x(x-4)-2(x-4)=(x-2)(x-4)$

$x^2-9=x^2-3=(x-3)(x+3)$

Substituting each polynomial, we get:

[(x + 3)(x – 2)][(x – 3)(x – 4)]/[(x – 2)(x – 4)][(x – 3)(x + 3)]

= 1 [Since all factors cancel]

75. Simplify the following rational expression assuming that the expression in the denominator are not equal to zero:

$\frac{p^4-16}{p^2-4p+4}$

Ans. : Factorising numerator:

p^4-16

$=\left(p^2-4\right)\left(p^2+4\right)\left[U\operatorname{sing} a^2-b^2=(a+b)(a-b)\right]$

$=(p-2)(p+2)\left(p^2+4\right)$ [Again using $a^2-b^2=(a+b)(a-b)$]

Factorising denominator:

p^2-4p+4

$=p^2-2(p)(2)+2^2$

$=(p-2)^2$ [Using $a^2-2ab+b^2=(a-b)^2$]

So, $\left[(p-2)(p+2)\left(p^2+4\right)\right]/(p-2)^2$

$=(p+2)\left(p^2+4\right)/(p-2)$ [Cancelling common factor $(p-2)$].

76. Use suitable identity to find the following product:

$\left(\frac{7}{2}k-\frac{2}{3}m\right)^3$

Ans. : [Using the identity $(a-b)^3=a^3-3a^2b+3ab^2-b^3$]

Here, a = 7k/2, b = 2m/3

So, $a^3=(7k/2)^3=343k^3/8$

$3a^2b=3\times\left(49k^2/4\right)\times(2m/3)=49k^2m/2$

$3ab^2=3\times(7k/2)\times\left(4m^2/9\right)=14km^2/3$

$b^3=(2m/3)^3=8m^3/27$

Therefore, $(7/2k-2/3m)^3$

$=343k^3/8-49k^2m/2+14km^2/3-8m^3/27$.

77. Find the value using suitable identity:

$(-107)^3$

Ans. : $(-107)^3=-\left(107^3\right)$

Now, 107 = 100 + 7

$107^3=(100+7)^3$

$=100^3+3\times100^2\times7+3\times100\times7^2+7^3$ [Using identity: $(a+b)^3=a^3+3a^2b+3ab^2+b^3$
]

= 1000000 + 210000 + 14700 + 343

= 1225043

So, $(-107)^3=-1225043$

78. Find the value using suitable identity:

$(-299)^3$

Ans. : $(-299)^3=-\left(299^3\right)$

Here, 299 = 300 − 1

So, $299^3=(300-1)^3$

$=300^3-3\times300^2\times1+3\times300\times1^2-1$ [Using identity: $(a-b)^3=a^3-3a^2b+3ab^2-b^3$]

= 27000000 − 270000 + 900 − 1

= 26730900 − 1

= 26730899

So, $(-299)^3=-26730899$

79. Factor the following algebraic expression:

$9x^3-\frac{8}{3}y^3+\frac{z^3}{3}+6xyz$

Ans. : $9x^3-8/3y^3+z^3/3+6xyz$

$=(1/3)\left[9x^3-8y^3+z^3+18xyz\right]$

$= (1/3)\left[(3x)^3 + (-2y)^3 + z^3 - 3(3x)(-2y)z\right]$

$= (1/3)(3x - 2y$

$+ z)\left[(3x)^2 + (-2y)^2 + z^2 - (3x)(-2y) - (-2y)(z) - (z)(3x)\right]$

[Using $a^3 + b^3 + c^3 - 3abc = (a+b+c)\left(a^2 + b^2 + c^2 - ab - bc - ca\right)$]

$= (1/3)(3x - 2y + z)\left[9x^2 + 4y^2 + z^2 + 6xy + 2yz - 3zx\right]$

80. Simplify the following:

$\frac{4x^2+4x+1}{4x^2-1}$

Ans. : Factorising numerator:

$4x^2 + 4x + 1$

$= (2x)^2 + 2(2x)(1) + 1^2$

$= (2x+1)^2$ [Using $a^2 + 2ab + b^2 = (a+b)^2$]

Factorising denominator:

$4x^2 - 1 = (2x+1)(2x-1)$

Now the expression: $\left(4x^2 + 4x + 1\right) / \left(4x^2 - 1\right)$

$= (2x+1)^2/[(2x+1)(2x-1)]$

= (2x + 1)/(2x − 1)

81. Simplify the following:

$\frac{9\left(3a^3-24b^3\right)}{9a^2-36b^2}$

Ans. : First simplify:

