STD 9 Maths [Manjari]
Sequence and progression

*** Choose the right answer from the given options. [1 Marks Each]** **[40]**

1. What is the next number in the sequence 4, 8, 12, 16, ___?
   (A) 18 (B) 20 (C) 22 (D) 24

   Ans. : (B) 20

2. Which sequence follows addition of 5?
   (A) 2, 7, 12, 17 (B) 3, 6, 12, 24 (C) 1, 4, 9, 16 (D) 5, 15, 30, 60

   Ans.: (A) 2, 7, 12, 17

3. What is the next number in the pattern 1, 3, 6, 10, ___?
   (A) 12 (B) 14 (C) 15 (D) 16

   Ans. : (C) 15

4. Find the missing term: 2, 4, 8, 16, ___
   (A) 24 (B) 28 (C) 30 (D) 32

   Ans. : (D) 32

5. Which of the following is an arithmetic sequence?
   (A) 5, 10, 20, 40 (B) 2, 4, 6, 8 (C) 1, 2, 4, 8 (D) 3, 9, 27, 81

   Ans. : (B) 2, 4, 6, 8

6. What is the common difference in the sequence 9, 14, 19, 24?
   (A) 3 (B) 4 (C) 5 (D) 6

   Ans. : (C) 5

7. What is the next number in the sequence 1, 8, 27, 64, ___?
   (A) 81 (B) 100 (C) 125 (D) 144

   Ans. : (C) 125

8. Find the missing term: 5, 10, 17, 26, ___
   (A) 35 (B) 36 (C) 37 (D) 38

   Ans. : (C) 37

9. Which sequence follows multiplication by 3?
   (A) 1, 3, 9, 27 (B) 2, 4, 6, 8 (C) 5, 10, 15, 20 (D) 3, 6, 12, 18

   Ans.: (A) 1, 3, 9, 27

10. What is the next term in 50, 45, 40, 35, ___?
    (A) 25 (B) 28 (C) 30 (D) 32

Ans. : (C) 30

11. What is the common difference in 20, 25, 30, 35?

    (A) 3 (B) 4 (C) 5 (D) 6

    Ans. : (C) 5

12. Which sequence represents cube numbers?

    (A) 1, 4, 9, 16 (B) 1, 8, 27, 64 (C) 2, 4, 8, 16 (D) 5, 10, 15, 20

    Ans. : (B) 1, 8, 27, 64

13. Find the next number in 3, 7, 13, 21, ___

    (A) 29 (B) 30 (C) 31 (D) 32

    Ans. : (C) 31

14. What is the next term in the sequence 8, 16, 24, 32, ___?

    (A) 36 (B) 40 (C) 42 (D) 48

    Ans. : (B) 40

15. Find the missing number: 6, 11, 16, 21, ___

    (A) 24 (B) 25 (C) 26 (D) 27

    Ans. : (C) 26

16. What is the next number in the sequence 7, 14, 21, 28, ___?

    (A) 30 (B) 32 (C) 35 (D) 42

    Ans. : (C) 35

17. Which of the following is not an arithmetic sequence?

    (A) 3, 6, 9, 12 (B) 10, 20, 30, 40 (C) 1, 2, 4, 8 (D) 5, 10, 15, 20

    Ans. : (C) 1, 2, 4, 8

18. Find the next term in 9, 18, 27, 36, ___

    (A) 40 (B) 42 (C) 45 (D) 54

    Ans. : (C) 45

19. 4 groups in a class were asked to come up with an arithmetic progression (AP). Shown below are their responses:

Group	Arithmetic progression
(i)	$4, 2, 0, -2, \ldots$
(ii)	$41, 38.5, 36, 33.5, \ldots$
(iii)	$-19, -21, -23, -25, \ldots$
(iv)	$-3, -3, -3, -3, \ldots$

Which of these groups correctly came up with an AP?

(A) only groups (i) and (iii)

(B) (b) only groups (ii) and (iii)

(C) (c) only groups (i), (ii) and (iii)

(D) all groups (i), (ii), (iii) and (iv)

Ans. : (D) all groups (i), (ii), (iii) and (iv)

20. If 5^{th} term of an AP is $\frac{1}{4}$ and 4^{th} term is $\frac{1}{5}$, then its 20^{th} term is:

(A) 0 (B) 1 (C) -1 (D) 3

Ans. : (B) 1

21. If n^{th} term of an A.P. is $7n-4$, then the common difference of the AP is:

(A) 7 (B) $7n$ (C) -4 (D) 4

Ans. : (C) -4

22. If the common difference of an AP is 3, then $a_{20} - a_{15}$ is

(A) 5 (B) 3 (C) 15 (D) 20

Ans. : (C) 15

23. If the common difference of an AP is 7, then the value of $a_7 - a_4$ is:

(A) 7 (B) 14 (C) 21 (D) 24

Ans. : (C) 21

24. If the n^{th} term of an AP is $(2n+1)$, then the sum of its first three terms is:

(A) $6n+3$ (B) 15 (C) 12 (D) 21

Ans. : (B) 15

25. If the n^{th} term of an AP is given by $3+\frac{2}{3}n$, then the sum of the first 31 terms =

(A) $\frac{1371}{3}$ (B) $\frac{1271}{3}$ (C) $\frac{1351}{3}$ (D) $\frac{1971}{3}$

Ans. : (B) $\frac{1271}{3}$

26. If the sum of n terms of an A.P. is $2n^2+5n$, then which of its term is 143?

(A) 32 (B) 35 (C) 38 (D) 185

Ans. : (B) 35

27. If the sum of n terms of an A.P. is $3n^2+5n$, then which of its term is 164?

(A) 25^{th} (B) 26^{th} (C) 27^{th} (D) 28^{th}

Ans. : (C) 27^{th}

28. If the sum of n terms of an arithmetic progression $S_n = n^2 - n$ then its third term is:

(A) 2 (B) 4 (C) 6 (D) 9

Ans. : (B) 4

29. In an AP, if $a = 1, a_n = 20$ and $S_n = 399$, then n is:

(A) 19 (B) 21 (C) 38 (D) 42

Ans. : (C) 38

30. In an AP, $S_p = q, S_q = p$, then S_{p+q} is equal to :

(A) $-(p+q)$ (B) 1 (C) 0 (D) pq

Ans.: (A) $-(p+q)$

31. Sum of all multiples of 7 lying between 500 and 900 is :

(A) 35600 (B) 39900 (C) 38600 (D) 37500

Ans. : (B) 39900

32. The 10^{th} term of the AP 5, 8, 11, 14, ... is:

(A) 32 (B) 35 (C) 38 (D) 185

Ans.: (A) 32

33. The 21^{st} term of the AP whose first two terms are -3 and 4 is:

(A) 17 (B) 137 (C) 143 (D) -143

Ans. : (B) 137

34. The list of numbers -10, -6, -2, 2, ... is:

(A) an AP with $d=-16$ (B) an AP with $d=4$

(C) an AP with $d=-4$ (D) not an AP

Ans. : (B) an AP with $d=4$

35. The sum of first 16 terms of the AP 11, 6, 2, ... is:

(A) -424 (B) 320 (C) -352 (D) -400

Ans.: (A) -424

36. The sum of first five multiples of 3 is:

(A) 45 (B) 55 (C) 65 (D) 75

Ans.: (A) 45

37. The sum of first n terms of an AP is $3n^2+4n$ and its common difference is 6, then its first term is:

(A) 7 (B) 4 (C) 6 (D) 3

Ans.: (A) 7

38. The value of middle most term(s) of the AP: -11, -7, -3, ..., 49 is:

(A) 12, 13 (B) 15, 16 (C) 17, 21 (D) 25, 26

Ans. : (C) 17, 21

39. Total number of terms in the AP: 7, 16, 25, ..., 349 =

(A) 35 (B) 36 (C) 37 (D) 39

Ans. : (D) 39

40. The first four terms of an AP, whose first term is -2 and the common difference is -2, are:

(A) $-2,0,2,4$ (B) $-2,-4,-8,-16$ (C) $-2,-4,-6,-8$ (D) $-2,-4,-8,-16$

Ans. : (C) $-2,-4,-6,-8$

*** A statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option.** **[5]**

Directions: In each question below, there are two statements labelled as Assertion (A) and Reason (R). Choose the correct answer as per the options given below:
(a) Both (A) and (R) are true and (R) is the correct explanation of (A)
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A)
(c) (A) is true but (R) is false
(d) (A) is false but (R) is true

41. Assertion (A) : Sum of first 15 terms of $2+5+8\ldots$ is 345.
Reason (R) : Sum of fist n terms in an A.P. is given by the formula: $S_n = \frac{n}{2}[2a+(n-1)d]$

Ans. : D

42. Assertion (A) : Sum of natural numbers from 1 to 100 is 5050
Reason (R) : Sum of n natural numbers is $\frac{n(n+1)}{2}$

Ans. : C

43. Assertion (A) : The common difference of $5,4,3,2,\ldots$ A.P. is -1.
Reason (R): The constant difference between any two terms of an AP is commonly known as common difference.

Ans. : A

44. Assertion (A): The sequence $-1,-1,-1,..,-1$ is an AP.
Reason (R): In an AP $a_n - a_n - 1$ is constant where $n \geq 2$ and $n \in N$.

Ans. : A

45. Assertion (A): The value of n, if $a=10, d=5, a_n=95$ is 16.
Reason (R): The formula of general term a_n is $a_n = a+(n-1)d$.

