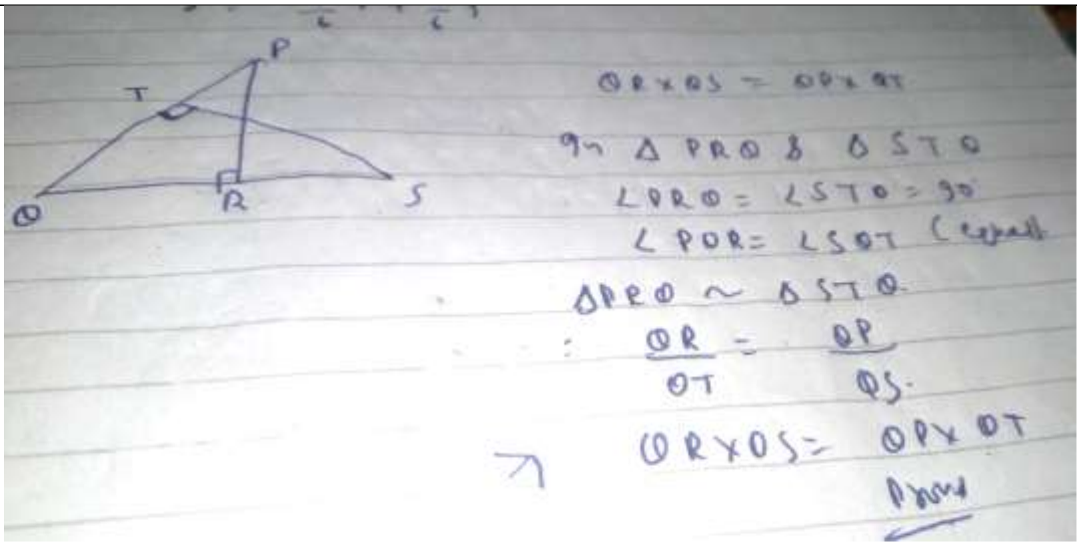
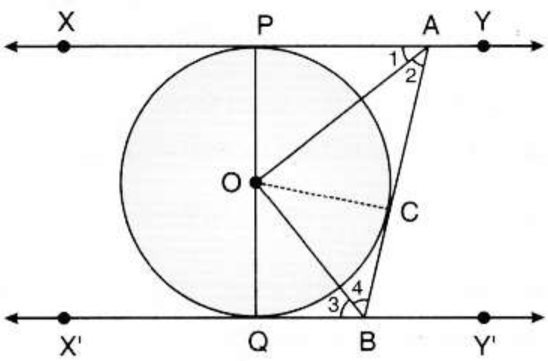
ANSWER KEY

SECTION A	
Q.NO.	**ANSWERS**
1	d
2	d
3	d
4	a
5	c
6	c
7	d
8	b
9	b
10	c
11	c
12	c
13	b
14	a
	SECTION B
15	$$\left(\frac{a}{b}+\frac{b}{a}\right)=\left(\frac{a^2+\ b^2}{ab}\right)=\frac{(a+b)^2-2ab}{ab}=\frac{-13}{6}$$
16	 $QR \times QS = QP \times QT$ In ΔPRQ & ΔSTQ $\angle PRQ = \angle STQ = 90$ $\angle PQR = \angle SQT$ (common) $\Delta PRQ \sim \Delta STQ$ $\therefore \frac{QR}{QT} = \frac{QP}{QS}$ $\Rightarrow QR \times QS = QP \times QT$ Proved
17	Join OC In ΔOPA and ΔOCA OP = OC (radii of same circle) PA = CA (length of two tangents from an external point) 

$$AO = AO \quad \text{(Common)}$$

$$\therefore \quad \Delta OPA \cong \Delta OCA$$

(By SSS congruency criterion)

Hence, $\angle 1 = \angle 2$ (cpct)

Similarly $\angle 3 = \angle 4$

Now,

$$\angle PAB + \angle QBA = 180^\circ$$

$$\Rightarrow \quad 2\angle 2 + 2\angle 4 = 180^\circ$$

$$\Rightarrow \quad \angle 2 + \angle 4 = 90^\circ$$

$$\Rightarrow \quad \angle AOB = 90^\circ$$

(Angle sum property)

Or

Given, $OM \perp PQ$, bisects PQ.

$\therefore$ $PM = 8$ cm (as $PQ = 16$ cm.)