$9\left(3a^3 - 24b^3\right) = 27\left(a^3 - 8b^3\right)$

and $9a^2 - 36b^2 = 9\left(a^2 - 4b^2\right)$

So, $9\left(3a^3 - 24b^3\right) / \left(9a^2 - 36b^2\right)$

$= 3\left(a^3 - 8b^3\right) / \left(a^2 - 4b^2\right)$

$= 3(a-2b)\left(a^2 + 2ab + 4b^2\right)$

$/\left(a^2 - 4b^2\right)$ [Since $a^3 - 8b^3 = a^3 - (2b)^3 = (a-2b)\left(a^2 + 2ab + 4b^2\right)$]

$= 3(a-2b)\left(a^2 + 2ab + 4b^2\right)/(a-2b)(a+2b)$ [Since $a^2 - 4b^2 = (a-2b)(a+2b)$]

$= 3\left(a^2 + 2ab + 4b^2\right)/(a+2b)$ [Cancelling the common factor $(a-2b)$]

82. Simplify the following:

$\frac{s^3+125t^3}{s^2-2st-35t^2}$

Ans. : Factorising numerator:

$s^3 + 125t^3 = s^3 + (5t)^3$

$= (s+5t)\left(s^2 - 5st + 25t^2\right)$ [Using $a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$]

Factorising denominator:

$s^2 - 2st - 35t^2$

$s^2 - 7st + 5st - 35t^2$

= s(s − 7t) + 5t(s − 7t)

= (s + 5t)(s − 7t)

Now the given expression:

$\left(s^3+125t^3\right) / \left(s^2-2st-35t^2\right)$

$=(s+5t)\left(s^2-5st+25t^2\right)/(s+5t)(s-7t)$

$=\left(s^2-5st+25t^2\right)/(s-7t)$ [Cancelling common factor $(s+5t)$].

83. Find possible expressions for the length and breadth of each of the following rectangles whose areas are given by the following expressions in square units.

(i) $25a^2-30ab+9b^2$

(ii) $36s^2-49t^2$

Ans. : (i) This is a perfect square:

$25a^2-30ab+9b^2$

$=(5a)^2-2(5a)(3b)+(3b)^2$

$=(5a-3b)^2$ [Using $a^2-2ab+b^2=(a-b)^2$]

So possible length and breadth are (5a − 3b) and (5a − 3b).

(ii) $36s^2-49t^2$

$=(6s)^2-(7t)^2$

$=(6s+7t)(6s-7t)\left[U\operatorname{sing} a^2-b^2=(a+b)(a-b)\right]$

So possible length and breadth are (6s + 7t) and (6s − 7t).

84. Find possible expressions for the length, breadth, and heights of each of the following cuboids whose volumes are given by the following expressions in cubic units.

(i) $6a^2-24b^2$

(ii) $3ps^2-15ps+12p$

Ans. : (i) Take common factor:

$6a^2-24b^2$

$=6\left(a^2-4b^2\right)$

$=6\left[a^2-(2b)^2\right]$

$=6(a+2b)(a-2b)$ [Using $a^2-b^2=(a+b)(a-b)$]

So possible dimensions are 6, (a + 2b) and (a − 2b).

(ii) $3ps^2-15ps+12p$

$=3p\left(s^2-5s+4\right)$ [Taking common factor]

$=3p\left(s^2-4s-1s+4\right)$

= 3p[s(s − 4) − 1(s − 4)]

= 3p(s − 1)(s − 4)

So, the possible dimensions are 3p, (s − 1) and (s − 4).

85. The village playground is shaped as a square of side 40 metres. A path of width s metres is created around the playground for people to walk. Find an expression for the area of the path in terms of s.

Ans. : Side of playground = 40 m
Since a path of width s metres is made all around the outside, the side of the outer square becomes:
40 + 2s
Area of outer square $= (40 + 2s)^2$
Area of playground $= 40^2 = 1600$
Therefore, area of the path
$= (40 + 2s)^2 - 1600$
$= \left(1600 + 160s + 4s^2\right) - 1600$
$= 4s^2 + 160s$
Hence, the required expression: Area of path $= 4s^2 + 160s$ square metres.

86. If a number plus its reciprocal equals $\frac{10}{3}$, find the number.

Ans. : Let the number be x.
Then, according to question: x + 1/x = 10/3
Multiply both sides by 3x:
$3x^2 + 3 = 10x$
$\Rightarrow 3x^2 - 10x + 3 = 0$
$\Rightarrow 3x^2 - 9x - x + 3 = 0$
⇒ 3x(x − 3) − 1(x − 3) = 0
⇒ (3x − 1)(x − 3) = 0
So, 3x − 1 = 0 or x − 3 = 0
Hence, x = 1/3 or x = 3
Therefore, the number is 3 or 1/3.