Ans. : D

*** Answer the following short questions. [2 Marks Each]** **[80]**

46. In an AP:
given $a=5, d=3, a_n=50$, find n and S_n

Ans. : We have, $a_n = 50$

$\Rightarrow a+(n-1)d = 50 \Rightarrow 5+(n-1)(3) = 50 \Rightarrow 5+3n-3 = 50$

$\Rightarrow 3n+2=50 \quad \Rightarrow 3n=48 \quad \Rightarrow n=16$

Now, $S_n=\frac{n}{2}[a+a_n] \Rightarrow S_{16}=\frac{16}{2}(5+50)=8\times 55=440$.

47. In an AP:

given $a=7, a_{13}=35$, find d and S_{13}.

Ans. : We have, $a_{13}=35 \Rightarrow a+(13-1)d=35$

$\Rightarrow 7+12(d)=35 \quad \Rightarrow 12d=28 \quad \Rightarrow d=\frac{28}{12}=\frac{7}{3}$

Now, $S_n=\frac{n}{2}(a+l) \Rightarrow S_{13}=\frac{13}{2}(7+35)=\frac{13}{2}\times 42=273$.

48. Find the sum of the following APs:

-37, -33, -29, ... to 12 terms

Ans. : Here, $a=-37, d=-33-(-37)=4$ and $n=12$

Now, $S_n=\frac{n}{2}[2a+(n-1)d]$

$\Rightarrow \; S_{12}=\frac{12}{2}[2(-37)+(12-1)4] \Rightarrow S_{12}=6[-74+11(4)]=6(-30)=$
-180

49. Find the sum of the following APs:

$\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$, ... to 11 terms.

Ans. : Here, $a=\frac{1}{15}, d=\frac{1}{12}-\frac{1}{15}=\frac{5-4}{60}=\frac{1}{60}$ and $n=11$

Now, $S_n=\frac{n}{2}[2a+(n-1)d]$

$\Rightarrow \; S_{11}=\frac{11}{2}\left[2\left(\frac{1}{15}\right)+(11-1)\left(\frac{1}{60}\right)\right]=\frac{11}{2}\times\frac{9}{30}=\frac{33}{20}$.

50. Check whether 301 is a term of the list of numbers $5, 11, 17, 23, \ldots\ldots$

Ans. : Here $a=5; d=11-5=6$

Let 301 be the nth term of the AP. Then,

$a_n=301 \Rightarrow a+(n-1)d=301$

$\Rightarrow 5+(n-1)(6)=301 \Rightarrow 6n-1=301 \Rightarrow 6n=302 \Rightarrow n=\frac{302}{6}=\frac{151}{3}$

But n should be a positive integer, so 301 is not a term of the given list of numbers.

51. Find the common difference of the AP:

$\frac{1}{3q}, \frac{1-6q}{3q}, \frac{1-12q}{3q}, \ldots$

Ans. : Here, $\quad a_1=\frac{1}{3q}; \quad a_2=\frac{1-6q}{3q}; \quad a_3=\frac{1-12q}{3q}$

Common difference $(d)=a_2-a_1=\frac{1-6q}{3q}-\frac{1}{3q}=\frac{1-6q-1}{3q}=\frac{-6q}{3q}=-2$.

52. Write the next terms of the $AP: \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots\ldots$

Ans. : Here,

$a_1=\sqrt{8}=\sqrt{2\times 4}=2\sqrt{2}, a_2=\sqrt{18}=\sqrt{2\times 9}=3\sqrt{2}, a_3=\sqrt{32}$
$=\sqrt{2\times 16}=4\sqrt{2}$

Now, first term $(a_1)=2\sqrt{2}$ and common difference $(d)=a_2-a_1=3\sqrt{2}-2\sqrt{2}=\sqrt{2}$.

$\therefore$ The next two terms are $4\sqrt{2}+\sqrt{2}=5\sqrt{2}$ and $5\sqrt{2}+\sqrt{2}=6\sqrt{2}$.

53. In Sierpinski Carpet, find the number of squares produced in (i) step 5 (ii) step 8.

Ans. : (i) 4096

(ii) 8^7

54. Show that the progression 0.4, 0.8, 1.6, ... is a G.P. Write its 7^{th} term.

Ans. : $a_7 = \frac{123}{5}$

55. Show that the progression 0.4, 0.8, 1.6, ... is a G.P. Write its common ratio

Ans. : $r = 2$

56. Show that the progression 0.4, 0.8, 1.6, ... is a G.P. Write its n^{th} term

Ans. : $a_n = \frac{2^n}{5}$

57. Show that the progression $-\frac{3}{4}, \frac{1}{2}, -\frac{2}{3}, \frac{4}{9}$, ... is a G.P. Write its 6^{th} term.

Ans. : $a_6 = \frac{8}{81}$

58. Show that the progression $-\frac{3}{4}, \frac{1}{2}, -\frac{2}{3}, \frac{4}{9}$, ... is a G.P. Write its common ratio

Ans. : $r = \frac{-2}{3}$

59. Show that the progression $-\frac{3}{4}, \frac{1}{2}, -\frac{2}{3}, \frac{4}{9}$, ... is a G.P. Write its n^{th} term

Ans. : $a_n = \frac{2^{n-3}}{(3)^{n-2}}$

60. Show that the progression $2, 2\sqrt{2}, 4, 4\sqrt{2}$, ... is a G.P. Write its 11^{th} term

Ans. : $a_{11} = 64$

61. Show that the progression -27, 9, -3, 1, $-\frac{1}{3}$, ... is a G.P. Write its common ratio

Ans. : $r = \frac{-1}{3}$

62. Show that the progression $625, 125, 25, 5, \frac{1}{5}$, ... is a G.P. Write its common ratio

Ans. : $r = \frac{1}{5}$

63. Find the 4^{th} term from the end of the G.P. $\frac{2}{81}, \frac{2}{27}, \frac{2}{9}$, ..., 54.

Ans. : 2

64. Find the 6^{th} term from the end of the G.P. 16, 8, 4, 2, ..., $\frac{1}{512}$.

Ans. : $a_6 = \frac{1}{16}$

65. Show that the progression 2, 6, 18, 54, 162, ... is a G.P. Write its common ratio

Ans. : $r = 3$

66. Show that the progression 2, 6, 18, 54, 162, ... is a G.P. Write its first term

Ans. : $a = 2$

67. Find the sum of all natural numbers less than 200 which are divisible by 5.

Ans. : 3900

68. Find the sum of natural numbers less than 500 and divisible by both 3 and 7.

Ans. : 5796

69. Find the sum of all natural numbers between 200 and 300 which are divisible by 4.

Ans. : 6000

70. Find the sum of all two digits odd positive numbers.

Ans. : 2475

71. Find the sum of the integers between 1 and 200 which are Multiples of 3 or 7

Ans. : 8530

72. Find the sum of the first m natural numbers.

Ans. : $\frac{n(n+1)}{2}$

73. In an AP:
$a_3 = 15, S_{10} = 125$, find d and

Ans. : $d = -1, a_{10} = 8$

74. In an AP:
$d = 5, S_9 = 75$, find a and a_9

Ans. : $a = \frac{-35}{3}, a_9 = \frac{85}{3}$

75. In an AP:
$a = 5, a_2 = 9$, find d and a_3.

Ans. : 4; 13

76. In an AP:
$a = 6, d = 3$, find a_5 and S_5.

Ans. : 18; 60

77. Find the sum of the following APs:
$a, (a+b), (a+2b), \ldots.$ up to n terms

Ans. : $an + \frac{bn^2 - bn}{2}$

78. Find the sum of the following APs:
$(a-b)^2, \left(a^2+b^2\right), (a+b)^2, \ldots$ upto n terms.

Ans. : $a^2n + b^2n - 3abn + abn^2$

79. If the ath term of an AP is given by $a_n = 7n + 1$, find the sum of first 30 terms of the A.P.

Ans. : 3285

80. If the nth term of an AP is given by $a_n = 5 - 2n$, find the sum of first 25 terms of an AP.

Ans. : -525

81. If the nth term of an AP is given by $a_n = 5n - 3$, find the sum of first 20 terms of the AP.

Ans. : 990

82. If the nth term of an AP is given by $a_n = 9 - 5n$, find the sum of first 15 terms.

Ans. : -465

83. Represent the following APs graphically, whose :
first term, $a = 4$, common difference, $d = -1$

Ans. : $a = 4, d = -1$
$n = 1 : a_1 = 4$
$n = 2 : a_2 = 4 + (2-1)(-1) = 3$
$n = 3 : a_3 = 4 + (3-1)(-1) = 2$
$n = 4 : a_4 = 4 + (4-1)(-1) = 1$

84. Write first four terms of the AP, when the first term a and the common difference d are given as follows:
$a = -1.25, d = -0.25$

Ans. : $-1.25, -1.50, -1.75, -2.0$

85. Write first four terms of the AP, when the first term a and the common difference d are given as follows:
$a = -2, d = 0$

Ans. : $-2, -2, -2, -2$

*** Answer the following questions. [3 Marks Each]** **[120]**

86. Find the smallest value of n such that the sum of the first n natural numbers is greater than 1,000.

Ans. : Sum of first n natural numbers:

$S_n = n(n+1)/2$

We need: $n(n+1)/2$ > 1000

$\Rightarrow n(n+1) > 2000$

Now checking:

$44 \times 45 = 1980$, so $S_{44} = 990$

$45 \times 46 = 2070$, so $S_{45} = 1035$

Since 1035 > 1000,

Therefore, the smallest value of n is 45.