In rt. ΔOMP, $OM = \sqrt{10^2 - 8^2}$

$$= \sqrt{100 - 64}$$

$$= \sqrt{36}$$

$$= 6 \text{ cm}$$

$$\angle TPM + \angle MPO = 90^\circ$$

Also, $\angle TPM + \angle PTM = 90^\circ$

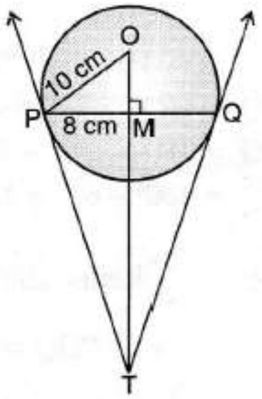
$$\angle MPO = \angle PTM$$

$$\angle TMP = \angle OMP = 90^\circ$$

$$\Delta TMP \sim \Delta PMO \ (AA)$$

or, $$\frac{TP}{PO} = \frac{MP}{MO}$$

$$\frac{TP}{10} = \frac{8}{6}$$



or, $$TP = \frac{80}{6}$$

	SECTION C
18	Let A(x1,y1), B(x2,y2), C(x3,y3) be the vertices of ΔABC Let D(3, 4), E(4, 6) and F(5, 7) be the midpoints of BC, CA and AB. Since, D is the midpoint of BC. $\therefore \frac{x_2+x_3}{2}=3 \text{ and } \frac{y_2+y_3}{2}=4$ $\Rightarrow x_2+x_3=6 \text{ and } y_2+y_3=8 \quad \dots\dots\dots(i)$ Since, E is the midpoint of CA $\therefore \frac{x_1+x_3}{2}=4 \text{ and } \frac{y_1+y_3}{2}=6$ $\therefore x_1+x_3=8 \text{ and } y_1+y_3=12 \quad \dots\dots\dots(ii)$ Since F is the mid-point of AB $\frac{x_1+x_2}{2}=5 \text{ and } \frac{y_1+y_2}{2}=7$ $\Rightarrow x_1+x_2=10 \text{ and } y_1+y_3=14 \quad \dots\dots\dots(iii)$ From (i), (ii) and (iii), we get $x_2+x_3+x_1+x_3+x_1+x_2=6+8+10$ $x_1+x_2+x_3=12 \quad \dots\dots\dots(iv)$ And $y_2+y_3+y_1+y_3+y_1+y_2=8+12+14$ $y_1+y_2+y_3=17 \quad \dots\dots\dots(iv)$ From (i) and (iv) $x_1+6=12, y_1+8=17$ $x_1=6, y_1=9$ From (ii) and (iv) $x_2+8=12, y_2+12=17$ $x_2=4, y_2=5$ From (iii) and (iv) $x_3+10=12, y_3+14=17$ $x_3=2, y_3=3$ Hence the vertices of triangle ABC are (6,9);(4,5);(2,3)

19	Given quadratic equation is $p(x - 4)(x - 2) + (x - 1)^2 = 0$ $\Rightarrow p(x^2 - 4x - 2x + 8) + (x^2 + 1 - 2x) = 0$ $\Rightarrow px^2 - 6px + 8p + x^2 + 1 - 2x = 0$ $\Rightarrow x^2(p + 1) - 2x(3p + 1) + (8p + 1) = 0$ Comparing the above equation with $ax^2 + bx + c = 0$, we get $a = p + 1$, $b = -2(3p + 1)$ and $c = 8p + 1$ For real and equal roots $D = 0$ i.e., $b^2 - 4ac = 0$ $\therefore [-2(3p + 1)]^2 - 4(p + 1)(8p + 1) = 0$ $\Rightarrow 4(3p + 1)^2 - 4(8p^2 + 9p + 1) = 0$ $\Rightarrow 4(9p^2 + 1 + 6p) - 32p^2 - 36p - 4 = 0$ $\Rightarrow 36p^2 + 4 + 24p - 32p^2 - 36p - 4 = 0$ $\Rightarrow 4p^2 - 12p = 0$ $\Rightarrow 4p(p - 3) = 0$ $\Rightarrow p = 0$ or $p = 3$ Hence, for $p = 0$ or $p = 3$, the given quadratic equation has real and equal roots.
20	Given, last term, $l = 119$ No. of terms in A.P. = 30 8^{th} term from the end = 91 Let d be a common difference and assume that the first term of A.P. is 119 (from the end) Since the n^{th} term of AP is $a_n = l + (n - 1)d$ $\therefore a_8 = 119 + (8 - 1)d$ $\Rightarrow 91 = 119 + 7d$ $\Rightarrow 7d = 91 - 119$ $\Rightarrow 7d = -28$ $\Rightarrow d = -4$

Now, this common difference is from the end of A.P.