87. A rectangular pool has area $2x^2 + 7x + 3$ square hastas. If its width is $2x + 1$ hastas, find its length. Hasta was a unit used to measure length.

Ans. : Area of pool $= 2x^2 + 7x + 3$
Width = 2x + 1
Length = Area/Width
$= \left(2x^2 + 7x + 3\right)/(2x + 1)$
$= \left(2x^2 + 6x + x + 3\right)/(2x + 1)$
= [2x(x + 3) + 1(x + 3)]/(2x + 1)
= (2x + 1)(x + 3)/(2x + 1)
= x + 3
Therefore, the length of the pool is x + 3 hastas.

88. If both $x - 2$ and $x - \frac{1}{2}$ are factors of $px^2 + 5x + r$, show that $p = r$.

Ans. : Since x − 2 and x − 1/2 are factors, the quadratic polynomial can be written as:
$px^2 + 5x + r = k(x - 2)(x - 1/2)$

$\Rightarrow (x-2)(x-1/2) = k\left[x^2-(5/2)x+1\right]$

$\Rightarrow x^2-(1/2)x-2x+1 = k\left[x^2-(5/2)x+1\right]$

$\Rightarrow x^2-(5/2)x+1 = k\left[x^2-(5/2)x+1\right]$

Comparing the coefficients on both sides, we get:

Coefficient of x^2 gives: $k = p$

Constant term gives: k = r

Therefore, p = r

Hence proved.

89. Find the value of

$x^3+y^3-12xy+64$, when $x+y=-4$

Ans. : Since x + y = −4, we have:

$(x+y)^3 = (-4)^3 = -64$

Using identity: $(x+y)^3 = x^3+y^3+3xy(x+y)$

So,

$-64 = x^3+y^3+3xy(-4)$

$\Rightarrow -64 = x^3+y^3-12xy$

Therefore, $x^3+y^3-12xy = -64$

Now, $x^3+y^3-12xy+64 = -64+64 = 0$

Hence, the value is 0.

*** Questions with calculation. [4 Marks Each]** **[36]**

90. Factor completely:

(i) $9x^2+24xy+16y^2$

(ii) $4s^2+20st+25t^2$

Ans. : (i) We know that:

$9x^2 = (3x)^2$

$16y^2 = (4y)^2$

24xy = 2 × 3x × 4y

So, $9x^2+24xy+16y^2$

$= (3x)^2+2\times 3x\times 4y+(4y)^2$

$= (3x+4y)^2$ [Using $a^2+2ab+b^2 = (a+b)^2$]

(ii) We can write:

$4s^2 = (2s)^2$

$25t^2 = (5t)^2$

20st = 2 × 2s × 5t

So, $4s^2+20st+25t^2$

$= (2s)^2+2\times 2s\times 5t+(5t)^2$

$= (2s+5t)^2$ [Using $a^2+2ab+b^2 = (a+b)^2$]

91. Factor completely:

(i) $49x^2 + 28xy + 4y^2$

(ii) $64p^2 + \frac{32}{3}pq + \frac{4}{9}q^2$

Ans. : (i) $49x^2 = (7x)^2$

$4y^2 = (2y)^2$

28xy = 2 × 7x × 2y

So, $49x^2 + 28xy + 4y^2$

$= (7x)^2 + 2 \times 7x \times 2y + (2y)^2$

$= (7x + 2y)^2$ [Using $a^2 + 2ab + b^2 = (a + b)^2$]

(ii) $64p^2 = (8p)^2$

$(4/9)q^2 = (2/3q)^2$

Middle term:

2 × 8p × (2/3 q) = 32/3 pq

So, $64p^2 + (32/3)pq + (4/9)q^2$

$= (8p)^2 + 2 \times 8p \times (2/3q) + (2/3q)^2$

$= (8p + 2/3q)^2 \left[U \operatorname{sing} a^2 + 2ab + b^2 = (a + b)^2\right]$

92. Factor completely:

(i) $3a^2 + 4ab + \frac{4}{3}b^2$

(ii) $\frac{9}{5}s^2 + 6sv + 5v^2$

Ans. : (i) Take common factor 3:

$= 3\left[a^2 + (4/3)ab + (4/9)b^2\right]$

Now, $a^2 = (a)^2$

$(4/9)b^2 = (2/3b)^2$

(4/3)ab = 2 × a × (2/3 b)

