Mind Curve Mid-Term Test Series 2026–27

Class XII | Test 02 – Answer Key

Chapters 3–4: Matrices & Determinants

Q.No	Step-by-Step Solution	Marks per Step
	SECTION – A (Q1–10, 1 mark each)	
1	**Q1** Scalar matrix ⇒ off-diagonal entries are 0, so $b = 0$ and $c = 0$ Scalar matrix ⇒ diagonal entries are equal, so $a = d = 5$ $a + 2b + 3c + 4d = 5 + 0 + 0 + 20 = 25$ **Answer: (d) 25**	*1 mark for the correct option.*
2	**Q2** $A = \begin{vmatrix} -2 & 0 & 0 \\ 1 & 2 & 3 \\ 5 & 1 & -1 \end{vmatrix}$ $\|A\| = -2 \times [(2)(-1) - (3)(1)]$ (expanding along row 1, using the two zeros) $\|A\| = -2 \times (-2 - 3) = -2 \times (-5) = 10$ $A(\text{adj}A) = \|A\| I$, so $\|A(\text{adj}A)\| = \|A\|^n$ for an $n \times n$ matrix Here $n = 3$, so $\|A(\text{adj}A)\| = 10^3 = 1000$ **Answer: (d) 1000**	*1 mark for the correct option.*
3	**Q3** Apply $R2 \to R2 - R1$ and $R3 \to R3 - R1$ Determinant becomes $\begin{vmatrix} 1 & 1 & 1 \\ 0 & \sin x & 0 \\ 0 & 0 & \cos x \end{vmatrix}$ Expanding along column 1: value $= \sin x \cdot \cos x = (1/2)\sin 2x$ Maximum value of $\sin 2x$ is 1, so the maximum value of the determinant is 1/2 **Answer: (b) 1/2**	*1 mark for the correct option.*
4	**Q4** $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $A^2 = A \cdot A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ so $A^2 = 2A$. By induction, $A^n = 2^{(n-1)} A$ $A^{10} = 2^9 A = 512A$ **Answer: (a) 512A**	*1 mark for the correct option.*
5	**Q5** For a symmetric matrix, $A^T = A$ For a skew-symmetric matrix, $A^T = -A$ If both hold: $A = -A \Rightarrow 2A = 0 \Rightarrow A = 0$ **Answer: (b) Zero square matrix**	*1 mark for the correct option.*

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	SECTION – A (Q1–10, 1 mark each)	
6	**Q6** (a) A row matrix has only one row — true (b) A diagonal matrix only needs its off-diagonal entries to be zero; the diagonal entries can be any value, so this statement is false (c) A symmetric matrix is a square matrix satisfying $A^T = A$ — true (d) A skew-symmetric matrix does have all diagonal entries zero — true **Answer: (b) A diagonal matrix has all diagonal elements equal to zero**	*1 mark for the correct option.*
7	**Q7** $\det(kA) = k^n \det(A)$ for an $n\times n$ matrix; here $n = 3$ $\det(4MN) = 4^3 \times \det(M) \times \det(N)$ $= 64 \times 2 \times (-3) = -384$ **Answer: (d) −384**	*1 mark for the correct option.*
8	**Q8** $A(\mathrm{adj}A) = \|A\| I$, and here $A(\mathrm{adj}A) = \mathrm{diag}(-3,-3,-3)$, so $\|A\| = -3$ $\|\mathrm{adj}A\| = \|A\|^{(n-1)} = \|A\|^2$ for $n = 3$, so $\|\mathrm{adj}A\| = (-3)^2 = 9$ $\|A\| - \|\mathrm{adj}A\| = -3 - 9 = -12$ **Answer: (b) −12**	*1 mark for the correct option.*
9	**Q9** Assertion: $(A+B)^T = A^T+B^T$ holds for any two matrices of the same order — TRUE Reason: the correct identity is $(AB)^T = B^TA^T$, not A^TB^T — the given Reason is FALSE **Answer: (c) A is true but R is false**	*1 mark for the correct option.*
10	**Q10** $A = \begin{vmatrix} 1 & 3 & k+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{vmatrix}$ Expand $\|A\|$ along row 1: $\|A\| = 1(4\times10-8\times5) - 3(2\times10-8\times3) + (k+2)(2\times5-4\times3)$ $= 1(0) - 3(-4) + (k+2)(-2) = 12 - 2k - 4 = 8 - 2k$ For A singular, $\|A\| = 0 \Rightarrow 8 - 2k = 0 \Rightarrow k = 4$, so Assertion is TRUE Reason correctly states that a singular matrix has $\|A\| = 0$, and correctly explains why $k = 4$ — Reason is TRUE and is the correct explanation **Answer: (a) Both A and R are true, and R is the correct explanation of A**	*1 mark for the correct option.*
	SECTION – B (Q11–13, 2 marks each)	
11(A)	**Q11(A)** $(ABA^T)^T = (A^T)^T B^T A^T = A B^T A^T$ Since B is skew-symmetric, $B^T = -B$ So $(ABA^T)^T = A(-B)A^T = -(ABA^T)$ **Answer: ABA^T is skew-symmetric, since $(ABA^T)^T = -(ABA^T)$**	*1 mark for computing $(ABA^T)^T$* *1 mark for the conclusion.*
11(B)	**Q11(B)**	*1 mark for identifying A is symmetric*

