Mind Curve Mid-Term Test Series 2026-27

Class XII | Test 05 – Answer Key

Chapters 7–8: Integration & Application of Integrals

Q.No	Step-by-Step Solution	Marks per Step
SECTION – A (Q1–10, 1 mark each)		
1	**Q1** Let $I = \int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right) dx$ Using $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ with $a = \frac{\pi}{2}$: since $\sin\left(\frac{\pi}{2}-x\right) = \cos x$ and $\cos\left(\frac{\pi}{2}-x\right) = \sin x$ $I = \int_0^{\pi/2} \log\left(\frac{4+3\cos x}{4+3\sin x}\right) dx = -I$ (since $\log(1/y) = -\log y$) $2I = 0 \Rightarrow I = 0$ **Answer: (c) 0**	*1 mark for the correct option.*
2	**Q2** $\cos x \geq 0$ on $\left[0, \frac{\pi}{2}\right]$ and $\cos x \leq 0$ on $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$, so the area must be split at $x = \frac{\pi}{2}$ $\int_0^{\pi/2} \cos x\, dx = [\sin x]_0^{\pi/2} = 1$ $\int_{\pi/2}^{3\pi/2} \cos x\, dx = [\sin x]_{\pi/2}^{3\pi/2} = -1-1 = -2$, so this contributes $\lvert -2 \rvert = 2$ Total area $= 1 + 2 = 3$ sq units **Answer: (c) 3 sq. unit**	*1 mark for the correct option.*
3	**Q3** $\int \frac{x^9}{(4x^2+1)^6} dx$ — dividing numerator and denominator by x^{12}: $= \int x^{-3}\left(4 + \frac{1}{x^2}\right)^{-6} dx$ Let $u = 4 + \frac{1}{x^2}$, so $du = -\frac{2}{x^3}dx \Rightarrow x^{-3}dx = -\frac{du}{2}$ $= \int u^{-6}\left(-\frac{du}{2}\right) = -\frac{1}{2} \times \frac{u^{-5}}{-5} = \frac{1}{10}u^{-5} + C = \frac{1}{10}\left(4 + \frac{1}{x^2}\right)^{-5} + C$ **Answer: (d)** $\frac{1}{10}\left(4 + \frac{1}{x}\right)^{-5} + C$	*1 mark for the correct option.*
4	**Q4** The upper y-limit was left blank in the source paper; based on the answer options, this is the standard problem with limits $y = -1$ to $y = 1$ Area $= \int_{-1}^{1} (2y+3)\, dy = \left[y^2 + 3y\right]_{-1}^{1} = (1+3) - (1-3) = 4 - (-2) = 6$ **Answer: (c) 6 sq. unit**	*1 mark for the correct option.*
5	**Q5**	*1 mark for the correct option.*

