Mind Curve Mid-Term Test Series 2026–27

Class XII | Test 03 – Answer Key

Chapters 5–6: Continuity & Differentiability, Application of Derivatives

Q.No	Step-by-Step Solution	Marks per Step
SECTION – A (Q1–10, 1 mark each)		
1	**Q1** $f(x)=\begin{cases}\lambda x^2+8x, & x\le 2\\ 2x+6, & x>2\end{cases}$ For continuity at x=2: $\lim_{x\to 2^-} f(x)=\lim_{x\to 2^+} f(x)$ Left side: $\lambda(2)^2+8(2) = 4\lambda+16$. Right side: $2(2)+6 = 10$ $4\lambda+16=10 \ \Rightarrow\ \lambda=-\frac{3}{2}$ **Answer: (d) −3/2**	*1 mark for the correct option.*
2	**Q2** $y=\log(\cos(e^x))$ $\frac{dy}{dx}=\frac{1}{\cos(e^x)}\cdot(-\sin(e^x))\cdot e^x=-e^x\tan(e^x)$ **Answer: (d) $-e^x\tan(e^x)$**	*1 mark for the correct option.*
3	**Q3** $y=\log_x e=\frac{1}{\ln x}$ $\frac{dy}{dx}=-\frac{1}{x(\ln x)^2}$ $\left.\frac{dy}{dx}\right\vert_{x=e}=-\frac{1}{e(1)^2}=-\frac{1}{e}$ **Answer: (d) −1/e**	*1 mark for the correct option.*
4	**Q4** $V=\frac{4}{3}\pi r^3 \ \Rightarrow\ \frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$ $\frac{dV}{dt}=4\pi(6)^2(2)=288\pi\ \text{cm}^3/\text{sec}$ **Answer: (a) 288π cm^3/sec**	*1 mark for the correct option.*
5	**Q5** $A=\frac{\sqrt{3}}{4}s^2 \ \Rightarrow\ \frac{dA}{dt}=\frac{\sqrt{3}}{2}s\frac{ds}{dt}$ $\frac{dA}{dt}=\frac{\sqrt{3}}{2}(10)(2)=10\sqrt{3}\ \text{cm}^2/\text{s}$ **Answer: (a) $10\sqrt{3}$ cm^2/s**	*1 mark for the correct option.*
6	**Q6** $y = x^3+x^2+x+1$	*1 mark for the correct option.*

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	$\frac{dy}{dx} = 3x^2 + 2x + 1$ $\text{Discriminant} = 4 - 12 = -8 < 0, \text{ coefficient of } x^2 > 0$ Since the discriminant is negative and the leading coefficient is positive, dy/dx > 0 for all real x — the function is strictly increasing everywhere **Answer: (c) neither has a local minimum nor local maximum**	
7	**Q7** y = x^2. y-coordinate changing six times as fast as x-coordinate means dy/dt = 6 dx/dt $\frac{dy}{dt} = 2x\frac{dx}{dt}$ $2x\frac{dx}{dt} = 6\frac{dx}{dt} \Rightarrow x = 3, \ y = 9$ **Answer: (c) (3, 9)**	*1 mark for the correct option.*
8	**Q8** x sin(a+y) = sin y. Differentiating implicitly w.r.t. x: $\sin(a+y) + x\cos(a+y)\frac{dy}{dx} = \cos y\,\frac{dy}{dx}$ From the original equation: $x = \frac{\sin y}{\sin(a+y)}$ $\cos y - x\cos(a+y) = \frac{\cos y\sin(a+y) - \sin y\cos(a+y)}{\sin(a+y)} = \frac{\sin a}{\sin(a+y)}$ $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ **Answer: (a) [$\sin^2$(a+y)]/sin a**	*1 mark for the correct option.*
9	**Q9** Assertion: $f(x) = e^{-3x}$ always decreases $f'(x) = -3e^{-3x} < 0 \text{ for all } x$ Since f'(x) is always negative, f(x) is indeed always decreasing — Assertion is TRUE Reason: a function is decreasing wherever dy/dx<0 — this is the correct general condition, TRUE, and it's exactly the reasoning used above to confirm the Assertion **Answer: (a) Both A and R are true, and R is the correct explanation of A**	*1 mark for the correct option.*
10	**Q10** Assertion: $f(x) = x + 2e^{-x}$ on $(-1, \infty)$ is strictly increasing $f'(x) = 1 - 2e^{-x}$ $f'(x) > 0 \iff e^{-x} < \frac{1}{2} \iff x > \ln 2$ So f is strictly increasing only on (ln2, ∞), not on all of (−1, ∞) $f'(0) = 1 - 2 = -1 < 0, \text{ and } 0 \in (-1, \infty)$ This confirms f is NOT increasing throughout (−1,∞) — Assertion is FALSE Reason (the general first-derivative test for strictly increasing) is a correct statement — TRUE **Answer: (d) A is false and R is true**	*1 mark for the correct option.*
SECTION – B (Q11–13, 2 marks each)		

