MIND CURVE — Half Yearly Maths Practice Paper Series 2026-27

CLASS X — TERM 1 — PRACTICE PAPER 03

ANSWER KEY WITH FULL STEP-WISE SOLUTIONS (MM: 80, Time: 3 Hrs)

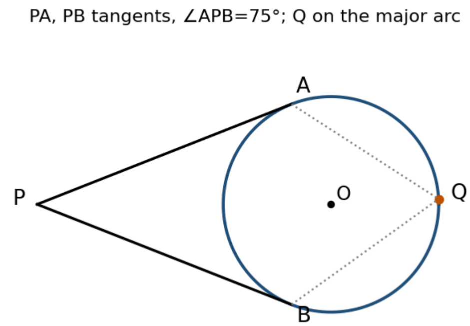
Section A — MCQs (1 mark each)

Q.No	Answer	Reasoning / Working
1	(d) 2	$a1/a2=b1/b2\neq c1/c2 \Rightarrow 3/(2k-1)=1/(k-1) \Rightarrow 3(k-1)=2k-1 \Rightarrow k=2$. Check c-ratio $1/5 \neq 1$ ✓ confirms inconsistency.
2	(d) $3\sqrt{3}$ cm	Half-angle=30°; in right ΔOTP: $\tan30°=r/L \Rightarrow L=r/\tan30°=3/(1/\sqrt{3})=3\sqrt{3}$ cm.
3	(a)/(c) 8 cm	$AB/DE=BC/EF \Rightarrow EF=BC\times DE/AB=6\times4/3=8$ cm.
4	(c) (2,-1)	Midpoint PR = Midpoint QS ⇒ (0,1)=midpoint of Q,S ⇒ S=(2(0)-(-2), 2(1)-3)=(2,-1).
5	(c) 35	$HCF=p^2q^3$ (m=2,n=3); $LCM=p^3q^4$ (r=3,s=4). $(m+n)(r+s)=5\times7=35$.
6	(b) $x^2-(p+1)x+p=0$	Factors of prime p: 1,p. Sum=1+p, product=p.
7	(b) 2	Squaring $\sin\theta+\cos\theta=\sqrt{2}$: $1+2\sin\theta\cos\theta=2 \Rightarrow \sin\theta\cos\theta=1/2$. $\tan\theta+\cot\theta=1/(\sin\theta\cos\theta)=2$.
8	(b) tan30°	$x\sqrt{3}(1/2)=(\sqrt{3}/2)(1/\sqrt{3}) \Rightarrow x\sqrt{3}/2=1/2 \Rightarrow x=1/\sqrt{3}=\tan30°$.
9	(b) 2/3	Sum=2/p, product=3. Equal ⇒ 2/p=3 ⇒ p=2/3.
10	(d) 18 cm	Perimeter DEF=6+9+12=27; ratio AB/DE=4/6=2/3; perimeter ABC=(2/3)×27=18 cm.
11	(c) x = ay/(a+b)	ΔADE~ΔABC ⇒ AE/AC=DE/BC ⇒ a/(a+b)=x/y ⇒ x=ay/(a+b).
12	(c) 8cm	ΔAOD~ΔCOB ⇒ AO/OC=AD/BC ⇒ 1/2=4/BC ⇒ BC=8cm.
13	(c) (0,0)	Midpoint = ((2-2)/2,(4-4)/2) = (0,0).
14	(c) -77	a30=10+29(-3)=10-87=-77.
15	(a) 1/3	Divide num/denom by cosβ: (5tanβ-2)/(5tanβ+2)=(4-2)/(4+2)=2/6=1/3.
16	(d) 16th	an=3+(n-1)5=5n-2=78 ⇒ n=16.
17	(b) -12	Midpoint x: (-6-2)/2=-4=p/3 ⇒ p=-12.
18	(c) 50°	AC diameter ⇒ ∠ABC=90° ⇒ ∠BAC=180-90-50=40°. ∠BAT=∠OAT-∠OAB=90-40=50° (tangent-chord = angle in alternate segment = ∠ACB directly = 50°).
19	(a) Both true, R is the correct explanation	DE (midpoints) = $\sqrt{[(3+3)^2+(5+3)^2]}=\sqrt{100}=10$; BC=2×DE=20 by the midpoint theorem — confirms both A and R.
20	(b) Both true, R not the correct explanation	LCM=product/HCF=5780/17=340 (A true); R (HCF divides LCM) is a true general fact but is not the actual formula used to compute 340 — the real reason is 'product of two numbers = HCF×LCM'.

