

# Mind Curve Mid-Term Test Series 2026–27

Class X | Test 04 – Answer Key

## Chapters 8–9: Trigonometry & Some Applications of Trigonometry

| Q.No                                    | Step-by-Step Solution                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | Marks per Step                 |
|-----------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------|
| <b>SECTION – A (Q1–10, 1 mark each)</b> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |                                |
| 1                                       | <p><b>Q1</b></p> <p>Given <math>3 \tan x = 4 \Rightarrow \tan x = \frac{4}{3}</math></p> <p>In a right triangle with <math>\tan x = \frac{4}{3}</math>: opposite = <math>4k</math>, adjacent = <math>3k</math>, hypotenuse = <math>5k</math></p> <p>So <math>\sin x = \frac{4}{5}</math> and <math>\cos x = \frac{3}{5}</math></p> <p><math>\sin x + \cos x = \frac{4}{5} + \frac{3}{5} = \frac{7}{5}</math></p> <p><b>Answer: (c) 7/5</b></p>                                                                                  | 1 mark for the correct option. |
| 2                                       | <p><b>Q2</b></p> <p><math>\tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow \tan^2 30^\circ = \frac{1}{3}</math></p> <p><math>\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}} = \frac{\frac{2}{3}}{\frac{4}{3}} = \frac{1}{2}</math></p> <p><b>Answer: (a) 1/2</b></p>                                                                                                                                                                                                                       | 1 mark for the correct option. |
| 3                                       | <p><b>Q3</b></p> <p>Squaring both sides: <math>(\sin \theta + \cos \theta)^2 = 2</math></p> <p><math>\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta = 2 \Rightarrow 1 + 2 \sin \theta \cos \theta = 2</math></p> <p><math>\sin \theta \cos \theta = \frac{1}{2}</math></p> <p><math>\tan \theta + \cot \theta = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} = \frac{1}{\frac{1}{2}} = 2</math></p> <p><b>Answer: (b) 2</b></p> | 1 mark for the correct option. |
| 4                                       | <p><b>Q4</b></p> <p><math>\tan \beta = \frac{4}{5}</math></p> <p>Dividing numerator and denominator by <math>\cos \beta</math>: <math>\frac{5 \sin \beta - 2 \cos \beta}{5 \sin \beta + 2 \cos \beta} = \frac{5 \tan \beta - 2}{5 \tan \beta + 2}</math></p> <p><math>= \frac{5 \times \frac{4}{5} - 2}{5 \times \frac{4}{5} + 2} = \frac{4 - 2}{4 + 2} = \frac{2}{6} = \frac{1}{3}</math></p> <p><b>Answer: (a) 1/3</b></p>                                                                                                    | 1 mark for the correct option. |
| 5                                       | <p><b>Q5</b></p> <p>Let height = <math>h</math>, shadow = <math>\sqrt{3}h</math></p> <p><math>\tan \theta = \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ</math></p> <p><b>Answer: (a) 30°</b></p>                                                                                                                                                                                                                                                                                                     | 1 mark for the correct option. |