So, the common difference from the beginning = –d

= –(–4) = 4

Thus, a common difference of the A.P. is 4.

Now, using the formula

$l = a + (n - 1)d$

$\Rightarrow 119 = a + (30 - 1)4$

$\Rightarrow 119 = a + 29 \times 4$

$\Rightarrow 119 = a + 116$

$\Rightarrow a = 3$

Hence, using the formula for the sum of n terms of an A.P.

i.e., $S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{30} = \frac{30}{2}[2 \times 3 + (30 - 1) \times 4]$

$= 15 (6 + 29 \times 4)$

$= 15 (6 + 116)$

$= 15 \times 122$

$= 1830$

Therefore, the sum of 30 terms of an A.P. is 1830.

21

(i) $\Delta ABC \sim \Delta PQR$ (Given

So, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$ (1)

and $\angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R$ (2)

But AB = 2 AM and PQ = 2 PN

(As CM and RN are medians)

So, from (1), $\frac{2AM}{2PN} = \frac{CA}{RP}$

i.e., $\frac{AM}{PN} = \frac{CA}{RP}$ (3)

Also, $\angle MAC = \angle NPR$ [From (2)] (4)

So, from (3) and (4),

$\Delta AMC \sim \Delta PNR$ (SAS similarity) (5)

(ii) From (5), $\frac{CM}{RN} = \frac{CA}{RP}$ (6)

But $\frac{CA}{RP} = \frac{AB}{PQ}$ [From (1)] (7)

Therefore, $\frac{CM}{RN} = \frac{AB}{PQ}$ [From (6) and (7)] (8)

(iii) Again, $\frac{AB}{PQ} = \frac{BC}{QR}$ [From (1)]

Therefore, $\frac{CM}{RN} = \frac{BC}{QR}$ [From (8)] (9)

Also, $\frac{CM}{RN} = \frac{AB}{PQ} = \frac{2\,BM}{2\,QN}$

i.e., $\frac{CM}{RN} = \frac{BM}{QN}$ (10)

i.e CM/RN=BC/QR=BM/QN [From (9) and (10)]
Therefore, ΔCMB ~ ΔRNQ (SSS similarity)

[Note: You can also prove part (iii) by following the same method as used for proving part (i)

Or

In ΔBED and ΔACB, we have:

$\angle BED = \angle ACB = 90°$

$\because \angle B + \angle C = 180°$

$\therefore$ BD || AC

$\angle EBD = \angle CAB$ (*Alternate angles*)

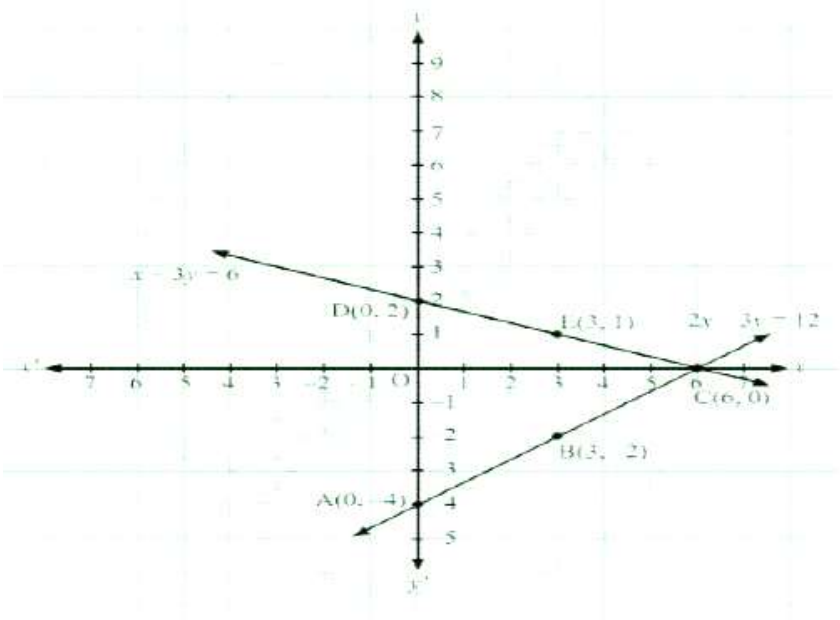
Therefore, by AA similarity theorem, we get :

Δ BED ~ Δ ACB

$$\Rightarrow \frac{BE}{AC} = \frac{DE}{BC}$$

$$\Rightarrow \frac{BE}{DE} = \frac{AC}{BC}$$

This completes the proof.