Q.No	Step-by-Step Solution	Marks per Step
	$I=\int_0^{2\pi}\frac{dx}{e^{\sin x}+1}$ Using $\int_0^{2a} f(x)dx=\int_0^{a}[f(x)+f(2a-x)]dx$ with $a=\pi$: $f(2\pi-x)=\frac{1}{e^{\sin(2\pi-x)}+1}=\frac{1}{e^{-\sin x}+1}=\frac{e^{\sin x}}{1+e^{\sin x}}$ $f(x)+f(2\pi-x)=\frac{1}{e^{\sin x}+1}+\frac{e^{\sin x}}{e^{\sin x}+1}=1$ $I=\int_0^{\pi}1\,dx=\pi$ **Answer: (a) p (i.e. π)**	
6	**Q6** $\int\frac{\sin^2 x-\cos^2 x}{\sin^2 x\cos^2 x}dx=\int\left[\frac{1}{\cos^2 x}-\frac{1}{\sin^2 x}\right]dx=\int[\sec^2 x-\operatorname{cosec}^2 x]\,dx$ $=\tan x-(-\cot x)+C=\tan x+\cot x+C$ **Answer: (c) tanx + cotx + C**	*1 mark for the correct option.*
7	**Q7** $\cos 2x=\cos^2 x-\sin^2 x=(\cos x-\sin x)(\cos x+\sin x)$ $\frac{\cos 2x}{(\sin x+\cos x)^2}=\frac{(\cos x-\sin x)(\cos x+\sin x)}{(\sin x+\cos x)^2}=\frac{\cos x-\sin x}{\sin x+\cos x}$ Let $u=\sin x+\cos x$, so $du=(\cos x-\sin x)dx$ $\int\frac{du}{u}=\log\lvert u\rvert+C=\log\lvert\sin x+\cos x\rvert+C$ **Answer:** $\log\lvert\sin x+\cos x\rvert+C$ (c)	*1 mark for the correct option.*
8	**Q8** Let $t=\tan x$, so $dt=\sec^2 x\,dx=\frac{dx}{\cos^2 x}$ $\sin x\cos x=\tan x\cos^2 x=t\cos^2 x$, so $\frac{dx}{\sin x\cos x}=\frac{1}{t}\times\frac{dx}{\cos^2 x}=\frac{dt}{t}$ $\int\sqrt{t}\times\frac{dt}{t}=\int t^{-1/2}dt=2\sqrt{t}+C=2\sqrt{\tan x}+C$ **Answer: (a)** $2\sqrt{\tan x}$	*1 mark for the correct option.*
9	**Q9** Let $u=x+\frac{1}{x}$, so $du=\left(1-\frac{1}{x^2}\right)dx$ $\int\left(x+\frac{1}{x}\right)^3\left(1-\frac{1}{x^2}\right)dx=\int u^3\,du=\frac{u^4}{4}+C=\frac{1}{4}\left(x+\frac{1}{x}\right)^4+C$ — matches Assertion, so (A) is TRUE Reason (R) is the standard power-rule for integration ($n\neq -1$) — a TRUE statement, and it is exactly what was applied to u^3 after the substitution **Answer: (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).**	*1 mark for the correct option.*
10	**Q10** Try $f(x)=\frac{1}{x+1}$, so $f'(x)=\frac{-1}{(x+1)^2}$	*1 mark for the correct option.*

Q.No	Step-by-Step Solution	Marks per Step
	$f(x)+f'(x)=\dfrac{1}{x+1}-\dfrac{1}{(x+1)^2}=\dfrac{(x+1)-1}{(x+1)^2}=\dfrac{x}{(x+1)^2}$ — exactly the integrand's non-exponential factor By the standard rule $\int e^x[f(x)+f'(x)]dx=e^x f(x)+C$: $\int e^x\dfrac{x}{(x+1)^2}dx=\dfrac{e^x}{x+1}+C$ — matches Assertion, so (A) is TRUE Reason (R) states exactly this standard rule — TRUE, and it is the correct explanation **Answer: (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).**	
	SECTION – B (Q11–13, 2 marks each)	
11	**Q11** **(A)** $\int \dfrac{dx}{(x-4)^2-1^2}=\dfrac{1}{2(1)}\ln\left\|\dfrac{(x-4)-1}{(x-4)+1}\right\|+C$ $=\dfrac{1}{2}\ln\left\|\dfrac{x-5}{x-3}\right\|+C$ **(B)** $9x^2+6x+10=9\left(x^2+\dfrac{2}{3}x\right)+10=9\left[\left(x+\dfrac{1}{3}\right)^2-\dfrac{1}{9}\right]+10$ $=9\left(x+\dfrac{1}{3}\right)^2-1+10=9\left[\left(x+\dfrac{1}{3}\right)^2+1\right]$ $\int\dfrac{dx}{9x^2+6x+10}=\dfrac{1}{9}\int\dfrac{dx}{(x+\frac{1}{3})^2+1}=\dfrac{1}{9}\tan^{-1}\left(x+\dfrac{1}{3}\right)+C$ **Answer:** $\dfrac{1}{9}\tan^{-1}\left(x+\dfrac{1}{3}\right)+C$	*2 marks for a correct, complete solution to either part.*
12	**Q12** Write $\cos x=\cos[(x-a)+a]=\cos(x-a)\cos a-\sin(x-a)\sin a$ $\dfrac{\cos x}{\cos(x-a)}=\cos a-\sin a\tan(x-a)$ $\int[\cos a-\sin a\tan(x-a)]\,dx=x\cos a-\sin a\times(-\log\|\cos(x-a)\|)+C$ $=x\cos a+\sin a\log\|\cos(x-a)\|+C$ **Answer:** $x\cos a+\sin a\,\log\|\cos(x-a)\|+C$	*1 mark for the split $\cos a-\sin a\tan(x-a)$; 1 mark for the final integral.*
13	**Q13** **(A)** $y=x^2$ and $y=16$ meet where $x^2=16\Rightarrow x=\pm4$ Area $=\int_{-4}^{4}(16-x^2)\,dx=2\int_0^4(16-x^2)\,dx=2\left[16x-\dfrac{x^3}{3}\right]_0^4$ $=2\left[64-\dfrac{64}{3}\right]=2\times\dfrac{128}{3}=\dfrac{256}{3}$ **Answer:** $\dfrac{256}{3}$ **sq units** **(B)**	*1 mark for setting up the integral; 1 mark for the final area.*

