Answer Key

Maths - X Mathematics | Practice Sheet-03 (Standard)

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SECTION – A

Q1. If n is a natural number, then 12^n ends with an even digit except:

Answer: (a) 0

$12^n = 2^n \times 6^n$, and its prime factorisation is $2^{(2n)} \times 3^n$ — it has no factor of 5.

So 12^n is never divisible by 10, and can never end in 0. (Last digit cycles through 2, 4, 8, 6.)

Q2. If 18, a, b, −3 are in A.P., find a + b:

Answer: (d) 15

Common difference $d = (-3 - 18)/3 = -7$

$a = 18 + d = 11$, $b = 18 + 2d = 4 \Rightarrow a + b = \mathbf{15}$

Q3. Find HCF of Kx^2, $2Kx$, $4(xK)^2$ and K^2x:

Answer: (c) Kx

Coefficients: 1, 2, 4, 1 → HCF = 1

Powers of K: K^1, K^1, K^2, K^2 → lowest power K^1

Powers of x: x^2, x^1, x^2, x^1 → lowest power x^1

⇒ HCF = **Kx**

Q4. The zeroes of the polynomial $x^2 + kx + k$, $k \neq 0$:

Answer: (d) both cannot be positive

Sum of zeroes $= -k$, Product of zeroes $= k$

If both zeroes were positive: sum $> 0 \Rightarrow k < 0$, but product $> 0 \Rightarrow k > 0$ — a contradiction.

So the zeroes can never both be positive (for $k > 0$ both are negative; for $k < 0$ they have opposite signs).

Q5. Distance between the points (a cos A, −b sin A) and (a sin A, b cos A):

Answer: (d) $\sqrt{(a^2 + b^2)}$

Using the distance formula and the identity $\sin^2A + \cos^2A = 1$, the cross terms cancel out,

leaving distance$^2 = a^2 + b^2$, so distance $= \sqrt{(a^2 + b^2)}$

Q6. Value of k for which the zeroes of $kx^2 + 3x + 5$ are reciprocal of each other:

Answer: (d) 5

If zeroes are α and 1/α, product of zeroes = 1

Product of zeroes $= \frac{5}{k} = 1 \Rightarrow k = \mathbf{5}$

Q7. Value of p for which $(2p-1)x + (p-1)y = 2p+1$ and $3x + y - 1 = 0$ has no solution:

Answer: (c) 2

For no solution: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$(2p-1)/3 = (p-1)/1 \Rightarrow 2p-1 = 3p-3 \Rightarrow p = \mathbf{2}$

Check: $c_1/c_2 = (2p+1)/1 = 5$, while $a_1/a_2 = 1$ — since $1 \neq 5$, the condition for no solution is satisfied.

Q8. Assertion (A): Polynomial $x^2 + 4x$ has two real zeroes. Reason (R): Zeroes of $x^2 + ax$ ($a \neq 0$) are 0 and −a.

Answer: (a) Both A and R are true and R is the correct explanation of A

$x^2 + 4x = x(x+4) \Rightarrow$ zeroes are 0 and −4 — both real, so A is true.

In general, $x^2 + ax = x(x+a) \Rightarrow$ zeroes 0 and −a, so R is true and directly explains A (with a = 4).

SECTION – B

Q9. Least number which, when increased by 4, is completely divisible by 10, 15, 20 and 25:

Answer: LCM(10, 15, 20, 25) = 300

Required number = 300 − 4 = **296**

OR — Greatest number that divides 445, 572 and 699 leaving remainders 4, 5 and 6 respectively:

Answer: 445 − 4 = 441, 572 − 5 = 567, 699 − 6 = 693

$441 = 3^2 \times 7^2$, $567 = 3^4 \times 7$, $693 = 3^2 \times 7 \times 11$

HCF = $3^2 \times 7$ = **63**

Q10. Prove that the reciprocal of (√3 + 2√4) is an irrational number, given √3 is irrational:

Answer: Note: 2√4 = 2 × 2 = 4 (rational), so √3 + 2√4 = √3 + 4

Now suppose the reciprocal $\frac{1}{\sqrt{3}+4}$ is rational, say = r.

Then √3+4 = 1/r, which is rational

(since r ≠ 0 and r is rational) — again contradicting that √3+4 is irrational.

Hence **the reciprocal of (√3 + 2√4) is irrational.**

Q11. Split 207 into three parts in A.P. such that the product of the two smaller parts is 4623:

Answer: Let the parts be (a−d), a, (a+d). Sum = 3a = 207 ⇒ a = 69

Product of two smaller parts: (a−d)(a) = 4623 ⇒ 69(69−d) = 4623 ⇒ 69−d = 67 ⇒ d = 2

The parts are **67, 69 and 71**

SECTION – C

Q12. Train problem — find the length of the journey:

Answer: Let usual speed = x km/hr, usual time =y hours,

so distance D = xy

(x+6)(y−4) = D ⇒ 6y − 4x − 24 = 0 ⇒ 3y − 2x = 12 ... (i)

(x−6)(y+6) = D ⇒ 6x – 6y − 36 = 0 ⇒ x − y = 6 ⇒ y = x − 6 ... (ii)

Substituting (ii) in (i): 3(x−6) − 2x = 12 ⇒ x − 18 = 12 ⇒ x = 30, so y = 24

Length of journey = D = xy = 30 × 24 = **720 km**

Q13. P(x, y) is equidistant from A(a+b, b−a) and B(a−b, a+b). Prove that bx = ay:

Answer: $PA^2 = PB^2$

$(x-(a+b))^2 + (y-(b-a))^2 = (x-(a-b))^2 + (y-(a+b))^2$

Expanding both sides and cancelling the common x^2, y^2, $(a+b)^2$, $(a-b)^2$ terms, and simplifying the remaining linear terms gives: −4bx + 4ay = 0

