STD 9 Maths [Manjari]

Measuring Spaces: Perimeter and Area

*** Choose the right answer from the given options. [1 Marks Each]** **[37]**

1. The perimeter of a square with side 9 cm is:

 (A) 18 cm (B) 27 cm (C) 36 cm (D) 81 cm

 Ans. : (C) 36 cm

 Perimeter of a square = 4 × side = 4 × 9 = 36 cm.

2. The perimeter of a rectangle with length 14 cm and breadth 6 cm is:

 (A) 20 cm (B) 40 cm (C) 84 cm (D) 120 cm

 Ans. : (B) 40 cm

 Perimeter of rectangle = 2(length + breadth) = 2(14 + 6) = 40 cm.

3. The circumference of a circle with radius 14 cm is:

 (A) 44 cm (B) 66 cm (C) 88 cm (D) 176 cm

 Ans. : (C) 88 cm

 Circumference = 2πr = 2 × 22/7 × 14 = 88 cm.

4. The circumference of a circle is directly proportional to its:

 (A) Area (B) Diameter (C) Radius squared (D) Chord

 Ans. : (B) Diameter

 Circumference = πd, where d is the diameter.

5. The length of a semicircular arc of radius 21 cm is:

 (A) 33 cm (B) 44 cm (C) 66 cm (D) 132 cm

 Ans. : (C) 66 cm

 Semicircular arc length = πr = 22/7 × 21 = 66 cm.

6. The length of a quarter-circle arc of radius 14 cm is:

 (A) 11 cm (B) 22 cm (C) 44 cm (D) 88 cm

 Ans. : (B) 22 cm

 Quarter-circle arc length $= (1/4) \times 2\pi r = \pi r/2 = 22cm$.

7. The area of a rectangle with length 18 cm and breadth 7 cm is:

 (A) $25cm^2$ (B) $50cm^2$ (C) $126cm^2$ (D) $252cm^2$

 Ans. : (C) $126cm^2$

 Area $=$ length × breadth $= 18 \times 7 = 126cm^2$.

8. The area of a parallelogram with base 15 cm and height 8 cm is:

 (A) $60cm^2$ (B) $120cm^2$ (C) $240cm^2$ (D) $360cm^2$

Ans. : (B) $120cm^2$

Area of parallelogram $=$ base × height $= 15 \times 8 = 120cm^2$.

9. The area of a triangle with base 20 cm and height 9 cm is:

(A) $90cm^2$ (B) $120cm^2$ (C) $180cm^2$ (D) $360cm^2$

Ans. : (A) $90cm^2$

Area of triangle $= 1/2\times$ base × height $= 1/2 \times 20 \times 9 = 90cm^2$.

10. A median of a triangle divides it into:

(A) Two congruent triangles (B) Two triangles of equal area

(C) Two right triangles (D) Two similar triangles

Ans. : (B) Two triangles of equal area

A median divides a triangle into two triangles having equal areas.

11. The semi-perimeter of a triangle with sides 5 cm, 12 cm, and 13 cm is:

(A) 15 cm (B) 20 cm (C) 25 cm (D) 30 cm

Ans. : (A) 15 cm

Semi-perimeter = (5 + 12 + 13) / 2 = 15 cm.

12. Using Heron's formula, the area of a triangle with sides 5 cm, 12 cm, and 13 cm is:

(A) $30cm^2$ (B) $60cm^2$ (C) $78cm^2$ (D) $90cm^2$

Ans. : (A) $30cm^2$

Semi-perimeter $= 15$.

Area $\surd[15(15-5)(15-12)(15-13)]$

$\surd(15 \times 10 \times 3 \times 2)$

$= \sqrt{900}$

$= 30cm^2$.

13. A quadrilateral whose vertices lie on a circle is called:

(A) Rhombus (B) Trapezium

(C) Cyclic quadrilateral (D) Kite

Ans. : (C) Cyclic quadrilateral

A quadrilateral with all four vertices on the same circle is called a cyclic quadrilateral.

14. Brahmagupta's formula is used to find the area of a:

(A) Triangle (B) Circle

(C) Rectangle (D) Cyclic quadrilateral

Ans. : (D) Cyclic quadrilateral

Brahmagupta's formula gives the area of a cyclic quadrilateral using the lengths of its sides.

15. The value of π is approximately equal to:

(A) 2.14 (B) 3.14 (C) 4.13 (D) 22.7

Ans. : (B) 3.14

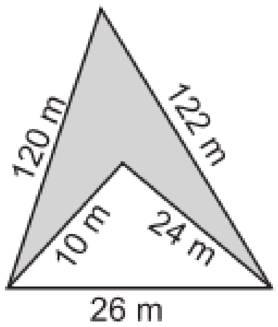
The approximate value of π is 3.14.

16. An isosceles right triangle has area $8cm^2$. The length of its hypotenuse is:

(A) $\sqrt{32}cm$ (B) $\sqrt{96}cm$ (C) $\sqrt{112}cm$ (D) $\sqrt{24}cm$

Ans.: (A) $\sqrt{32}cm$

17. Area of the shaded region in the given figure is:



(A) $1400m^2$ (B) $1500m^2$ (C) $1550m^2$ (D) $1440m^2$

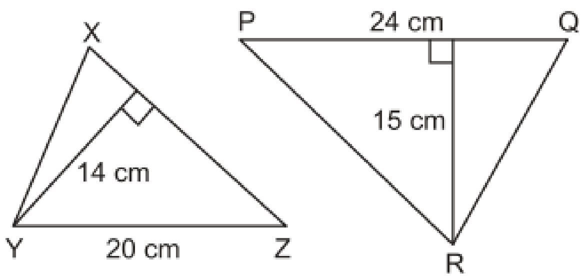
Ans. : (D) $1440m^2$

18. A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are $26cm, 28cm$ and 30 cm and the parallelogram stands on the base 28 cm , then height of the parallelogram is:

(A) 8 cm (B) 10 cm (C) 12 cm (D) 14 cm

Ans. : (C) 12 cm

19.



Two triangles are shown below.

Which of the following is true?

(A) Area of both the triangles can be calculated, area of $\triangle XYZ = 140cm^2$ and area of $\triangle PQR = 180cm^2$.

(B) Area of only triangle XYZ can be calculated. area of $\triangle XYZ = 140cm^2$

(C) Area of only triangle PQR can be calculated, area of $\triangle PQR = 180cm^2$

(D) Area of both the triangles cannot be calculated.

Ans. : (C) Area of only triangle PQR can be calculated, area of $\triangle PQR = 180 cm^2$

20. If a, b and c are the sides of a triangle and $s =$ semi perimeter, then is s is equal:

(A) $\frac{a+b+c}{2}$ (B) $2(a+b+c)$ (C) $2(a-b+c)$ (D) $\frac{a+b-c}{2}$

Ans.: (A) $\frac{a+b+c}{2}$

21. If each side of a triangle is doubled, then its area increased by:

(A) 100% (B) 200% (C) 300% (D) 350%

Ans. : (C) 300%

22. If the area of a sector of a circle is $\frac{7}{20}$ of the area of the circle, then the angle at the centre is equal to:

(A) 110° (B) 130° (C) 100° (D) 126°

Ans. : (D) 126°

23. If the difference between the circumference and radius of a circle is 37 cm , then circumference (in cm) of the circle is:

(A) 155 (B) 44 (C) 10 (D) 14

Ans. : (B) 44

24. If the length of an arc of a circle of diameter 84 cm , is 88 cm , then the angle subtended by the arc at the centre of the circle is:

(A) 120° (B) 90° (C) 60° (D) 30°

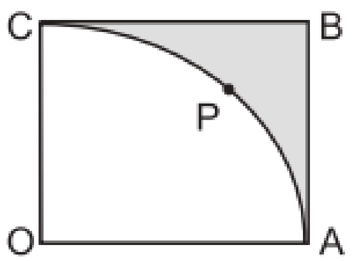
Ans.: (A) 120°

25. If the sides of a triangular board are $56 cm, 60 cm$ and 52 cm long, then its area is:

(A) $12.24 cm^2$ (B) $1344 cm^2$ (C) $1444 cm^2$ (D) $1554 cm^2$

Ans. : (B) $1344 cm^2$

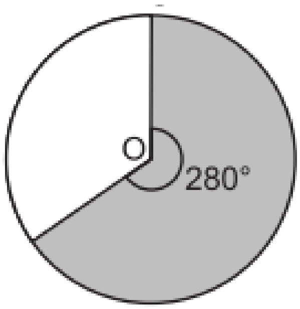
26. In the figure, OABC is a square of side 7 cm . If OAPC is a quadrant of a circle with centre O , then the area of the shaded region is:



(A) $\frac{17}{2} cm^2$ (B) $\frac{19}{2} cm^2$ (C) $\frac{21}{2} cm^2$ (D) $\frac{23}{2} cm^2$

Ans. : (C) $\frac{21}{2} cm^2$

27. In the figure, there is a circle with centre O . The shaded sector has an angle of 280° and area $A cm^2$. Which of these is the area of the unshaded sector?



(A) $\frac{2}{7} A cm^2$ (B) $\frac{1}{3} A cm^2$ (C) $\frac{2}{3} A cm^2$ (D) $\frac{7}{9} A cm^2$

Ans.: (A) $\frac{2}{7} A cm^2$

28. The area of an isosceles triangle having base 2 cm and the length of one of the equal sides 4 cm is:

(A) $\sqrt{18} cm^2$ (B) $\sqrt{15} cm^2$ (C) $\sqrt{12} cm^2$ (D) $\sqrt{8} cm^2$

Ans. : (B) $\sqrt{15} cm^2$

29. The area of a quadrant of a circle, where the circumference of circle is 44 cm is:

(A) $32.5 cm^2$ (B) $35.5 cm^2$ (C) $36.5 cm^2$ (D) $38.5 cm^2$

Ans. : (D) $38.5 cm^2$

30. The area of a triangle, two sides of which are 8 cm and 11 cm and the perimeter is $32 cm =$

(A) $7\sqrt{30} cm^2$ (B) $8\sqrt{30} cm^2$ (C) $9\sqrt{30} cm^2$ (D) $11\sqrt{30} cm^2$

Ans. : (B) $8\sqrt{30} cm^2$

31. The area of right-angled triangle with sides $17 cm, 15 cm$ and 8 cm is:

(A) $25 cm^2$ (B) $28 cm^2$ (C) $60 cm^2$ (D) $35 cm^2$

Ans. : (C) $60 cm^2$

32. The base of an isosceles triangle measures 24 cm and its area is $192 cm^2$. The length of one of equal side is:

(A) 20 cm (B) 22 cm (C) 24 cm (D) 28 cm

Ans.: (A) 20 cm

33. The base of a right triangle is 16 cm and hypotenuse is 34 cm . Its area will be:

(A) $240 cm^2$ (B) $180 cm^2$ (C) $160 cm^2$ (D) $150 cm^2$

Ans.: (A) $240 cm^2$

34. The perimeter of an equilateral triangle is 60 cm . Its height is:

(A) $10\sqrt{3} cm$ (B) $9\sqrt{3} cm$ (C) $\frac{10}{\sqrt{3}} cm$ (D) $20\sqrt{3} cm$

Ans.: (A) $10\sqrt{3}cm$

35. The sides of a triangle are $17cm, 25cm$ and 26 cm . The length of the altitude to the longest side correct upto two places of decimal is:

(A) 15.69 (B) 13.59 cm (C) 12.23 cm (D) 10.59 cm

Ans.: (A) 15.69

36. The sides of a triangle are in the ratio $5:12:13$ and its perimeter is 150 m . The area of the triangle is:

(A) $700m^2$ (B) $720m^2$ (C) $750m^2$ (D) $800m^2$

Ans. : (C) $750m^2$

37. The area of an equilateral triangle with side $2\sqrt{3}cm$ is:

(A) $2\sqrt{3}cm^2$ (B) $12\sqrt{3}cm^2$ (C) $4\sqrt{3}cm^2$ (D) $3\sqrt{3}cm^2$

Ans. : (D) $3\sqrt{3}cm^2$

*** A statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option.** **[7]**

Directions: In each question below, there are two statements labelled as Assertion (A) and Reason (R). Choose the correct answer as per the options given below:
(a) Both (A) and (R) are true and (R) is the correct explanation of (A)
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A)
(c) (A) is true but (R) is false
(d) (A) is false but (R) is true

38. **Assertion (A) :** A bicycle wheel makes 5000 revolutions in covering 11 km . Then diameter of the wheel is 35 cm .

Reason (R) : If a wire of length 22 cm is bent in the shape of a circle, then area of the circle so formed is $38.5cm^2$.

Ans. : D

39. **Assertion (A) :** A chord of a circle of radius 14 cm makes a right angle at the centre.The area of the minor segment is $56cm^2$.

Reason (R) : Angle described by the minute hand in 5 minutes $= 6^\circ$

Ans. : C

40. **Assertion (A) :** Area of rhombus with each edge 6 cm and one diagonal of length 6 cm is $18\sqrt{3}cm^2$.

Reason (R) : Area of equilateral triangle $= \frac{\sqrt{3}}{4}(\text{ side })^2$.

Ans. : A

41. **Assertion (A) :** If the circumference of two circles are in the ratio $2:3$, then ratio of their areas is $4:9$.

Reason (R) : The circumference of a circle of radius r is $2\pi r$ and its area is πr^2.

Ans. : A

42. **Assertion (A) :** The area of a triangle whose sides are $13cm, 14cm$ and 15 cm is $84cm^2$.

Reason (R) : Area of triangle $= \frac{1}{2} \times$ base × height

Ans. : B

43. **Assertion (A) :** The area of a triangle with base 4 cm and height 6 cm is $12cm^2$.

Reason (R) **:** Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

Ans. : B

44. **Assertion (A) :** The semi perimeter of a triangle is 30 cm , with sides $20cm, 25cm$ and 15 cm.

