

XIIth

MATHS



Practice Sheet - 05

Syllabus : Unit 1 Relations and Functions(NCERT Chapter 1-2)

Unit 2 Algebra (NCERT Chapter 3-4)

Unit 3 Calculus (NCERT Chapter 5 ,6 and 7)

SUBJECT: MATHEMATICS

Code : 041

MAX. MARKS : 60

DURATION : 120 min

1. This Question Paper has 5 Sections A-E.
2. **Section A** has 14 MCQs carrying 1 mark each.
3. **Section B** has 3 questions carrying 02 marks each.
4. **Section C** has 4 questions carrying 03 marks each.
5. **Section D** has 2 questions carrying 04 marks each.
6. **Section E** has 4 questions carrying 05 marks each .

Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated

SECTION – A

Questions 1 to 14 carry 1 mark each.

- 1.If $4\cos^{-1}x + \sin^{-1}x = \pi$, then the value of x is

(a) $\frac{3}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{2}{\sqrt{3}}$

2. If $y = |\cos x| + |\sin x|$, then $\frac{dy}{dx}$ at $x = \frac{2\pi}{3}$ is

(a) $\frac{1-\sqrt{3}}{2}$ (b) 0 (c) $\frac{\sqrt{3}-1}{2}$ (d) None of these

- 3.If $f(x) = \log_x (\ln x)$, then $\frac{df}{dx}$ at $x = e$ is equal to

(a) 2 (b) -2 (c) 1 (d) -1

- 4.The function $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 4 + 3 \cos x$ is

(a) Bijective (b) one one but not onto
(c) onto but not one one (d) neither one one nor onto

5. If a set A containing 5 elements and the set B containing 6 elements, then the number of one one and onto mappings from A to B is

(a) 720 (b) 120 (c) 0 (d) None of these

- 6.The value of expressions $\sin\left(\frac{1}{4} \sin^{-1} \frac{\sqrt{63}}{8}\right)$ is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{2\sqrt{2}}$ (d) $\frac{1}{3\sqrt{3}}$

7. If a matrix $\begin{bmatrix} 2k+3 & 4 & 5 \\ -4 & 0 & -6 \\ -5 & 6 & -2k-3 \end{bmatrix}$ is skew symmetric , then the value of k is

- (a) $3/2$ (b) $-3/2$ (c) $1/2$ (d) none of these

8. If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

- (a) A (b) $I + A$ (c) $I - A$ (d) I

9. Let $\int \frac{\sec^2 x \, dx}{1 + 4 \tan^2 x} = f(x) + C$, then $f(x)$ equals

- (a) $\tan^{-1}(2 \tan x)$ (b) $\frac{1}{2} \tan^{-1}(\tan^2 x)$ (c) $\frac{1}{4} \tan^{-1}(2 \tan x)$ (d) $\frac{1}{2} \tan^{-1}(2 \tan x)$

10. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 27$, then the value α is

- (a) ± 1 (b) ± 2 (c) $\pm \sqrt{5}$ (d) $\pm \sqrt{7}$

11. $\int_0^1 \log_e \left(\frac{1}{x} - 1 \right) dx =$

- (a) 0 (b) 1 (c) 2 (d) 3

12. The order and degree of the D.E. : $\left(\frac{dy}{dx} \right)^3 + \left(\frac{d^3y}{dx^3} \right)^3 + 5x = 0$ are, respectively

- (a) 3; 6 (b) 3; 3 (c) 3; 9 (d) 6; 3

13. Assertion: : If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$, then $x^{2022} + y^{2023} + z^{2024} = 1$.

Reason: The greatest value of $\cos^{-1} x$ is π at $x = -1$

- (a) Both A and R are true and R is correct explanation of A.
 (b) Both A and R are true but R is NOT the correct explanation of A.
 (c) A is true but R is false.
 (d) A is false and R is true.