| Q.No | Step-by-Step Solution                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Marks per Step                 |
|------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------|
| 6    | <p><b>Q6</b></p> <p>Let height of tower = <math>h</math></p> <p>Shadow at <math>30^\circ</math>: <math>\tan 30^\circ = \frac{h}{s_1} \Rightarrow s_1 = h\sqrt{3}</math></p> <p>Shadow at <math>60^\circ</math>: <math>\tan 60^\circ = \frac{h}{s_2} \Rightarrow s_2 = \frac{h}{\sqrt{3}}</math></p> <p><math>s_1 - s_2 = h\sqrt{3} - \frac{h}{\sqrt{3}} = h\left(\frac{3-1}{\sqrt{3}}\right) = \frac{2h}{\sqrt{3}}</math></p> <p><math>\frac{2h}{\sqrt{3}} = 70 \Rightarrow h = \frac{70\sqrt{3}}{2} = 35\sqrt{3}</math></p> <p><b>Answer: (b) <math>35\sqrt{3}</math> m</b></p>                                                  | 1 mark for the correct option. |
| 7    | <p><b>Q7</b></p> <p>Squaring: <math>(\sin \theta + \operatorname{cosec} \theta)^2 = 4</math></p> <p><math>\sin^2 \theta + \operatorname{cosec}^2 \theta + 2 \sin \theta \operatorname{cosec} \theta = 4</math></p> <p>Since <math>\sin \theta \cdot \operatorname{cosec} \theta = 1</math>: <math>\sin^2 \theta + \operatorname{cosec}^2 \theta + 2 = 4</math></p> <p><math>\sin^2 \theta + \operatorname{cosec}^2 \theta = 2</math></p> <p><b>Answer: (d) 2</b></p>                                                                                                                                                              | 1 mark for the correct option. |
| 8    | <p><b>Q8</b></p> <p><math>\cot^4 \theta - \operatorname{cosec}^4 \theta = (\cot^2 \theta - \operatorname{cosec}^2 \theta)(\cot^2 \theta + \operatorname{cosec}^2 \theta)</math></p> <p>Since <math>\operatorname{cosec}^2 \theta - \cot^2 \theta = 1</math>, we get <math>\cot^2 \theta - \operatorname{cosec}^2 \theta = -1</math></p> <p>So <math>\cot^4 \theta - \operatorname{cosec}^4 \theta = -(\cot^2 \theta + \operatorname{cosec}^2 \theta)</math></p> <p>Expression <math>= -(\cot^2 \theta + \operatorname{cosec}^2 \theta) + \cot^2 \theta + \operatorname{cosec}^2 \theta = 0</math></p> <p><b>Answer: (a) 0</b></p> | 1 mark for the correct option. |
| 9    | <p><b>Q9</b></p> <p>Given <math>\sin A = \frac{1}{3}</math></p> <p><math>\cos A = \sqrt{1 - \sin^2 A} = \sqrt{1 - \frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}</math></p> <p>Reason (R) — the identity <math>\sin^2 \theta + \cos^2 \theta = 1</math> — is a true, standard identity</p> <p>Since (A) is false and (R) is true</p> <p><b>Answer: (a) Both A and R are true, but R is the correct explanation of A</b></p>                                                                                                                                                                                              | 1 mark for the correct option. |
| 10   | <p><b>Q10</b></p> <p><math>\tan \theta = \frac{3}{4}</math> in a right triangle: opposite = <math>3k</math>, adjacent = <math>4k</math>, hypotenuse = <math>5k</math></p> <p><math>\sin \theta = \frac{3}{5}</math> — Assertion (A) is TRUE</p> <p>Reason (R) states <math>\sin \theta \leq 1</math> for <math>0^\circ \leq \theta \leq 90^\circ</math> — this is TRUE in general, but it does not explain why <math>\tan \theta = \frac{3}{4}</math> gives <math>\sin \theta = \frac{3}{5}</math></p> <p><b>Answer: (b) Both A and R are true, but R is not the correct explanation of A</b></p>                                 | 1 mark for the correct option. |

SECTION – B (Q11–13, 2 marks each)