87. Find the 10^{th} and 15^{th} terms of the sequence $t_n = 5n - 3$ for $n \geq 1$.

Ans. : Given: $t_n = 5n - 3$

10th term:

$t_{10} = 5(10) - 3$

= 50 − 3

= 47

15th term:

$t_{15} = 5(15) - 3$

= 75 − 3

= 72

Therefore:

10th term = 47

15th term = 72.

88. How many moves are required for 12 discs? Comment on whether it is practical to complete.

Ans. : 4095

89. Find the minimum moves for 10 discs.

Ans. : 1023

90. In Sierpinski Gasket, find the number of triangles left a steps 6th step.

Ans. : 243

91. Which term of the G.P. 5, 10, 20, 40, ... is 640?

Ans. : 8th term

92. The 4^{th},6^{th} and last term of a geometric progression are 10, 40 and 640 respectively. If the common ratio is positive, find the first term, common ratio and the number of terms of the series.

Ans. : $\frac{5}{4};2;10$

93. Find the G.P. whose 4^{th} and 7^{th} terms are $\frac{1}{18}$ and $-\frac{1}{486}$ respectively.

Ans. : $\frac{-3}{2}, \frac{1}{2}, \frac{-1}{6}, \frac{1}{18}, \ldots\ldots$

94. In a Geometric Progression (G.P.), the first term is 24 and the fifth term is 8. Find the ninth term of the G.P.

Ans. : $\frac{8}{3}$

95. Find the G.P. whose 5^{th} and 8^{th} terms are 80 and 640 respectively.

Ans. : 5, 10, 20, 40, 80, ...

96. Which term of the AP 5, 9, 13, ..., is 81?Also, find the sum 5 + 9 + 13 + ... +81?

Ans. : 20860

97. Find the sum:
$1+3+5+7+\ldots+199$

Ans. : 10000

98. Find the sum:
$18+15\frac{1}{2}+13+\ldots+\left(-49\frac{1}{2}\right)$

Ans. : -441

99. Find the sum:
$\left(1-\frac{1}{n}\right)+\left(1-\frac{2}{n}\right)+\left(1-\frac{3}{n}\right)\ldots$ up to n terms.

Ans. : $\frac{1}{2}(n-1)$

100. The first and last terms of an AP are 4 and 81 respectively. If the common difference is 7, how many terms are there in the AP and what is their sum?

Ans. : 12, 510

101. Find the $20th$ term of the AP whose $7th$ term is 24 less than the $11th$ term, first term being 12.

Ans. : 126

102. $26th, 11th$ and the last term of an AP are, 0, 3, and $-\frac{1}{5}$ respectively. Find the common difference and the number of terms of the AP.

Ans. : $d=-\frac{1}{5}, n=27$

103. Determine the AP whose fifth term is 19 and the difference of the eighth term from the thirteenth term is 20.

Ans. : 3, 7, 11, 15 ....

104. Find the $12th$ term and nth term of the AP 16, 9, 2, -5, ...

Ans. : $-61, 23-7n$

105. The sides of a right-angled triangle are in AP. Show that they are in the ratio 3 : 4 : 5.

Ans. : $x-d \rightarrow 4d-d=3d$
$x \rightarrow 4d$

$x+d \rightarrow 4d+d=5d$

The sides are 3d, 4d, and 5d. Expressed as a ratio, this is 3 : 4 : 5.

106. Find the value of the middle most term(s) of the AP: -11, -7, -3, ... 49

Ans. : 17, 21

107. Find the:

$29th$ term of the AP: $2\frac{3}{4}, 3\frac{1}{4}, 3\frac{3}{4}, 4\frac{1}{4}, \ldots\ldots$

Ans. : $16\frac{3}{4}$

108. Which term of the AP. 3, 14, 25, 26, ... will be 99 more than its $25th$ term?

Ans. : 34

109. Find the AP whose fourth term is 9 and the sum of its sixth term and thirteenth term is 40.

Ans. : 3, 5, 7, ...

110. The sum of $4th$ and $8th$ terms of an AP is 24 and the sum of $6th$ and $10th$ terms is 44. Find AP.

Ans. : -13, -8, -3, ...

111. If the $9th$ term of an AP is zero, prove that its $29th$ term is twice its $19th$ term.

Ans. : since $20d = 2 \times 10d$, it is proved that $a_{29} = 2 \times a_{19}$.

112. 14, 21, 28, 35, ... and 26, 39, 52, 65, ... are two arithmetic progressions such that the pth term of the first arithmetic progression is the same as the q term of the second arithmetic progression. Derive a relationship between p and q.

Ans. : $7p - 13q = -6$

113. A child saves ₹10 on day 1 , ₹12 on day 2 , ₹14 on day 3 , and so on. How much money will he save on 365th day?

Ans. : ₹738

114. A man starts repaying a loan as first installment of ₹100. If he increases the installment by ₹ 5 every month, what amount he will pay in the 30 th installment?

Ans. : ₹245

115. In a garden there are 30 mango trees in the first row, twenty seven in the second, twenty four in the third and so on. There are six trees in the last row. How many rows are there of mango trees?

Ans. : 9

116. Mohit was appointed as a lecturer. He was offered monthly salary of ₹15000, with annual increment of ₹500 . In the tenth year what would be his salary?

Ans. : ₹19,500

117. Riya saves ₹50 in the first week and increases her saving by ₹10 every week. How much will she save in the 15th week?

Ans. : ₹190

118. Rohan is appointed at an initial salary of ₹5000 with an increment of ₹ 500 every year. What will be his salary in the 20th year of his service?

Ans. : ₹14,500

119. Sandeep started work in 1995 at an annual salary of ₹5000 and received an increment of ₹200 each year. In which year did his income reach ₹7000 ?

Ans. : 11th year

120. Find the sums given below:

$7+10\frac{1}{2}+14+\ldots+84$

Ans. : Here, $a=7; d=10\frac{1}{2}-7=\frac{21}{2}-\frac{7}{1}=\frac{21-14}{2}=\frac{7}{2}$ and $l\left(a_n\right)=84$

We have, $a+(n-1)d=a_n$

$\Rightarrow \quad 7+(n-1)\left(\frac{7}{2}\right)=84 \quad \Rightarrow \quad 7+\frac{7}{2}n-\frac{7}{2}=84 \quad \Rightarrow \quad \frac{14+7n-7}{2}=84$

$\Rightarrow \quad 7n+7=168 \Rightarrow 7n=168-7 \quad \Rightarrow \quad n=\frac{161}{7}=23$

Now, $S_n=\frac{n}{2}(a+l) \Rightarrow S_{23}=\frac{23}{2}(7+84)=\frac{23}{2}\times 91=\frac{2093}{2}=1046\frac{1}{2}$

(ii) Here, $a=-5; d=-8-(-5)=-8+5=-3$ and $l\left(a_n\right)=-230$

We have, $a+(n-1)d=a_n$

$\Rightarrow(-5)+(n-1)(-3)=-230 \Rightarrow -5-3n+3=-230 \Rightarrow -3n-2=$
-230

$\Rightarrow -3n=-230+2 \Rightarrow -3n=-228 \Rightarrow n=76$

Now,

$S_n=\frac{n}{2}(a+l) \Rightarrow S_{76}=\frac{76}{2}[(-5)+(-230)]=\frac{76}{2}\times(-235)=38\times(-235)$
$=-8$

(iii) Here, $a=34, d=32-34=-2$ and $l\left(a_n\right)=10$

We have, $a+(n-1)d=a_n$

$\Rightarrow 34+(n-1)(-2)=10 \Rightarrow 34-2n+2=10$

$\Rightarrow -2n+36=10 \Rightarrow -2n=-26 \Rightarrow n=13$

Now, $S_n=\frac{n}{2}(a+l) \Rightarrow S_{13}=\frac{13}{2}(34+10)=\frac{13}{2}\times 44=13\times 22=286$.

121. Find: $\left(4-\frac{1}{n}\right)+\left(4-\frac{2}{n}\right)+\left(4-\frac{3}{n}\right)+\ldots$ to n terms.

Ans. : Here, $a=4-\frac{1}{n}, d=\left(4-\frac{2}{n}\right)-\left(4-\frac{1}{n}\right)=4-\frac{2}{n}-4+\frac{1}{n}=\frac{1}{n}-\frac{2}{n}=\frac{-1}{n}$

$S_n=\frac{n}{2}[2a+(n-1)d]=\frac{n}{2}\left[2\left(4-\frac{1}{n}\right)+(n-1)\left(\frac{-1}{n}\right)\right]$

$=\frac{n}{2}\left[8-\frac{2}{n}-1+\frac{1}{n}\right]$

$=\frac{n}{2}\left(7+\frac{1}{n}-\frac{2}{n}\right)=\frac{n}{2}\left(\frac{7n+1-2}{n}\right)=\frac{n}{2}\times\frac{7n-1}{n}=\frac{7n-1}{2}$.

122. The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Ans. : Here, $a=17, l\left(a_n\right)=350$ and $d=9$

We have, $a_n=350 \Rightarrow a+(n-1) d=350$

$\Rightarrow \quad 17+(n-1)(9)=350 \quad \Rightarrow \quad 17+9 n-9=350$

$\Rightarrow \quad 9 n+8=350 \Rightarrow 9 n=342 \Rightarrow n=\frac{342}{9}=38$

Now, $S_n=\frac{n}{2}(a+l) \Rightarrow S_{38}=\frac{38}{2}(17+350)=19 \times 367=6973$.