Q.No	Step-by-Step Solution	Marks per Step
	$y = x^2 \geq 0$ throughout $[-1, 2]$, so no sign splitting is needed Area $= \int_{-1}^{2} x^2\,dx = \left[\frac{x^3}{3}\right]_{-1}^{2} = \frac{8}{3} - \left(-\frac{1}{3}\right) = \frac{9}{3} = 3$ **Answer: 3 sq units**	
	SECTION – C (Q14–15, 3 marks each)	
14	**Q14** **(A)** Let $I = \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$ Using $\int_0^a f(x)dx = \int_0^a f(a-x)dx$: $I = \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x} dx = \pi \int_0^{\pi} \frac{\sin x}{1+\cos^2 x} dx - I$ $2I = \pi \int_0^{\pi} \frac{\sin x}{1+\cos^2 x} dx$. Let $t = \cos x$, $dt = -\sin x\,dx$ $\int_0^{\pi} \frac{\sin x}{1+\cos^2 x} dx = \int_{-1}^{1} \frac{dt}{1+t^2} = [\tan^{-1} t]_{-1}^{1} = \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$ $2I = \pi \times \frac{\pi}{2} = \frac{\pi^2}{2} \Rightarrow I = \frac{\pi^2}{4}$ **Answer:** $\frac{\pi^2}{4}$ **(B)** $x^2 - 2x = x(x-2)$, which is ≤ 0 on $[1, 2]$ and ≥ 0 on $[2, 3]$ $\int_1^3 \lvert x^2 - 2x \rvert\,dx = \int_1^2 (2x - x^2)\,dx + \int_2^3 (x^2 - 2x)\,dx$ $\int_1^2 (2x - x^2)dx = \left[x^2 - \frac{x^3}{3}\right]_1^2 = \left(4 - \frac{8}{3}\right) - \left(1 - \frac{1}{3}\right) = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}$ $\int_2^3 (x^2 - 2x)dx = \left[\frac{x^3}{3} - x^2\right]_2^3 = (9-9) - \left(\frac{8}{3} - 4\right) = 0 + \frac{4}{3} = \frac{4}{3}$ Total $= \frac{2}{3} + \frac{4}{3} = 2$ **Answer: 2**	*3 marks for a correctly-justified, step-wise evaluation of either integral.*
15	**Q15** $y = \sqrt{1-x^2}$ on $[-1, 1]$ is the upper half of the unit circle $x^2 + y^2 = 1$ Area under this curve $= \int_{-1}^{1} \sqrt{1-x^2}\,dx = \left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1} x\right]_{-1}^{1}$ $= \left(0 + \frac{1}{2} \times \frac{\pi}{2}\right) - \left(0 + \frac{1}{2} \times \left(-\frac{\pi}{2}\right)\right) = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$ **Answer: TRUE — the computed area exactly equals $\frac{\pi}{2}$, matching the statement (this is also just the area of a semicircle of radius 1, $\frac{1}{2}\pi r^2 = \frac{\pi}{2}$).**	*1 mark for identifying the curve/setup; 1 mark for evaluating the integral; 1 mark for the true/false conclusion.*
	SECTION – D (Q16–17, 4 marks each)	
16	**Q16** Integrals of the form $\int \frac{dx}{a \sin x + b \cos x + c}$ are evaluated using the Weierstrass substitution $t = \tan\frac{x}{2}$, giving $\sin x = \frac{2t}{1+t^2}$, $\cos x = \frac{1-t^2}{1+t^2}$, $dx = \frac{2\,dt}{1+t^2}$ (the steps described in the question)	*(i) 2 marks for either part (1 mark for the substitution/setup; 1 mark for the final log form). (ii) 2 marks similarly.*