Section B — 2 marks each

Q21 *[2 marks]* **O is the centre; PA, PB are tangents; ∠APB=75°. Find ∠AQB (Q on the circle).**

Solution



PA, PB tangents from P; Q lies on the major arc AB

Step 1: In quadrilateral OAPB: $\angle OAP = \angle OBP = 90°$ (radius ⊥ tangent).

Step 2: Angle sum of quadrilateral: $\angle AOB = 360° - 90° - 90° - \angle APB = 360-180-75 = 105°$.

Step 3: Q lies on the major arc, so ∠AQB = half the central angle AOB standing on the minor arc AB: $\angle AQB = 105°/2$.

Answer: **∠AQB = 52.5°.**

Q22 *[2 marks]* **Solve 49x+51y=499 and 51x+49y=501.**

Solution

Step 1: Add the two equations: $100x+100y = 1000 \Rightarrow x+y = 10$.

Step 2: Subtract the first from the second: $2x-2y = 2 \Rightarrow x-y = 1$.

Step 3: Adding these: $2x = 11 \Rightarrow x = 5.5$; then $y = 10-5.5 = 4.5$.

Answer: **x = 5.5, y = 4.5.**

Q23 *[2 marks]* **Sum of the first 25 terms of an AP with nth term an=2-3n (or an=7-3n).**

an = 2-3n

Step 1: $a_1 = 2-3 = -1$, and since a_n is linear in n with slope -3, the common difference $d = -3$.

Step 2: $S_{25} = 25/2\,[2a_1+24d] = 25/2\,[2(-1)+24(-3)] = 25/2 \times (-74)$.

Answer: **$S_{25} = -925$.**

OR — an = 7-3n

Step 1: $a_1 = 7-3 = 4$, $d = -3$.

Step 2: $S_{25} = 25/2\,[2(4)+24(-3)] = 25/2 \times (-64)$.

Answer: **$S_{25} = -800$.**

Q24 *[2 marks]* **AD/AE = AC/BD and ∠1=∠2. Show ΔBAE ~ ΔCAD.**

Solution

Step 1: Given AD/AE = AC/AB,

Step 2: Given ∠1 = ∠2 gives AB=BD By isosceles property

Step 3: By the SAS similarity criterion (equal included angle, proportional sides) **AD/AE = AC/AB**, ΔBAE ~ ΔCAD.

Q25 *[2 marks]* **sin(A+B)=1, cos(A-B)=√3/2, 0°<A+B≤90°, A>B — find A, B.**

Solution

Step 1: $\sin(A+B)=1 \Rightarrow A+B = 90°$ (only value in the given range with sine = 1).

Step 2: $\cos(A-B)=\sqrt{3}/2 \Rightarrow A-B = 30°$ (since A>B, A-B is positive, and $\cos 30°=\sqrt{3}/2$).

Step 3: Solve $A+B=90°$, $A-B=30°$: adding gives $2A=120° \Rightarrow A=60°$; then $B=90-60=30°$.

Answer: **A = 60°, B = 30°.**

OR — Find acute θ with $(\cos\theta-\sin\theta)/(\cos\theta+\sin\theta) = (1-\sqrt{3})/(1+\sqrt{3})$

Step 1: Divide numerator and denominator on the left by $\cos\theta$: $(1-\tan\theta)/(1+\tan\theta) = (1-\sqrt{3})/(1+\sqrt{3})$.