Q.No	Step-by-Step Solution	Marks per Step
11(A)	**Q11(A)** $x = a(1-\cos\theta), \quad y = a(\theta+\sin\theta)$ $\frac{dx}{d\theta} = a\sin\theta, \quad \frac{dy}{d\theta} = a(1+\cos\theta)$ $\frac{dy}{dx} = \frac{1+\cos\theta}{\sin\theta}$ Differentiating again w.r.t. θ: $\frac{d}{d\theta}\left(\frac{1+\cos\theta}{\sin\theta}\right) = -\frac{1+\cos\theta}{\sin^2\theta}$ $\frac{d^2y}{dx^2} = \frac{-(1+\cos\theta)/\sin^2\theta}{a\sin\theta} = -\frac{1+\cos\theta}{a\sin^3\theta}$ $\left.\frac{d^2y}{dx^2}\right\|_{\theta=\pi/2} = -\frac{1+0}{a(1)} = -\frac{1}{a}$ **Answer: d^2y/dx^2 at $\theta=\pi/2$ is $-1/a$**	*1 mark for dy/dx* *1 mark for d^2y/dx^2 at $\theta=\pi/2$.*
11(B)	**Q11(B)** Differentiate $\sin^2x$ with respect to e^(cos x): let $u=\sin^2x$, v=e^(cos x) $u = \sin^2 x, \qquad v = e^{\cos x}$ $\frac{du}{dx} = 2\sin x\cos x = \sin 2x$ $\frac{dv}{dx} = -\sin x\, e^{\cos x}$ $\frac{du}{dv} = \frac{\sin 2x}{-\sin x\, e^{\cos x}} = -2\cos x\, e^{-\cos x}$ **Answer: du/dv = −2cos x · e^(−cos x)**	*1 mark for du/dx and dv/dx* *1 mark for du/dv.*
12(A)	**Q12(A)** f(x) = sin2x + 5 $f'(x) = 2\cos 2x = 0 \;\Rightarrow\; x = \frac{\pi}{4} + \frac{n\pi}{2}$ At $x = \frac{\pi}{4}$: $f'' < 0,\; f\left(\frac{\pi}{4}\right) = 1+5 = 6$ (max) At $x = \frac{3\pi}{4}$: $f'' > 0,\; f\left(\frac{3\pi}{4}\right) = -1+5 = 4$ (min) **Answer: Maximum value = 6, minimum value = 4**	*1 mark for the critical points* *1 mark for both max and min values.*
12(B)	**Q12(B)** $\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt} = 8 \;\Rightarrow\; \frac{dr}{dt} = \frac{2}{\pi r^2}$ $\frac{dS}{dt} = 8\pi r\frac{dr}{dt} = 8\pi r\cdot\frac{2}{\pi r^2} = \frac{16}{r}$ $\left.\frac{dS}{dt}\right\|_{r=12} = \frac{16}{12} = \frac{4}{3}\ \text{cm}^2/\text{s}$ **Answer: 4/3 cm²/s**	*1 mark for dr/dt* *1 mark for dS/dt.*
13	**Q13** $\cos(xy) = \sin\left(\frac{y}{x}\right)$ Differentiating implicitly w.r.t. x: $-\sin(xy)\left(y + x\frac{dy}{dx}\right) = \cos\left(\frac{y}{x}\right)\cdot\frac{x\frac{dy}{dx} - y}{x^2}$	*1 mark for correct implicit differentiation* *1 mark for the simplified dy/dx.*

