Answer Key

Maths - X Mathematics | PBMT10S2 (Standard)

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SECTION – A

Q1. LCM of two prime numbers p and q (p > q) is 221. Find 3p − q:

Answer: (c) 38

Since p, q are distinct primes, HCF(p,q) = 1, so LCM = p × q = 221 = 13 × 17

As p > q: p = 17, q = 13 ⇒ 3p − q = 51 − 13 = **38**

Q2. Number of zeroes for the polynomial p(x) whose graph is given in the figure:

Answer: (b) 3

The number of zeroes of a polynomial equals the number of points where its graph crosses/touches the x-axis. The given curve crosses the x-axis at 3 distinct points, so the polynomial has **3 zeroes**.

Q3. If x − y = 2 and 2/(x+y) = 1/5, find x and y:

Answer: (c) x = 6, y = 4

$\frac{2}{x+y} = \frac{1}{5}$ ⇒ 2(5) = 1(x+y) ⇒ x + y = 10

Also x − y = 2. Adding: 2x = 12 ⇒ x = 6, then y = 10 − 6 = **4**

Q4. If one zero of $3x^2 + 8x + k$ is the reciprocal of the other, find k:

Answer: (a) 3

If the zeroes are α and 1/α, their product = 1

Product of zeroes = $\frac{k}{3}$ = 1 ⇒ k = **3**

Q5. The 10th term from the end of the A.P. 4, 9, 14, ..., 254:

Answer: (a) 209

First term a = 4, common difference d = 5, last term l = 254

nth term from the end = l − (n−1)d = 254 − 9(5) = 254 − 45 = **209**

Q6. The equation of a line whose graph passes through the origin:

Answer: (b) 2x + 3y = 0

A line ax + by = c passes through the origin (0,0) only if c = 0. Only option (b) has c = 0.

Q7. Which equation has the sum of its roots as 3?

Answer: (b) $-x^2 + 3x - 3 = 0$

For (a) $2x^2-3x+6=0$: sum = 3/2 — not 3

For (b) $-x^2+3x-3=0$: sum = $\frac{-3}{-1}$ = **3** ✓ — this is the answer

(The other options give sums of 3/2 or 1, not 3.)

Q8. Assertion (A): $25x^2 - 40x + 16 = 0$ has repeated roots. Reason (R): $ax^2 + bx + c$ has repeated roots when D = 0.

Answer: (a) Both A and R are true and R is the correct explanation of A

D = $(-40)^2 - 4(25)(16)$ = 1600 − 1600 = 0, so the equation does have repeated (equal) roots — A is true.

R correctly states the general condition for repeated roots, and directly explains why A holds. So R is true and explains A.

SECTION – B

Q9. Find p and q for which 4x+5y=2 and (2p+7q)x+(p+8q)y=2q−p+1 have infinitely many solutions:

Answer: For infinitely many solutions: $\frac{4}{2p+7q} = \frac{5}{p+8q} = \frac{2}{2q-p+1}$

From the first two ratios: $4(p+8q) = 5(2p+7q) \Rightarrow 4p+32q = 10p+35q \Rightarrow -6p = 3q \Rightarrow q = -2p$ ...(i)
From the first and third ratios: $4(2q-p+1) = 2(2p+7q) \Rightarrow 8q-4p+4 = 4p+14q \Rightarrow -8p+4 = 6q$ ...(ii)
Substituting (i) into (ii): $-8p+4 = 6(-2p) = -12p \Rightarrow 4p = -4 \Rightarrow p = \mathbf{-1}$, so $q = -2(-1) = \mathbf{2}$
Check: $a_1/a_2 = 4/12 = 1/3$, $b_1/b_2 = 5/15 = 1/3$, $c_1/c_2 = 2/6 = 1/3$ — all equal ✓

Q10. Solve for x: $9x^2 - 6px + (p^2 - q^2) = 0$:

Answer: Using the quadratic formula: $D = 36p^2 - 36(p^2-q^2) = 36q^2$, so $\sqrt{D} = 6q$
$x = \frac{6p \pm 6q}{18} = \frac{p \pm q}{3}$
$\Rightarrow\ x = \frac{p+q}{3}$ or $x = \frac{p-q}{3}$