Step 2: Comparing the two sides directly (same pattern $(1-x)/(1+x)$), $\tan\theta = \sqrt{3}$.

Answer: **θ = 60°.**

Section C — 3 marks each

Q26 *[3 marks]* Train problem (or chocolates problem) — internal choice.

Train covers a distance at uniform speed x km/h, time t h

Step 1: If speed is $(x+6)$, time is $(t-4)$: $xt = (x+6)(t-4) = xt-4x+6t-24 \Rightarrow 4x-6t+24=0 \Rightarrow 2x-3t=-12$ ...(i)

Step 2: If speed is $(x-6)$, time is $(t+6)$: $xt = (x-6)(t+6) = xt+6x-6t-36 \Rightarrow 6x-6t=36 \Rightarrow x-t=6$, i.e. $t=x-6$ ...(ii)

Step 3: Substitute (ii) into (i): $2x-3(x-6)=-12 \Rightarrow 2x-3x+18=-12 \Rightarrow -x=-30 \Rightarrow x=30$. Then $t=24$.

Answer: **Length of journey = xt = 30×24 = 720 km.**

OR — Anuj's chocolates: lot A at Rs 2/3 chocolates, lot B at Rs 1/chocolate, total Rs 400

Step 1: Let lot A have a chocolates, lot B have b chocolates: $(2/3)a + b = 400 \Rightarrow 2a+3b = 1200$ ...(i)

Step 2: Second scenario: lot A at Rs 1/chocolate, lot B at Rs 4/5 chocolates: $a + (4/5)b = 460 \Rightarrow 5a+4b = 2300$ ...(ii)

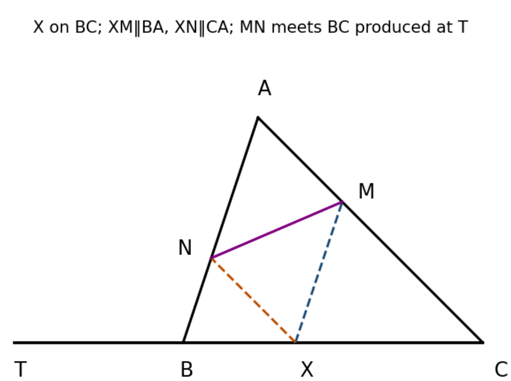
Step 3: Multiply (i) by 4 and (ii) by 3: $8a+12b=4800$ and $15a+12b=6900$. Subtract: $7a=2100 \Rightarrow a=300$.

Step 4: From (i): $2(300)+3b=1200 \Rightarrow 3b=600 \Rightarrow b=200$.

Answer: **Total chocolates = a+b = 300+200 = 500.**

Q27 *[3 marks]* X on BC; XM∥BA, XN∥CA (M on CA, N on BA); MN meets BC produced at T. Prove $TX^2=TB\times TC$.

Solution



XM∥AB, XN∥AC; line NM extended meets BC produced at T

Step 1: Since T, B, X lie on the same line (BC produced) and T, N, M lie on the same line (given), ray TX = ray TB and ray TM = ray TN.

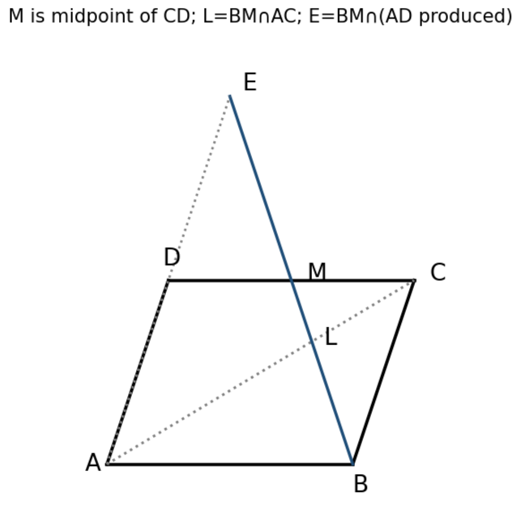
Step 2: Since XM ∥ AB (so XM ∥ the part BN of AB): ΔTXM ~ ΔTBN (AA — common angle at T; XM∥BN gives equal corresponding angles). So TX/TB = TM/TN. ...(i)