| Q.No                                      | Step-by-Step Solution                                                                                                                                                                                                                                                                                                                                                                                                                                                    | Marks per Step                                                       |
|-------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------|
| 11<br>(A)                                 | <b>Q11 (A)</b><br>$\cot 45^\circ = 1 \Rightarrow 4 \cot^2 45^\circ = 4$<br>$\sec 60^\circ = 2 \Rightarrow \sec^2 60^\circ = 4$<br>$\sin 60^\circ = \frac{\sqrt{3}}{2} \Rightarrow \sin^2 60^\circ = \frac{3}{4}$<br>$4 - 4 + \frac{3}{4} + p = \frac{3}{4} \Rightarrow p = 0$<br><b>Answer: p = 0</b>                                                                                                                                                                    | 1 mark for substituting standard values; 1 mark for the value of p.  |
| 11<br>(B)                                 | <b>Q11 (B)</b><br>$\cos 45^\circ = \frac{1}{\sqrt{2}} \Rightarrow \cos^2 45^\circ = \frac{1}{2} \Rightarrow 2 \cos^2 45^\circ = 1$<br>$\tan 45^\circ = 1 \Rightarrow \tan^2 45^\circ = 1 \Rightarrow 3 \tan^2 45^\circ = 3$<br>$\sin 90^\circ = 1 \Rightarrow \sin^2 90^\circ = 1 \Rightarrow 4 \sin^2 90^\circ = 4$<br>$1 - 3 + 4 = 2$<br><b>Answer: 2</b>                                                                                                              | 1 mark for substituting standard values; 1 mark for the final value. |
| 12                                        | <b>Q12</b><br>$\sin(A + B) = 1 \Rightarrow A + B = 90^\circ$<br>$2 \cos(A - B) = \sqrt{3} \Rightarrow \cos(A - B) = \frac{\sqrt{3}}{2} \Rightarrow A - B = 30^\circ$<br>Adding the two equations: $2A = 120^\circ \Rightarrow A = 60^\circ$<br>Then $B = 90^\circ - 60^\circ = 30^\circ$<br><b>Answer: A = 60°, B = 30°</b>                                                                                                                                              | 1 mark for forming both equations; 1 mark for the values of A and B. |
| 13<br>(A)                                 | <b>Q13 (A)</b><br>$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}}$<br>$= \frac{\frac{5}{6}}{\frac{5}{6}} = 1$<br>$\tan(A + B) = 1 \Rightarrow A + B = 45^\circ$<br><b>Answer: A + B = 45°</b>                                                                                                                                                                                             | 1 mark for substituting values; 1 mark for the final angle.          |
| 13<br>(B)                                 | <b>Q13 (B)</b><br>(i) $\sec^2 A - \tan^2 A = 1$ for every angle A (standard identity), independent of the value of $\cos A$<br>(ii) $\cos A = \frac{4}{5} \Rightarrow \sin A = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$ (using the 3–4–5 triangle)<br>$\sin^2 A = \frac{9}{25}$<br>$\frac{1 - \sin^2 A}{1 + \sin^2 A} = \frac{1 - \frac{9}{25}}{1 + \frac{9}{25}} = \frac{\frac{16}{25}}{\frac{34}{25}} = \frac{16}{34} = \frac{8}{17}$<br><b>Answer: (i) 1 (ii) 8/17</b> | 1 mark for part (i); 1 mark for part (ii).                           |
| <b>SECTION – C (Q14–15, 3 marks each)</b> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                      |
| 14                                        | <b>Q14</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                               | 1 mark for dividing by $\cos \theta$ ; 1 mark for                    |