Q11. One-fourth of a herd of camels was in the forest, twice the square root went to the mountain, and 15 remained by the river. Find the herd size:

Answer: Let herd size = N, and let $\sqrt{N} = t$, so $N = t^2$
$\frac{N}{4} + 2\sqrt{N} + 15 = N \ \Rightarrow\ \frac{t^2}{4} + 2t + 15 = t^2$
Multiplying by 4: $t^2 + 8t + 60 = 4t^2 \Rightarrow 3t^2 - 8t - 60 = 0$
$t = [8 \pm \sqrt{(64+720)}]/6 = [8 \pm 28]/6 \Rightarrow t = 6$ (rejecting the negative root)
$N = t^2 =$ **36 camels**

OR — A trader bought articles for Rs. 900; 5 were damaged; sold the rest at Rs. 2 more than cost each, making Rs. 80 profit. Find the number of articles:

Answer: Let number of articles = x. Cost per article = $\frac{900}{x}$
Selling price: $(x-5) \times (\frac{900}{x} + 2) = 900 + 80 = 980$
$900 - 4500/x + 2x - 10 = 980 \Rightarrow 2x - 4500/x = 90 \Rightarrow 2x^2 - 90x - 4500 = 0 \Rightarrow x^2 - 45x - 2250 = 0$
$x = [45 \pm \sqrt{(2025+9000)}]/2 = [45 \pm 105]/2 \Rightarrow x = 75$ (rejecting the negative root)
He bought **75 articles**

SECTION – C

Q12. If α, β are the zeroes of $f(x) = x^2 + x - 2$, find the value of $1/\alpha - 1/\beta$:

Answer: Sum $\alpha+\beta = -1$, Product $\alpha\beta = -2$. Factoring: $x^2+x-2 = (x+2)(x-1)$, so zeroes are -2 and 1.
$1/\alpha - 1/\beta = \frac{\beta - \alpha}{\alpha\beta}$
$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta = 1 + 8 = 9 \Rightarrow \alpha-\beta = \pm 3$
Taking $\alpha = 1, \beta = -2$: $1/\alpha - 1/\beta = 1 - (-1/2) = \frac{3}{2}$
(Taking $\alpha = -2, \beta = 1$ instead gives $-\frac{3}{2}$.) So the value is **± 3/2**, depending on which root is labelled α.

Q13. Rectangle: area reduces by 9 if length −5 & breadth +3; area increases by 67 if length +3 & breadth +2. Find length and breadth:

Answer: Let length = x, breadth = y.
$(x-5)(y+3) = xy - 9 \Rightarrow 3x - 5y - 15 = -9 \Rightarrow 3x - 5y = 6$ ...(i)
$(x+3)(y+2) = xy + 67 \Rightarrow 2x + 3y + 6 = 67 \Rightarrow 2x + 3y = 61$ ...(ii)
Solving (i) × 3 and (ii) × 5, then adding: $19x = 323 \Rightarrow x = 17$, and from (ii): $y = 9$
Length = **17 units**, Breadth = **9 units**

OR — Students in rows: 4 extra per row ⇒ 2 rows less; 4 less per row ⇒ 4 rows more. Find total students:

Answer: Let students per row = x, number of rows = y. Total N = xy.
$(x+4)(y-2) = xy \Rightarrow -2x+4y-8 = 0 \Rightarrow x = 2y-4$ ...(i)
$(x-4)(y+4) = xy \Rightarrow 4x-4y-16 = 0 \Rightarrow x = y+4$ ...(ii)
Equating: $2y-4 = y+4 \Rightarrow y = 8$, so $x = 12$
Total students = $xy = 12 \times 8 =$ **96**

Q14. Find the roots of $1/[(x-1)(x-2)] + 1/[(x-2)(x-3)] = 2/3,\ x \neq 1, 2, 3$:

Answer: Using partial fractions: $\frac{1}{(x-1)(x-2)} = \frac{1}{x-2} - \frac{1}{x-1}$, $\frac{1}{(x-2)(x-3)} = \frac{1}{x-3} - \frac{1}{x-2}$