Reason (R) : Semi perimeter $= \frac{a+b-c}{2}$

Ans. : C

*** Answer the following short questions. [2 Marks Each]** **[50]**

45. The perimeter of a circle is 44 cm. What is its radius?

Ans. : Perimeter of circle = Circumference = 44 cm

Formula for circumference C = 2πr

$\Rightarrow 44 = 2 \times 22/7 \times r[$ Using $\pi = 22/7]$

$\Rightarrow 44 = 44r/7$

$\Rightarrow r = 44 \times 7/44$

⇒$\Rightarrow r = 7cm$

Therefore, the radius of the circle is 7 cm.

46. Find the area of a sector of a circle with radius 7 cm if the angle of the sector is 60°.

Ans. : Area of sector $= \theta/360 \times \pi r^2$

Here,

$r = 7cm$

$\theta = 60^\circ$

Area $= 60/360 \times 22/7 \times 7^2$

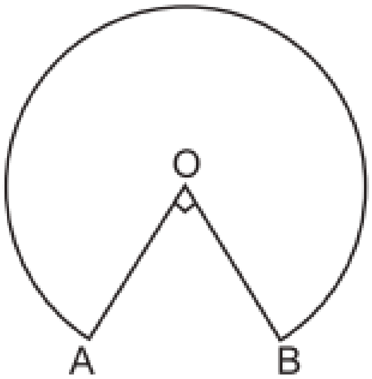
$= 1/6 \times 22/7 \times 49$

$= 77/3cm^2$

Therefore, area of the sector $= 77/3cm^2$.

47. In the given figure, the shape of the top of a table is that of a sector of a circle with centre O and $\angle AOB = 90^\circ$. If $AO = OB = 42cm$,

then find the perimeter of the top of the table. $\left[\text{Use } \pi = \frac{22}{7}\right]$.



Ans. :

We have, $r = 42cm, \theta = (360^\circ - 90^\circ) = 270^\circ$

Length of major arc $= 2\pi r \times \frac{\theta}{360^\circ} = 2 \times \frac{22}{7} \times 42 \times \frac{270}{360} cm = 198cm$

Now, perimeter of the top of the table $=$ length of the major arc $+2r$

$= [198 + 2(42)]cm = (198 + 84)cm = 282cm$.

48. The minute hand of a wallclock is 14 cm . Find the area swept by the minute hand in 1 minute.

Ans. : Angle described by minute hand in 60 minutes $= 360^\circ$

⇒ Angle described by minute hand in 1 minute $= \frac{360^\circ}{60} = 6^\circ$

$\therefore \theta = 6^\circ$ and radius, $r = 14cm$

Now, area swept by the minute hand in 1 minute $= \pi r^2 \times \frac{\theta}{360^\circ}$

$= \frac{22}{7} \times 14 \times 14 \times \frac{6}{360} cm^2 = 10\frac{4}{15} cm^2$

49. The circumference of a circle is 132 cm. Find its area.

Ans. : Let r be the radius of the circle.

Then, $2\pi r = 132$

$\Rightarrow 2 \times \frac{22}{7} \times r = 132 \Rightarrow r = \frac{132 \times 7}{2 \times 22} = 21cm$

Area of the circle $= \pi r^2$

$= \frac{22}{7} \times 21 \times 21cm^2 = 1386cm^2$.

50. The area of a circle is $154cm^2$. Find its diameter.

Ans. : Let r be the radius of the circle.

Then, $\pi r^2 = 154$

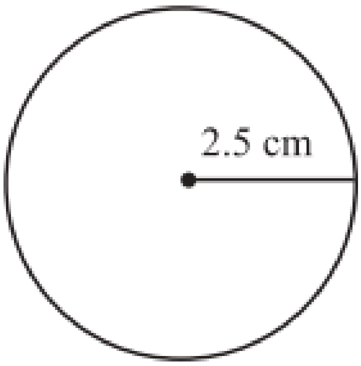
$\Rightarrow \frac{22}{7} \times r^2 = 154 \Rightarrow r^2 = \frac{154 \times 7}{22} = 49 \Rightarrow r \times r = 7 \times 7$

$\Rightarrow r = 7cm$

∴ Diameter of the circle $= (2 \times 7)cm = 14cm$.

51. How many bangles with radius 2.5 cm can be built from 44 m long golden string?

Ans. :



Radius of a bangle $= r = 2.5cm = \frac{5}{2}cm$

So, circumference of 1 bangle $= 2\pi r$

$= 2 \times \frac{22}{7} \times \frac{5}{2}cm = \frac{110}{7}cm$.

Thus, $\frac{110}{7}cm$ long string is needed for 1 bangle.

So, 44 m or 4400 cm long string will make $= \frac{7}{110} \times 4400$ bangles $= 280$ bangles.

52. Diameter of a circular park is 105 m. How much time will Akhilesh take cover five rounds of the park, if he walks at a speed of 3 km/hour.

Ans. : Circumference $= \pi \times d = \frac{22}{7} \times 105m = 330m$

Distance walked in 1 round $= 330m$

Distance walked in 5 rounds $= 330m \times 5 = 1650m$.

Time taken to cover 3 km or $3000m = 1$ hour $= 60$ minutes

Time taken to cover $1650m = \frac{60 \times 1650}{3000}$ minutes $= 33$ minutes.

53. The semi-perimeter of a triangle is 132 cm. The product of the differences of semi-perimeter and its respective sides is 13200 cm^3. Find the area of the triangle.

Ans. : We have, $s = 132cm$ and $(s-a)(s-b)(s-c) = 13200cm^3$

Now, area of triangle $= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{132 \times 13200}cm^2$

$= \sqrt{132 \times 132 \times 100}cm^2 = 132 \times 10cm^2 = 1320cm^2$.

54. A rectangle 10 cm $\times$ 6 cm, find the area by using Brahmagupta's Formula.

Ans. : A rectangle $10cm \times 6cm$, find the area by using Brahmagupta's Formula.

Given : a = 10 cm, b = 6 cm, c = 10 cm, d = 6cm

$s = \frac{32}{2} = 16$

Area $= \sqrt{(16-10)(16-6)(16-10)(16-6)}\ cm^2$

$= \sqrt{(6)(10)(6)(10)}cm^2 = \sqrt{3600}cm^2 = 60cm^2$

55. A square of side 6 cm, find the area of square by using Brahmagupta's Formula.

Ans. : Given : a = b = c = d = 6cm

$s = \frac{24}{2} = 12cm$

Area $= \sqrt{(12-6)(12-6)(12-6)(12-6)}\ cm^2 = \sqrt{(12-6)^2 \times (12-6)^2}\ cm^2$

$= \sqrt{36 \times 36} cm^2 = 36 cm^2$

This matches the known area : $6 \times 6 = 36 cm^2$

56. Find the area of the quadrilateral if, sides are 4 cm, 5 cm, 7 cm and 8 cm.

Ans. : $s = \frac{4+5+7+8}{2} = \frac{24}{2} = 12 cm$

Area $= \sqrt{(12-4)(12-5)(12-7)(12-8)}\ cm^2$

$= \sqrt{(8)(7)(5)(4)} cm^2 = \sqrt{1120} cm^2 \approx 33.47 cm^2$

57. Find the area of a cyclic quadrilateral with sides: a = 5 cm, b = 6 cm, c = 7 cm and d = 8 cm.

Ans. : Semi-perimeter $s = \frac{5+6+7+8}{2} = \frac{26}{2} = 13 cm$

Area $= \sqrt{(13-5)(13-6)(13-7)(13-8)}\ cm^2$

$= \sqrt{(8)(7)(6)(5)} cm^2 = \sqrt{1680} cm^2 = 41 cm^2$ (approx.)

58. The diameter of a circular garden is 220 m . Find the area of the garden. [Take $\pi = 3.14$]

Ans. : 37,994 sq. m

59. Find the radius of a circle whose area is $1386 cm^2$.

Ans. : 21 cm

60. The circumference of a circle is 110 cm. Find its area.

Ans. : 962.5 sq. cm

61. The ratio of radii of the two circles is 3: 5. Find the ratio of their areas.

Ans. : $9 : 25$

62. A circle is inscribed in a square of side 28 cm. Find the area of the circle.

Ans. : 616 sq. cm

63. Find the area of a triangle whose sides are 150 cm, 120 cm and 200 cm long.

Ans. : $8966.56 cm^2$

64. Find the area of a rectangle of dimensions 10 cm $\times$ 6 cm using Brahmagupta's formula.

Ans. : 60 sq. cm.

65. Find the area of a square of side 15 cm using Brahmagupta's formula.

Ans. : 225 sq. cm

66. A rectangle is always cyclic. Verify Brahmagupta's formula for a 15 cm $\times$ 8 cm rectangle.

Ans. : 120 sq. cm

67. A cyclic quadrilateral has sides 10 cm, 12 cm, 8 cm and 6 cm. Find its area.

Ans. : $24\sqrt{10} cm^2$

68. Sides of a cyclic quadrilateral are 5 cm, 5 cm, 8 cm and 8 cm. Find the area.

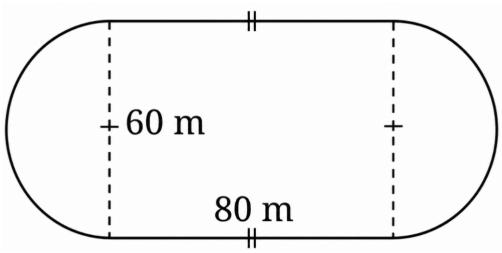
Ans. : 40 sq. cm

69. Find the area of a cyclic quadrilateral with sides 6 cm, 8 cm, 9 cm and 7 cm.

Ans. : $36\sqrt{5}cm^2$

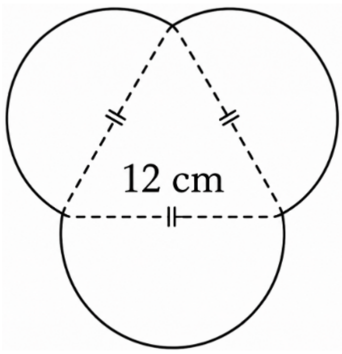
*** Answer the following questions. [3 Marks Each]** **[120]**

70. Find the perimeters of the following shapes (taking the arcs to be quarter or half or three-quarters of a circle, as appropriate).



Ans. : The shape has:
Two straight parts of 80 m each
Two semicircles of diameter 60 m
So, Perimeter
= Two straight parts of 80 m each + Two semicircles of diameter 60 m
$= 80 + 80 + \pi \times 60$ [Two semicircles make one full circle]
$= 160 + (22/7 \times 60)$
= 348.57 m

71. Find the perimeters of the following shapes (taking the arcs to be quarter or half or three-quarters of a circle, as appropriate)

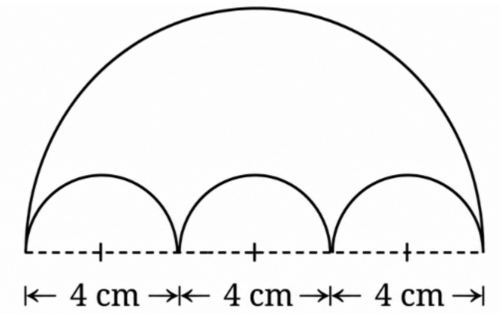


Ans. : The figure has 3 semicircles, each with diameter 12 cm.
Radius of each semicircle = 6 cm
Perimeter $= 3\times$ semicircle length
$= 3 \times \pi r$
$= 3 \times \pi \times 6$
$= 18\pi$
$= 18 \times 22/7$

= 396/7

= 56.57 cm

72. Find the perimeters of the following shapes (taking the arcs to be quarter or half or three-quarters of a circle, as appropriate).



Ans. : For Large semicircle:

Diameter = 12 cm

Radius = 6 cm

There are 3 small semicircles, each with diameter 4 cm.

Radius of each small semicircle = 2 cm

Perimeter = large semicircle + 3 small semicircles

$= \pi(6) + 3 \times \pi(2)$

$= 6\pi + 6\pi$

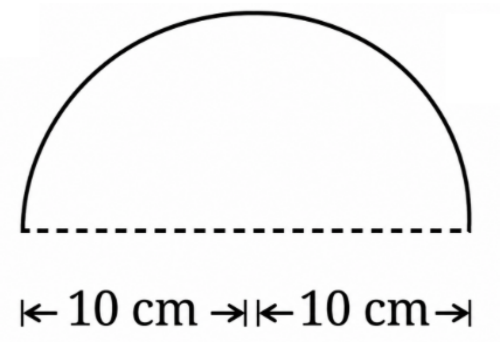
$= 12\pi$

$= 12 \times 22/7$

$= 264/7$

= 37.71 cm

73. Find the perimeters of the following shapes (taking the arcs to be quarter or half or three-quarters of a circle, as appropriate).



Ans. : For Large semicircle:

Diameter = 20 cm

Radius = 10 cm

There are 2 small semicircles, each with diameter 10 cm.

Radius of each small semicircle = 5 cm

Perimeter = large semicircle + 2 small semicircles

$= \pi(10) + 2 \times \pi(5)$

$= 10\pi + 10\pi$

$= 20\pi$

$= 20 \times 22/7$

= 440/7

= 62.86 cm

74. The sides of a triangular plot are in the ratio 3 : 5 : 7; its perimeter is 300 m. Find its area.

Ans. : Let the sides be 3x, 5x and 7x.

Perimeter: $3x + 5x + 7x = 300$

$\Rightarrow 15x = 300$

$\Rightarrow x = 20$

Now, the sides of triangle are 60 m, 100 m and 140 m

So, the semi-perimeter s = 300/2 = 150 m

Using Heron's formula:

Area $= \sqrt{[150(150-60)(150-100)(150-140)]}$

$= \sqrt{[150 \times 90 \times 50 \times 10]}$

$= \sqrt{6750000}$

$= 1500\sqrt{3}m^2$

Therefore, area $= 1500\sqrt{3}m^2$.

75. O is any point on the diagonal PR of a parallelogram PQRS. Prove that the areas of triangles PSO and PQO are equal.

Ans. : In parallelogram PQRS, diagonal PR is drawn.

Point O lies on PR.

Triangles PSO and PQO have the same base PO.