123. Which term of the AP $3,15,27,39, \ldots . .$ will be 120 more than its 21 st term?

Ans. : Here $a=3, d=15-3=12$

Let nth term $\left(a_n\right)$ of the AP be 120 more than its 21st term $\left(a_{21}\right)$. Then,

$a_n=a_{21}+120 \Rightarrow a+(n-1) d=a+(21-1) d+120$

$\Rightarrow(n-1) \times 12=20 \times 12+120 \Rightarrow 12 n-12=240+120$

$\Rightarrow 12 n-12=360 \Rightarrow 12 n=372 \Rightarrow n=\frac{372}{12}=31$

Hence, 31st term is 120 more than 21st term.

124. Determine the AP whose 3^{rd} term is 5 and the 7^{th} term is 9.

Ans. : We have, $a_3=5 \Rightarrow a+(3-1) d=5 \Rightarrow a+2 d=5$ ...(i)

Also, $a_7=9 \Rightarrow a+(7-1) d=9 \Rightarrow a+6 d=9$ ...(ii)

Subtracting (ii) from (i) we get, $-4 d=-4 \Rightarrow d=1$

Putting the value of $d=1$ in (i) we get, $a+2(1)=5 \Rightarrow a=5-2=3$

Hence, the required AP is: $a, a+d, a+2 d, \ldots .$

$=3,3+1,3+2(1), \ldots \ldots$ or $3,4,5, \ldots \ldots$

125. How many natural numbers are there between 200 and 500 which are divisible by 7 ?

Ans. : The list of natural numbers between 200 and 500 , which are divisible by 7 are $203,210,217, \ldots \ldots, 497$

Here, $a=203, d=210-203=7$ and $l=497\left(a_n\right)$

Now,

$a_n=497 \Rightarrow a+(n-1) d=497 \Rightarrow 203+(n-1)(7)=497 \Rightarrow 203+7 n-7$
$=497$

$\Rightarrow 7 n+196=497 \Rightarrow 7 n=497-196 \Rightarrow 7 n=301 \Rightarrow n=43$

Hence, there are 43 numbers between 200 and 500 which are divisible by 7 .

* Questions with calculation. [4 Marks Each] [160]

126. How many 2-digit numbers are divisible by 3? What is the sum of all these 2-digit numbers?

Ans. : The 2-digit numbers divisible by 3 are 12, 15, 18, ..., 99

We can observe that this is an AP where:

First term, a = 12

Common difference, d = 15 – 12 = 3

Last term = 99

Using the formula: $t_n=a+(n-1) d$, we have

99 = 12 + (n − 1)3

$\Rightarrow 99-12=3(n-1)$

$\Rightarrow 87=3(n-1)$

$\Rightarrow 87=3(n-1)$

$\Rightarrow 29=n-1$

$\Rightarrow n=30$

So, there are 30 two-digit numbers divisible by 3.

Now, sum of these numbers: $S_n=n/2\times$ (first term + last term)

$\Rightarrow S_{30}=30/2\times(12+99)$

$=15\times111$

= 1665

Therefore:

Number of 2-digit numbers divisible by 3 = 30

Sum of all these numbers = 1665.

127. Find the 12^{th} term of a GP with common ratio 2, whose 8^{th} term is 192.

Ans. : Given: Common ratio, r = 2

8th term = 192

So, $t_8=ar^7=192$ [Using $t_n=ar^{n-1}$]

$\Rightarrow a(2)^7=192$ [Since $r=2$]

$\Rightarrow a\times128=192$

$\Rightarrow a=192/128=3/2$

$\Rightarrow a=3/2$

Now, $t_{12}=\mathrm{ar}^{11}$ [Using $t_n=$ ar $^{n^{-1}}$]

$=3/2\times(2)^{11}$

$=3\times2^{10}$

$=3\times1024$

= 3072

Therefore, the 12th term is 3072.

128. Which term of the GP: 2, 6, 18, ... is 4374? Write the explicit formula as well as the recursive formula for the nth term.

Ans. : Given GP: 2, 6, 18, ...

First term, a = 2

Common ratio, $r=a_2/a_1=6/2=3$

Explicit formula: $t_n=ar^{n-1}$

$\Rightarrow t_n=2\times3^{n-1}$[As $a=2$ and $r=3$]

Let $t_n=4374$

$\Rightarrow 2\times3^{n-1}=4374$

$\Rightarrow 3^{n-1}=4374/2$

$\Rightarrow 3^{n^{-1}} = 2187$

$\Rightarrow 3^{n-1} = 3^7$ [Since $2187 = 3^7$]

$\Rightarrow n - 1 = 7$

$\Rightarrow n = 8$

Therefore, 4374 is the 8th term.

Recursive formula:

$t_1 = 2$

$t_n = 3t_{n-1}$, for $n \geq 2$.

129. A ball is dropped from a height of 80 metres. After hitting the ground, it bounces back to 60% of the height from which it fell. It continues bouncing in this way—each time rising to 60% of the previous height.
What is the total vertical distance the ball has travelled by the time it hits the ground for the 6th time?

Ans. : The ball first falls 80 m to hit the ground for the 1st time.

Then it rises and falls after each bounce.

Heights after bounces:

1st bounce = 48 m

2nd bounce = 28.8 m

3rd bounce = 17.28 m

4th bounce = 10.368 m

5th bounce = 6.2208 m

To hit the ground for the 6th time, it travels the total distance

= 80 + 2(48 + 28.8 + 17.28 + 10.368 + 6.2208)

= 80 + 2(110.6688)

= 80 + 221.3376

= 301.3376 m

Therefore, the total vertical distance travelled is 301.3376 m.

130. Which term of the sequence 2 , $2\sqrt{2}, 4, \ldots$ is 128?

Ans. : Given sequence: $2, 2\sqrt{2}, 4, \ldots$

This is a GP.

First term, a = 2

Common ratio: $r = a_2/a_1 = 2\sqrt{2}/2 = \sqrt{2}$

So, $t_n = 2(\sqrt{2})^{n^{-1}}$ [Using the formula $t_n = ar^{n^{-1}}$]

Let $t_n = 128$

$\Rightarrow 2(\sqrt{2})^{n-1} = 128$

$\Rightarrow (\sqrt{2})^{n-1} = 64$

$\Rightarrow \left(2^{1/2}\right)^{n-1} = 2^6$ [Since $64 = 2^6$ and $\sqrt{2} = 2^{1/2}$]

$\Rightarrow 2^\wedge(n-1)/2 = 2^6$

$\Rightarrow (n-1)/2 = 6$ [Comparing the powers of 2]
$\Rightarrow n-1 = 12$
$\Rightarrow n = 13$
Therefore, 128 is the 13th term.

131. Find the 31^{st} term of an AP whose 11^{th} term is 38 and 16^{th}term is 73.

Ans. : Let first term = a and common difference = d
Given:
$\text{11th term} \Rightarrow a+10d = 38...(1)$
16th term ⇒ $a+15d = 73...(2)$
Subtracting (1) from (2), we get:
$(a+15d)-(a+10d) = 73-38$
$\Rightarrow 5d = 35$
$\Rightarrow d = 7$
Substituting d = 7 in (1), we have:
$a+10(7) = 38$
$\Rightarrow a+70 = 38$
$\Rightarrow a = -32$
Now, 31st term: $t_{31} = a+30d$
$= -32+30(7)$
= -32 + 210
= 178
Therefore, the 31st term is 178.

132. Determine the AP whose third term is 16 and whose 7^{th} term exceeds the 5^{th} term by 12.

Ans. : Let first term = a and common difference = d
Third term: $a+2d = 16...(1)$
According to question:
7th term – 5th term = 12 [Since 7th term exceeds 5th term by 12]
$\Rightarrow (a+6d)-(a+4d) = 12$
$\Rightarrow 2d = 12$
$\Rightarrow d = 6$
Substituting d = 6 in (1), we get:
$a+2(6) = 16$
$\Rightarrow a+12 = 16$
$\Rightarrow a = 4$
Thus, AP: 4, 10, 16, 22, 28, ...
Therefore: First term = 4 and common difference = 6.

133. How many multiples of 4 lie between 10 and 250?
(Hint: All multiples of 4 form an AP. Find the smallest and largest multiples of 4

between 10 and 250.)

Ans. : The smallest multiple of 4 greater than 10 is 12.
The largest multiple of 4 less than 250 is 248.
So, the required AP: 12, 16, 20, ..., 248
Here: First term, a = 12
Common difference, d = 4
Last term = 248
Let $t_n = 994$
$\Rightarrow 248 = 12 + (n-1)4$ [Using Formula $t_n = a + (n-1)d$]
$\Rightarrow 248 - 12 = 4(n-1)$
$\Rightarrow 236 = 4(n-1)$
$\Rightarrow 59 = n - 1$
$\Rightarrow n = 60$
Therefore, 60 multiples of 4 lie between 10 and 250.

134. The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of the 2^{nd} hour, 4^{th} hour, n^{th} hour?

Ans. : Initial number of bacteria = 30
The bacteria double every hour.
At the end of 1 st hour $= 30 \times 2 = 60$
At the end of 2nd hour:
$= 30 \times 2^2$
$= 30 \times 4$
= 120
At the end of 4th hour:
$= 30 \times 2^4$
$= 30 \times 16$
= 480
At the end of nth hour $= 30 \times 2^n$
Therefore:
At the end of 2nd hour = 120 bacteria
At the end of 4th hour = 480 bacteria
At the end of nth hour $= 30 \times 2^n$ bacteria.

135. Which term of the GP: 2, 8, 32, ... is 131072? Write the explicit formula as well as the recursive formula for the n^{th} term.