Step 3: Since XN ∥ AC (so XN ∥ the part CM of AC): ΔTXN ~ ΔTCM (AA — common angle at T; XN∥CM gives equal corresponding angles). So TX/TC = TN/TM. ...(ii)

Step 4: Multiply (i) and (ii): (TX/TB)×(TX/TC) = (TM/TN)×(TN/TM) = 1.

Answer: **So $TX^2/(TB \times TC) = 1$, i.e. $TX^2 = TB \times TC$, as required.**

OR — M is midpoint of CD (parallelogram ABCD); BM meets AC at L and AD produced at E. Prove EL=2BL.



AB∥DC; M is the midpoint of CD

Step 1: AB ∥ DC and M is the midpoint of CD, so DM = MC = AB/2 (since DC=AB in a parallelogram).

Step 2: Since AB ∥ MC: ΔABL ~ ΔCML (vertically opposite angles at L, alternate angles from AB∥DC) ⇒ BL/ML = AB/MC = 2 ⇒ ML = BL/2.

Step 3: Since AB ∥ DM: ΔABE ~ ΔDME (common vertex E, AB∥DM) ⇒ EB/EM = AB/DM = 2 ⇒ EM = EB/2, i.e. M is the midpoint of BE.

Step 4: Since the order along the line is B, L, M, E: BM = BL+LM = BL + BL/2 = (3/2)BL. Since M is the midpoint of BE, BE = 2×BM = 3BL.

Step 5: EL = BE - BL = 3BL - BL.

Answer: **EL = 2BL, as required.**

Q28 *[3 marks]* Given √3 is irrational, prove 5+2√3 is irrational.

Solution

Step 1: Assume, for contradiction, that 5+2√3 is rational, say equal to r.

Step 2: Then 2√3 = r-5 ⇒ √3 = (r-5)/2, which would be rational (since r is rational).

Step 3: This contradicts the given fact that √3 is irrational.

Answer: **Hence 5+2√3 is irrational.**

Q29 *[3 marks]* Zeroes of x^2+px+q are double the zeroes of $2x^2-5x-3$. Find p, q.

Solution

Step 1: Factorise $2x^2-5x-3 = (2x+1)(x-3) = 0$ ⇒ zeroes are -1/2 and 3.

Step 2: Zeroes of x^2+px+q are double these: 2×(-1/2) = -1, and 2×3 = 6.

Step 3: Sum of new zeroes = -1+6 = 5 = -p ⇒ p = -5. Product = -1×6 = -6 = q.

Answer: **p = -5, q = -6.**

Q30 *[3 marks]* Prove $[(1+\sin\theta)^2+(1-\sin\theta)^2]/(2\cos^2\theta) = (1+\sin^2\theta)/(1-\sin^2\theta)$.

Solution

Step 1: Expand the numerator: $(1+\sin\theta)^2+(1-\sin\theta)^2 = 2(1+\sin^2\theta)$ [using $(a+b)^2+(a-b)^2=2a^2+2b^2$ with a=1, b=sinθ].

Step 2: LHS = $2(1+\sin^2\theta)/(2\cos^2\theta) = (1+\sin^2\theta)/\cos^2\theta$.

Step 3: RHS = $(1+\sin^2\theta)/(1-\sin^2\theta) = (1+\sin^2\theta)/\cos^2\theta$, since $1-\sin^2\theta = \cos^2\theta$.

Answer: LHS = RHS, hence proved.

Q31 *[3 marks]* **Prove $\tan^3\theta/(1+\tan^2\theta) + \cot^3\theta/(1+\cot^2\theta) = \sec\theta \operatorname{cosec}\theta - 2\sin\theta\cos\theta$.**

Solution

Step 1: $1+\tan^2\theta=\sec^2\theta$, so $\tan^3\theta/\sec^2\theta = \tan^3\theta \cos^2\theta = \sin^3\theta/\cos\theta$.