Also, S and Q lie on opposite sides of PR in the parallelogram.

Their perpendicular distances from line PR are equal.

So, triangles PSO and PQO have:

- Same base PO
- Equal heights

  Therefore, area $(\triangle PSO) = \text{area}(\triangle PQO)$

  Hence proved.

76. The sides of a quadrilateral ABCD are 6 cm, 8 cm, 12 cm and 14 cm (taken in order) respectively, and the angle between the first two sides is a right angle. Find its area.

Ans. : $24(\sqrt{6}+1)cm^2$

77. A advertising board is in the form of an isosceles triangle with its sides $12m, 10m$ and 10 m long. Find the cost of painting it at ₹2 per sq. metre.

Ans. : 96

78. The perimeter of a triangular field is 240 dm . If two of its sides are 50 dm and 78 dm , find the length of the perpendicular on the side of length 50 dm from the opposite vertex. Also calculate the cost of watering it at ₹ 3.50 per $100 sqdm.$.

Ans. : 67.2 dm; ₹58.80

79. One side of an equilateral triangle is 8 cm. Find its area using Heron's formula. Also, find the altitude.

Ans. : $16\sqrt{3} cm^2, 4\sqrt{3} cm$

80. Two sides of a triangular field are 85 m and 154 m in length and perimeter is 324 m. Find the area of the triangle.

Ans. : $2772 m^2$

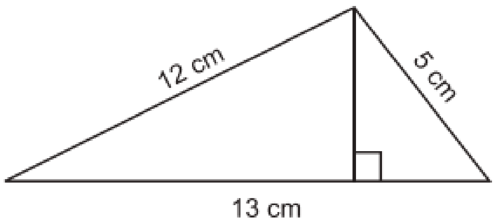
81. Find the length of hypotenuse of an isosceles right angled triangle, having an area of $200 cm^2$ (Take $\sqrt{2} = 1.414$)

Ans. : 28.28 cm

82. The area of a rhombus is $240 cm^2$ If one of the diagonals is 30 cm, find the length of the other diagonal.

Ans. : 16 cm

83. Show that the sides 5 cm, 12 cm and 13 cm make a right angled triangle. Find the length of the altitude on the longest side.

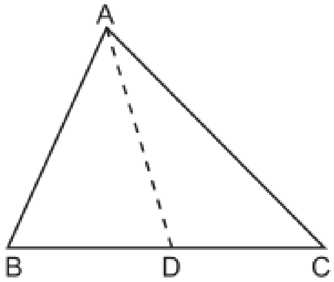


Ans. : $4\frac{8}{13} cm$ or 4.6 cm

84. Find the area of a right triangle whose one side is 5 cm and hypotenuse is 13 cm.

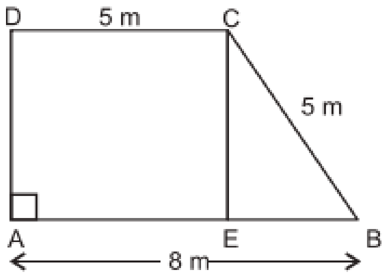
Ans. : 30 sq. cm

85. In the figure, AD is the median of the triangle. If area of $\triangle ADC = 24$ cm^2 and BC = 15 cm, find the height of the triangle ABC corresponding to the base BC.



Ans. : 6.4 cm

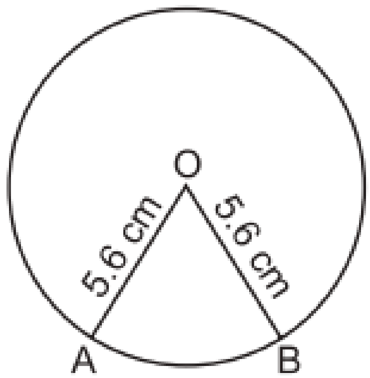
86. In the given figure, ABCD is a quadrilateral in which AB || DC, and AD $\perp$ AB. Also AB = 8 m, DC = 5 m. Find the area of the quadrilateral ABCD in two ways :
(i) taking ABCD as a trapezium
(ii) taking quadrilateral ABCD as sum of rectangle AECD and triangle BEC.



Ans. : $26cm^2$

87. The perimeter of a sector of a circle of radius 5.6 cm is 27.2. cm. Find the area of the sector.

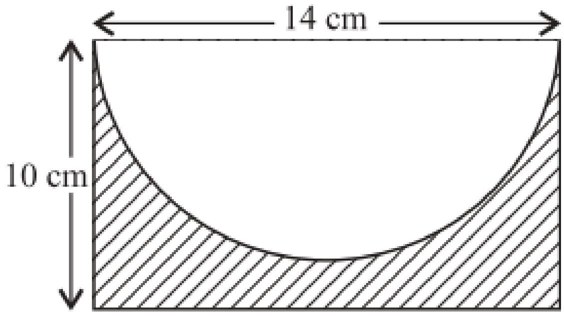
Ans. :



Let OAB be the given sector.
Let r be the radius of the given sector. Then $r=5.6cm$
Given, perimeter of sector $OABO=27.2cm$
$\Rightarrow OA+OB+\operatorname{arc} AB=27.2cm$
$\Rightarrow (5.6+5.6)cm+\operatorname{arc} AB=27.2cm$
$\Rightarrow \operatorname{arc} AB=16cm$
Let l be the length of $\operatorname{arc} AB$. Then, $l=16cm$
Area of sector $OABO=\frac{1}{2}lr=\frac{1}{2}\times16cm\times5.6cm=44.8cm^2$
Hence, the area of the sector is $44.8cm^2$.

88. In the figure, find the area of the shaded portion.

Ans. :



Area of the rectangle $= 14 \times 10 cm^2 = 140 cm^2$

Also, radius of the semi-circle $= \frac{14}{2} cm = 7 cm$

$\therefore$ Area of the semi-circle $= \frac{1}{2} \pi r^2$

$= \frac{1}{2} \pi \times 7 \times 7 cm^2$

$= \frac{1}{2} \times \frac{22}{7} \times 49 = 77 cm^2$

Area of the shaded portion $=$ Area of the rectangle - Area of the semi-circle $= (140 - 77) cm^2 = 63 cm^2$.

89. Shazli took a wire of length 44 cm and bent it into the shape of a circle. Find the radius of that circle. Also, find its area. If the same wire is bent into the shape of a square, what will be the length of each of its sides? Which figure encloses more area, the circle or the square ? (Take $\pi = \frac{22}{7}$)

Ans. : Circumference of the circle $= 2\pi r$

$\Rightarrow 44 = 2 \times \frac{22}{7} \times r$

$\Rightarrow r = \frac{44 \times 7}{2 \times 22} = 7 cm$

Hence, radius of the circle $= 7 cm$.

Area of the circle $= \pi r^2$

$= \frac{22}{7} \times 7 \times 7 cm^2 = 154 cm^2$

If the wire is bent into a square, then perimeter of the square $= 44 cm$

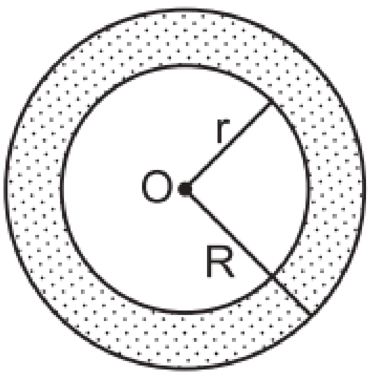
⇒ side of the square $= \frac{44}{4} cm = 11 cm$

Area of the square $= 11^2 cm^2 = 121 cm^2$.

Since $154 cm^2 > 121 cm^2$, so the circle encloses more area.

90. A race track is in the form of a ring whose inner and outer circumferences are $437 m$ and $503 m$ respectively. Find the width of the track.

Ans. :



Let the inner and outer radii of the circular track be r and R respectively. Then

$2\pi r = 437m$ and $2\pi R = 503m$

$\Rightarrow 2 \times \frac{22}{7} \times r = 437m$ and $2 \times \frac{22}{7} \times R = 503m$

$\Rightarrow r = \frac{437\times 7}{2\times 22}m$ and $R = \frac{503\times 7}{2\times 22}m$

Width of the circular track $= R - r$

$= \frac{503\times 7}{2\times 22} - \frac{437\times 7}{2\times 22} = \frac{7}{2\times 22} \times (503 - 437) = \frac{7}{2\times 22} \times 66 = \frac{21}{2} = 10.5$

Hence, the width of the track is 10.5 m .

91. The diameter of a wheel of a car is 63 cm . Find the distance travelled by the car during the period in which the wheel makes 1000 revolutions.

Ans. : Diameter of the wheel $= 63cm$

⇒ Radius of the wheel $= \left(\frac{63}{2}\right) cm$ circumference $= 2\pi r$

Circumference $= \left(2 \times \frac{22}{7} \times \frac{63}{2}\right) = 198cm = \frac{198}{100}m = 1.98m$

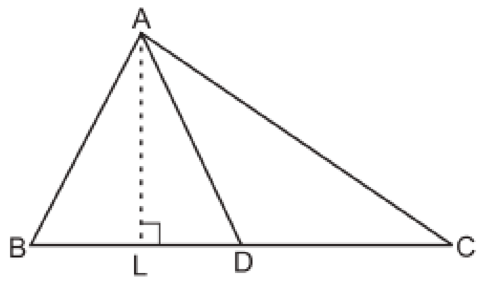
Distance covered by the wheel in 1 revolution $= 1.98m$

Distance covered by the wheel in 1000 revolutions $= (1.98 \times 1000)m = 1980m$

Hence, the distance covered by the car $= 1980m$.

92. Prove that a median divides a triangle into two triangles of equal area.

Ans. :



Given a $\triangle ABC$ in which AD is a median, i.e.,

$BD = DC$

We have to prove area of $\triangle ABD =$ area of $\triangle ADC$.

Draw $AL \perp BC$.

Area of $\triangle ABD = \frac{1}{2} \times BD \times AL$ [Area of a triangle $= \frac{1}{2} \times$ base × height]

$= \frac{1}{2} \times DC \times AL \quad [BD = DC]$

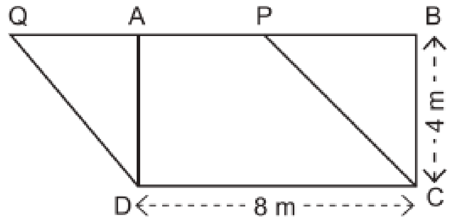
$=$ area of $\triangle ADC$

$\therefore$ area of $\triangle ABD =$ area of $\triangle ADC = \frac{1}{2}$ area of $\triangle ABC$.

Hence, we can say that the median of a triangle divides it into two triangles of equal area.

93. ABCD is a rectangle 8 m long and 4 m broad. P is the mid-point of AB. BA is produced to Q such that AQ = BP. Find the area of DCPQ.

Ans. :



$PQ = PA + AQ$

$= PA + BP$ [Since $AQ = BP$]

$= AB = CD$

PQ is parallel and equal to CD.

So, PQDC is a parallelogram.

So, area of parallelogram DCPQ $= Base \times Height$

$= CD \times AD = 8m \times 4m = 32m^2$.

94. The base of an isosceles triangle is 24 cm and its area is $192cm^2$. Find its perimeter.

Ans. : Let each equal side be a cm.

Given : Base, b = 24cm

$\therefore$ Area $= \frac{1}{4}b\sqrt{4a^2 - b^2} = \frac{1}{4} \times 24 \times \sqrt{4a^2 - 576} = 12 \times \sqrt{a^2 - 144}$

$\therefore 12 \times \sqrt{a^2 - 144} = 192 \Rightarrow \sqrt{a^2 - 144} = 16$

$\Rightarrow a^2 - 144 = 256 \Rightarrow a^2 = 400 \Rightarrow a = 20$.

Thus, $a = 20cm$ and $b = 24cm$.

Hence, perimeter $= (2a + b)cm = (2 \times 20 + 24)cm = 64cm$.

95. The minute hand of a clock is 14 cm long. Find the area on the face of the clock described by the minute hand in 5 minutes.

Ans. : $51.33 sq.cm$

96. The length of the minute hand of a clock is 5 cm . Find the area swept by the minute hand during the time period 6.05 am and 6.40 am .

Ans. : 45.83 sq. cm

97. A horse is placed for grazing inside a rectangular field 70 m by 52 m. It is tethered to one corner by a rope 21 m long. On how much area can it graze? How much area is left ungrazed?

Ans. : 346.5 sq. m, 3293.5 sq. m

98. The minute hand of a clock is $\sqrt{21} cm$ long. Find the area described by the minute hand on the face of the clock between 6 a.m. and 6.05 a.m.

Ans. : $5.5 cm^2$

99. A decorative window is made in the shape of a minor segment of a circle. The radius of the circle is 14 cm , and the central angle is 90°.
(i) Find the area of the corresponding sector.
(ii) Find the area of the triangle formed by the two radii.
(iii) Hence, find the area of the minor segment.

Ans. : (i) 154 sq. cm
(ii) $98 sq. cm$
(iii) $56 sq. cm$

100. In a circle of radius 21 cm , an arc subtends an angle of 60° at the centre. Find (i) the length of the arc, (ii) the area of the sector bounded by the arc and (iii) the area of the segment made by this arc.

Ans. : (i) 22 cm (ii) $231 cm^2$ (iii) $40.5 cm^2$

101. The perimeter of a sector of a circle of radius 7 cm is 25 cm. Find the area of the sector.

Ans. : 38.5 sq. cm

102. Semi-circular lawns are attached to both edges along the breadth of a rectangular lawn measuring 56 m by 35 m . Find the total area enclosed.[Hots]

Ans. : 2922.5 sq. m

103. Find the area of a ring whose inner and outer radii are 15 cm and 18 cm respectively.[Take $\pi=3.14$]

Ans. : 310.86 sq. cm

104. The length of the minute hand in the clock tower is 28 cm . Calculate the area, it covers in 30 minutes. Take $\pi=\frac{22}{7}$.[Hots]

Ans. : 1232 sq. cm

105. Find the circumference of the circle whose radius is:
(i) 21 cm
(ii) 28 cm
(iii) 56 cm

Ans. : (i) 132 cm
(ii) 176 cm
(iii) 352 cm

106. A rectangle and a parallelogram are drawn between the same parallel lines on a common base of 10 cm. If the perimeter of the rectangle is 36 cm. Find the area of the parallelogram.