So, $3a^2 + 4ab + (4/3)b^2$

$= 3\left[a^2 + (4/3)ab + (4/9)b^2\right]$

$= 3\left[a^2 + 2 \times a \times (2/3b) + (2/3b)^2\right]$

$= 3(a + 2/3b)^2$ [Using $a^2 + 2ab + b^2 = (a + b)^2$]

(ii) Take common factor 1/5:

$= (1/5)\left[9s^2 + 30sv + 25v^2\right]$

Now, $9s^2 = (3s)^2$

$25v^2 = (5v)^2$

30sv = 2 × 3s × 5v

So, $(9/5)s^2 + 6sv + 5v^2$

$= (1/5)\left[9s^2 + 30sv + 25v^2\right]$

$= (1/5)\left[(3s)^2 + 2 \times 3s \times 5v + (5v)^2\right]$

$= (1/5)(3s + 5v)^2$ [Using $a^2 + 2ab + b^2 = (a + b)^2$]

93. Factor using suitable identities:

$\frac{m^2}{9}+\frac{mk}{3}+\frac{k^2}{4}+3nk+2mn+9n^2$

Ans. : Grouping the terms as:

$m^2/9+mk/3+k^2/4+2mn+3nk+9n^2$

We have:

$m^2/9=(m/3)^2$

$k^2/4=(k/2)^2$

$9n^2=(3n)^2$

Now checking the cross terms:

2 × (m/3) × (k/2) = mk/3

2 × (m/3) × (3n) = 2mn

2 × (k/2) × (3n) = 3nk

So, $m^2/9+mk/3+k^2/4+3nk+2mn+9n^2$

$=m^2/9+k^2/4+9n^2++mk/3+3nk+2mn$ [Rearranging the terms]

$=(m/3)^2+(k/2)^2+(3n)^2+2\times(m/3)\times(k/2)+2\times(m/3)\times(3n)+2$
$\times(k/2)\times(3n)$

$=(m/3+k/2$
$+3n)^2\left[U\operatorname{sing}a^2+b^2+c^2+2ab+2bc+2ca=(a+b+c)^2\right]$

94. Expand the following using the identity $(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca:$

(i) $(p+3q+7r)^2$

(ii) $(3x-2y+4z)^2$

Ans. : (i) Here, a = p, b = 3q, c = 7r

Using the identity:

$(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$

So, $(p+3q+7r)^2$

$=p^2+(3q)^2+(7r)^2+2(p)(3q)+2(3q)(7r)+2(p)(7r)$

$=p^2+9q^2+49r^2+6pq+42qr+14pr$

(ii) Write it as:

$[3x+(-2y)+4z]^2$

Here,

a = 3x, b = −2y, c = 4z

Using the identity:

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

So, $(3x-2y+4z)^2$

$=(3x)^2+(-2y)^2+(4z)^2+2(3x)(-2y)+2(-2y)(4z)+2(3x)(4z)$

$=9x^2+4y^2+16z^2-12xy-16yz+24xz$

95. Fill in the blanks to complete the following identities:

(i) $s^2-11s+24=0$ (__________) (__________)

(ii) (__________) $(x+1)=\left(3x^2-4x-7\right)$

Ans. : (i) We need two numbers whose:

Sum = 11 and Product = 24

These are 3 and 8.

So, $s^2-11s+24$

$=s^2-8s-3s+24$

= s(s – 8) – 3(s – 8)

= (s – 3)(s – 8)

So, $s^2-11s+24==(s-3)(s-8)$

(ii) RHS: $3x^2-4x-7$

$=3x^2-7x+3x-7$

= x(3x – 7) + 1(3x – 7)

= (3x – 7)(x + 1)

So, $(3x-7)(x+1)=\left(3x^2-4x-7\right)$

96. Simplify the following rational expression assuming that the expression in the denominator are not equal to zero:

$\frac{w^3-v^3+x^3+3wvx}{w^2+v^2+x^2-2wv-2vx+2wx}$

Ans. : Factorising the numerator:

$w^3-v^3+x^3+3wvx$

$=w^3+(-v)^3+x^3-3w(-v)x$

$=(w-v+x)\left(w^2+v^2+x^2+wv+vx-wx\right)$

$\left[\right.$ Using $\left.a^3+b^3+c^3-3abc=(a+b+c)\left(a^2+b^2+c^2-ab-bc-ca\right)\right]$

Now denominator:

$w^2+v^2+x^2-2wv-2vx+2wx$

$=w^2+(-v)^2+x^2+2w(-v)+2(-v)x+2xw)$

$=(w-v+x)^2\left[U\operatorname{sing} a^2+b^2+c^2+2ab+2bc+2ca=(a+b+c)^2\right]$

Therefore, $\left(w^3-v^3+x^3+3wvx\right)/\left(w^2+v^2+x^2-2wv-2vx+2wx\right)$

$=\left[(w-v+x)\left(w^2+v^2+x^2+wv+vx-wx\right)\right]/(w-v+x)^2$

$=\left(w^2+x^2+v^2-wx+vx-wv\right)/(w+x-v)$ [Canceling the common factor]