| Q.No                                      | Step-by-Step Solution                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | Marks per Step                                                                                   |
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|                                           | $\frac{1 - \tan \theta}{1 + \tan \theta} = \frac{1 - \sqrt{3}}{1 + \sqrt{3}}$ <p>Dividing numerator and denominator of the LHS by <math>\cos \theta</math>:</p> <p>Comparing both sides: <math>\tan \theta = \sqrt{3}</math></p> <p><math>\Rightarrow \theta = 60^\circ</math></p> <p><b>Answer: <math>\theta = 60^\circ</math></b></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | <p>comparing; 1 mark for the final angle.</p>                                                    |
| 15                                        | <p><b>Q15</b></p> <p>Let the distance between the two buildings be <math>d</math>, and the height of the second building be <math>h</math></p> <p>From the foot of the second building, the angle of elevation of the top of the first building (30 m) is <math>60^\circ</math>: <math>\tan 60^\circ = \frac{30}{d}</math></p> $\sqrt{3} = \frac{30}{d} \Rightarrow d = \frac{30}{\sqrt{3}} = 10\sqrt{3} \text{ m}$ <p>From the foot of the first building, the angle of elevation of the top of the second building is <math>30^\circ</math>: <math>\tan 30^\circ = \frac{h}{d}</math></p> $\frac{1}{\sqrt{3}} = \frac{h}{10\sqrt{3}} \Rightarrow h = \frac{10\sqrt{3}}{\sqrt{3}} = 10 \text{ m}$ <p><b>Answer: Distance between buildings = <math>10\sqrt{3} \text{ m} \approx 17.3 \text{ m}</math>; height of second building = 10 m</b></p>                                                                                                                                                                                                                                                                                                                                    | <p>1 mark for setting up the first equation; 1 mark for the distance; 1 mark for the height.</p> |
| <b>SECTION – D (Q16–17, 4 marks each)</b> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                                                                                                  |
| 16                                        | <p><b>Q16</b></p> <p><b>(i) Height of section B</b></p> <p>Let the height of the top of section B above the ground be <math>h_B</math></p> $\tan 30^\circ = \frac{h_B}{36}$ $h_B = 36 \tan 30^\circ = 36 \times \frac{1}{\sqrt{3}} = \frac{36}{\sqrt{3}} = 12\sqrt{3} \text{ m} \approx 20.8 \text{ m}$ <p><b>Answer: Height of section B = <math>12\sqrt{3} \text{ m} \approx 20.8 \text{ m}</math></b></p> <p><b>(ii) Length of the wire from O to the top of section B</b></p> $\cos 30^\circ = \frac{36}{OB}$ $OB = \frac{36}{\cos 30^\circ} = \frac{36}{\frac{\sqrt{3}}{2}} = \frac{72}{\sqrt{3}} = 24\sqrt{3} \text{ m} \approx 41.6 \text{ m}$ <p><b>Answer: Length of wire O to top of B = <math>24\sqrt{3} \text{ m} \approx 41.6 \text{ m}</math></b></p> <p><b>(iii)(A) Height of section A</b></p> <p>Total height of the tower (top of section A) above the ground:</p> $\tan 45^\circ = \frac{h_A}{36} \Rightarrow h_A = 36 \text{ m}$ <p>Height of section A alone = <math>h_A - h_B = 36 - 12\sqrt{3} \text{ m} \approx 15.2 \text{ m}</math></p> <p><b>Answer: Height of section A <math>\approx (36 - 12\sqrt{3}) \text{ m} \approx 15.2 \text{ m}</math></b></p> | <p>(i) 1 mark. (ii) 1 mark. (iii)(A)/(B) 2 marks.</p>                                            |