Ans. : $80cm^2$

107. The lengths of the parallel sides of a trapezium are 56 cm, 40 cm and the length of the other sides are 28 cm and 30 cm. Find the area of the trapezium.

Ans. : $1260cm^2$

108. The parallel sides of a trapezium are 60 cm and 77 cm and the other sides are 25 cm and 26 cm. Find the area of the trapezium.

Ans. : $1644cm^2$

109. A farmer's field is in the shape of a quadrilateral. The four sides are: $25m, 26m$, 27 m and 28 m . a surveyor measures one diagonal as 35 m .
(i) Find the area of each triangle using Heron's formula.
(ii) Find the total area of the field.
(iii) If wheat is sown at ₹ 50 per m^2, calculate the total cost of sowing.

Ans. : (i) $324.45m^2, 371.08m^2$,
(ii) $695.53m^2$
(iii) ₹34,776.50

*** Questions with calculation. [4 Marks Each]** **[228]**

110. Calculate, correct to 3 significant figures, the circumference of a circle with: (i) radius 7 cm (ii) radius 10 cm (iii) radius 12 cm.

Ans. : (i) Radius r = 7 cm
We know that C = 2πr
$\Rightarrow C = 2 \times 22/7 \times 7$
= 44 cm
Therefore, circumference = 44.0 cm
(ii) Radius = 10 cm
We know that C = 2πr
$\Rightarrow C = 2 \times 22/7 \times 10$
= 440/7
= 62.857...
Correct to 3 significant figures: C = 62.9 cm
(iii) Radius = 12 cm
We know that C = 2πr
$\Rightarrow C = 2 \times 22/7 \times 12$
= 528/7
= 75.428...
Correct to 3 significant figures: C = 75.4 cm

111. Calculate the length of the arc of a circle if: (i) the radius is 3.5 cm and the angle at the centre is 60°, and (ii) the radius is 6.3 m and the angle at the centre is

120°.

Ans. : (i) Radius $= 3.5cm$, angle $= 60^{\circ}$

We know that Arc Length $L = \theta/360 \times 2\pi r$

$\Rightarrow L = 60/360 \times 2 \times 22/7 \times 3.5$

$= 1/6 \times 22$

$= 11/3cm$

= 3.67 cm

Therefore, arc length L = 3.67 cm

(ii) Radius $= 6.3m$, angle $= 120^{\circ}$

We know that Arc Length L $= \theta/360 \times 2\pi r$

$\Rightarrow L = 120/360 \times 2 \times 22/7 \times 6.3$

$= 1/3 \times 2 \times 22/7 \times 6.3$

$= 1/3 \times 39.6$

$= 13.2m$

Therefore, arc length = 13.2 m

112. Find the perimeter of a sector (i.e., the curved portion as well as the two straight portions) of a circle of radius 14 cm and sector angle 75°.

Ans. : Radius = 14 cm

Sector angle $= 75^{\circ}$

We know that the Perimeter of Sector = L + 2r

Now the Arc length $L = \theta/360 \times 2\pi r$

$= 75/360 \times 2 \times 22/7 \times 14$

$= 75/360 \times 88$

$= 5/24 \times 88$

$= 55/3cm$

Therefore the Perimeter $= 55/3 + 2 \times 14$

$= 55/3 + 28$

$= 55/3 + 84/3$

= 139/3 cm

= 46.33 cm

Therefore, the perimeter of the sector is 139/3 cm or 46.33 cm.

113. If the diameter of a car tyre is 56 cm, then: (i) How far does the car need to travel for the tyre to complete one revolution? (ii) How many revolutions does the tyre make if the car travels 10 km?

Ans. : (i) Distance in one revolution = Circumference of tyre

$C = 2\pi r$

$= 2 \times 22/7 \times 28$

$= 22 \times 8$

= 176 cm

Therefore, the car travels 176 cm in one revolution.

(ii) Total distance $= 10km = 10 \times 1000 \times 100 = 1,000,000cm$

Number of revolutions = Total distance / Distance per revolution

= 1,000,000 / 176

$= 5681.82 \approx 5682$ revolutions

Therefore, the tyre makes approximately 5682 revolutions.

114. The parallel sides of a trapezium are 40 cm and 20 cm. If its non-parallel sides are both equal, each being 26 cm, find the area of the trapezium.

Ans. : Difference of parallel sides = 40 − 20 = 20 cm

Since non-parallel sides are equal:

Half difference = $20/2 = 10cm$

Using Pythagoras theorem:

Height $^2 = 26^2 - 10^2$

= 676 − 100

= 576

⇒ Height = 24 cm

Area of trapezium

$= 1/2 \times$ sum of parallel sides × height

$= 1/2 \times (40 + 20) \times 24$

$= 30 \times 24$

$= 720cm^2$

Therefore, area $= 720cm^2$.

115. In $\triangle ABC$, the midpoint of BC is D (Fig.) Median AD is drawn. P is any point on AD. Show that area $\triangle ABP$ = area $\triangle ACP$.

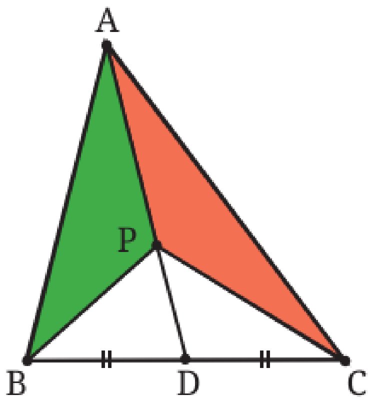


Fig.

Ans. : Given: D is the midpoint of BC.

So, BD = DC

AD is a median and P is any point on AD.

Join PB and PC.

Now consider triangle $\triangle PBD$ and $\triangle PCD$.

They have BD = DC and the same height from P to line BC.

Therefore, area $(\triangle PBD) = \text{area}(\triangle PCD) \ldots (1)$

Now consider triangles $\triangle ABD$ and $\triangle ACD$.

They have, BD = DC and the same height from A to line BC.

Therefore, area $(\triangle ABD) = \text{area}(\triangle ACD) \ldots (2)$

Subtracting equation (1) from (2), we get

$\text{area}(\triangle ABD) - \text{area}(\triangle PBD) = \text{area}(\triangle ACD) - \text{area}(\triangle PCD)$

$\Rightarrow \text{area}(\triangle ABP) = \text{area}(\triangle ACP)$

Hence proved.

116. A chord of a circle of radius 10 cm subtends 90° at the centre. Find the area of the corresponding: (i) minor sector that subtends 90° at the centre, and (ii) major sector that subtends 270° at the centre. (Use $\pi \approx 3.14$.)

Ans. : Radius = 10 cm

(i) Area of minor sector:

Area $= 90/360 \times \pi r^2$

$= 1/4 \times 3.14 \times 10^2$

$= 1/4 \times 314$

$= 78.5 cm^2$

Therefore, area of minor sector $= 78.5 cm^2$.

(ii) Area of major sector:

Area $= 270/360 \times \pi r^2$

$= 3/4 \times 3.14 \times 10^2$

$= 3/4 \times 314$

$= 235.5 cm^2$

Therefore, area of major sector $= 235.5 cm^2$.

117. A car has two wipers which do not overlap. Each wiper has a blade of length 28 cm and sweeps through an angle of 120°. Find the total area cleaned at each sweep of the blades.

Ans. : Each wiper cleans a sector of radius 28 cm and angle 120°.

Area cleaned by one wiper:

$= 120/360 \times \pi r^2$

$= 1/3 \times 22/7 \times 28^2$

$= 1/3 \times 22/7 \times 784$

$= 2464/3 cm^2$

Since there are two wipers and they do not overlap:

Total area cleaned:

$= 2 \times 2464/3$

$= 4928/3 cm^2$

Therefore, total area cleaned $= 4928/3 cm^2$.

118. A chord of a circle of radius r subtends an angle of 60° at the centre of the circle. Show that the area of the corresponding minor segment of the circle is equal to $r^2(\pi/6 - \sqrt{3}/4)$.

Ans. : Let O be the centre of the circle and AB be the chord.

Given: OA = OB = r

$\angle AOB = 60^\circ$

Since $OA = OB$ and $\angle AOB = 60^\circ$, triangle OAB is equilateral.

So, AB = r

Area of minor sector OAB:

$= 60/360 \times \pi r^2$

$= \pi r^2/6$

Area of equilateral triangle $OAB = \sqrt{3}/4 \times r^2$

Area of minor segment = Area of sector − Area of triangle

$= \pi r^2/6 - \sqrt{3}r^2/4$

$= r^2(\pi/6 - \sqrt{3}/4)$

Hence proved.

119. A hexagon is inscribed in a circle of radius r. Show that the ratio of the area of the hexagon to the area of the circle is equal to $3\sqrt{3/2\pi} \approx 0.827$. Can you see why the answer is exactly twice the answer to Question 8?

Ans. : A regular hexagon inscribed in a circle can be divided into 6 equilateral triangles.

Each equilateral triangle has side r, because the radius of the circle is r.

Area of one equilateral triangle $= \sqrt{3}/4 \times r^2$

Area of 6 such triangles:

$= 6 \times \sqrt{3}/4 \times r^2$

$= 3\sqrt{3}/2 \times r^2$

So, Area of hexagon $= (3\sqrt{3}/2)r^2$

Area of circle $= \pi r^2$

Therefore, Area of hexagon : Area of circle

$= \left[(3\sqrt{3}/2)r^2\right]/\pi r^2$

$= 3\sqrt{3}/2\pi$

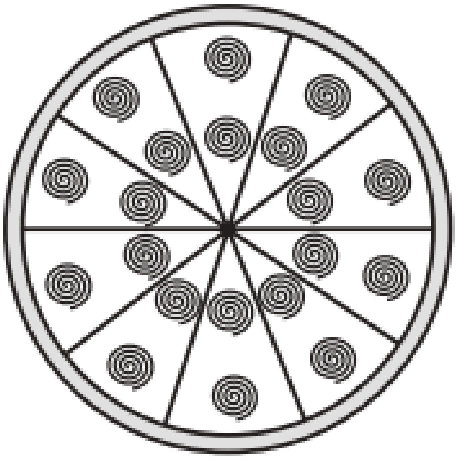
Hence, the ratio is $3\sqrt{3}/2\pi \approx 0.827$.

120. A brooch is made with silver wire in the form of a circle with diameter 35 mm . The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in the figure. Find

(i) the total length of the silver wire required.

(ii) the area of each sector of the brooch.

Ans. :



(i) We have, $r=\frac{35}{2} mm$

Now, total length of the silver wire

$=$ Circumference of the circle of radius $\frac{35}{2} mm+$ Length of five diameters

$=\left(2 \pi \times \frac{35}{2}+5 \times 35\right) mm=\left(2 \times \frac{22}{7} \times \frac{35}{2}+175\right) mm=285 mm$

(ii) The circle is divided into 10 equal sectors. Therefore,

Area of each sector of the brooch $=\frac{1}{10}$ (Area of the circle)

$=\left[\frac{1}{10} \times \pi \times\left(\frac{35}{2}\right)^2\right] mm^2=\left(\frac{1}{10} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2}\right) mm^2=\frac{385}{4} mm^2$.

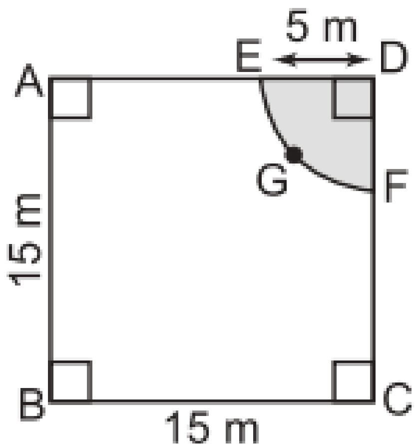
121. A horse is tied to a peg at one corner of a square shaped grass field of side $15 m$ by means of a $5 m$ long rope. Find

(i) The area of that part of the field in which the horse can graze.

(ii) The increase in the grazing area if the rope were 10 m long instead of 5 m .

[Take $\pi=3.14$]

Ans. :



Let $ABCD$ be the grass field and horse is tied at the corner D.

We have, $r=5 m$ and $\theta=90^{\circ}$

(i) Required area $=$ Area of quadrant EGFDE

$=\frac{1}{4} \times \pi r^2=\left(\frac{1}{4} \times 3.14 \times 5^2\right) m^2=19.625 m^2$.

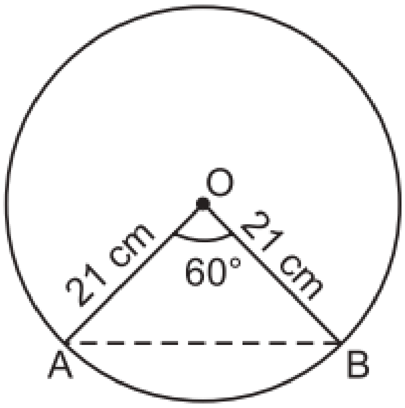
(ii) When rope is 10 m long, i.e., radius of the quadrant $=10 m$

$\therefore$ Area that can be grazed by the horse $= \frac{1}{4} \times 3.14 \times 10^2 m^2 = 78.5 m^2$

Now, increase in grazing area $= 78.5 m^2 - 19.625 m^2 = 58.875 m^2$.

122. In a circle of radius 21 cm , an arc subtends an angle of 60° at the centre. Find:
(i) the length of the arc. (ii) area of the sector formed by the arc,
(iii) area of the segment formed by the corresponding chord.