Q.No	Step-by-Step Solution	Marks per Step
	(i) Evaluate $\int \frac{dx}{1+2\sin x}$ With $t=\tan\frac{x}{2}$: $1+2\sin x = 1+\frac{4t}{1+t^2} = \frac{t^2+4t+1}{1+t^2}$ $\int \frac{dx}{1+2\sin x} = \int \frac{2\,dt}{t^2+4t+1}$ $t^2+4t+1=(t+2)^2-3$, so $\int \frac{2\,dt}{(t+2)^2-(\sqrt{3})^2} = \frac{2}{2\sqrt{3}}\log\left\lvert\frac{t+2-\sqrt{3}}{t+2+\sqrt{3}}\right\rvert + C$ **Answer:** $\frac{1}{\sqrt{3}}\log\left\lvert\frac{\tan(x/2)+2-\sqrt{3}}{\tan(x/2)+2+\sqrt{3}}\right\rvert + C$ **(i) [OR] Evaluate $\int \frac{dx}{1+2\cos x}$** With $t=\tan\frac{x}{2}$: $1+2\cos x = 1+\frac{2(1-t^2)}{1+t^2} = \frac{3-t^2}{1+t^2}$ $\int \frac{dx}{1+2\cos x} = \int \frac{2\,dt}{3-t^2} = \frac{2}{2\sqrt{3}}\log\left\lvert\frac{\sqrt{3}+t}{\sqrt{3}-t}\right\rvert + C$ **Answer:** $\frac{1}{\sqrt{3}}\log\left\lvert\frac{\sqrt{3}+\tan(x/2)}{\sqrt{3}-\tan(x/2)}\right\rvert + C$ **(ii) Evaluate $\int \frac{dx}{\sin x+\cos x}$** With $t=\tan\frac{x}{2}$: $\sin x+\cos x = \frac{2t+1-t^2}{1+t^2}$, so $\int \frac{dx}{\sin x+\cos x} = \int \frac{2\,dt}{1+2t-t^2} = -2\int \frac{dt}{(t-1)^2-2}$ $= -2\times\frac{1}{2\sqrt{2}}\log\left\lvert\frac{t-1-\sqrt{2}}{t-1+\sqrt{2}}\right\rvert + C = \frac{1}{\sqrt{2}}\log\left\lvert\frac{t-1+\sqrt{2}}{t-1-\sqrt{2}}\right\rvert + C$ (equivalently, writing $\sin x+\cos x=\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$ gives the compact form $\frac{1}{\sqrt{2}}\log\left\lvert\tan\left(\frac{x}{2}+\frac{\pi}{8}\right)\right\rvert + C$) **Answer:** $\frac{1}{\sqrt{2}}\log\left\lvert\frac{\tan(x/2)-1+\sqrt{2}}{\tan(x/2)-1-\sqrt{2}}\right\rvert + C$	
17	**Q17** O(0,0) is the centre and A$(4\sqrt{2},0)$ lies on the circle, so the radius is $4\sqrt{2}$ **(i) Equation of the circle** $x^2+y^2=(4\sqrt{2})^2=32$ **Answer:** $x^2+y^2=32$ **(ii) Coordinates of B** B is where $y=x$ meets the circle in the first quadrant: substitute into $x^2+y^2=32$ $2x^2=32 \Rightarrow x^2=16 \Rightarrow x=4$ (taking the positive root) **Answer:** **B = (4, 4)** **(iii) Area of the shaded region**	*(i) 1 mark. (ii) 1 mark. (iii) 2 marks for the area.*