	SECTION D
22	**I)** **48** **II)** **2 second** **III)** $-8t^2 + 8t + 48$ or K is 3
23	**i)** **18** **ii)** hockey cards requires 3 pages baseball cards requires 4 pages and basketball cards requires 5 pages **Iii)** **1080**
	SECTION E
24	 From the graph, it is clear that, the given lines intersect at (6, 0). So, the solution of the given system of equation is (6, 0). The vertices of the triangle formed by the system of equations and y-axis are (0, 2), (6, 0) and (0, -4). Area (ΔDAC) $= \frac{1}{2} \times DA \times OC$ $= \frac{1}{2} \times 6 \times 6$ = 18 sq. units Hence, the vertices of the triangle are (0, 2), (6, 0) and (0, -4) and its area is 18 sq. units. **Or**

Let the unit digit be x and ten's digit be y

So, original number is $x + 10y$

By reversing the digit, we get $y + 10x$

According to the question,

$(x + 10y) + (y + 10x) = 66$

$11x + 11y = 66$

$\Rightarrow x + y = 6$.(1)

And, since the digits differ by 2, so $x - y = 2$ ....(2)

Adding eqns(1) and (2) we get

$2x = 8$ or $x = 4$

And put $x = 4$ in eqn(1) we get

$4 + y = 6$

$\therefore y = 6 - 4 = 2$

$\therefore$ The number is given is by putting value of x and y.

Hence the number is $(4 + 10 \times 2) = 24$

Thus, the numbers can be 42 or 24

Hence there are 2 such numbers.

25

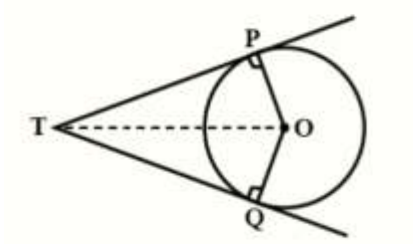
Given : PT and TQ are two tangents drawn from an external point T to the circle C(o, r)

To prove : PT = TQ

Proof : We know that a tangent to the circle is $\perp$ to the radius through the point of contact. So, $\angle OPT = \angle OQT$,

$OT = OT$ (common)

$\angle OPT = \angle OQT = 90^{\circ}$ (Tangent and radius are perpendicular at point of contact)



$OP = OQ$ = radius

$\therefore \Delta OPT \cong \Delta OQT$ (RHS congruence)

$\therefore PT = TQ$ (by c.p.c.t)

As tangents drawn from an external point to a circle are equal in length.
So, therefore, we get AP = AQ (tangents from A)

BP = BR (tangent from B)

CQ = CR (tangent from C)

It is given that ABC is an isosceler triangle with sides AB = AC

⇒ AB – AP = AC – AP

⇒ AB – AP = AC – AQ

⇒ BR = CQ

⇒ BR = CR

So, therefore, BR = CR that imples BC is bisected at the point of contact.

26

As we know that —

$$\boxed{S_n = \frac{n}{2} \times 2a + (n-1)d}$$

Therefore,

$$180 = \frac{n}{2} \times 2(45) + (n-1)(-6)$$

$$180 = \frac{n}{2} \times 90 - 6n + 6$$

$$180 = \frac{n}{2} \times 90 - 6n + 6$$

$$180 \times 2 = n(96 - 6n)$$

$$360 = 96n - 6n^2$$

$$6n^2 - 96n + 360 = 0$$

$$6(n^2 - 16n + 60) = 0$$

$$n^2 - 10n - 6n + 60 = 0 \quad \text{(factorizing)}$$

$$n(n-10) - 6(n-10) = 0$$

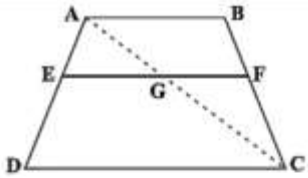
$$(n-6)(n-10) = 0$$

$$n - 6 = 0 \text{ or } n - 10 = 0$$

$$\boxed{n = 6} \text{ or } \boxed{n = 10}$$

Therefore, 6 or 10 terms must be taken so, that their sum is 180.

27 **i) Prove BPT**

	ii)  Fig. 6.15 AB || DC and EF || AB (Given) So, EF || DC (Lines parallel to the same line are parallel to each other) Now, in Δ ADC, EG $\parallel$ DC (As EF $\parallel$ DC) So, $\frac{AE}{ED} = \frac{AG}{GC}$ (Theorem 6.1) (1) Similarly, from Δ CAB, $\frac{CG}{AG} = \frac{CF}{BF}$ i.e., $\frac{AG}{GC} = \frac{BF}{FC}$ (2) Therefore, from (1) and (2), $\frac{AE}{ED} = \frac{BF}{FC}$

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