14. Assertion: If $f(x)$ is an even differentiable function, then $\frac{df}{dx}$ is an odd function.

Reason : If $f(x)$ and $g(x)$ are real differentiable functions, then

$$\frac{d}{dx} \{f(g(x))\} = \frac{d}{d(g(x))} \{f(g(x))\} \times \frac{d}{dx} \{g(x)\}.$$

- (a) Both A and R are true and R is correct explanation of A.
 (b) Both A and R are true but R is NOT the correct explanation of A.
 (c) A is true but R is false.
 (d) A is false and R is true.

SECTION – B

Questions 15 to 17 carry 2 mark each.

15. Show that the function defined by $f(x) = x - [x]$, where $[x]$ denotes the greatest integer less than or equal to x is discontinuous at all integer points.

Or

Find the relationship between 'a' and 'b' so that the function f defined by

$$f(x) = \begin{cases} ax + 1, & \text{if } x \leq 3 \\ bx + 3, & \text{if } x > 3 \end{cases} \text{ is continuous at } x = 3.$$

16. Prove that: $\cot^{-1} \left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} \right) = \frac{x}{2}, x \in \left(0, \frac{\pi}{4}\right)$.
17. How many reflexive relations are possible in a set A whose $n(A) = 3$.

SECTION – C
Questions 18 to 21 carry 3 mark each.

18. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, for $-1 < x < 1$, then show that $\frac{dy}{dx} = -\frac{1}{(1+x^2)}$

Or

If $x^m y^n = (x+y)^{m+n}$, then prove that $\frac{dy}{dx} = \frac{y}{x}$.

19. Find : $\int \sec^3 x \, dx$.

20. Evaluate : $\int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} \, dx$.

OR

Evaluate : $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x) \, dx$.

21. Consider $f: \mathbb{R} \rightarrow [-9, \infty]$ given by $f(x) = 5x^2 + 6x - 9$. Prove that f is invertible.

SECTION – D
Questions 22 to 23 carry 4 mark each.

22. If a relation between x and y is such that y cannot be expressed in terms of x , then y is called an implicit function of x . When a given relation expresses y as an implicit function of x and we want to find $\frac{dy}{dx}$, then we differentiate every term of the given relation w.r.t. x , remembering that a term in y is first differentiated w.r.t. y and then multiplied by $\frac{dy}{dx}$.

Based on the above information, find the value of $\frac{dy}{dx}$ in each of the following questions

(i) $x^3 + x^2 y + xy^2 + y^3 = 81$

(ii) $y = (\sqrt{x})^{(\sqrt{x})}$

23. Read the following passage and answer the questions given below.

A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank. A tap is connected to such a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uniform rate of $2 \text{ cm}^3/\text{sec}$

The semi-vertical angle of the conical tank is 45°

(i) Find the volume of water in the tank in terms of its radius r .

(ii) Find rate of change of radius at an instant when $r = 2\sqrt{2} \text{ cm}$.

(iii) Find the rate at which the wet surface of the conical tank is decreasing at an instant when radius $r = 2\sqrt{2} \text{ cm}$.

OR

Find the rate of change of height 'h' at an instant when slant height is 4 cm .

SECTION – E
Questions 24 to 27 carry 5 mark each.

24. Find the value of p and q for which

$$f(x) = \begin{cases} \frac{1-\sin^3 x}{3 \cos^2 x} & , \text{ if } x < \frac{\pi}{2} \\ p & , \text{ if } x = \frac{\pi}{2} \\ \frac{q(1-\sin x)}{(\pi-2x)^2} & , \text{ if } x > \frac{\pi}{2} \end{cases} \text{ is continuous at } \frac{\pi}{2}$$

25. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations:

$$X - Y + 2Z = 1, 2Y - 3Z = 1, 3X - 2Y + 4Z = 2$$

26. Find $\frac{dy}{dx}$, if $Y = e^{\sin^2 x} \left\{ 2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right\}$.

27. Find the area enclosed between by the circle $x^2 + y^2 = 16$ and the lines $\sqrt{3}y = x, x = 0$ in the first quadrant using integration.

OR

Using integration, find the area of the region bounded in first quadrant by $4x^2 + 9y^2 = 36$.

End

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