| Q.No                                      | Step-by-Step Solution                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | Marks per Step                                                                                                                                                                                        |
|-------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                           | <p><b>(iii)(B) Length of the wire from O to the top of section A</b></p> $\cos 45^\circ = \frac{36}{OA}$ $OA = \frac{36}{\cos 45^\circ} = \frac{36}{\frac{1}{\sqrt{2}}} = 36\sqrt{2} \text{ m} \approx 50.9 \text{ m}$ <p><b>Answer: Length of wire O to top of A = <math>36\sqrt{2} \text{ m} \approx 50.9 \text{ m}</math></b></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |                                                                                                                                                                                                       |
| 17                                        | <p><b>Q17</b></p> <p><b>(i) Distance of the first position of the pigeon from the eyes of the boy</b></p> <p>Height of the pigeon above the boy's eye level = <math>54 - 4 = 50 \text{ m}</math></p> $\sin 60^\circ = \frac{50}{PB_1}$ $PB_1 = \frac{50}{\sin 60^\circ} = \frac{50}{\frac{\sqrt{3}}{2}} = \frac{100}{\sqrt{3}} = \frac{100\sqrt{3}}{3} = \frac{100 \times 1.73}{3} \approx 57.7 \text{ m}$ <p><b>Answer: <math>\approx 57.7 \text{ m}</math></b></p> <p><b>(ii) Distance between the boy and the first pole (before the pigeon flies)</b></p> $\tan 60^\circ = \frac{50}{x_1}$ $x_1 = \frac{50}{\tan 60^\circ} = \frac{50}{\sqrt{3}} = \frac{50\sqrt{3}}{3} = \frac{50 \times 1.73}{3} \approx 28.8 \text{ m}$ <p><b>Answer: <math>\approx 28.8 \text{ m}</math></b></p> <p><b>(iii)(A) Distance the pigeon covers in 8 seconds</b></p> <p>After flying, <math>\tan 45^\circ = \frac{50}{x_2} \Rightarrow x_2 = 50 \text{ m}</math></p> <p>Horizontal distance covered = <math>x_2 - x_1 = 50 - 28.8 = 21.2 \text{ m}</math></p> <p><b>Answer: <math>\approx 21.2 \text{ m}</math></b></p> <p><b>(iii)(B) Speed of the pigeon in m/s</b></p> $\text{Speed} = \frac{\text{distance covered}}{\text{time taken}} = \frac{21.2}{8} \approx 2.65 \text{ m/s}$ <p><b>Answer: <math>\approx 2.65 \text{ m/s}</math></b></p> | <p>(i) 1 mark. (ii) 1 mark.<br/>(iii)(A)/(B) 2 marks.</p>                                                                                                                                             |
| <b>SECTION – E (Q18–19, 5 marks each)</b> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                                       |
| 18<br>(A)                                 | <p><b>Q18 (A)</b></p> <p>From <math>\tan A = n \tan B</math>: <math>\tan B = \frac{\tan A}{n}</math></p> <p>From <math>\sin A = m \sin B</math>: <math>\sin B = \frac{\sin A}{m}</math></p> $\cos B = \frac{\sin B}{\tan B} = \frac{\sin A/m}{\tan A/n} = \frac{n \sin A}{m \tan A} = \frac{n \sin A \cos A}{m \sin A} = \frac{n \cos A}{m}$ <p>Using <math>\sin^2 B + \cos^2 B = 1</math>: <math>\left(\frac{\sin A}{m}\right)^2 + \left(\frac{n \cos A}{m}\right)^2 = 1</math></p> $\sin^2 A + n^2 \cos^2 A = m^2$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | <p>1 mark for expressing <math>\tan B</math>, <math>\sin B</math>; 2 marks for <math>\cos B</math>; 1 mark for substituting in <math>\sin^2 B + \cos^2 B = 1</math>; 1 mark for the final result.</p> |