Ans. : Given GP: 2, 8, 32, ...
First term, a = 2
Common ratio, $r = 8/2 = 4$
Explicit formula:

$t_n = ar^{n-1}$

$t_n = 2 \times 4^{n-1}$

Now: $2 \times 4^{n-1} = 131072$

$\Rightarrow 4^{n-1} = 131072/2$

$\Rightarrow 4^{n-1} = 65536$

Since: $65536 = 4^8$

So: $n - 1 = 8$

⇒ n = 9

Therefore, 131072 is the 9th term.

Recursive formula:

$t_1 = 2$

$t_n = 4t_{n-1}$, for $n \geq 2$.

136. The sum of the first three terms of a GP is $\frac{13}{12}$ and their product is –1. Find the common ratio and the terms.

Ans. : Let the three terms of the GP be a/r, a, ar

Product: $(a/r) \times a \times ar = a^3$

According to question, product = -1

$\Rightarrow a^3 = -1$

$\Rightarrow a = -1$

So, the three terms are $-1/r, -1, -r$

Sum: $-1/r - 1 - r = 13/12$

$\Rightarrow -12 - 12r - 12r^2 = 13r$

$\Rightarrow 12r^2 + 25r + 12 = 0$

$\Rightarrow 12r^2 + 16r + 9r + 12 = 0$

$\Rightarrow 4r(3r + 4) + 3(3r + 4) = 0$

$\Rightarrow (4r + 3)(3r + 4) = 0$

So: $r = -3/4$ or $r = -4/3$

If $r = -3/4$

Terms are 4/3, -1, 3/4

If r = -4/3:

Terms are $3/4, -1, 4/3$

Therefore, the common ratio is $-3/4$ or $-4/3$, and the terms are $4/3, -1, 3/4$ or $3/4, -1, 4/3$.

137. Determine whether 97 and 172 are terms of the sequence $t_n = 5n - 3$ for $n \geq$ 1.

Ans. : Given: $t_n = 5n - 3$

For 97:

Let $t_n = 97$

$\Rightarrow 5n - 3 = 97$

$\Rightarrow 5n = 100$

$\Rightarrow n = 20$

Since n = 20 is a natural number, 97 is a term of the sequence.

So, 97 is the 20th term.

For 172:

Let $t_n = 172$

$\Rightarrow 5n - 3 = 172$

$\Rightarrow 5n = 175$

$\Rightarrow n = 35$

Since n = 35 is a natural number, 172 is also a term of the sequence.

So, 172 is the 35th term.

138. Find the n^{th} term of the AP: 11, 8, 5, 2, ... Write the recursive rule for this AP.

Ans. : Given AP: 11, 8, 5, 2, ...

First term, a = 11

Common difference, d = 8 − 11 = −3

Formula: $t_n = a + (n-1)d$

$t_n = 11 + (n-1)(-3)$ [Putting the values of a and d]

= 11 − 3n + 3

= 14 − 3n

Therefore, nth term: $t_n = 14 - 3n$

Recursive rule:

$t_1 = 11$

$t_n = t_{n-1} - 3$, for $n \geq 2$.

139. If the 4th and 7th terms of a G.P. are 54 and 1458 respectively, then find its:

(i) common ratio, r

(ii) first term, a

(iii) nth term, a_n

(iv) 6th term, a_6

Ans. : Let a be the first term and r be the common ratio of the given G.P.

Then, $a_4 = 54 \Rightarrow ar^3 = 54$ ... (i)

and $a_7 = 1458 \Rightarrow ar^6 = 1458$ ...(ii)

On dividing the corresponding sides of (ii) and (i), we get:

$\frac{ar^6}{ar^3} = \frac{1458}{54} \Rightarrow r^3 = 27 = 3^3 \Rightarrow r = 3$.

Thus, (i) Common ratio, r = 3.

(ii) Putting r = 3 in (i), we get:

$a \times 3^3 = 54 \Rightarrow a \times 27 = 54 \Rightarrow a = \frac{54}{27} = 2$.

∴ First term, $a = 2$

(iii) nth term, $a_n = ar^{n-1} = \left(2 \times 3^{n-1}\right)$.

(iv) 6th term, $a_6 = \left\{2 \times 3^{(6-1)}\right\} = \left(2 \times 3^5\right) = (2 \times 243) = 486$.

140. Show that the progression 81, 27, 9, 3, 1, ... is a G.P. Write its

(i) first term

(ii) common ratio

(iii) nth term

(iv) 11th term

Ans. : The given progression is 81, 27, 9, 3, 1, ...

We have, $\frac{27}{81}=\frac{9}{27}=\frac{3}{9}=\frac{1}{3}$ (constant).

So, the given progression is a G.P.

We have

(i) first term, $a=81$.

(ii) common ratio, $r=\frac{27}{81}=\frac{1}{3}$.

(iii) nth term, $a_n=ar^{n-1}$

$\Rightarrow a_n=81\times\left(\frac{1}{3}\right)^{n-1}=\frac{81}{3^{n-1}}$ ...(1)

(iv) Putting n = 11 in (1), we get:

$a_{11}=\frac{81}{3^{(11-1)}}=\frac{81}{3^{10}}=\frac{3^4}{3^{10}}=\frac{1}{3^6}=\frac{1}{729}$.

141. Show that the progression 3, 6, 12, 24, 48, ... is a G.P. Write its:

(i) first term

(ii) common ratio

(iii) nth term

(iv) 10th term

Ans. : The given terms are 3, 6, 12, 24, 48, ...

Clearly, we have $\frac{6}{3}=\frac{12}{6}=\frac{24}{12}=\frac{48}{24}=2$ (a constant).

So, the given numbers form a G.P.

Clearly, we have

(i) first term, a = 3

(ii) common ratio, $r=\frac{6}{3}=2$.

(iii) nth term, $a_n=ar^{n-1}\Rightarrow a_n=\left(3\times 2^{n-1}\right)$ ...(1)

(iv) Putting $n=10$, in (1) we get:

10th term, $a_{10}=\left[3\times 2^{(10-1)}\right]=\left(3\times 2^9\right)=(3\times 512)=1536$.

142. Show that $a_1, a_2, \ldots a_n$ form an AP where a_n is defined as below. Also find S_{15} in each case.

$a_n=3+4n$

Ans. : We have, $a_n=3+4n$

$\Rightarrow \quad a_1=3+4(1)=3+4=7$

$a_2=3+4(2)=3+8=11$

$\Rightarrow \quad a_3=3+4(3)=3+12=15$

Here, $a_2-a_1=a_3-a_2=4$

$\therefore a_1, a_2, \ldots a_n$ form an AP with common difference $d=4$.

$\therefore S_n = \frac{n}{2}[2a+(n-1)d] \Rightarrow S_{15} = \frac{15}{2}[2(7)+(15-1)(4)]$

$\Rightarrow = \frac{15}{2}[14+56] = \frac{15}{2} \times 70 = 15 \times 35 = 525$.

143. Show that $a_1, a_2, \ldots a_n$ form an AP where a_n is defined as below. Also find S_{15} in each case.

$a_n = 9 - 5n$

Ans. : We have, $a_n = 9 - 5n$

$\Rightarrow \quad a_1 = 9 - 5(1) = 9 - 5 = 4$

$\Rightarrow \quad a_2 = 9 - 5(2) = 9 - 10 = -1$

$\Rightarrow \quad a_3 = 9 - 5(3) = 9 - 15 = -6$

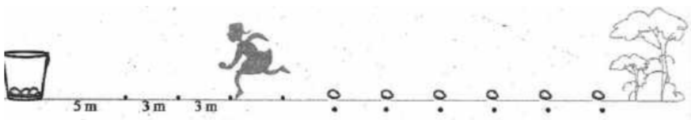
Here, $a_2 - a_1 = a_3 - a_2 = -5$

$\therefore a_1, a_2, \ldots, a_n$ form an AP with common difference $d = -5$.

$\therefore S_n = \frac{n}{2}[2a+(n-1)d] \Rightarrow S_{15} = \frac{15}{2}[2(4)+(15-1)(-5)]$

$= \frac{15}{2}(8-70) = \frac{15}{2} \times (-62) = 15 \times (-31) = -465$.

144. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line, as shown in the figure.



A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

Ans. : Distance covered to pick up the 1st potato $= 2(5m) = 10m$

Distance between two successive potatoes $= 3m$

Distance covered to pick up the 2nd potato $= 2(5+3)m = 16m$

Distance covered to pick up the 3rd potato $= 2(5+3+3)m = 22m$

$10, 16, 22, \ldots..$ is an AP, where $a = a_1 = 10, a_2 = 16, a_3 = 22, \ldots$,

So, $d = a_2 - a_1 = 16 - 10 = 6$ and $n = 10$

$\therefore$ Total distance the competitor has to run $= S_{10}$

$S_n = \frac{n}{2}[2a+(n-1)d] \Rightarrow S_{10} = \frac{10}{2}[2(10)+(10-1)6] = 5[20+54] = 5$
$\times 74 = 370$

Hence, the competitor has to run 370 m in all.

145. In an $A.P.$, if $S_n = 3n^2 + 5n$ and $a_k = 164$, find the value of k.

Ans. : Let a be the first term and d be the common difference of the A.P.