Step 2: $1+\cot^2\theta=\operatorname{cosec}^2\theta$, so $\cot^3\theta/\operatorname{cosec}^2\theta = \cot^3\theta \sin^2\theta = \cos^3\theta/\sin\theta$.

Step 3: LHS $= \sin^3\theta/\cos\theta + \cos^3\theta/\sin\theta = (\sin^4\theta+\cos^4\theta)/(\sin\theta\cos\theta)$.

Step 4: $\sin^4\theta+\cos^4\theta = (\sin^2\theta+\cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta = 1-2\sin^2\theta\cos^2\theta$.

Step 5: LHS $= (1-2\sin^2\theta\cos^2\theta)/(\sin\theta\cos\theta) = 1/(\sin\theta\cos\theta) - 2\sin\theta\cos\theta = \sec\theta\operatorname{cosec}\theta - 2\sin\theta\cos\theta$.

Answer: LHS = RHS, hence proved.

Section D — 5 marks each

Q32 *[5 marks]* **Prove the Basic Proportionality Theorem, then use it for a trapezium's diagonals.**

Proof (Basic Proportionality / Thales' Theorem)

Step 1: In ΔABC, let $DE \parallel BC$ meet AB at D and AC at E. Join BE and CD; draw $EF\perp AB$ and $DG\perp AC$.

Step 2: ar(ADE)/ar(BDE) = AD/DB [triangles with the same height EF, bases AD, DB]; ar(ADE)/ar(DEC) = AE/EC [similarly].

Step 3: ΔBDE and ΔDEC lie on the same base DE, between the same parallels DE and BC, so ar(BDE) = ar(DEC).

Answer: Therefore AD/DB = AE/EC, proving the theorem.

Application — trapezium ABCD, AB∥CD, diagonals meet at O; line through O parallel to AB meets AD at P, BC at Q. Prove AP/PD = BQ/QC

Step 1: Since AB∥CD, $\Delta OAB \sim \Delta OCD$ (AA: vertical angles at O, alternate angles) ⇒ OA/OC = OB/OD.

Step 2: In ΔABD, P is on AD and O is on BD with PO∥AB: by the theorem just proved, AP/PD = BO/OD.

Step 3: In ΔABC, Q is on BC and O is on AC with QO∥AB: by the theorem, BQ/QC = AO/OC.

Step 4: Since OA/OC = OB/OD (from the first step), the two ratios AP/PD and BQ/QC are equal (both equal this common ratio).

Answer: Hence AP/PD = BQ/QC — the line through the diagonals' intersection, parallel to the base, divides the non-parallel sides in the same ratio.

Q33 *[5 marks]* **Adding 1 to numerator and subtracting 1 from denominator gives 1; adding 1 only to denominator gives 1/2. Find the fraction.**

Solution

Step 1: Let the fraction be x/y. $(x+1)/(y-1) = 1 \Rightarrow x+1 = y-1 \Rightarrow x = y-2$ ...(i)

Step 2: $x/(y+1) = 1/2 \Rightarrow 2x = y+1$ ...(ii)

Step 3: Substitute (i) into (ii): $2(y-2) = y+1 \Rightarrow 2y-4 = y+1 \Rightarrow y = 5$. Then $x = 5-2 = 3$.

Answer: The fraction is 3/5 (check: (3+1)/(5-1)=1 ✓; 3/(5+1)=1/2 ✓).

Q34 *[5 marks]* **Two pipes fill a pool: 4h (larger) + 9h (smaller) = half the pool; smaller takes 10h more than larger. Find each pipe's time.**

Solution

Step 1: Let the larger pipe take x hours alone; the smaller takes (x+10) hours alone.