Ans. :



We have, $r = 21 cm$ and $\theta = 60^\circ$

Now, (i) Length of the arc $= 2\pi r \times \frac{\theta}{360^\circ} = \pi r \times \frac{\theta}{180^\circ}$

$= \left(\frac{60^\circ}{180^\circ} \times \frac{22}{7} \times 21\right) cm = 22 cm$

(ii) Area of the sector formed by the $\text{arc} = \pi r^2 \times \frac{0}{360^\circ}$

$= \left(\frac{22}{7} \times 21 \times 21 \times \frac{60^\circ}{360^\circ}\right) cm^2 = \frac{22 \times 21 \times 3}{6} cm^2 = 21 \times 11 cm^2 = 231 cm^2$

(iii) Area of segment formed by the corresponding chord

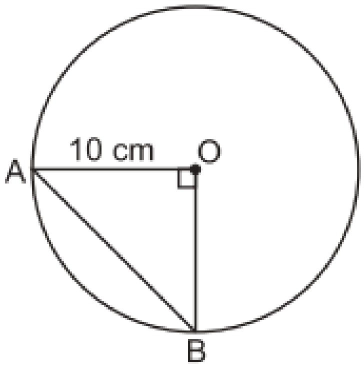
$=$ Area of sector - Area of equilateral $\triangle OAB$

$= 231 cm^2 - \frac{\sqrt{3}}{4}(21)^2 cm^2 = \left(231 - \frac{\sqrt{3}}{4} \times 21 \times 21\right) cm^2$.

$= \left(231 - \frac{441\sqrt{3}}{4}\right) cm^2$

123. A chord of a circle of radius $10 cm$ subtends a right angle at the centre. Find the area of the corresponding:
(i) minor segment,
(ii) maior sector. (Use $\pi = 3.14$)

Ans. :



We have, $r = 10cm$ and $\theta = 90^\circ$

Now, (i) Area of minor sector $AOBA = \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{90^\circ}{360} \times 3.14 \times 10 \times 10cm^2 = 78.5cm^2$

And, area of $\triangle AOB = \frac{1}{2} \times b \times h = \frac{1}{2} \times 10 \times 10cm^2 = 50cm^2$

Now, area of minor segment $=$ Area of minor sector - Area of $\triangle AOB$

$= (78.5 - 50)cm^2 = 28.5cm^2$

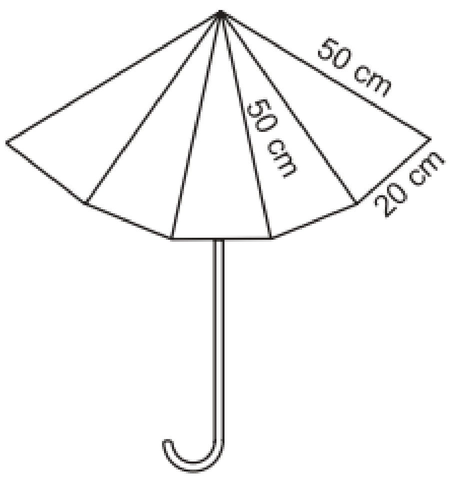
(ii) Area of the major sector $=$ Area of the circle - Area of minor sector

$= \pi r^2 - 78.5cm^2$ [From (i) we have area of minor sector $= 78.5cm^2$]

$= \left(3.14 \times 10^2 - 78.5\right) cm^2 = (3.14 \times 100 - 78.5)cm^2 = 235.5cm^2$.

124. An umbrella is made by stitching 10 triangular pieces of cloth of two different colour, each piece measuring 20 cm, 50 cm and 50 cm. How much cloth of each colour is required for the umbrella?

Ans. :



Let $a = 20cm, b = 50cm$ and $c = 50cm$. Then

$s = \frac{a+b+c}{2} = \frac{20+50+50}{2}cm = \frac{120}{2}cm = 60cm$

Now, area of one triangular piece $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{60(60-20)(60-50)(60-50)}\ cm^2$

$= \sqrt{60 \times 40 \times 10 \times 10}\ cm^2$

$= \sqrt{10 \times 10 \times 10 \times 10 \times 4 \times 6}\ cm^2$

$= 10 \times 10 \times 2\sqrt{6}cm^2 = 200\sqrt{6}cm^2$

$\therefore$ Required area of cloth of each colour $= 5 \times 200\sqrt{6}cm^2 = 1000\sqrt{6}cm^2$

125. The base of an isosceles triangle is 16 cm and length of equal sides is 10 cm. Find the area of the triangle using Heron's formula.

Ans. : Let $a = 16cm, b = 10cm, c = 10cm$

$s = \frac{a+b+c}{2} = \frac{16+10+10}{2}cm = \frac{36}{2}cm = 18cm$.

Now, area $= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{18(18-16)(18-10)(18-10)}\ cm^2$

$= \sqrt{18 \times 2 \times 8 \times 8}\ cm^2 = \sqrt{2 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}\ cm^2$

$= \sqrt{2\times2\times2\times2\times2\times2\times2\times2\times3\times3}\ cm^2$ [Rearranging the factors]

$= 2\times2\times2\times2\times3cm^2 = 48cm^2$.

126. The sides of a triangle are 8 cm, 15 cm and 17 cm. Find its area.

Ans. : Let $a = 8cm, b = 15cm$ and $c = 17cm$. Then,

$s = \frac{a+b+c}{2} = \frac{8+15+17}{2}cm = \frac{40}{2}cm = 20cm$

Now, area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{20(20-8)(20-15)(20-17)}\ cm^2 = \sqrt{20\times12\times5\times3}\ cm^2$

$= \sqrt{2\times2\times5\times2\times2\times3\times5\times3}\ cm^2$

$= \sqrt{\underline{2\times2\times2\times2\times3\times3\times5\times5}}\ cm^2$

$= 2\times2\times3\times5cm^2 = 60cm^2$

Alternative Method

We have, $a = 8cm, b = 15cm$ and $c = 17cm$.

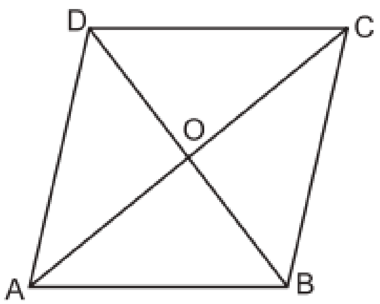
Now, $a^2 + b^2 = 8^2 + 15^2 = 64 + 225 = 289cm^2 = c^2$

The triangle is a right angled triangle with legs 8 cm and 15 cm .[By converse of Baudhayana-Pythagoras Theorem]

∴ Area of the right triangle $= \frac{1}{2}\times$ Product of its lengths $= \frac{1}{2}\times8\times15cm^2 = 60cm^2$

127. Diagonals of a rhombus are 8 m and 6 m. Find the side of the rhombus and its area also.

Ans. :



$AC = 8m \Rightarrow AO = \frac{8m}{2} = 4m$

$BD = 6m \Rightarrow BO = \frac{6m}{2} = 3m$

⇒ AOB is a right angle triangle.

We have, $AB^2 = AO^2 + OB^2$ [Baudhayana-Pythagoras Theorem]

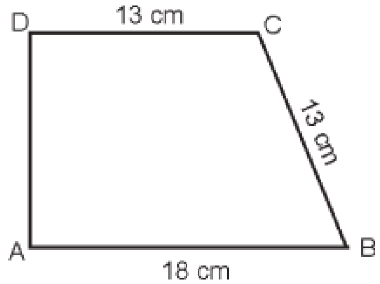
$= (4m)^2 + (3m)^2 = 16m^2 + 9m^2 = 25m^2 = 5m\times5m$

$\Rightarrow AB = 5m$

Area of rhombus $ABCD = \frac{1}{2}\times AC\times BD$

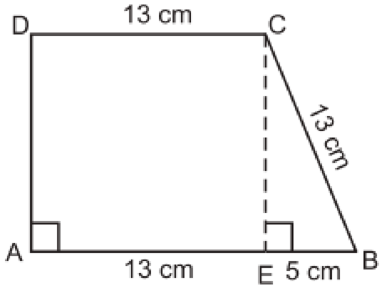
$= \frac{1}{2}\times8m\times6m = 24m^2$

128.



In the figure, ABCD is a trapezium in which AB || DC. If DA ⊥ AB, AB = 18 cm, BC = 13 cm and CD = 13 cm, find the area of the trapezium.

Ans. :



Draw $CE \perp AB$

Now, $BE = (18 - 13)cm = 5cm$

In right triangle BEC ,

$BC^2 = BE^2 + CE^2$

[Using Baudhayana-Pythagoras Theorem]

$\Rightarrow \quad 13^2 = 5^2 + CE^2$

$\Rightarrow \quad CE^2 = 169 - 25 = 144$

$\Rightarrow \quad CE = \sqrt{144} = 12cm$

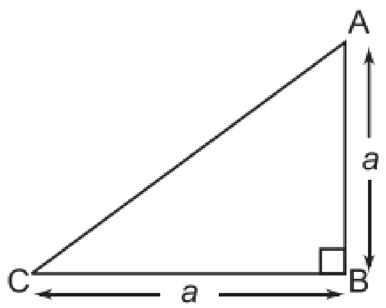
Now, area of trapezium ABCD $= \frac{1}{2}(AB + DC) \times CE$

$= \frac{1}{2}(18 + 13) \times 12cm^2$

$= 31 \times 6cm^2 = 186cm^2$

129. An isosceles right triangle has area $8cm^2$. Find the length of its hypotenuse.

Ans. :



Let AB = BC = a cm

We have, area of triangle $= 8cm^2 \Rightarrow \frac{1}{2} \times a \times a = 8cm^2$

$\Rightarrow a^2 = 16cm^2 \Rightarrow a = 4cm$

Now, $AC = \sqrt{AB^2 + BC^2}$ [Using Baudhayana-Pythagoras Theorem]

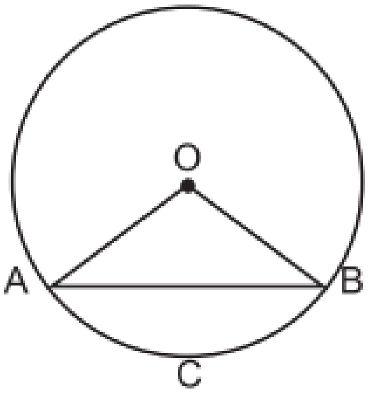
$\Rightarrow AC = \sqrt{a^2 + a^2} = \sqrt{4^2 + 4^2} = \sqrt{32} cm$

Hence, length of hypotenuse $= \sqrt{32} cm = 4\sqrt{2} cm$

130. The perimeter of sector OACB of the circle centered at O and of radius 24 cm is 73.12 cm .

(i) Find the central angle $\angle AOB$.

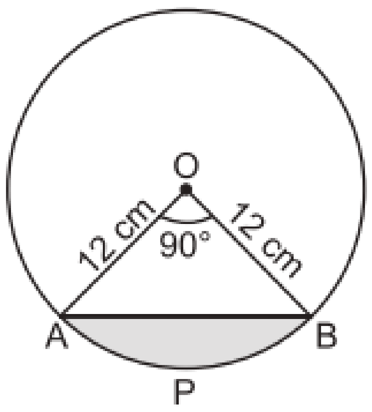
(ii) Find the area of the minor segment ACB . (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)



Ans. : (i) 60°

(ii) $176.88 sq.cm$

131. In a circle with centre O, AB is a chord of the circle. $AO = 12cm$ and angle $AOB = 90^\circ$. Find the area of the shaded region.(Use $\pi = 3.14$).



Ans. : $41.04 sq.cm$

132. The minute hand of a clock is 10 cm long. Find the area on the face of the clock described by the minute hand between 8 am and 8.25 am.

Ans. : 131 sq. cm (appprox.)

133. A circular park has a radius of 21 m . A 60° sector is converted into a flower garden.

(i) Find the area of the flower garden (sector).

(ii) Find the remaining area of the park.

Ans. : (i) 231 sq. m

(ii) $1155 sq \cdot m$

134. A pizza has a radius of 14 cm . It is cut into 8 equal slices.

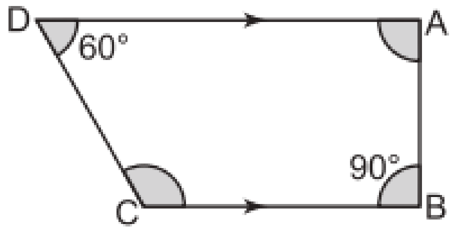
(i) Find the area of one slice.

(ii) If each slice is sold at ₹120, find the price per cm^2 of pizza.

Ans. : (i) $77 sq.cm$

(ii) ₹1.56 per sq. cm

135. ABCD is a field in the shape of a trapezium in which $AD \| BC, \angle ABC=90^{\circ}$ and $\angle ADC=60^{\circ}$. Four sectors are formed with centres A, B, C and D as shown in figure. The radius of each sector is 14 m . Find the total area of four sectors.



Ans. : 616 sq. cm

136. The outer circumference of a circular track is 194.68 m and the track is everywhere 6 m wide. Find the cost of levelling the track at the rate of 25 paise per m^2.[Take $\pi=3.14$]

Ans. : ₹263.76

137. A circular garden has a radius of 21 m . Around it, a circular walking path of width 3 m is constructed.

(i) Find the area of the walking path.

(ii) If paving costs ₹120 per m^2, calculate the total cost of paving the path.

Ans. : (i) 424.29 sq. m

(ii) ₹50,914.80

138. A circular flower bed has a radius of 7 m . A gardener wants to spread manure over it.

(i) Find the area of the flower bed.

(ii) If manure costs ₹35 per m^2, find the total expense.

Ans. : (i) 154 sq. m

(ii) ₹5390

139. A circular pond has a radius of 3.5 m . A protective net is placed over its surface.

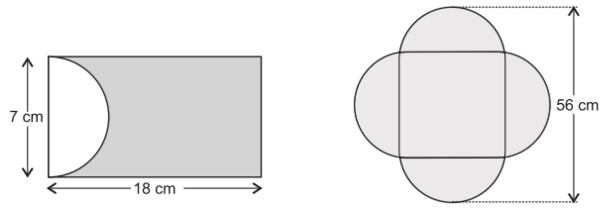
(i) Find the area of the pond

(ii) If the net costs ₹120 per m^2, find the total cost.