Q.No	Step-by-Step Solution	Marks per Step
	The shaded region OBA (bounded by line OB, the arc BA, and the x-axis back to O) splits into the triangle OMB (M = (4,0)) plus the circular-arc strip from $x = 4$ to $x = 4\sqrt{2}$ Triangle part: $\int_0^4 x\,dx = \left[\frac{x^2}{2}\right]_0^4 = 8$ Arc part: $\int_4^{4\sqrt{2}} \sqrt{32 - x^2}\,dx = \left[\frac{x}{2}\sqrt{32 - x^2} + 16\sin^{-1}\frac{x}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$ At $x = 4\sqrt{2}$: $0 + 16 \times \frac{\pi}{2} = 8\pi$. At $x = 4$: $2 \times 4 + 16 \times \frac{\pi}{4} = 8 + 4\pi$ Arc part $= 8\pi - (8 + 4\pi) = 4\pi - 8$ Total shaded area $= 8 + (4\pi - 8) = 4\pi$ **Answer: 4π sq units**	
SECTION – E (Q18–19, 5 marks each)		
18	**Q18** P(3,1), Q(9,3), R(5,7). Line PQ: slope $= \frac{3-1}{9-3} = \frac{1}{3} \Rightarrow y = \frac{x}{3}$ Line QR: slope $= \frac{7-3}{5-9} = -1 \Rightarrow y = 12 - x$ Line PR: slope $= \frac{7-1}{5-3} = 3 \Rightarrow y = 3x - 8$ Since R is the topmost vertex, PR lies above PQ on $[3, 5]$ and QR lies above PQ on $[5, 9]$ Area $= \int_3^5 \left[(3x - 8) - \frac{x}{3}\right] dx + \int_5^9 \left[(12 - x) - \frac{x}{3}\right] dx$ $= \int_3^5 \left(\frac{8x}{3} - 8\right) dx + \int_5^9 \left(12 - \frac{4x}{3}\right) dx$ First integral $= \left[\frac{4x^2}{3} - 8x\right]_3^5 = \left(-\frac{20}{3}\right) - (-12) = \frac{16}{3}$ Second integral $= \left[12x - \frac{2x^2}{3}\right]_5^9 = 54 - \frac{130}{3} = \frac{32}{3}$ Total area $= \frac{16}{3} + \frac{32}{3} = \frac{48}{3} = 16$ **Answer: 16 sq units**	*1 mark for each line equation; 2 marks for setting up the split integral; 1 mark for the final area.*
19	**Q19** **(A)** $x\sin(\pi x)$ is positive on $(-1, 1)$ and negative on $(1, 3/2)$ (checked from the sign of x and $\sin(\pi x)$ in each sub-interval), so $\int_{-1}^{3/2} \lvert x\sin(\pi x)\rvert\,dx = \int_{-1}^{1} x\sin(\pi x)\,dx - \int_{1}^{3/2} x\sin(\pi x)\,dx$ By parts, $\int x\sin(\pi x)\,dx = -\frac{x\cos(\pi x)}{\pi} + \frac{\sin(\pi x)}{\pi^2} + C = F(x)$ $F(1) = \frac{1}{\pi}, \quad F(-1) = -\frac{1}{\pi}, \quad F(3/2) = -\frac{1}{\pi^2}$ $\int_{-1}^{1} x\sin(\pi x)dx = F(1) - F(-1) = \frac{2}{\pi}$ $\int_{1}^{3/2} x\sin(\pi x)dx = F(3/2) - F(1) = -\frac{1}{\pi^2} - \frac{1}{\pi}$ Total $= \frac{2}{\pi} - \left(-\frac{1}{\pi^2} - \frac{1}{\pi}\right) = \frac{3}{\pi} + \frac{1}{\pi^2}$	*5 marks for a correctly-justified, step-wise evaluation of either integral.*

Q.No	Step-by-Step Solution	Marks per Step
	Answer: $\frac{3}{\pi} + \frac{1}{\pi^2} = \frac{3\pi + 1}{\pi^2}$ **(B) [OR]** $\sqrt{\cot x} + \sqrt{\tan x} = \frac{\cos x + \sin x}{\sqrt{\sin x \cos x}}$ Let $t = \sin x - \cos x$, so $dt = (\cos x + \sin x)\, dx$, and $t^2 = 1 - 2 \sin x \cos x \Rightarrow \sin x \cos x = \frac{1 - t^2}{2}$ $\int \frac{dt}{\sqrt{(1-t^2)/2}} = \sqrt{2} \int \frac{dt}{\sqrt{1-t^2}} = \sqrt{2} \sin^{-1} t + C$ **Answer:** $\sqrt{2}\, \sin^{-1}(\sin x - \cos x) + C$	

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