Q.No	Step-by-Step Solution	Marks per Step
	SECTION – A (Q1–10, 1 mark each)	
	$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ The symmetric part is $P = 1/2(A+A^T)$ and the skew-symmetric part is $Q = 1/2(A-A^T)$ Since A is already symmetric ($A^T = A$), the symmetric part equals A itself, and the skew-symmetric part $Q = 1/2(A-A^T)$ is the zero matrix **Answer:** $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$	*1 mark for stating the symmetric/skew-symmetric parts.*
12	**Q12** Given matrix $= \begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix}$ Subtract $= \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$ Row 1: $9-1=8$, $-1-2=-3$, $4-(-1)=5$ Row 2: $-2-0=-2$, $1-4=-3$, $3-9=-6$ **Answer:** $A = \begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & -6 \end{bmatrix}$	*1 mark for setting up A = given − given* *1 mark for the correct matrix A.*
13(A)	**Q13(A)** Given A and B are symmetric: $A^T = A$, $B^T = B$ $(AB+BA)^T = (AB)^T+(BA)^T = B^TA^T+A^TB^T = BA+AB = AB+BA \Rightarrow$ symmetric $(AB-BA)^T = (AB)^T-(BA)^T = BA-AB = -(AB-BA) \Rightarrow$ skew-symmetric **Answer: AB+BA is symmetric and AB−BA is skew-symmetric, as shown above**	*1 mark for proving AB+BA is symmetric* *1 mark for proving AB−BA is skew-symmetric.*
13(B)	**Q13(B)** $B^TA^T = (AB)^T$, so first compute AB, then transpose it $AB = \begin{bmatrix} 75 & 9 & 42 \\ 56 & 4 & 22 \\ 71 & 13 & 58 \end{bmatrix}$ Transposing AB gives B^TA^T **Answer:** $B^TA^T = \begin{bmatrix} 75 & 56 & 71 \\ 9 & 4 & 13 \\ 42 & 22 & 58 \end{bmatrix}$	*1 mark for computing AB correctly* *1 mark for transposing to get B^TA^T.*
	SECTION – C (Q14–15, 3 marks each)	
14(A)	**Q14(A)** Let X be a 2×2 matrix, so that X times the given 2×3 matrix gives a 2×3 result $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ Comparing entries: $a+4b=-7$, $2a+5b=-8$, $3a+6b=-9$ Solving $a+2b=-3$ and $a+4b=-7$ gives $b=-2$, $a=1$ Similarly $c+4d=2$, $2c+5d=4$, $3c+6d=6$ gives $d=0$, $c=2$ **Answer:** $X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$	*1 mark for setting up X as a 2×2 matrix* *1 mark for the correct matrix X.*