We have, $\quad S_n = 3n^2 + 5n$

And, $\quad S_{n-1} = 3(n-1)^2 + 5(n-1) = 3\left(n^2 + 1 - 2n\right) + 5(n-1)$

$= 3n^2 - 6n + 3 + 5n - 5 = 3n^2 - n - 2$

Now, $\quad a_n = S_n - S_{n-1} = 3n^2 + 5n - \left(3n^2 - n - 2\right)$

$= 3n^2 + 5n - 3n^2 + n + 2 = 6n + 2$

$\Rightarrow \quad a_k = 6k + 2$

It is given that $a_k = 164 \Rightarrow 6k + 2 = 164 \Rightarrow 6k = 162 \Rightarrow k = 27$

146. The first term of an AP is -5 and the last term is 45. If the sum of the terms of the AP is 120, then find the number of terms and the common difference.

Ans. : Now, $\quad l = 45 \Rightarrow a_n = 45$

147. The first term of an AP is 3, the last term is 83 and the sum of all its terms is 903. Find the number of terms and the common difference of the AP.

Ans. : We have, $a = 3 \quad l = 83, S_n = 903$

Let the AP have n terms. Then

$l = a + (n-1)d \Rightarrow 83 = 3 + (n-1)d$

$\Rightarrow (n-1)d = 80$ ....(i)

Also, $S_n = \frac{n}{2}(a + l)$

$\Rightarrow 903 = \frac{n}{2}(3 + 83) \Rightarrow 1806 = 86n \Rightarrow n = 21$

From (i), $(21-1)d = 80 \Rightarrow d = 4$

Hence, number of terms $= 21$ and common difference $= 4$.

148. The 19 th term of an AP is equal to three times its 6 th term. If its 9 th term is 19 , find the AP.

Ans. : Let a be the first term and d be the common difference of the AP.

We have, $\quad a_{19} = 3a_6$

$\Rightarrow a + 18d = 3(a + 5d) \Rightarrow a + 18d = 3a + 15d \Rightarrow -2a + 3d = 0$ ...(i)

Also, $a_9 = 19 \Rightarrow a + 8d = 19 \Rightarrow a = 19 - 8d$ ...(ii)

Substituting the value of (ii) in (i), we get

$-2(19 - 8d) + 3d = 0 \Rightarrow -38 + 16d + 3d = 0$

$\Rightarrow -38 + 19d = 0 \Rightarrow 19d = 38 \Rightarrow d = 2$

Putting the value of $d = 2$ in (ii), we get, $a = 19 - 8 \times 2 = 19 - 16 = 3$

Hence the required AP is: $a, a + d, a + 2d, \ldots.$. or i.e., $3, 3 + 2, 3 + 2(2), \ldots.$. or $3, 5, 7, \ldots.$.

149. Which term of the AP $-7, -12, -17, -22, \ldots.$ will be -82 ? Is -100 any term of the AP?

Ans. : Given AP is $-7, -12, -17, -22, \ldots.$

Here, $a = -7, d = -12 - (-7) = -5$

Let nth term of the AP be -82 .

Then, $a_n = a + (n-1)d \Rightarrow -82 = -7 + (n-1)(-5)$

$\Rightarrow -75 = (n-1) \times (-5) \Rightarrow n - 1 = 15 \Rightarrow n = 16$

Hence, 16th term of the AP is -82 .

Let $a_n = -100$.

Then, $a_n = a + (n-1)d$

$\Rightarrow -100 = -7 + (n-1) \times (-5) \Rightarrow -93 = (n-1) \times (-5) \Rightarrow n - 1 = \frac{93}{5}$
$\Rightarrow n = \frac{98}{5}$
But n cannot be a fractional number as it denotes number of terms.
Hence, -100 cannot be the term of the given AP.

150. Which term of the AP $21, 18, 15, \ldots\ldots$ is -96 ? Also, check whether 0 is a term of the AP.

Ans. : The given AP is $21, 18, 15, \ldots$
Here $a = 21, d = 18 - 21 = -3$
Let nth term of the AP be -96 .
Then $a_n = -96 \Rightarrow a + (n-1)d = -96 \Rightarrow 21 + (n-1)(-3) = -96$
$\Rightarrow -3n + 24 = -96 \Rightarrow -3n = -96 - 24 = -120 \Rightarrow n = \frac{-120}{-3} = 40$
Hence, 40th term of the AP is -96 .
Let m th term of the AP be 0 .
Then $a_m = 0 \Rightarrow a + (m-1)d = 0$
$\Rightarrow 21 + (m-1) \times (-3) = 0 \Rightarrow m - 1 = \frac{-21}{-3} = 7 \Rightarrow m = 7 + 1 = 8$.
So, 8th term of the AP is 0 .

151. Two APs have the same common difference. The difference between their $100th$ terms is 123456. What is the difference between their millionth terms?

Ans. : Let a_1, b_1 be the first terms of the two APs and d be the common difference of each AP.
Then nth term of 1st $AP = a_n = a_1 + (n-1)d$
nth term of 2nd $AP = b_n = b_1 + (n-1)d$
$a_n - b_n = a_1 + (n-1)d - b_1 - (n-1)d = a_1 - b_1$
Thus, $a_n - b_n$ is independent of n
Now, $a_{100} - b_{100} = 123456$
$= a_1 - b_1$
$\Rightarrow\ a_m - b_n = a_1 - b_1 = 123456$
$\Rightarrow\ a_{1,000,000} - b_{1,000,000} = a_1 - b_1 = 123456$
Hence, the difference between their millionth terms is same as difference between their 100th terms.

152. In a given AP, if pth term is q and qth term is p, show that nth term is $(p + q - n)$.

Ans. : Let a be the first term and d be the common difference of the AP.
We have, $a_p = q \Rightarrow a + (p-1)d = q$ ...(i)
Also, $a_q = p \quad \Rightarrow a + (q-1)d = p$ ...(ii)
Subtracting (ii) from (i), we get $(p-1)d - (q-1)d = q - p$
$\Rightarrow d[p - 1 - q + 1] = q - p \Rightarrow d(p-q) = -(p-q) \Rightarrow d = -1$
Putting the value of d in (i), we get $a + (p-1)(-1) = q$
$\Rightarrow a - p + 1 = q \Rightarrow a = p + q - 1$

Now, $a_n = a + (n-1)d = p + q - 1 + (n-1)(-1)$
$= p + q - 1 - n + 1 = p + q - n$.

153. Find the 12^{th} term from the end of the AP: –2, –4, –6, ..., –100.

Ans. : Here, $a = -2; d = -4 - (-2) = -4 + 2 = -2$ and, $l\,(a_n) = -100$
$\Rightarrow a_n = -100 \Rightarrow a + (n-1)d = -100 \Rightarrow -2 + (n-1)(-2) = -100$
$\Rightarrow -2 - 2n + 2 = -100 \Rightarrow -2n = -100 \Rightarrow n = 50$.
∴ There are 50 terms in the given AP.
Now, 12th term from the end $= (50 - 12 + 1)$ th term $= 39$ th term
$\therefore a_{39} = a + (39-1)d = -2 + (39-1)(-2) = -2 - 76 = -78$.
Alternative Method
nth term from the end $= l - (n-1)d$
$\therefore 12$ th term from the end $= -100 - (12-1)(-2) = -100 + 22 = -78$.

154. Represent the following GPs graphically, whose :
first term, $a = 1$, common ratio, $r = 2$.

Ans. : GP where $a = 1$ and $r = 2$
First, let's calculate the first five terms:
$n = 1 : a_1 = 1$
$n = 2 : a_2 = 2$
$n = 3 : a_3 = 4$
$n = 4 : a_4 = 8$
$n = 5 : a_5 = 16$

155. Represent the following GPs graphically, whose :
first term $a = 4$, common ratio, $r = -2$

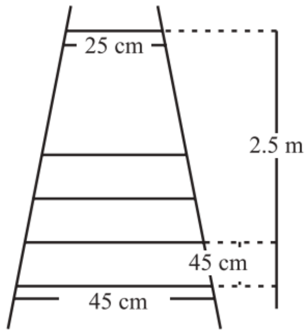
Ans. : GP where $a = 4$ and $r = -2$
First, let's calculate the first five terms:
$n = 1 : a_1 = 4$
$n = 2 : a_2 = -8$
$n = 3 : a_3 = 16$
$n = 4 : a_4 = -32$
$n = 5 : a_5 = 64$

156. Raghav buys a shop for ₹120000. He pays half of the amount in cash and agrees to pay the balance in 12 annual instalments of ₹ 5000 each. If the rate of interest is 12% per annum and he pays with the instalment the interest due on the unpaid amount, find the total cost of the shop.

Ans. : 166800

157. A ladder has rungs 25 cm apart (as shown in the given figure). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and the bottom rungs are $12\frac{1}{2}m$ apart, what is the length of the wood

required for the rungs?



Ans. : 3.85 m

158. The 16th term of an AP is five times its third term. If its 10th term is 41, then find the sum of its first 15 terms.

Ans. : 495

159. The first and last term of an AP are 8 and 65 respectively. If the sum of all its terms is 730, find its common difference.

Ans. : 3

160. In an AP $S_5+S_7=167$ and $S_{10}=235$. Find AP.

Ans. : $1, 6, 11, \ldots$

161. The sum of first m terms of an AP is $\frac{3m^2+7m}{2}$. Find its mth term.

Ans. : $3m+2$

162. Show that $(a-b)^2, \left(a^2+b^2\right)$ and $(a+b)^2$ are in AP.

Ans. : 1. Calculate the difference between the second and first term $\left(d_1\right)$:

$(a-b)^2=a^2-2ab+b^2$

$d_1=a^2+b^2-\left(a^2-2ab+b^2\right)$

$d_1=a^2+b^2-a^2+2ab-b^2$

$d_1=2ab$

2. Calculate the difference between the third and second term $\left(d_2\right)$:

$(a+b)^2=a^2+2ab+b^2$

$d_2=\left(a^2+2ab+b^2\right)-\left(a^2+b^2\right)$

$d_2=a^2+2ab+b^2-a^2-b^2$

$d_2=2ab$

163. In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

(i)The fee charged from a student every month by a school for the whole session when the monthly fee is ₹400.