97. If $a+b+c=5$ and $ab+bc+ca=10$, then prove that $a^3+b^3+c^3-3abc=-25$.

Ans. : a + b + c = 5

ab + bc + ca = 10

First finding $a^2+b^2+c^2$ using:

$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

So, $5^2=a^2+b^2+c^2+2(10)$

$\Rightarrow 25=a^2+b^2+c^2+20$

$\Rightarrow a^2+b^2+c^2=5$

Now, $a^2+b^2+c^2-ab-bc-ca=5-10=-5$

Using the identity: $a^3+b^3+c^3-3abc=(a+b+c)\left(a^2+b^2+c^2-ab-bc-ca\right)$

$a^3+b^3+c^3-3abc$

= (5)(−5)

= −25

Hence proved $a^3+b^3+c^3-3abc=-25$.

98. By factoring the expression, check that n^3-n is always divisible by 6 for all natural numbers n. Give reasons.

Ans. : We have: n^3-n

$=n\left(n^2-1\right)$

= n(n − 1)(n + 1)

So, $n^3-n=n(n-1)(n+1)$

Thus, n^3-n is the product of three consecutive natural numbers: $(n-1), n,(n+1)$

Among any three consecutive natural numbers:

- one is always divisible by 3
- at least one is always even, so divisible by 2

Therefore, their product is always divisible by: 2 × 3 = 6

Hence, n^3-n is always divisible by 6 for all natural numbers n .

*** Answer the following questions. [5 Marks Each]** **[10]**

99. Is this an identity?

$(a+b-c)^2+(a-b+c)^2+(a-b-c)^2=2a^2+2b^2+2c^2$

Ans. : To check whether this is an identity, expand the left-hand side.

First Part: $(a+b-c)^2$

$=a^2+b^2+c^2+2ab-2ac-2bc$

Second Part: $(a-b+c)^2$

$=a^2+b^2+c^2-2ab+2ac-2bc$

Third Part: $(a-b-c)^2$

$=a^2+b^2+c^2-2ab-2ac+2bc$

Now adding all three:

$LHS=\left(a^2+b^2+c^2+2ab-2ac-2bc\right)$

$+\left(a^2+b^2+c^2-2ab+2ac-2bc\right)$

$+\left(a^2+b^2+c^2-2ab-2ac+2bc\right)$

$=3a^2+3b^2+3c^2-2ab-2ac-2bc$

This is NOT equal to $2a^2+2b^2+2c^2$

Hence, the given statement is not true for all values of a, b and c.

So, it is not an identity.

100. Find the value of

$x^3 - 8y^3 - 36xy - 216$, when $x = 2y + 6$

Ans. : Given: x − 2y + 6 = 0

So, x − 2y = −6

Now use the identity: $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

We have: $x^3 - 8y^3 - 36xy - 216$

$= x^3 - 8y^3 + 3(x)(-2y)(-6) + (-6)^3$

So it matches: $a^3 + b^3 + c^3 - 3abc$

with a = x, b = −2y, c = −6

Using identity: $a^3 + b^3 + c^3 - 3abc$

$= (a+b+c)\left(a^2 + b^2 + c^2 - ab - bc - ca\right)$

Thus, $x^3 - 8y^3 - 36xy - 216$

$= x^3 - 8y^3 - 216 - 36xy$

$= x^3 + (-2y)^3 + (-6) - 3(x)(-2y)(-6)$

$= (x - 2y - 6)\left[x^2 + 4y^2 + 36^2 + 2xy - 12y + 18x\right]$

But from the question:

x − 2y + 6 = 0, which gives x − 2y = −6, so x − 2y − 6 = −12, not zero.

So this expression does not become 0 under the given condition as written.

Using x = 2y − 6 from the condition, Substitute into the expression:

$x^3 - 8y^3 - 36xy - 216$

$= (2y-6)^3 - 8y^3 - 36(2y-6)y - 216$

$= \left(8y^3 - 72y^2 + 216y - 216\right) - 8y^3 - 72y^2 + 216y - 216$

$= -144y^2 + 432y - 432$

$= -144\left(y^2 - 3y + 3\right)$

Therefore, under the given condition, the value is $-144\left(y^2 - 3y + 3\right)$.

So, the expression does not reduce to a constant unless there is a typo in the question.

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