Q.No	Step-by-Step Solution	Marks per Step
	SECTION – A (Q1–10, 1 mark each)	
14(B)	**Q14(B)** $\begin{vmatrix} 0 & \cos\theta & \sin\theta \\ \cos\theta & \sin\theta & 0 \\ \sin\theta & 0 & \cos\theta \end{vmatrix}$ Expand along row 1: $D = 0\cdot C_{11} - \cos\theta(\cos^2\theta) + \sin\theta(-\sin^2\theta)$ $D = -(\cos^3\theta + \sin^3\theta)$ Given $\cos2\theta=0 \Rightarrow \cos^2\theta-\sin^2\theta=0 \Rightarrow \cos^2\theta=\sin^2\theta$, and since $\cos^2\theta+\sin^2\theta=1$, each equals 1/2 So $\cos\theta=\sin\theta=1/\sqrt{2}$, giving $\cos^3\theta=\sin^3\theta=1/2\sqrt{2}$ $D = -(1/2\sqrt{2} + 1/2\sqrt{2}) = -1/\sqrt{2}$ $D^2 = 1/2$, as required **Answer: $D^2 = 1/2$ (hence proved)**	*1 mark for expanding the determinant* *1 mark for using $\cos2\theta=0$* *1 mark for the final result.*
15	**Q15** Area $= (1/2)\lvert x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\rvert$ $= (1/2)\lvert 3(3-7) + 9(7-1) + 5(1-3)\rvert$ $= (1/2)\lvert -12+54-10\rvert = (1/2)(32) = 16$ sq units Equation of line PQ: $\begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix}$ $= 0 \Rightarrow x(1-3) - y(3-9) + (9-9) = 0 \Rightarrow -2x+6y=0$ Simplifying: $x - 3y = 0$ **Answer: Area of ΔPQR = 16 sq units; equation of line PQ: $x - 3y = 0$**	*1 mark for setting up the area determinant* *1 mark for the area* *1 mark for the equation of line PQ.*
	SECTION – D (Q16–17, 4 marks each)	
16	**Q16** **(i)** Condition 1: $(x-20)(y+30) = xy+1400 \Rightarrow 30x-20y=2000 \Rightarrow 3x-2y=200$ Condition 2: $(x-50)(y+50) = xy \Rightarrow 50x-50y=2500 \Rightarrow x-y=50$ **Answer: $3x - 2y = 200$ and $x - y = 50$** **(ii)** Writing the two equations as a matrix equation AX = B $A = \begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \end{bmatrix}$ $B = \begin{bmatrix} 200 \\ 50 \end{bmatrix}$ **Answer: AX = B, with A, X, B as above** **(iii)(a)** $A = \begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix}$ $A^T = \lceil\ 3 \quad 1\ \rceil$	*1 mark for the equations* *1 mark for the matrix form* *2 marks for AA^T (or PQ and the QP reasoning).*

Q.No	Step-by-Step Solution	Marks per Step
	SECTION – A (Q1–10, 1 mark each)	
	⌊ -2 -1 ⌋ AA^T: row1·col1=9+4=13, row1·col2=3+2=5, row2·col1=3+2=5, row2·col2=1+1=2 **Answer:** $AA^T = \begin{bmatrix} 13 & 5 \\ 5 & 2 \end{bmatrix}$ **(iii)(b) — OR** $P = \begin{bmatrix} 1 & -1 \\ 3 & -2 \end{bmatrix}$ $Q = \begin{bmatrix} 200 \\ 50 \end{bmatrix}$ Q is 2×1, P is 2×2. PQ: row1 = 1(200)+(−1)(50)=150; row2 = 3(200)+(−2)(50)=500 QP would need the number of columns of Q (1) to equal the number of rows of P (2) — it doesn't, so QP is not defined **Answer:** $PQ = \begin{bmatrix} 150 \\ 500 \end{bmatrix}$	
17	**Q17** **(i)** Let x, y, z be the tons of first, second, and third product Total production: $x+y+z=45$ Third exceeds first by 8: $z-x=8$, i.e. $-x+z=8$ Sum of first and third is twice the second: $x+z=2y$, i.e. $x-2y+z=0$ **Answer: $x+y+z=45$, $-x+z=8$, $x-2y+z=0$** **(ii)** we have given $A^{-1}=\frac{1}{6}\begin{pmatrix} 2 & 2 & 2 \\ -3 & 0 & 3 \\ 1 & -2 & 1 \end{pmatrix}$ where $A=\begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & -2 \\ 1 & 1 & 1 \end{pmatrix}$ We have matrix system of part(i) given as $A^tX = B$ $X = (A^t)^{-1}B$ $X = (A^{-1})^tB$ $X = \left[\frac{1}{6}\begin{pmatrix} 2 & 2 & 2 \\ -3 & 0 & 3 \\ 1 & -2 & 1 \end{pmatrix}\right]^t \begin{pmatrix} 45 \\ 8 \\ 0 \end{pmatrix}$ $X = \frac{1}{6}\left[\begin{pmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 1 & 1 \end{pmatrix}\right] \begin{pmatrix} 45 \\ 8 \\ 0 \end{pmatrix}$ $x = (1/6)(2\times45 - 3\times8 + 0) = (1/6)(66) = 11$ $y = (1/6)(2\times45 + 0 - 0) = (1/6)(90) = 15$ $z = (1/6)(2\times45 + 3\times8 + 0) = (1/6)(114) = 19$ **Answer: x = 11 tons, y = 15 tons, z = 19 tons**	*1 mark for the system of equations* *3 marks for correctly solving by the matrix method.*
	SECTION – E (Q18–19, 5 marks each)	
18	**Q18** $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \end{bmatrix}$	*2 marks for A^2* *2 marks for A^3*