(ii)The fee charged every month by a school from classes I to XII when the monthly fee for class I is 250, and it increases by 50 for the next higher class.

(iii)The amount of money in the account of Varun at the end of every year when ₹1000 is deposited at simple interest of 10% per annum.

Ans. : (i) Yes; $a_{k+1} - a_k =$ Constant

(ii) Yes : $a_{k+1} - a_k =$ Constant

(iii) Yes; $a_{k+1} - a_k =$ Constant

(iv) No; $a_{k+1} - a_k \neq$ Constant

164. In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

(i) The taxi fare after each km when the fare is 24 for the first km and 10 for each additional km.

(ii)The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air in the cylinder at a time.

(iii)The cost of digging a well after every metre of digging when it costs 150 for the first metre and rises by 50 for each subsequent metre.

(iv)The amount of money in the account every year when ₹10000 is deposited at compound interest at 8% per annum.

Ans. : (i) Yes, $a_{k+1} - a_k =$ Constant

(ii) No, $a_{k+1} - a_k \neq$ Constant

(iii) Yes, $a_{k+1} - a_k =$ Constant

(iv) No, $a_{k+1} - a_k \neq$ Constant

165. Two arithmetic progressions have the same first term. The common difference of one progression is 4 more than the other progression. 124th term of the first arithmetic progression is the same as 42th term of the second.
Find one set of possible values of the common differences.

Ans. : $-2, -6$, or 2, 6

*** Answer the following questions. [5 Marks Each]** **[120]**

166. Which term of the AP: 21, 18, 15, ... is −81? Also, is 0 a term of this AP? Give reasons for your answer.

Ans. : Given AP: 21, 18, 15, ...

First term, a = 21

Common difference, d = 18 − 21 = −3

Formula: $t_n = a + (n-1)d$

So, $t_n = 21 + (n-1)(-3)$ [Putting the values of a and d]

= 21 − 3n + 3

= 24 − 3n

Now, checking for −81:

Let $t_n = 81$

$\Rightarrow 24 - 3n = -81$

$\Rightarrow -3n = -105$
$\Rightarrow n = 35$
Therefore, −81 is the 35th term.
Now, checking whether 0 is a term:
Let $t_n = 0$
$\Rightarrow 24 - 3n = 0$
$\Rightarrow 3n = 24$
$\Rightarrow n = 8$
Since n = 8 is a natural number, 0 is a term of this AP.
Therefore, 0 is the 8th term.

167. An AP consists of 50 terms in which the 3^{rd} term is 12 and the last term is 106. Find the 29^{th} term.
(Hint: If ‘a’ is the first term and ‘d’ the common difference, then we arrive at the equations a + 2d = 12 and a + 49d = 106. Solve this pair of linear equations for ‘a’ and ‘d’.)

Ans. : Let the first term be a and common difference be d.
Given: 3rd term = 12
$\text{So}, a + 2d = 12 \ldots (1)$
The AP has 50 terms and last term = 106.
So, 50th term = 106
$\Rightarrow a + 49d = 106 \ldots (2)$
Subtracting equation (1) from equation (2), we get:
$(a + 49d) - (a + 2d) = 106 - 12$
$\Rightarrow 47d = 94$
$\Rightarrow d = 2$
Now, Substituting d = 2 in equation (1), we have:
$a + 2(2) = 12$
$\Rightarrow a + 4 = 12$
$\Rightarrow a = 8$
Find the 29th term:
$t_{29} = a + (29 - 1)d$
$= 8 + 28 \times 2$
= 8 + 56
$= 64$
Therefore, the 29th term is 64.

168. A sequence is given by the recursive rule $t_1 = 2, t_{n+1} = 3t_n - 2$ for $n \geq 1$. Which term of the sequence is 730 ?

Ans. : Given: $t_1 = 2$
$t_{n+1} = 3t_n - 2$
Finding terms step by step:

$t_1 = 2$
$t_2 = 3t_1 - 2$ [Using $t_{n+1} = 3t_n - 2$]
$= 3(2) - 2$
= 4
Similarly, $t_3 = 3t_2 - 2$
$= 3(4) - 2$
= 10
$t_4 = 3t_3 - 2$
$= 3(10) - 2$
= 28
$t_5 = 3t_4 - 2$
$= 3(28) - 2$
= 82
$t_6 = 3t_5 - 2$
$= 3(82) - 2$
= 244
$t_7 = 3t_6 - 2$
$= 3(244) - 2$
= 730
Therefore, 730 is the 7th term of the sequence.

169. A ball is dropped from a height of 80 metres. After hitting the ground, it bounces back to 60% of the height from which it fell. It continues bouncing in this way—each time rising to 60% of the previous height.
What height does the ball reach after the 5th bounce?

Ans. : Initial height = 80 m
Bounce ratio $= 60\% = 0.6$
Height after 1st bounce:
$= 80 \times 0.6$
= 48 m
Height after 2nd bounce:
$= 48 \times 0.6$
= 28.8 m
Height after 3rd bounce:
$= 28.8 \times 0.6$
= 17.28 m
Height after 4th bounce:
$= 17.28 \times 0.6$
= 10.368 m
Height after 5th bounce:
$= 10.368 \times 0.6$

= 6.2208 m

Therefore, the ball reaches 6.2208 m after the 5th bounce.

170. Fig.shows Stages 0 to 3 of the Sierpiński square carpet. Stage 0 of this fractal is a square sheet of paper. To construct Stage 1, each side of the square is trisected and the points of trisection of opposite sides are joined to obtain nine smaller squares. The centre square is then removed and the 8 smaller squares are retained, leaving a square hole in the centre. The same process is repeated on the eight smaller shaded squares to obtain Stage 2 and so on.

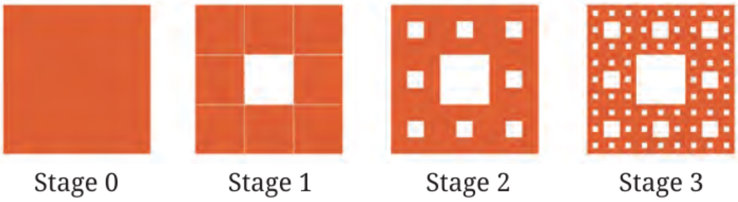


Fig. : Stages 0, 1, 2 and 3 of the Sierpiński square carpet

Look at Fig. and try to answer the following questions.

(i) How many red squares are there in Stages 0 to 3?

(ii) Can you predict the number of red squares in Stages 4 and 5?

(iii) Can you find a rule for the number of red squares at the n^{th} stage? Write the explicit formula as well as the recursive formula for the number of red squares at any stage.

(iv) Suppose the area of the square in Stage 0 is 1 square unit. What is the area of the red region in Stages 1, 2 and 3? What will be the area of the red region in Stages 4 and 5? Find the explicit as well as the recursive formula for the area of the red region at the nth stage. What happens to this area as n, the number of stages, goes on increasing?

Ans. : (i) Stage 0: 1 red square

Stage 1: 8 red squares

Stage 2: $8^2 = 64$ red squares

Stage 3: $8^3 = 512$ red squares

Therefore: Stages 0 to 3 have 1, 8, 64 and 512 red squares respectively.

(ii) The number of red squares is multiplied by 8 at each stage.

Stage $4 = 8^4 = 4096$

Stage $5 = 8^5 = 32768$

Therefore:

Stage 4 has 4096 red squares.

Stage 5 has 32768 red squares.

(iii) At each stage, every red square is replaced by 8 smaller red squares.

So, the number of red squares forms the sequence:

1, 8, 64, 512, ...

Explicit formula: $t_n = 8^n$

Here, Stage 0 corresponds to n = 0.

Recursive formula:
$t_0 = 1$
$t_n = 8t_{n-1}$, for $n \geq 1$
(iv) At each stage, the square is divided into 9 equal parts and the centre part is removed.
So, the remaining red area becomes 8/9 of the previous area.
Stage 0 area = 1 square unit
Stage 1 area = 8/9
Stage 2 area $= (8/9)^2 = 64/81$
Stage 3 area $= (8/9)^3 = 512/729$
Stage 4 area $= (8/9)^4 = 4096/6561$
Stage 5 area $= (8/9)^5 = 32768/59049$
Explicit formula $A_n = (8/9)^n$
Here, Stage 0 corresponds to n = 0.
Recursive formula:
$A_0 = 1$
$A_n = (8/9)A_{n-1}$, for $n \geq 1$
As n increases, the area keeps decreasing and gets closer and closer to 0, but it never becomes exactly 0.