Step 2: $4/x + 9/(x+10) = 1/2$. Multiply by $2x(x+10)$: $8(x+10)+18x = x(x+10)$.

Step 3: $8x+80+18x = x^2+10x \Rightarrow 26x+80 = x^2+10x \Rightarrow x^2-16x-80 = 0$.

Step 4: $x = [16\pm\sqrt{(256+320)}]/2 = [16\pm24]/2 \Rightarrow x = 20$ (rejecting the negative root).

Answer: Larger pipe: 20 hours; smaller pipe: 30 hours.

OR — 600 km flight; speed reduced by 200 km/h; time increased by 30 min. Find scheduled duration.

Step 1: Let usual speed = x km/h, so scheduled time = 600/x hours. Reduced speed = x-200, actual time = 600/(x-200).
Step 2: 600/(x-200) - 600/x = 1/2. Multiply by 2x(x-200): 1200x - 1200(x-200) = x(x-200).
Step 3: $1200\times200 = x^2-200x \Rightarrow x^2-200x-240000 = 0$.
Step 4: $x = [200\pm\sqrt{(40000+960000)}]/2 = [200\pm1000]/2 \Rightarrow x = 600$ (rejecting the negative root).

Answer: **Scheduled duration = 600/600 = 1 hour.**

Q35 *[5 marks]* Trigonometric identities.

(i) Prove $\tan\theta/(1-\cot\theta) + \cot\theta/(1-\tan\theta) = 1+\tan\theta+\cot\theta$

Step 1: Let $t=\tan\theta$, so $\cot\theta=1/t$. $\tan\theta/(1-\cot\theta) = t/(1-1/t) = t^2/(t-1)$.
Step 2: $\cot\theta/(1-\tan\theta) = (1/t)/(1-t) = -1/[t(t-1)]$ [since $1-t=-(t-1)$].
Step 3: LHS = $t^2/(t-1) - 1/[t(t-1)] = [t^3-1]/[t(t-1)] = (t-1)(t^2+t+1)/[t(t-1)] = (t^2+t+1)/t = t+1+1/t$.

Answer: **LHS = $\tan\theta+1+\cot\theta = 1+\tan\theta+\cot\theta$, hence proved.**

(ii) Prove $\mathrm{cosec}^6\theta = \cot^6\theta + 3\cot^2\theta\,\mathrm{cosec}^2\theta + 1$

Step 1: Let $c=\cot\theta$, so $\mathrm{cosec}^2\theta=1+c^2$. $\mathrm{cosec}^6\theta = (1+c^2)^3 = 1+3c^2+3c^4+c^6$.
Step 2: RHS = $c^6 + 3c^2(1+c^2) + 1 = c^6+3c^2+3c^4+1$.

Answer: **Both sides equal $c^6+3c^4+3c^2+1$, hence proved.**

OR — If $\cot\theta = 1/\sqrt{3}$, find $(1-\cos^2\theta)/(2-\sin^2\theta)$

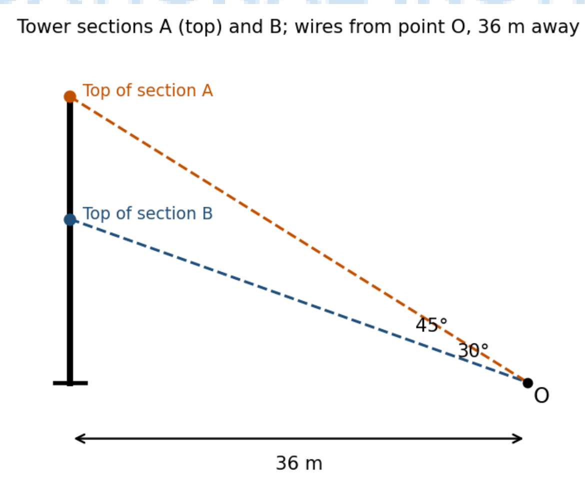
Step 1: $1-\cos^2\theta = \sin^2\theta$. Also $2-\sin^2\theta = 1+(1-\sin^2\theta) = 1+\cos^2\theta$, so the expression = $\sin^2\theta/(1+\cos^2\theta)$.
Step 2: $\cot^2\theta = 1/3 \Rightarrow \sin^2\theta = 1/(1+\cot^2\theta) = 1/(4/3) = 3/4$, and $\cos^2\theta = 1-\sin^2\theta = 1/4$.
Step 3: Expression = (3/4)/(1+1/4) = (3/4)/(5/4) = 3/5.