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	$\dfrac{dy}{dx}=\dfrac{y\left[\cos(y/x)-x^2\sin(xy)\right]}{x\left[x^2\sin(xy)+\cos(y/x)\right]}$ **Answer:** $\dfrac{dy}{dx}=\dfrac{y\left[\cos(y/x)-x^2\sin(xy)\right]}{x\left[x^2\sin(xy)+\cos(y/x)\right]}$	
	SECTION – C (Q14–15, 3 marks each)	
14(A)	**Q14(A)** $f(x)=\sin x+\cos x,\quad 0\le x\le 2\pi$ $f'(x)=\cos x-\sin x=\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)$ $f'(x)>0$ on $\left[0,\frac{\pi}{4}\right)\cup\left(\frac{5\pi}{4},2\pi\right]$ $f'(x)<0$ on $\left(\frac{\pi}{4},\frac{5\pi}{4}\right)$ **Answer: Increasing on [0,π/4)∪(5π/4,2π]; decreasing on (π/4,5π/4)**	*1 mark for f'(x)* *1 mark each for the increasing/decreasing intervals.*
14(B)	**Q14(B)** $f(x)=\sin^4 x+\cos^4 x,\quad 0\le x\le \frac{\pi}{2}$ $f'(x)=4\sin^3 x\cos x-4\cos^3 x\sin x=4\sin x\cos x(\sin^2 x-\cos^2 x)$ $f'(x)=-2\sin 2x\cos 2x=-\sin 4x$ $f'(x)<0$ on $\left(0,\frac{\pi}{4}\right),\qquad f'(x)>0$ on $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$ **Answer: Decreasing on (0,π/4); increasing on (π/4,π/2)**	*1 mark for f'(x) = −sin4x* *2 marks for the increasing/decreasing intervals.*
15	**Q15** $x=a\cos\theta+b\sin\theta,\qquad y=a\sin\theta-b\cos\theta$ $\dfrac{dx}{d\theta}=-a\sin\theta+b\cos\theta=-y,\qquad \dfrac{dy}{d\theta}=a\cos\theta+b\sin\theta=x$ $y'=\dfrac{dy}{dx}=\dfrac{dy/d\theta}{dx/d\theta}=\dfrac{x}{-y}=-\dfrac{x}{y}$ $y\,y'=-x\ \Rightarrow\ x+y\,y'=0$ Differentiating (*) again w.r.t. x: $y''=\dfrac{d}{dx}\left(-\dfrac{x}{y}\right)=\dfrac{xy'-y}{y^2}$ $y^2y''=xy'-y\ \Rightarrow\ y^2y''-xy'+y=0$ **Answer: $y^2y''-xy'+y=0$ (hence proved)**	*1 mark for dx/dθ, dy/dθ* *1 mark for y'=−x/y* *1 mark for the final result.*
	SECTION – D (Q16–17, 4 marks each)	
16	**Q16** **(a) & (b)** $f(x)=\begin{cases}\dfrac{\sin(a+1)x+\sin x}{x}, & x<0\\ c, & x=0\\ \dfrac{\sqrt{x+bx^2}-\sqrt{x}}{b\,x^{3/2}}, & x>0\end{cases}$ Left-hand limit as $x\to0^-$: $\lim_{x\to0^-}f(x)=\lim_{x\to0}\left[\dfrac{\sin(a+1)x}{x}+\dfrac{\sin x}{x}\right]=(a+1)+1=a+2$ Right-hand limit as $x\to0^+$: $\lim_{x\to0^+}f(x)=\lim_{x\to0^+}\dfrac{\sqrt{x}\left[\sqrt{1+bx}-1\right]}{b\,x^{3/2}}=\lim_{x\to0^+}\dfrac{\sqrt{1+bx}-1}{bx}$	*1 mark for the left-hand limit* *1 mark for the right-hand limit* *1 mark for b* *1 mark for a+c.*