Adding, the middle terms cancel: $\frac{1}{x-3} - \frac{1}{x-1} = \frac{2}{3}$

$\Rightarrow \frac{2}{(x-3)(x-1)} = \frac{2}{3} \Rightarrow (x-3)(x-1) = 3$

Expanding: $x^2 - 4x + 3 = 3 \Rightarrow x^2 - 4x = 0 \Rightarrow x(x-4) = 0$

$\Rightarrow x = \mathbf{0}$ or $x = \mathbf{4}$ (both satisfy $x \neq 1, 2, 3$)

Q15. Find the greatest 6-digit number exactly divisible by 24, 15 and 36:

Answer: LCM(24, 15, 36): $24 = 2^3\times3$, $15 = 3\times5$, $36 = 2^2\times3^2 \Rightarrow$ LCM $= 2^3\times3^2\times5 = \mathbf{360}$

Greatest 6-digit number = 999999. Dividing by 360: $999999 = 360 \times 2777 + 279$

Required number = 999999 − 279 = **999720**

SECTION – D

Q16. Sea shells in cells form an A.P. 3rd + 11th cell = 68; 11th cell is 24 more than the 3rd cell.

Answer: Let first term = a, common difference = d.

$(a+2d) + (a+10d) = 68 \Rightarrow a + 6d = 34$ ...(i)

$(a+10d) - (a+2d) = 24 \Rightarrow 8d = 24 \Rightarrow d = 3$

From (i): $a + 18 = 34 \Rightarrow a = 16$

(i) Difference between 19th and 20th cell = common difference d = **3**

(ii) Number of shells in the first cell = a = **16**

(iii) Sum of first 13 cells: $S_{13} = \frac{13}{2}[2a+12d] = \frac{13}{2}[32+36] = \frac{13}{2}(68) = \mathbf{442}$

OR — Find the number of sea shells in the 7th and 9th cell:

Answer: 7th cell = a+6d = 16+18 = **34**

9th cell = a+8d = 16+24 = **40**

SECTION – E

Q17. α, β are zeroes of $p(x) = x^2 - 2x + 3$. Find a quadratic polynomial with zeroes $(\alpha-1)/(\alpha+1)$ and $(\beta-1)/(\beta+1)$:

Answer: $\alpha+\beta = 2$, $\alpha\beta = 3$

Sum of new zeroes = $\frac{2\alpha\beta - 2}{\alpha\beta + (\alpha+\beta) + 1} = \frac{6-2}{3+2+1} = \frac{4}{6} = \frac{2}{3}$

Product of new zeroes = $\frac{\alpha\beta - (\alpha+\beta) + 1}{\alpha\beta + (\alpha+\beta) + 1} = \frac{3-2+1}{6} = \frac{2}{6} = \frac{1}{3}$

Required polynomial: $x^2 - \frac{2}{3}x + \frac{1}{3} \Rightarrow$ multiplying by 3: $\mathbf{k(3x^2 - 2x + 1), k \neq 0}$

Q18. If the roots of $(a^2+b^2)x^2 - 2(ac+bd)x + (c^2+d^2) = 0$ are equal, prove that ad = bc:

Answer: Equal roots ⇒ Discriminant = 0:

$[2(ac+bd)]^2 - 4(a^2+b^2)(c^2+d^2) = 0 \Rightarrow (ac+bd)^2 = (a^2+b^2)(c^2+d^2)$

$a^2c^2 + 2abcd + b^2d^2 = a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2$

$2abcd = a^2d^2 + b^2c^2 \Rightarrow a^2d^2 - 2abcd + b^2c^2 = 0 \Rightarrow (ad - bc)^2 = 0$

$\Rightarrow \mathbf{ad = bc}$ (Proved)

OR — If $(1+m^2)x^2 + 2mcx + (c^2-a^2) = 0$ has equal roots, prove that $c^2 = a^2(1+m^2)$:

Answer: Equal roots ⇒ Discriminant = 0:

$(2mc)^2 - 4(1+m^2)(c^2-a^2) = 0 \Rightarrow m^2c^2 = (1+m^2)(c^2-a^2)$

$m^2c^2 = c^2 - a^2 + m^2c^2 - m^2a^2 \Rightarrow 0 = c^2 - a^2 - m^2a^2$

$\Rightarrow \mathbf{c^2 = a^2(1+m^2)}$ (Proved)

— End of Answer Key —