⇒ **bx = ay** (Proved)

OR — C(−1, 2) divides AB in ratio 3:4, where A(2, 5). Find $x^2 + y^2$ for B(x, y):

Answer: By section formula: $(-1, 2) = \left(\frac{3x+8}{7}, \frac{3y+20}{7}\right)$

$\frac{3x+8}{7} = -1 \Rightarrow x = \mathbf{-5}$

$\frac{3y+20}{7} = 2 \Rightarrow y = \mathbf{-2}$

$x^2 + y^2 = 25 + 4 =$ **29**

Q14. Sum of first three terms of an AP is 48; product of 1st and 2nd exceeds 4 × 3rd term by 12. Find the AP:

Answer: Let terms be (a−d), a, (a+d). 3a = 48 ⇒ a = 16

(a−d)(a) = 4(a+d) + 12 ⇒ 16(16−d) = 4(16+d) + 12

256 − 16d = 76 + 4d ⇒ 180 = 20d ⇒ d = 9

AP: **7, 16, 25, ...**

OR — Divide 56 into four parts in A.P. with ratio of product of extremes : product of means = 5:6:

Answer: Let the parts be a−3d, a−d, a+d, a+3d. $4a = 56 \Rightarrow a = 14$

$\frac{a^2-9d^2}{a^2-d^2} = \frac{5}{6} \Rightarrow 6(a^2-9d^2) = 5(a^2-d^2) \Rightarrow a^2 = 49d^2$

$196 = 49d^2 \Rightarrow d^2 = 4 \Rightarrow d = 2$

Parts: 14−6, 14−2, 14+2, 14+6 = **8, 12, 16, 20**

Q15. A(−2,1), B(a,0), C(4,b), D(1,2) are vertices of parallelogram ABCD. Find a, b and the side lengths:

Answer: In a parallelogram, diagonals bisect each other, so midpoint of AC = midpoint of BD.

Midpoint AC = $\left(1, \frac{1+b}{2}\right)$; Midpoint BD = $\left(\frac{a+1}{2}, 1\right)$

$\frac{a+1}{2} = 1 \Rightarrow$ **a = 1** ; $\frac{1+b}{2} = 1 \Rightarrow$ **b = 1**

With a = 1, b = 1: A(−2,1), B(1,0), C(4,1), D(1,2)

$AB = \sqrt{(1+2)^2 + (0-1)^2} = \sqrt{10}$; $BC = \sqrt{(4-1)^2 + (1-0)^2} = \sqrt{10}$

All four sides work out equal, so ABCD is a rhombus with **each side = $\sqrt{10}$ units**

Q16. Honeycomb graph: parabola crosses x-axis at (−7, 0) and (7, 0).

Answer: (i) Number of zeroes = **2**

(ii) Zeroes of the polynomial: **x = −7 and x = 7**

(iii) If zeroes of $x^2 + (a+1)x + b$ are 2 and −3:

Sum = 2+(−3) = −1 = −(a+1) ⇒ a = **0**

Product = 2×(−3) = −6 = b ⇒ b = **−6**

OR — If the square of the difference of the zeroes of $x^2 + px + 45$ is 144, find p:

Answer: $\alpha+\beta = -p$, $\alpha\beta = 45$

$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta = p^2 - 180 = 144 \Rightarrow p^2 = 324$

p = **±18**

SECTION – E

Q17. α, β are zeroes of $f(x) = 3x^2 + 2x + 1$. Find a quadratic polynomial with zeroes $(1-\alpha)/(1+\alpha)$ and $(1-\beta)/(1+\beta)$:

Answer: $\alpha+\beta = -\frac{2}{3}$, $\alpha\beta = \frac{1}{3}$

Sum of new zeroes = $\frac{2-2\alpha\beta}{1+(\alpha+\beta)+\alpha\beta} = \frac{4}{3} \div \frac{2}{3} =$ **2**

Product of new zeroes = $\frac{1-(\alpha+\beta)+\alpha\beta}{1+(\alpha+\beta)+\alpha\beta} = 2 \div \frac{2}{3} =$ **3**

Required polynomial: $x^2 - (\text{sum})x + (\text{product}) = x^2 - 2x + 3$

General form: **$k(x^2 - 2x + 3)$, $k \neq 0$**

Q18. Rectangular park 50 m × 40 m; grass strip (uniform width) around the pond has area 1184 m². Find the pond's length and breadth:

Answer: Let border width = x. Pond dimensions: (50−2x) by (40−2x)

Park area − pond area = grass area: $2000 - (50-2x)(40-2x) = 1184$

$(50-2x)(40-2x) = 816 \Rightarrow 4x^2 - 180x + 2000 = 816 \Rightarrow x^2 - 45x + 296 = 0$

$x = [45 \pm \sqrt{(2025-1184)}]/2 = [45 \pm 29]/2 \Rightarrow x = 37$ (rejected) or $x = 8$

Length of pond = 50 − 16 = **34 m**, Breadth of pond = 40 − 16 = **24 m**

OR — Plane delayed by 0.5 hr; speed increased by 250 km/hr to cover 1500 km on time. Find the usual speed:

Answer: Let usual speed = x km/hr. Time saved = 0.5 hr

$\frac{1500}{x} - \frac{1500}{x+250} = 0.5$

$1500(x+250) - 1500x = 0.5x(x+250) \Rightarrow 375000 = 0.5x^2 + 125x \Rightarrow x^2 + 250x - 750000 = 0$

$x = [-250 \pm \sqrt{(62500 + 3000000)}]/2 = [-250 \pm 1750]/2 \Rightarrow x = 750$ (rejecting negative root)

Usual speed = **750 km/hr**

— End of Answer Key —