Answer: **The value is 3/5.**

Section E — Case-Based Questions (4 marks each)

Q36 *[4 marks]* Radio tower built in sections A (top) and B; wire from point O, 36 m from the base; elevation of top of B = 30°, of top of A = 45°.

(i) Height of section B



Sections A (top) and B; point O is 36 m from the base

Step 1: $\tan30° = (\text{height of B})/36 \Rightarrow$ height of B = $36\times(1/\sqrt{3}) = 36/\sqrt{3} = 12\sqrt{3}$ m.

Answer: **Height of section B = $12\sqrt{3}$ m ≈ 20.8 m.**

(ii) Length of the wire from O to the top of section B

Step 1: sin30° = (height of B)/(wire length) ⇒ wire length = 12√3/(1/2) = 24√3 m.

Answer: **Length of wire to top of B = 24√3 m ≈ 41.6 m.**

(iii) Height of section A

Step 1: tan45° = (total height to top of A)/36 ⇒ total height = 36 m.

Step 2: Height of section A alone = total height - height of B = 36 - 12√3.

Answer: **Height of section A = (36-12√3) m ≈ 15.2 m.**

OR — Length of the wire from O to the top of section A

Step 1: sin45° = (total height)/(wire length) ⇒ wire length = 36/(√2/2) = 36√2.

Answer: **Length of wire to top of A = 36√2 m ≈ 50.9 m.**

Q37 *[4 marks]* **Tessellation on the Cartesian plane (octagons, squares, triangles).**

Method (exact coordinates depend on the printed grid)

(i) $B = (1, 2)$

$F = (-2, 9)$

$$BF = \sqrt{(-2-1)^2 + (9-2)^2} = \sqrt{\mathbf{58}} \text{ units}$$

(ii) The center z of the figure is the point of intersection of the diagonals of quadrilateral WXOP. In a symmetric quadrilateral, the diagonals bisect each other at the center,z is the midpoint of either diagonal WO or XP.

Identify the coordinates of the vertices from the graph: (W = (-5, 1) and O = (5, 9)

$$Z = \left(\frac{0}{2}, \frac{10}{2}\right) = (\mathbf{0}, \mathbf{5})$$

(iii) **Coordinates of the point on the y-axis equidistant from A and G**

Identify the coordinates of points A = (-1, 2) and G = (-2, 7)

Any point lying on the y-axis has an x-coordinate 0. Let this point be P(0, y). Since P is equidistant from A and G, we have

PA = PG

The coordinates of the point on the y-axis are **(0, 4.8)**.

Q38 *[4 marks]* **Road roller production increases uniformly each year; 800 in year 6, 1130 in year 9.**

Setting up the AP

Step 1: Let a = production in year 1, d = yearly increase. a+5d = 800 (year 6); a+8d = 1130 (year 9).

Step 2: Subtract: 3d = 330 ⇒ d = 110. Then a = 800-5(110) = 250.

(i) Production in the first year

Answer: **a = 250 rollers.**

(ii) Yearly increase

Answer: **d = 110 rollers.**

(iii) Year in which production is 1350

Step 1: a+(n-1)d = 1350 ⇒ 250+(n-1)(110) = 1350 ⇒ (n-1)×110 = 1100 ⇒ n-1 = 10 ⇒ n = 11.

Answer: **The 11th year.**

OR — Production in the 8th year

Step 1: a8 = a+7d = 250+7(110) = 250+770.

Answer: **Production in the 8th year = 1020 rollers.**