Ans. : (i) 38.5 sq. m

(ii) ₹4620

140. Find the area of the shaded portion in each of the following figures :



Ans. : (i) 106.75 sq. cm
(ii) 2016 sq. cm

141. A wire is in a circular shape of radius 49 cm. It is bent into the shape of a square. Find the area of the square so formed.

Ans. : 5929 sq. m

142. Find the circumference of a circle with radius 56 cm and the perimeter of two semicircles with the same radius.

Ans. : 352 cm , 576 cm

143. Find the ratio between the circumferences of two circles whose radii are in the ratio 2: 3.

Ans. : $2:3$

144. How many revolutions will a wheel make to cover a distance of 132 m, if its radius is 60 cm?

Ans. : 35

145. Measure the diameters of one rupee coin and five rupee coin and find their circumferences.

Ans. : One Rupee Coin: Diameter = $\mathbf{2.5\ cm}$, Circumference = $\mathbf{7.85\ cm}$
Five Rupee Coin: Diameter $= 2.3\ \text{cm}$, Circumference $= 7.23\ \text{cm}$

146. A wheel covers 110 m in one revolution. What is its radius?

Ans. : 17.5 m

147. The radius of a bicycle wheel is 35 cm.
(i) Find the distance covered in one revolution.
(ii) Find the distance covered in 200 revolutions.

Ans. : (i) 220 m
(ii) 440 m

148. A circular stage has a radius of 3.5 m. Decorative ribbon is fixed around its boundary.
(i) Find the length of ribbon required.
(ii) If ribbon costs 40 per metre, find the total cost.

Ans. : (i) 22 m
(ii) ₹880

149. Find the difference between the perimeters of two circular plates, if one has a radius of 14 cm and the other has a diameter of 14 cm.

Ans. : 44 cm

150. A circular stadium has a diameter of 70 m.

(i) Find the length of one round.

(ii) How many rounds must an athlete run to complete 1.1 km?

Ans. : (i) 220 m

(ii) 5

151. A circular garden has a radius of 14 m. The owner wants to put fencing around it.

(i) Find the circumference of the garden.

(ii) If fencing costs ₹250 per metre, find the total cost.

Ans. : (i) 88 m

(ii) ₹22,000

152. An ancient king ordered a circular garden with radius 21 m .

(a) Archimedes used $\pi=\frac{22}{7}$

(b) Aryabhata used $\pi=3.1416$

(c) Zu Chongzhi used $\pi=\frac{355}{113}$

(i) Calculate the circumference using all three values.

(ii) Arrange the results in increasing order.

Ans. : (i) $132 m, 131.9472, 131.9469 m$

(ii) $131.9469<131.9472<132$

153. A circular road has a diameter of 70 m . Engineers use $\pi=\frac{22}{7}$ for planning.

(i) Find the length of the road.

(ii) If the actual value of π is 3.1416, estimate the difference in total length.

(iii) Discuss whether this difference would matter in large-scale construction.

Ans. : (i) 220 m

(ii) 0.088 m

154. A student calculates the circumference of a circular track with radius 5 m and gets:$C=10 \pi$

(i) Explain why the exact value of circumference cannot be written as a terminating or repeating decimal.

(ii) Why do we still use approximations in practical life?

Ans. :

The standard formula for the circumference (C) of a circle using its diameter (d) is:

$C=\pi d$

155. A circular fountain has a diameter of 14 m .

(i) Find its circumference using:(a) $\pi=\frac{22}{7}$ (b) $\pi=3.14$

(ii) Which approximation gives a larger value?
(iii) In real life construction, which value would you prefer and why?

Ans. : (i) (a) 44 m
(b) 43.96 m
(ii) 44 m

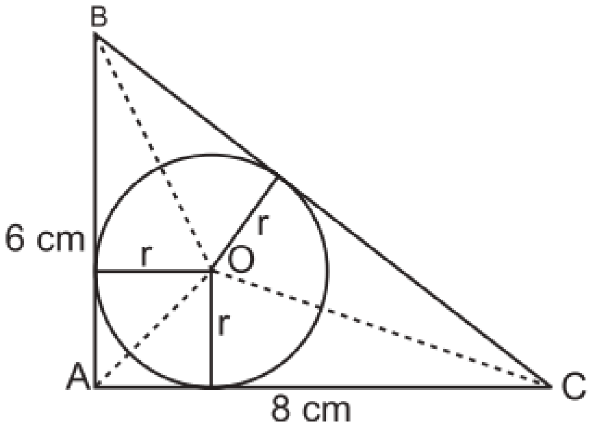
156. A bicycle wheel makes 5000 revolutions in moving 11 km. Find the diameter of the wheel.

Ans. : 70 cm

157. The inner circumference of a circular track is 220 m. The track is 7 m wide everywhere. Calculate the cost of putting up a fence along the outer circle at the rate of 2 per metre.(Use $\pi = \frac{22}{7}$)

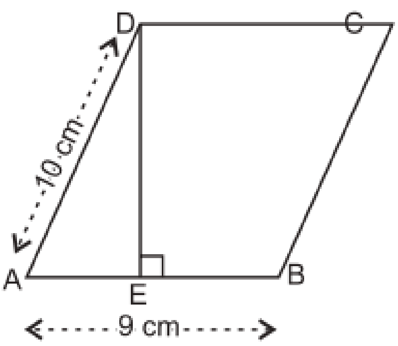
Ans. : 528

158. In the figure, ABC is a right triangular with AB 6 cm and AC = 8 cm. A circle with centre O is inscribed in the triangle. Calculate the value of r, the radius of the inscribed circle.



Ans. : $r = 2cm$

159. One side AB of a parallelogram is 9 cm and its area is $54m^2$. DE $\perp$ AB. If the second side of the parallelogram is 10 cm, find AE.



Ans. : 8 cm

160. The ratio of the parallel sides of a trapezium is 2 : 3. The distance between them is 15 cm. If the area of the trapezium is $600cm^2$, find the lengths of parallel

sides.

Ans. : 32 cm, 48 cm

161. Area of a trapezium is $400 cm^2$. One of its parallel sides is 20 m longer than the other and height is 10 m. Find its two parallel sides.

Ans. : 30 cm, 50 cm

162. One of the parallel sides of a trapezium is double the other and its height is 24 cm. If area of the trapezium is $360 cm^2$, find the length of the parallel sides.

Ans. : 10 cm, 20 cm

163. The perimeter of an isosceles triangle is 39 cm. The ratio of the equal side to its base is 4 : 5. Find the area of the triangle.

Ans. : $70.25 cm^2$

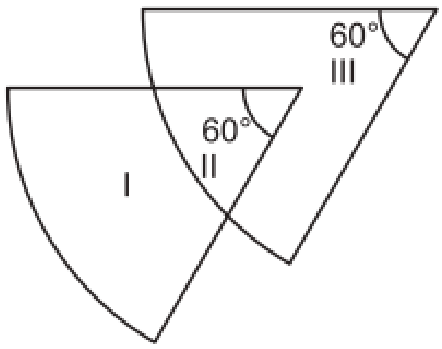
164. Find the area of an isosceles triangle each of whose equal sides measures 13 cm and whose base measures 20 cm.

Ans. : $82 cm^2$

165. Find the area of an isosceles triangle, each of whose equal sides is 13 cm and base 24 cm.

Ans. : $60 cm^2$

166. In the figure, there are two overlapping sectors of a circle. The radii of the sectors are 6 cm and 8 cm . The figure is divided into three regions- I, II and III.



Find the difference in the areas of regions I and III.

Ans. : $\frac{44}{3} cm^2$

*** Answer the following questions. [5 Marks Each]** **[125]**

167. Which term of the AP: $-2, -7, -12, \ldots$ will be -77? Find the sum of this AP up to the term -77.

Ans. : Let ath term (a_n) of the AP be -77. Then a_{n} = - 77

Here, $a = -2$ and $d = -7 - (-2) = -7 + 2 = -5$

We have, $a_n = -77 \Rightarrow a + (n-1)d = -77$

$\Rightarrow \quad (-2) + (n-1)(-5) = -77 \quad \Rightarrow \quad -2 - 5n + 5 = -77$

$\Rightarrow \quad -5n+3=-77 \quad \Rightarrow \quad -5n=-80 \Rightarrow n=16$

Hence, 16th term of the AP is -77.

Now, $\quad S_n=\frac{n}{2}[2a+(n-1)d]$

$\Rightarrow \quad S_{16}=\frac{16}{2}[2(-2)+(16-1)(-5)]$

$\Rightarrow \quad S_{16}=8(-4-75)=8\times(-79)=-632$.

168. Divya deposited ₹1000 at compound interest at the rate of 10% per annum. The amounts at the end of first year, second year, third year, form an AP. Justify your answer.

Ans. : Amount at the end of 1st year $=P\left(1+\frac{r}{100}\right)^n=₹1000\left(1+\frac{10}{100}\right)^1=₹1000\times\frac{11}{10}=₹1100$

Amount at the end of the 2 nd year $=₹1000\left(1+\frac{10}{100}\right)^2$

$=₹1000\left(\frac{11}{10}\right)^2=₹1000\times\frac{121}{100}=₹1210$

Amount at the end of the 3rd year $=₹1000\left(1+\frac{10}{100}\right)^3$

$=₹1000\left(\frac{11}{10}\right)^3=₹1000\times\frac{1331}{1000}=₹1331$

So, the amounts (in ₹) at the end of 1st year, 2nd year, 3rd year, are $1100, 1210, 1331$ respectively.

Here,

$a_2-a_1=1210-1100=110, a_3-a_2=1331-1210=121 a_2-a_1\neq a_3-a_2$. It does not form an AP.

169. Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

(i) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$.

(ii) $0, -4, -8, -12$

(iii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \ldots$.

Ans. : (i) Here, $a_1=3; a_2=3+\sqrt{2}, a_3=3+2\sqrt{2}, a_4=3+3\sqrt{2}, \ldots\ldots$

We have, $\quad a_2-a_1=3+\sqrt{2}-3=\sqrt{2}$

$a_3-a_2=(3+2\sqrt{2})-(3+\sqrt{2})=3+2\sqrt{2}-3-\sqrt{2}=\sqrt{2}$

$a_4-a_3=3+3\sqrt{2}-(3+2\sqrt{2})=3+3\sqrt{2}-3-2\sqrt{3}=\sqrt{2}$.

Since $a_{n+1}-a_n$ is same every time. So, the given numbers form an AP with common difference $d=\sqrt{2}$.

Now, next three terms are:

$a_5=3+3\sqrt{2}+\sqrt{2}=3+4\sqrt{2}, a_6=3+4\sqrt{2}+\sqrt{2}=3+5\sqrt{2}, a_7=3+5\sqrt{2}+\sqrt{2}=3+6\sqrt{2}$

(ii) Here, $a_1=0, a_2=-4, a_3=-8, a_4=-12, \ldots\ldots$

We have, $\quad a_2-a_1=-4-0=-4$

$a_3-a_2=-8-(-4)=-8+4=-4$

$a_4-a_3=-12-(-8)=-12+8=-4$

Since $a_{n+1}-a_n$ is same every time. So, the given numbers form an AP with common difference $d=-4$.

Now, next three terms are:

$a_5=-12+(-4)=-12-4=-16; a_6=-16+(-4)=-16-4=-20$

$a_7=-20+(-4)=-20-4=-24$

(iii) Here, $a_1=-\frac{1}{2}; a_2=-\frac{1}{2}; a_3=-\frac{1}{2}; a_4=-\frac{1}{2}, \ldots.$.

We have, $a_2-a_1=-\frac{1}{2}-\left(-\frac{1}{2}\right)=-\frac{1}{2}+\frac{1}{2}=0$

$a_3-a_2=-\frac{1}{2}-\left(\frac{1}{2}\right)=-\frac{1}{2}+\frac{1}{2}=0$

$a_4-a_3=-\frac{1}{2}-\left(\frac{1}{2}\right)=-\frac{1}{2}+\frac{1}{2}=0$

Since $a_{n+1}-a_n$ is same every time. So, the given numbers form an AP with common difference $d=0$.

Now, next three terms are : $a_5=-\frac{1}{2}+0=-\frac{1}{2}, a_6=-\frac{1}{2}+0=-\frac{1}{2}, a_7=-\frac{1}{2}+0=-\frac{1}{2}$.

170. The sum of first n terms of an AP is represented by $S_n=6n-n^2$ Find the common difference.

Ans. : -2

171. If $S_n=3n^2-4n$, then find the nth terms.

Ans. : $6n-7$

172. If $S_n=\frac{1}{2}\left(3n^2+7n\right)$, find nth term and 20th term.

Ans. : $3n+2; 62$

173. In an AP, the sum of first in terms is $\left(\frac{3n^2}{2}+\frac{5n}{2}\right)$. Find its 25^{th} term.

Ans. : (i) $a_{25}=76$

174. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees that each section of each class will plant, will be the same as the class in which they are studying, e.g., a section of class I will plant I tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. How many trees will be planted by the students?

Ans. : 234

175. The sequence 2, 9, 16, is given.

(i) Identify if the given sequence is an AP.

(ii) Find the 20th term of the sequence.

(iii) Find the difference between the sum of its first 22 and 25 terms.

(iv) Is the term 102 belong to this sequence?

(v) If k is added to each of the above terms, will the new sequence be in AP?

Ans. : (i) yes

(ii) 135

(iii) 489

(iv) no

(v) yes

176. If the mid-points of the sides of a 4-gon are joined in order, prove that the area of the parallelogram thus formed will be half of the area of the given 4-gon. (You may wonder whether the 4-gon thus formed is always a parallelogram, and if so, why? These questions will be tackled and answered in the chapter on quadrilaterals.)

Ans. : Let ABCD be the given 4-gon.

Let P, Q, R and S be the midpoints of sides AB, BC, CD and DA respectively.

Join P, Q, R and S in order.

So, PQRS is the parallelogram formed.