| Q.No      | Step-by-Step Solution                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | Marks per Step                                                                                                                       |
|-----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------|
|           | $(1 - \cos^2 A) + n^2 \cos^2 A = m^2$ $\cos^2 A(n^2 - 1) = m^2 - 1$ $\cos^2 A = \frac{m^2 - 1}{n^2 - 1} \text{ — Hence proved}$ <p><b>Answer: Proved: <math>\cos^2 A = (m^2 - 1)/(n^2 - 1)</math></b></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                                                                                                                      |
| 18<br>(B) | <p><b>Q18 (B)</b></p> <p>LHS numerator:<br/> <math>\cos^4 x - \sin^4 x = (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) = \cos^2 x - \sin^2 x = (\cos x - \sin x)(\cos x + \sin x)</math></p> <p>LHS denominator:<br/> <math>1 - \tan x = 1 - \frac{\sin x}{\cos x} = \frac{\cos x - \sin x}{\cos x}</math></p> $= \frac{(\cos x - \sin x)(\cos x + \sin x)}{\frac{\cos x - \sin x}{\cos x}} = (\cos x + \sin x) \cos x = \cos^2 x + \sin x \cos x$ <p>LHS</p> <p>RHS numerator:<br/> <math>\cot x + 1 = \frac{\cos x}{\sin x} + 1 = \frac{\cos x + \sin x}{\sin x}</math></p> <p>RHS denominator:<br/> <math>\sec x \operatorname{cosec} x = \frac{1}{\cos x \sin x}</math></p> $= \frac{\frac{\cos x + \sin x}{\sin x}}{\frac{1}{\cos x \sin x}} = (\cos x + \sin x) \cos x = \cos^2 x + \sin x \cos x$ <p>RHS</p> <p>LHS = RHS — Hence proved</p> <p><b>Answer: Proved: LHS = RHS = <math>\cos^2 x + \sin x \cos x</math></b></p> | <p>2 marks for simplifying LHS; 2 marks for simplifying RHS; 1 mark for concluding LHS = RHS.</p>                                    |
| 19<br>(A) | <p><b>Q19 (A)</b></p> <p>Let <math>t = \tan \theta</math>, so <math>\cot \theta = \frac{1}{t}</math></p> $\frac{\tan \theta}{1 - \cot \theta} = \frac{t}{1 - \frac{1}{t}} = \frac{t^2}{t - 1}$ $\frac{\cot \theta}{1 - \tan \theta} = \frac{\frac{1}{t}}{1 - t} = \frac{1}{t(1 - t)} = \frac{-1}{t(t - 1)}$ $\text{Sum} = \frac{t^2}{t - 1} - \frac{1}{t(t - 1)} = \frac{t^3 - 1}{t(t - 1)}$ <p>Since <math>t^3 - 1 = (t - 1)(t^2 + t + 1)</math>: Sum</p> $= \frac{(t - 1)(t^2 + t + 1)}{t(t - 1)} = \frac{t^2 + t + 1}{t} = t + 1 + \frac{1}{t}$ $= \tan \theta + \cot \theta + 1 \text{ — Hence proved}$ <p><b>Answer: Proved: LHS = <math>1 + \tan \theta + \cot \theta</math></b></p>                                                                                                                                                                                                                                       | <p>2 marks for converting to a single variable <math>t</math>; 2 marks for combining the fractions; 1 mark for the final result.</p> |
| 19<br>(B) | <p><b>Q19 (B)</b></p> $\tan^2 A - 1 = \frac{\sin^2 A - \cos^2 A}{\cos^2 A}, \text{ so } \frac{\tan^2 A}{\tan^2 A - 1} = \frac{\sin^2 A}{\sin^2 A - \cos^2 A}$ $\sec^2 A - \operatorname{cosec}^2 A = \frac{1}{\cos^2 A} - \frac{1}{\sin^2 A} = \frac{\sin^2 A - \cos^2 A}{\sin^2 A \cos^2 A}$ $\frac{\operatorname{cosec}^2 A}{\sec^2 A - \operatorname{cosec}^2 A} = \frac{1/\sin^2 A}{\frac{\sin^2 A - \cos^2 A}{\sin^2 A \cos^2 A}} = \frac{\cos^2 A}{\sin^2 A - \cos^2 A}$                                                                                                                                                                                                                                                                                                                                                                                                                                                   | <p>2 marks for simplifying each term; 2 marks for combining into a single fraction; 1 mark for the final result.</p>                 |

| Q.No | Step-by-Step Solution                                                                                                                                                                                                                                                                                            | Marks per Step |
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|      | $\text{Sum} = \frac{\sin^2 A + \cos^2 A}{\sin^2 A - \cos^2 A} = \frac{1}{\sin^2 A - \cos^2 A}$ <p>Since <math>\sin^2 A - \cos^2 A = (1 - \cos^2 A) - \cos^2 A = 1 - 2 \cos^2 A</math></p> $= \frac{1}{1 - 2 \cos^2 A} \text{ — Hence proved}$ <p><b>Answer: Proved: LHS = <math>1/(1 - 2\cos^2 A)</math></b></p> |                |

*Prepared by Deepika Bhati | Infinity Think Beyond*