Q.No	Step-by-Step Solution	Marks per Step
	SECTION – A (Q1–10, 1 mark each)	
	$\begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$ $A^2 = A\cdot A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$ $A^3 = A^2\cdot A = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$ Now combine $A^3 - 6A^2 + 9A - 4I$, entry by entry (using A, A^2, A^3 above) $A^3 - 6A^2 + 9A - 4I = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ **Answer: $A^3 - 6A^2 + 9A - 4I = O$ (hence proved)**	*1 mark for the final simplification to O.*
19(A)	**Q19(A)** First matrix = $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ Second matrix = $\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ Product = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Therefore, $AD = I$ $A^{-1} = D$ Given system $x+3z=9$, $x+2y-2z=4$, $2x-3y+4z=-3$ $A^t = \begin{pmatrix} 1 & 0 & 3 \\ 1 & 2 & -2 \\ 2 & -3 & 4 \end{pmatrix}, X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, B = \begin{pmatrix} 9 \\ 4 \\ -3 \end{pmatrix}$ Above system can be written as $A^tX = B$ $X = (A^t)^{-1}B$ $X = (A^{-1})^tB$ $X = (D)^tB$ $X = \left[\begin{pmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{pmatrix}\right]^t \begin{pmatrix} 9 \\ 4 \\ -3 \end{pmatrix}$ $X = \left[\begin{pmatrix} -2 & 9 & 6 \\ 0 & 2 & 1 \\ 1 & -3 & -2 \end{pmatrix}\right] \begin{pmatrix} 9 \\ 4 \\ -3 \end{pmatrix}$ **Answer: $x = 0, y = 5, z = 3$**	*1 mark for verifying the matrix product is I* *3 marks for correctly solving the system* *1 mark for the final answer.*
19(B)	**Q19(B)** $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 1 & 0 & 1 \end{bmatrix}$ $\det(A) = -6$	*2 marks for finding A^{-1}* *3 marks for solving x, y, z.*

Q.No	Step-by-Step Solution	Marks per Step
SECTION – A (Q1–10, 1 mark each)		
	$\text{adj}(A) = \begin{bmatrix} 3 & -3 & -3 \\ -2 & 0 & 2 \\ -5 & 3 & -1 \end{bmatrix}$ $6 \cdot A^{-1} = \begin{bmatrix} -3 & 3 & 3 \\ 2 & 0 & -2 \\ 5 & -3 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 5 \\ 1 \\ 8 \end{bmatrix}$ Solving $X=A^{-1}B$: $x=(1/6)(-15+3+24)=2$ $y=(1/6)(10+0-16)=-1$ $z=(1/6)(25-3+8)=5$ **Answer:** $X = \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}$	