Now, draw diagonal AC of the 4-gon.

In triangle ABC, P and Q are the midpoints of AB and BC.

Therefore, by midpoint theorem: $PQ \| AC$ and $PQ=1/2 AC$

In triangle ADC, S and R are the midpoints of AD and DC.

Therefore, $SR \| AC$ and $SR=1/2 AC$

So, $PQ \| SR$ and $PQ=SR$

Hence, PQRS is a parallelogram.

Now, the four corner triangles are $\triangle APS, \triangle BPQ, \triangle CQR$ and $\triangle DRS$.

Each of these triangles has half the base and half the height of the corresponding triangle into which the 4-gon is divided.

So, the total area of the four corner triangles is half the area of the original 4-gon.

Therefore, the remaining middle parallelogram PQRS has the other half of the area.

Hence, area(parallelogram PQRS) $=1/2 \times \operatorname{area}(4-$ gon $ABCD)$

Hence proved.

177. Given a square ABCD, let P be a point within it. Join PA, PB, PC, PD (Fig). What is the ratio of the areas of the red region ($\triangle PAB$ and $\triangle PCD$) and the green region ($\triangle PBC$ and $\triangle PDA$) ?

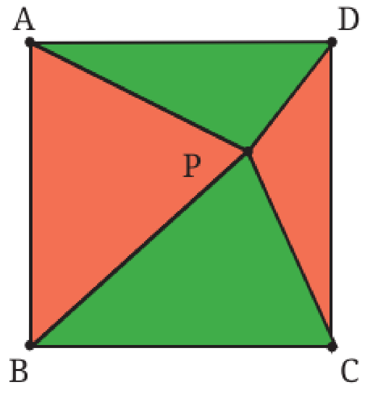


Fig.

Ans. : Let the side of the square ABCD be a.
The red region consists of $\triangle PAB$ and $\triangle PCD$
The green region consists of $\triangle PBC$ and $\triangle PDA$
Now, $AB \| CD$ and $AB = CD = a$.
Let the perpendicular distance of P from AB be h.
Then the perpendicular distance of P from CD will be a − h.
Area of $\triangle PAB = 1/2 \times a \times h$
Area of $\triangle PCD = 1/2 \times a \times (a - h)$
So, total red area:
$= 1/2 \times ah + 1/2 \times a(a - h)$
$= 1/2 \times a[h + a - h]$
$= 1/2 \times a^2$
Similarly, let the perpendicular distance of P from BC be y.
Then the perpendicular distance of P from AD will be a − y.
Area of $\triangle PBC = 1/2 \times a \times y$
Area of $\triangle PDA = 1/2 \times a \times (a - y)$
So, total green area:
$= 1/2ay + 1/2a(a - y)$
$= 1/2a[y + a - y]$
$= 1/2a^2$
Therefore, Red area = Green area
Hence, the required ratio:
Red region : Green region = 1 : 1.

178. In $\triangle ABC, D$ is the midpoint of $AB. P$ is any point on BC, and Q is a point on AB such that $CQ \| PD. PQ$ is joined (Fig). Prove that Area $\triangle BPQ) = 1/2$ Area(△ABC).

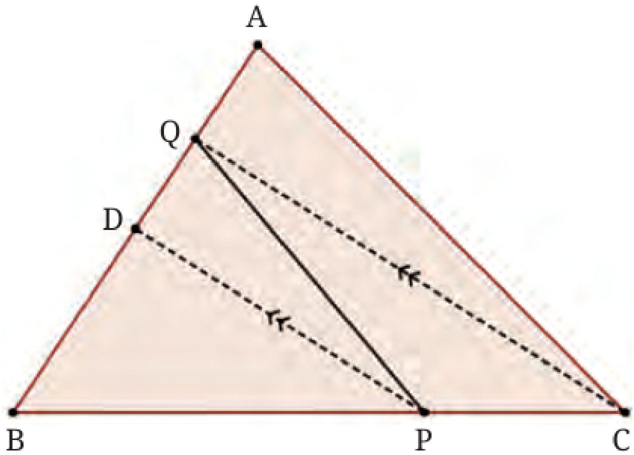


Fig.

Ans. : Given: D is the midpoint of AB .
So, $BD = DA$
Since P lies on BC, triangles $\triangle ABP$ and $\triangle ABC$ have the same altitude from A to line BC.

Therefore, $\text{Area}(\triangle ABP)/\text{Area}(\triangle ABC) = BP/BC \ldots (1)$
Now, in $\triangle ABP, D$ and Q lie on AB.
Since $CQ \| PD$ and P lies on BC , by the basic proportionality idea in the figure,
Q is positioned so that $\triangle BPQ$ has half the area of $\triangle ABC$.
As D is the midpoint of AB, so
$\text{Area}(\triangle BDP) = 1/2\,\text{Area}(\triangle ABP)$...(2)
This is because triangles $\triangle BDP$ and $\triangle ABP$ have bases BD and AB on the same line AB and they have the same height from P to AB.
Now, since $CQ \| PD$, triangles $\triangle BDP$ and $\triangle BPQ$ have the same base BP and lie between the same parallels BP and DQ . So, their corresponding heights equal.
$\text{Thus, Area}(\triangle BPQ) = \text{Area}(\triangle BDP) + \text{Area}(\triangle DQP)$
Using the parallel condition, the extra area exactly accounts for half of the remaining part of $\triangle ABC$.
$\text{Hence, Area}(\triangle BPQ) = 1/2\,\text{Area}(\triangle ABC)$
Therefore proved.

179. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre of the circle. Find the areas of the corresponding minor and major segments of the circle. (Use $\pi \approx 3.14$ and $\sqrt{3} \approx 1.73$.)

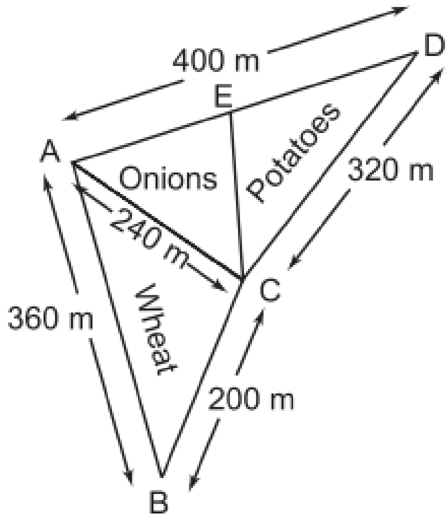
Ans. : Radius = 15 cm
Angle at centre = 60°
Area of minor sector:
$= 60/360 \times \pi r^2$
$= 1/6 \times 3.14 \times 15^2$
$= 1/6 \times 3.14 \times 225$
$= 117.75 cm^2$
Since the angle is 60° and both radii are equal, the triangle formed is equilateral.
Area of equilateral triangle:
$= \sqrt{}3/4\times$ side ^{2}
$= 1.73/4 \times 15^2$
$= 1.73/4 \times 225$
$= 97.3125 cm^2$
Area of minor segment:
= Area of minor sector − Area of triangle
= 117.75 − 97.3125
$= 20.4375 cm^2$
So, Minor segment area = $20.44 cm^2$ approximately.
Area of circle $= \pi r^2$
$= 3.14 \times 225$
$= 706.5 cm^2$

Area of major segment = Area of circle − Area of minor segment
= 706.5 − 20.4375
$= 686.0625 cm^2$
So, Major segment area $= 686.06 cm^2$ approximately.

180. If $a_n = 3 - 4n$, show that $a_1, a_2, a_3, \ldots..$ form an AP. Also, find S_{20}.

Ans. : We have, $a_n = 3 - 4n$
$\therefore a_1 = 3 - 4(1) = 3 - 4 = -1$
$a_2 = 3 - 4(2) = 3 - 8 = -5$
$a_3 = 3 - 4(3) = 3 - 12 = -9$
Here, $a_2 - a_1 = a_3 - a_2 = -4$
$\therefore a_1, a_2, a_3, \ldots$ form an AP with common difference $d = -4$.
Now, $S_n = \frac{n}{2}[2a + (n-1)d]$
$\Rightarrow \quad S_{20} = \frac{20}{2}[2(-1) + (20-1)(-4)] \Rightarrow = 10(-2-76) = 10 \times (-78) =$.
-780

181. Kamla has a triangular field with sides $240m$, $200m, 360m$, where she grew wheat. In another triangular field with sides $240m, 320m, 400m$ adjacent to the previous field, she wanted to grow potatoes and onions (see figure). She divided the field in two parts by joining the mid-point of the longest side to the opposite vertex and grew potatoes in one part and onions in the other part. How much area (in hectares) has been used for wheat, potatoes and onions? (1 hectare $= 10000m^2$).



Ans. : Let ABC be the field where wheat is grown. Also let ACD be the field which has been divided in two parts by joining C to the mid-point E of AD .
For $\triangle ABC$, Let $a = 200m, b = 240m$ and $c = 360m$. Then
$s = \frac{200+240+360}{2}m = 400m$
Now, area for growing wheat $=$ area of $\triangle ABC$
$= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{400(400-200)(400-240)(400-360)}\, m^2$
$= \sqrt{400 \times 200 \times 160 \times 40}\, m^2 = \sqrt{400 \times 100 \times 2 \times 16 \times 10 \times 4 \times 10}\, m^2$
$= \sqrt{400 \times 100 \times 10 \times 10 \times 16 \times 4 \times 2}\, m^2 = 20 \times 10 \times 10 \times 4 \times 2\sqrt{2} m^2$

$= 16000\sqrt{2}m^2 = 1.6 \times \sqrt{2}$ hectares $= 2.26$ hectares

For $\triangle ACD$ let $a = 240m, b = 320m, c = 400m$. Then,

$s = \frac{a+b+c}{2} = \frac{240+320+400}{2}m = 480m$

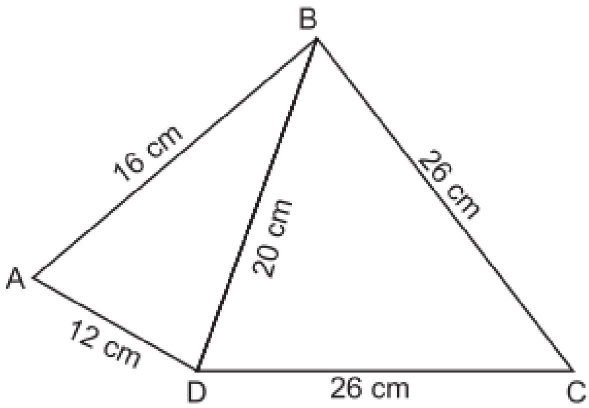
$\therefore$ Area of $\triangle ACD = \sqrt{480(480-240)(480-320)(480-400)}\ m^2$

$= \sqrt{480 \times 240 \times 160 \times 80}\, m^2 = \sqrt{2 \times 240 \times 240 \times 2 \times 80 \times 80}\ m^2$

$= \sqrt{2 \times 2 \times 80 \times 80 \times 240 \times 240}\ m^2 = 2 \times 80 \times 240m^2 = 38400m^2 = 3.84$ hectares

182. Find the area of a quadrilateral $ABCD$ in which $AB = 16cm, BC = CD = 26cm$, $AD = 12cm$ and the diagonal $BD = 20cm$.

Ans. :



Join diagonal BD,

For $\triangle ABD$, let $a = BD = 20cm, b = AD = 12cm$

and $c = AB = 16cm$

Then, $s = \frac{a+b+c}{2}$

$= \frac{20+12+16}{2}cm = \frac{48}{2}cm = 24cm$

Now, area of $\Delta ABD = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{24(24-20)(24-12)(24-16)}\ cm^2$

$= \sqrt{24 \times 4 \times 12 \times 8}\, cm^2$

$= \sqrt{2 \times 2 \times 2 \times 3 \times 2 \times 2 \times 2 \times 2 \times 3 \times 2 \times 2 \times 2}\ \ cm^2$

$= \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3}\ \ cm^2 = 2 \times 2 \times 2 \times 2 \times 2$

$\times 3cm^2 = 96cm^2$

Also, for $\triangle BCD$, let $a = BD = 20cm, b = BC = 26cm$ and $c = DC = 26cm$. Then,

$s = \frac{a+b+c}{2} = \frac{20+26+26}{2}cm = \frac{72}{2}cm = 36cm$

$\therefore$ Area of $\triangle BCD = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{36(36-20)(36-26)(36-26)}\ cm^2$

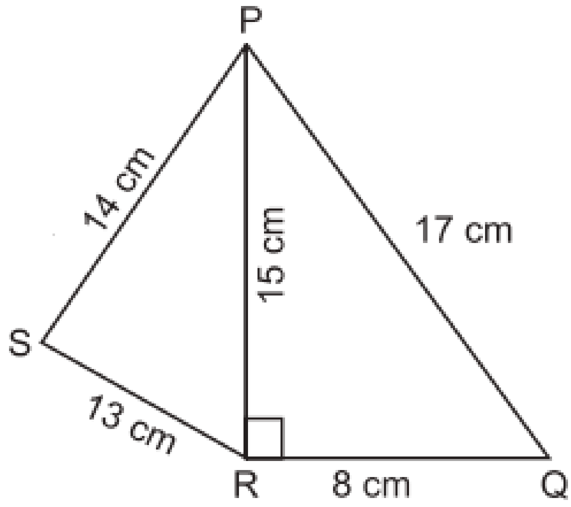
$= \sqrt{36 \times 16 \times 10 \times 10}\, cm^2 = 6 \times 4 \times 10cm^2 = 240cm^2$

Now, area of quadrilateral $ABCD =$ Area of $\triangle ABD +$ Area of $\triangle BCD$

$= 96cm^2 + 240cm^2 = 336cm^2$.

183. Find the perimeter and area of a quadrilateral $PQRS$ in which $PQ = 17cm$, $QR = 8cm, RS = 13cm$ and $PS = 14cm$ and $\angle PRQ = 90^\circ$.