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	Using the small-x approximation: $\sqrt{1+bx} \approx 1 + \frac{bx}{2} \Rightarrow \frac{\sqrt{1+bx}-1}{bx} \to \frac{1}{2}$ For continuity, all three values must be equal: $a + 2 = c = \frac{1}{2} \Rightarrow c = \frac{1}{2},\ a = -\frac{3}{2}$ $a + c = -\frac{3}{2} + \frac{1}{2} = -1$ **Answer: b can be any non-zero real number (it cancels out of the right-hand limit); a+c = −1**	
17	**Q17** **(i)** $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ Solving for y: $y = \frac{b}{a}\sqrt{a^2 - x^2}$ $A(x) = 4xy = \frac{4b}{a} x\sqrt{a^2 - x^2}$ **Answer: A(x) = (4b/a) x√(a²−x²)** **(ii)** $\frac{dA}{dx} = \frac{4b}{a} \cdot \frac{a^2 - 2x^2}{\sqrt{a^2 - x^2}}$ $a^2 - 2x^2 = 0 \Rightarrow x = \frac{a}{\sqrt{2}}$ **Answer: Critical point: x = a/√2** **(iii)(a)** For $x<a/\sqrt{2}$, $a^2-2x^2>0$ so $dA/dx>0$ (increasing); for $x>a/\sqrt{2}$, $dA/dx<0$ (decreasing) — so $x=a/\sqrt{2}$ gives a maximum $y = \frac{b}{a}\sqrt{a^2 - \frac{a^2}{2}} = \frac{b}{\sqrt{2}}$ $\text{Length } 2x = a\sqrt{2}, \qquad \text{Width } 2y = b\sqrt{2}$ **Answer: Length = a√2, width = b√2** **(iii)(b) — OR** $\left.\frac{d^2A}{dx^2}\right\|_{x=a/\sqrt{2}} = -\frac{16b}{a} < 0 \text{ (confirms maximum)}$ Since the second derivative is negative at the critical point, this confirms a maximum **Answer: Length = a√2, width = b√2**	*1 mark for the area function* *1 mark for the critical point* *2 marks for (iii)(A) or (iii)(B).*
SECTION – E (Q18–19, 5 marks each)		
18	**Q18** $(x-a)^2 + (y-b)^2 = c^2, \quad c > 0$ Differentiating once w.r.t. x: $(x-a) + (y-b)y' = 0$ Differentiating again:	*1 mark for differentiating twice* *2 marks for expressing (x−a) and (y−b) in terms of y′, y″*

Q.No	Step-by-Step Solution	Marks per Step
	$1+(y')^2+(y-b)y''=0 \;\Rightarrow\; (y-b)=-\dfrac{1+(y')^2}{y''}$ From the first derivative equation: $(x-a)=-(y-b)y'=\dfrac{[1+(y')^2]\,y'}{y''}$ Substituting both back into the original circle equation: $\left[\dfrac{(1+(y')^2)y'}{y''}\right]^2+\left[\dfrac{1+(y')^2}{y''}\right]^2=c^2$ $\dfrac{[1+(y')^2]^2\,[(y')^2+1]}{(y'')^2}=c^2 \;\Rightarrow\; \dfrac{[1+(y')^2]^3}{(y'')^2}=c^2$ Taking the square root of both sides: $\dfrac{[1+(dy/dx)^2]^{3/2}}{d^2y/dx^2}=\pm c$ **Answer: [1+(dy/dx)²]^(3/2) / (d²y/dx²) = ±c, a constant independent of a and b (hence proved)**	*2 marks for substituting back and simplifying to the final constant.*
19(A)	**Q19(A)** Let the two equal legs and the top all have length 10 cm, with each leg making angle θ with the base $\text{Base}=10+20\cos\theta,\quad \text{height}=10\sin\theta$ $A(\theta)=\dfrac{1}{2}(10+10+20\cos\theta)(10\sin\theta)=100\sin\theta(1+\cos\theta)$ $\dfrac{dA}{d\theta}=100[\cos\theta+\cos^2\theta-\sin^2\theta]=100[2\cos^2\theta+\cos\theta-1]$ $2\cos^2\theta+\cos\theta-1=0 \;\Rightarrow\; \cos\theta=\dfrac{1}{2}\text{ or }\cos\theta=-1$ $\cos\theta=\dfrac{1}{2}\;\Rightarrow\;\theta=60^\circ$ (the other root, cosθ=−1, is not valid for a trapezium) $A=100\times\dfrac{\sqrt{3}}{2}\times\dfrac{3}{2}=75\sqrt{3}\text{ cm}^2$ **Answer: Maximum area = 75√3 cm²**	*2 marks for setting up A(θ)* *2 marks for solving dA/dθ=0* *1 mark for the maximum area.*
19(B) — OR	**Q19(B) — OR** $V=\pi r^2h=100 \;\Rightarrow\; h=\dfrac{100}{\pi r^2}$ $S=2\pi r^2+2\pi rh=2\pi r^2+\dfrac{200}{r}$ $\dfrac{dS}{dr}=4\pi r-\dfrac{200}{r^2}=0 \;\Rightarrow\; r^3=\dfrac{50}{\pi}$ $r=\left(\dfrac{50}{\pi}\right)^{1/3}\text{ cm}$ $h=\dfrac{100}{\pi r^2}=2r\text{ (using }r^3=50/\pi)$ **Answer:** $r=\left(\dfrac{50}{\pi}\right)^{1/3}\text{ cm},\qquad h=2\left(\dfrac{50}{\pi}\right)^{1/3}\text{ cm}$	*1 mark for S(r)* *2 marks for dS/dr=0 and solving for r* *2 marks for h and confirming h=2r.*