Answer Key

Maths - X Mathematics | Practice Sheet-01 (Standard)

Prepared by Deepika Bhati

SECTION – A

Q1. The least number divisible by all from 1 to 5 is:

Answer: (b) 60

LCM(1, 2, 3, 4, 5) = **60**

Q2. Greatest number of branches (63 treadmills, 108 ellipticals, nothing left over):

Answer: (d) 9

Required number = HCF(63, 108). $63 = 9\times7$, $108 = 9\times12 \Rightarrow$ HCF = **9**

Q3. HCF(306, 657) = 9. Find LCM(306, 657):

Answer: (b) 22338

LCM = $\frac{306 \times 657}{9}$ = 22338

Q4. Least number which leaves remainder 1 when divided by 18, 24, 30, 42:

Answer: (c) 2521

LCM(18, 24, 30, 42): $18=2\times3^2$, $24=2^3\times3$, $30=2\times3\times5$, $42=2\times3\times7 \Rightarrow$ LCM = $2^3\times3^2\times5\times7 = 2520$

Required number = 2520 + 1 = **2521**

Q5. LCM of the smallest two-digit composite number (10) and the smallest composite number (4):

Answer: (c) 20

LCM(10, 4) = **20**

Q6. Assertion–Reason: HCF = 16, product = 3072, claimed LCM = 162:

Answer: (d) A is false but R is true

Actual LCM = $\frac{3072}{16}$ = 192, not 162 — so A is false.

The relation HCF × LCM = a × b is always true for two positive integers, so R is true.

SECTION – B

Q7. Greatest number dividing 85 and 72 leaving remainders 1 and 2 respectively:

Answer: $85 - 1 = 84$, $72 - 2 = 70$

Required number = HCF(84, 70) = **14**

($84 = 2^2\times3\times7$, $70 = 2\times5\times7 \Rightarrow$ HCF = 2×7 = 14)

OR — Least prime factor of p is 3, least prime factor of q is 5. Find least prime factor of (p + q):

Answer: Since the least prime factor of p is 3, p is odd. Since the least prime factor of q is 5, q is odd (not divisible by 2 or 3). Odd + Odd = Even, so (p+q) is even.

$\Rightarrow$ Least prime factor of (p + q) = **2**

Q8. Traffic lights change every 48s, 72s, 108s; when do they change together again (from 7:00 am)?

Answer: LCM(48, 72, 108): $48=2^4\times3$, $72=2^3\times3^2$, $108=2^2\times3^3 \Rightarrow$ LCM = $2^4\times3^3$ = 432 seconds = 7 min 12 sec

They will change together again at **7 : 07 : 12 am**

SECTION – C

Q9. Can two numbers have 18 as their HCF and 380 as their LCM?

Answer: No. HCF must always exactly divide LCM.

$380 \div 18 = \frac{380}{18} = 21.11$ (not a whole number), so 18 does not divide 380.
Hence two numbers cannot have HCF 18 and LCM 380.

Q10. Can 6^n (n a natural number) end with the digit 5?

Answer: No. $6^n = (2 \times 3)^n = 2^n \times 3^n$ — its only prime factors are 2 and 3.
For a number to end in 5, it must be divisible by 5 (i.e. 5 must be a prime factor).
Since 5 never appears in the factorisation of 6^n, **6^n can never end with the digit 5**.

OR — Explain why $(17\times5\times11\times3\times2 + 2\times11)$ is a composite number:

Answer: $17\times5\times11\times3\times2 + 2\times11 = 11\times(17\times5\times3\times2 + 2) = 11 \times 512 = 5632$
Since the number can be written as a product of two factors (11 and 512), each greater than 1,
it has factors other than 1 and itself. Hence it is a **composite number**.

SECTION – D

Q11. Shilpi has 36 diaries and 60 golden pens, distributed equally with nothing left over.

Answer: (i) Maximum guests = HCF(36, 60). $36 = 2^2\times3^2$, $60 = 2^2\times3\times5 \Rightarrow$ HCF = **12**
(ii) Each guest gets: diaries $= \frac{36}{12} =$ **3**, golden pens $= \frac{60}{12} =$ **5**
(iii) Adding 42 watches: HCF(36, 60, 42). $42 = 2\times3\times7 \Rightarrow$ HCF(36,60,42) = 6. **Maximum guests = 6**

OR — Shilpi adds 3 more watches (total 45) and removes 6 diaries (leaving 30):

Answer: New quantities: diaries = 36 − 6 = 30, pens = 60, watches = 42 + 3 = 45
Maximum guests = HCF(30, 60, 45) = **15**

SECTION – E

Q12. Prove that $\sqrt{3}$ is irrational; hence show that $5 + 2\sqrt{3}$ is also irrational.

Answer: Suppose, to the contrary, that $\sqrt{3}$ is rational.
Then $\sqrt{3} = \frac{a}{b}$, where a, b are co-prime integers and $b \neq 0$.
$\Rightarrow b\sqrt{3} = a \Rightarrow 3b^2 = a^2$ (after squaring both sides) ...(1)
$\Rightarrow a^2$ is a multiple of 3, so a is a multiple of 3. Let a = 3k, k an integer.
Substituting in (1): $3b^2 = (3k)^2 = 9k^2 \Rightarrow b^2 = 3k^2$
$\Rightarrow b^2$ is a multiple of 3, so b is also a multiple of 3.
As a and b are both multiples of 3, this contradicts that a and b are co-prime.
∴ Our assumption is wrong. Hence **$\sqrt{3}$ is irrational.**

Next, suppose $5 + 2\sqrt{3}$ is rational, say equal to r (a rational number).
Then $\sqrt{3} = \frac{r-5}{2}$
Since r is rational, $(r-5)/2$ is also rational, which would mean $\sqrt{3}$ is rational —
a contradiction, since $\sqrt{3}$ is irrational (proved above).
Hence **$5 + 2\sqrt{3}$ is irrational.**

Q13. LCM of two numbers is 14 times their HCF. Sum of LCM and HCF is 600. One number is 280; find the other.

Answer: Let HCF = y, then LCM = 14y. Given LCM + HCF = 600 $\Rightarrow$ 14y + y = 600 $\Rightarrow$ 15y = 600
$\Rightarrow y = \frac{600}{15} = 40$ = HCF, and LCM = 14 × 40 = 560
Using HCF × LCM = product of the two numbers: 40 × 560 = 280 × (other number)
Other number $= \frac{40 \times 560}{280} =$ **80**

OR — Smallest 3-digit number exactly divisible by 6, 8 and 12:

Answer: LCM(6, 8, 12) = 24
Smallest 3-digit multiple of 24: 100 ÷ 24 leaves remainder 4, so 100 + (24 − 4) = 100 + 20 = 120
Required number = **120**

— *End of Answer Key* —