Ans. :



In $\triangle PQR, PR = \sqrt{PQ^2 - QR^2}$ [Using Baudhayana-Pythagoras Theorem]

$\Rightarrow PR = \sqrt{(17)^2 - (8)^2}\, cm = \sqrt{289 - 64}\, cm = \sqrt{225} cm = 15cm$

Now, perimeter of $PORS = PO + OR + RS + PS$

$= (17 + 8 + 13 + 14)cm = 52cm$

Area of $\triangle PQR = \frac{1}{2} \times PR \times QR = \frac{1}{2} \times 15 \times 8 = 60cm^2$.

For $\triangle PRS$, let $a = PR = 15cm, b = PS = 14cm$ and $c = RS = 13cm$.

$\therefore s = \frac{a+b+c}{2} = \frac{15+14+13}{2} cm = \frac{42}{2} cm = 21cm$

$\therefore$ Area of $\triangle PRS = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-15)(21-14)(21-13)}\, cm^2 = \sqrt{21 \times 6 \times 7 \times 8}\, cm^2$

$= \sqrt{3 \times 7 \times 2 \times 3 \times 7 \times 2 \times 2 \times 2}\, cm^2$

$= \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 7}\, cm^2$

$= 2 \times 2 \times 3 \times 7 cm^2 = 84cm^2$

$\therefore$ Area of quadrilateral $PQRS =$ area of $\triangle PQR +$ area of $\triangle PQR = (60 + 84)cm^2 = 144cm^2$

184. The lengths of the sides of a triangle are in the ratio $5:4:3$ and its perimeter is 96 cm . Find
(i) the area of the triangle
(ii) the height of the triangle corresponding to the longest side.

Ans. : (i) Let $a = 5x, b = 4x$ and $c = 3x$.

We have, perimeter $= 96cm$

$\Rightarrow s = \frac{96}{2} cm = 48cm$

Also, $5x + 4x + 3x = 96 \Rightarrow 12x = 96 \Rightarrow x = 8$

Thus, the sides are $a = 5 \times 8cm = 40cm, b = 4 \times 8cm = 32cm$, and $c = 3 \times 8cm = 24cm$.

Now, area of the triangle

$= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{48(48-40)(48-32)(48-24)}\, cm^2$

$= \sqrt{48 \times 8 \times 16 \times 24}\, cm^2 = \sqrt{2 \times 24 \times 8 \times 8 \times 2 \times 24}\, cm^2$

$= \sqrt{2 \times 2 \times 8 \times 8 \times 24 \times 24}\ cm^2 = 2 \times 8 \times 24 cm^2 = 384 cm^2$

(ii) Let $h cm$ be the height of the triangle corresponding to the longest side, i.e., 40 cm .We have, area of triangle $= 384 cm^2$

$\Rightarrow \frac{1}{2} \times 40 \times h = 384 \Rightarrow h = \frac{384 \times 2}{40} = 19.2$

Hence, the height of the triangle corresponding to the longest side is 19.2 cm .

185. Find the percentage decrease in the area of a triangle, if each of its side is halved.

Ans. : Let a, b, c be the sides of a triangle. Then, $s = \frac{a+b+c}{2}$

$\therefore$ Area of the triangle $(A) = \sqrt{s(s-a)(s-b)(s-c)}$

Since, each side of the triangle is halved, therefore the sides of new triangle are $\frac{a}{2}, \frac{b}{2}$ and $\frac{c}{2}$.

Then,

$s' = \frac{\frac{a}{2} + \frac{b}{2} + \frac{c}{2}}{2} = \frac{a+b+c}{4} = \frac{s}{2}$

And, area of new triangle $(A') = \sqrt{s'\left(s' - \frac{a}{2}\right)\left(s' - \frac{b}{2}\right)\left(s' - \frac{c}{2}\right)}$

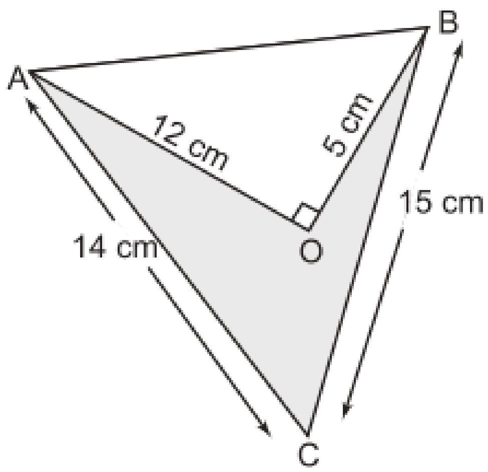
$= \sqrt{\frac{s}{2}\left(\frac{s}{2} - \frac{a}{2}\right)\left(\frac{s}{2} - \frac{b}{2}\right)\left(\frac{s}{2} - \frac{c}{2}\right)}$ [From (ii)]

$= \frac{1}{4}\sqrt{s(s-a)(s-b)(s-c)} = \frac{A}{4}$ [From (i)]

So, decrease in the area of the triangle $= A - A' = A - \frac{1}{4}A = \frac{3A}{4}$.

Hence, percentage decrease in area $= \left(\frac{\frac{3}{4}A}{A} \times 100\right)\% = 75\%$.

186. Calculate the area of the following shaded region.



Ans. : In $\triangle AOB, \angle AOB = 90^\circ$

$\therefore AB^2 = OA^2 + OB^2 \Rightarrow AB^2 = (12)^2 + (5)^2$

$\Rightarrow AB^2 = 144 + 25 \Rightarrow AB^2 = 169 \Rightarrow AB = 13 cm$

Let $a = 15 cm, b = 14 cm$ and $c = 13 cm$. Then

$s = \frac{a+b+c}{2} = \frac{15+14+13}{2} cm = \frac{42}{2} cm = 21 cm$

Area of $\triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-15)(21-14)(21-13)}\ cm^2 = \sqrt{21 \times 6 \times 7 \times 8}\ cm^2$

$= \sqrt{3 \times 7 \times 3 \times 2 \times 7 \times 2 \times 4}\ cm^2 = \sqrt{2 \times 2 \times 3 \times 3 \times 7 \times 4}\ cm^2$

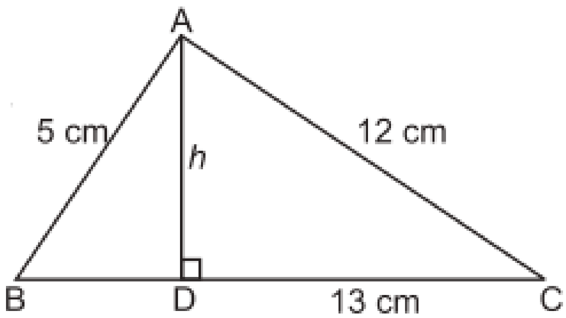
$= 2 \times 3 \times 7 \times 2 cm^2 = 84 cm^2$

And, area of $\triangle AOB = \frac{1}{2} \times OA \times OB = \frac{1}{2} \times 12 \times 5 cm^2 = 30 cm^2$

$\therefore$ Area of shaded region $=$ Area of $\triangle ABC -$ Area of $\triangle AOB = (84 - 30) cm^2 = 54 cm^2$.

187. The lengths of the sides of a triangle are 5 cm, 12 cm and 13 cm. Find the length of perpendicular from the opposite vertex to the side whose length is 13 cm. Use Heron's formula

Ans. :



Let a = 13cm b = 12cm and c = 5cm Then,

$s = \frac{a+b+c}{2} = \frac{13+12+5}{2} cm = \frac{30}{2} cm = 15 cm$.

$\therefore$ Area of given triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{15(15-13)(15-12)(15-5)}\ cm^2$

$= \sqrt{15 \times 2 \times 3 \times 10}\ cm^2$

$= \sqrt{5 \times 3 \times 2 \times 3 \times 2 \times 5}\ cm^2$

$= \sqrt{2 \times 2 \times 3 \times 3 \times 5 \times 5}\ cm^2 = 2 \times 3 \times 5 cm^2 = 30 cm^2$.

Again, let h be the length the perpendicular from opposite vertex A to the side BC.

We have, area of $\triangle ABC = 30 cm^2$

$\Rightarrow \frac{1}{2} \times 13 \times h = 30 \quad [$ Area of $\Delta = \frac{1}{2} \times b \times h]$

$\Rightarrow h = \frac{30 \times 2}{13} = \frac{60}{13} = 4.62$

Thus, required height $= 4.62 cm$.

188. Find the area of the ground having sides $51m, 37m$ and $20m$. Also find the cost of levelling the ground, if $1m^2$ costs ₹3.

Ans. : Let $a = 51m . b = 37m$ and $c = 20m$

$s = \frac{a+b+c}{2} = \frac{51+37+20}{2} m = \frac{108}{2} m = 54m$.

$\therefore$ Area of the ground $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{54(54-51)(54-37)(54-20)}\ m^2 = \sqrt{54 \times 3 \times 17 \times 34}\ m^2$

$= \sqrt{3 \times 3 \times 3 \times 2 \times 3 \times 17 \times 17 \times 2}\ m^2$

$= \sqrt{3 \times 3 \times 3 \times 3 \times 17 \times 17 \times 2 \times 2}\ m^2$

$= 3 \times 3 \times 17 \times 2 m^2 = 306 m^2$

Now, cost of leveling $1m^2 = ₹3$.

$\therefore$ Cost of leveling $306m^2 = ₹3 \times 306 = ₹918$.

189. Find the area of an isosceles triangle whose one side is 10 cm greater than its equal side and its perimeter is 100 cm.

Ans. : Let each of the equal sides of the triangle be x. Then

Third side $= x + 10$

We have, perimeter of the triangle $= 100cm$

$\Rightarrow x + x + x + 10 = 100 \Rightarrow 3x + 10 = 100$

$\Rightarrow 3x = 90 \Rightarrow x = 30$

$\therefore$ Hence, sides of the triangle are $30cm, 30cm$ and 40 cm .

Now, $s = \frac{100}{2}cm = 50cm$

$\therefore$ Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{50(50-30)(50-30)(50-40)}\ cm^2$

$= \sqrt{50 \times 20 \times 20 \times 10}\ cm^2 = \sqrt{5 \times 10 \times 10 \times 20 \times 20}\ cm^2$

$= 10 \times 20 \times \sqrt{5}cm^2 = 200\sqrt{5}cm^2$.

190. A race track is in the form of a ring whose inner circumference is 528 m and the outer circumference is 616 m. Find the width of the track.

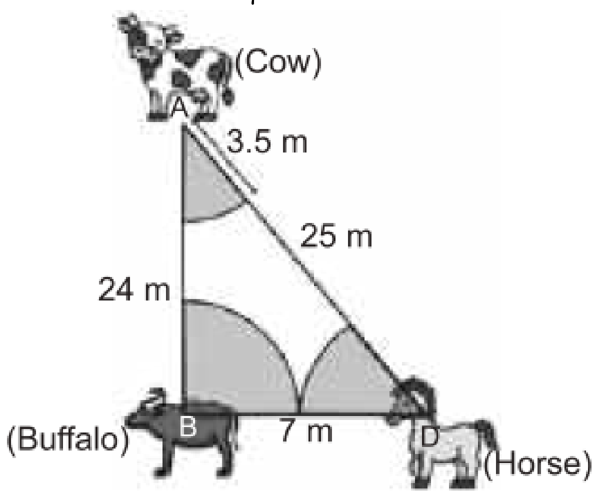
Ans. : 14 m

191. One circle has a radius of 98 cm and a second concentric circle has a radius of 1 m 26 cm. How much longer is the circumference of the second circle than that of first?

Ans. : 176 cm

*** Case study based questions.** **[12]**

192. A buffalo, a cow and a horse are tied to pegs at the corners of a right triangular field of sides $24m, 7m$ and 25 m by means of a 3.5 m long rope as shown in the figure. Use $\pi = \frac{22}{7}$.



Based on the above information, answer the following questions:

(i) What is the area of right triangular grass field?

(ii) Find the combined angle made by the grazing area of horse and cow.

(iii) Find the area of that part of field in which buffalo can graze.

(iv) Calculate the decrease in the grazing area, if the ropes were 3 m instead of 3.5 m.

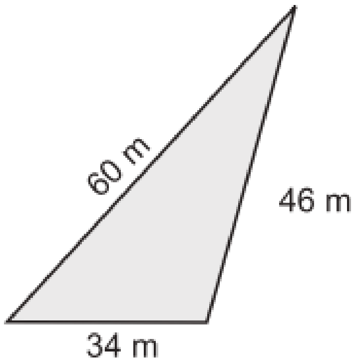
Ans. : (i) $84m^2$

(ii) 90°

(iii) $9.625m^2$

(iv) $5.11m^2$

193. A playground is made in the shape of a triangle. The dimensions of the triangular playground are $34m \times 46m \times 60m$. The city municipal corporation decided to level the ground. The cost of leveling the ground is ₹1000 per sq. m.



Based on this information, answer the following questions:

(i) If the playground is to be fenced with the wire, find the length of the wire.

(ii) Find the cost of leveling the ground. (Take $\sqrt{42} = 6.48$)

(iii) If the playground is an equilateral triangle with side 60 m , then find its area.

(iv) In Heron's formula, what does s stand for?

Ans. : (i) 140 m

(ii) ₹7,77,600

(iii) $900\sqrt{3}m^2$

(iv) semi-perimeter

194. Many buildings feature a double pitched roof. When viewed from one end, the roof edges meet at a common point to form a triangle. In many instances, these edges are equal in length. The roof trusses form an isosceles triangle. In the given figure there is a roof truss with two sides equal and unequal side as 12 m and its perimeter is 42 m.

Based on this information answer, the following questions:

(i) Find the dimensions of equal sides.

(ii) Find the semi-perimeter and values of (s-a), (s-b) and (s-c).

(iii) Find the area of triangle.

(iv) Area of a triangle having sides 2x, 2y and 2z is given by $\sqrt{(x+y+z)(x+y-z)(y+z-x)(x+z-y)}$. It is true?

Ans. : (i) $15m, 15m$

(ii) $21m; 6m, 6m, 9m$

(iii) $18\sqrt{21}m^2$

(iv) yes

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