171. Find a GP for which the sum of the first two terms is −4 and the fifth term is 4 times the third term.

Ans. : Let the first term of the GP be a and common ratio be r.
So, the required GP: a, ar, $ar^2, ar^3, ar^4, \ldots$
Given: Sum of first two terms = −4
$\Rightarrow a + ar = -4$
$\Rightarrow a(1+r) = -4 \ldots (1)$
Also, fifth term is 4 times the third term.
Fifth term $= ar^4$
Third term = ar^2
So, $ar^4 = 4ar^2$
$\Rightarrow r^2 = 4$ [Dividing by ar^2]
$\Rightarrow r = 2$ or $r = -2$
Case 1: r = 2
From (1): $a(1+2) = -4$
$\Rightarrow 3a = -4$
$\Rightarrow a = -4/3$
Hence, the GP: $-4/3, -8/3, -16/3, -32/3, -64/3, \ldots$
Case 2: r=-2
From (1): a(1 − 2) = −4
$\Rightarrow -a = -4$

$\Rightarrow a=4$

Hence, the GP: $4,-8,16,-32,64,\ldots$

Therefore, possible GPs are $-4/3,-8/3,-16/3,-32/3,\ldots$ and $4,-8,16,-32,\ldots$

172. Find all possible ways of expressing 100 as the sum of consecutive natural numbers.

Ans. : We need consecutive natural numbers whose sum is 100.

Possible lengths:

1 term: 100

So, 100 = 100

5 terms:

Let the numbers be: $x-2,x-1,x,x+1,x+2$

Sum = 5x

$\Rightarrow 5x=100$

$\Rightarrow x=20$

So, 100 = 18 + 19 + 20 + 21 + 22

8 terms:

Let the numbers be: $x,x+1,x+2,\ldots,x+7$

Sum $=8x+28$

$\Rightarrow 8x+28=100$

$\Rightarrow 8x=72$

$\Rightarrow x=9$

So, 100 = 9 + 10 + 11 + 12 + 13 + 14 + 15 + 16

Therefore, all possible ways are:

100 = 100

100 = 18 + 19 + 20 + 21 + 22

100 = 9 + 10 + 11 + 12 + 13 + 14 + 15 + 16.

173. The sum of the 4^{th} and 8^{th} terms of an AP is 24 and the sum of the 6^{th} and 10^{th} terms is 44. Find the first three terms of the AP.

Ans. : Let the first term be a and common difference be d.

4th term = a + 3d

8th term = a + 7d

So, the sum of 4th and 8th term:

$(a+3d)+(a+7d)=24$

$\Rightarrow 2a+10d=24$

$\Rightarrow a+5d=12\ldots(1)$

6th term = a + 5d

10th term = a + 9d

So, the sum of 6th and 10th term:

$(a+5d)+(a+9d)=44$

$\Rightarrow 2a+14d=44$

$\Rightarrow a+7d=22\ldots(2)$
Subtracting (1) from (2), we get:
2d = 10
$\Rightarrow d=5$
Substituting d = 5 in (1), we have:
$a+5(5)=12$
$\Rightarrow a+25=12$
$\Rightarrow a=-13$
First three terms: a, a + d, a + 2d
$=-13,-8,-3$
Therefore, the first three terms are $=-13,-8,-3$

174. The sum of the first three terms of a geometric progression is 26, and the sum of their squares is 364. Find the terms of the GP.

Ans. : Let the three terms be a, ar, ar^2
According to first condition:
$a+ar+ar^2=26$
$\Rightarrow a\left(1+r+r^2\right)=26\ldots(1)$
According to second condition:
$(a)^2+(ar)^2+\left(ar^2\right)^2=364$
$\Rightarrow a^2\left(1+r^2+r^4\right)=364\ldots(2)$
Now, dividing equation (2) by square of equation (1), we have
$a^2\left(1+r^2+r^4\right)/\left[a\left(1+r+r^2\right)\right]^2=364/26^2$
$\Rightarrow\left(1+r^2+r^4\right)/\left(1+r+r^2\right)^2=7/13$
$\Rightarrow 13\left(1+r^2+r^4\right)=7\left(1+r+r^2\right)^2$
$\Rightarrow 13\left(1+r^2+r^4\right)-7\left(r^2+r+1\right)^2=0$
$\Rightarrow 13\left(r^2+r+1\right)\left(r^2-r+1\right)-7\left(r^2+r+1\right)^2=0\left[U\operatorname{sing}1+r^2+\right.$
$\left.r^4=\left(r^2+r+1\right)\left(r^2-r+1\right)\right]$
$\Rightarrow\left[13\left(r^2-r+1\right)-7\left(r^2+r+1\right)\right]=0\left[\text{ Dividing by }\left(r^2+r+1\right)\right]$
$\Rightarrow\left[13r^2-13r+13-7r^2-7r-7\right]=0$
$\Rightarrow\left(6r^2-20r+6\right)=0$
$\Rightarrow 3r^2-10r+3=0$
$\Rightarrow(3r-1)(r-3)=0$
$\therefore r=1/3$ or $r=3$
If $r=1/3$, from (1), we have
$a\left[1+1/3+(1/3)^2\right]=26$
$\Rightarrow a[1+1/3+1/9]=26$
$\Rightarrow a[(9+3+1)/9=26$
$\Rightarrow a[13/9]=26$

$\Rightarrow a=18$

Hence, the three terms a, ar, a^2 are $18, 6, 2$.

If $r=3$, from (1), we have

$a\left[1+3+(3)^2\right]=26$

$\Rightarrow a[1+3+9]=26$

$\Rightarrow a[13]=26$

$\Rightarrow a=2$

Hence, the three terms a, ar, ar ^{2} are $2, 6, 18$.

So, the terms of the GP are 2, 6, 18.

175. Suppose $P_1=1, P_2=2$ and for $n>2, P_n=P_1,+P_2+\ldots+P_{n-1}+1$. Find the values of $P_1, P_2, \ldots, P_8$. Can you find a simpler recursive formula for P_n ? Can you give an explicit formula?

Ans. : Given:

$P_1=1$

$P_2=2$

For $n>2$: $P_n=P_1+P_2+\ldots+P_{n^{-1}}+1$

Now: $P_3=P_1+P_2+1$

= 1 + 2 + 1

= 4

$P_4=P_1+P_2+P_3+1$

= 1 + 2 + 4 + 1

= 8

$P_5=1+2+4+8+1$

= 16

$P_6=1+2+4+8+16+1$

= 32

$P_7=64$

$P_8=128$

So: $P_1, P_2, \ldots, P_8$ are $1, 2, 4, 8, 16, 32, 64, 128$

Simpler recursive formula:

$P_1=1$

$P_n=2P_{n^{-1}}$, for $n \geq 2$

Explicit formula $P_n=2^{n-1}$.

176. Suppose $W_1=1, W_2=2$ and for $n>2, W_n=W_1+W_2+\cdots+W_{n-2}+2$. Find the values of $W_1, W_2, \ldots, W_8$. Do you recognise this sequence?

Ans. : Given:

$W_1=1$

$W_2=2$

For $n>2$: $W_n=W_1+W_2+\ldots+W_{n-2}+2$

Now: $W_3=W_1+2$

= 1 + 2
= 3
$W_4 = W_1 + W_2 + 2$
= 1 + 2 + 2
= 5
$W_5 = W_1 + W_2 + W_3 + 2$
= 1 + 2 + 3 + 2
= 8
$W_6 = W_1 + W_2 + W_3 + W_4 + 2$
= 1 + 2 + 3 + 5 + 2
= 13
$W_7 = 1 + 2 + 3 + 5 + 8 + 2$
= 21
$W_8 = 1 + 2 + 3 + 5 + 8 + 13 + 2$
= 34
Therefore: $W_1, W_2, \ldots, W_8$ are $1, 2, 3, 5, 8, 13, 21, 34$
Yes, this is the Virahanka-Fibonacci sequence.

177. Find the first five terms of the sequence in which the n^{th} term is given by (i) $t_n = 3n - 4$, (ii) $t_n = 2 - 5n$, and (iii) $t_n = n^2 - 2n + 3$ for $n \geq 1$.

Ans. : (i) $t_n = 3n - 4$
$t_1 = 3(1) - 4 = -1$
$t_2 = 3(2) - 4 = 2$
$t_3 = 3(3) - 4 = 5$
$t_4 = 3(4) - 4 = 8$
$t_5 = 3(5) - 4 = 11$
So, the first five terms: $-1, 2, 5, 8, 11$
(ii) $t_n = 2 - 5n$
$t_1 = 2 - 5(1) = -3$
$t_2 = 2 - 5(2) = -8$
$t_3 = 2 - 5(3) = -13$
$t_4 = 2 - 5(4) = -18$
$t_5 = 2 - 5(5) = -23$
So, the first five terms: $-3, -8, -13, -18, -23$
(iii) $t_n = n^2 - 2n + 3$
$t_1 = 1^2 - 2(1) + 3 = 2$
$t_2 = 2^2 - 2(2) + 3 = 3$
$t_3 = 3^2 - 2(3) + 3 = 6$
$t_4 = 4^2 - 2(4) + 3 = 11$

$t_5 = 5^2 - 2(5) + 3 = 18$

So, the first five terms: 2, 3, 6, 11, 18.

178. A sequence is given by the recursive rule $t_1 = -5, t_{n+1} = t_n + 3$ for $n \geq 1$. Find the first five terms of the sequence. Is 52 a term of this sequence? If so, which term is it?

Ans. : Given: $t_1 = -5$ and $t_{n+1} = t_n + 3$

First five terms:

$t_1 = -5$

$t_2 = t_1 + 3 = -5 + 3 = -2$ [Using $t_{n+1} = t_n + 3$]

$t_3 = t_2 + 3 = -2 + 3 = 1$

$t_4 = t_3 + 3 = 1 + 3 = 4$

$t_5 = t_4 + 3 = 4 + 3 = 7$

Therefore, the first five terms: −5, −2, 1, 4, 7

This is an arithmetic sequence with first term a = −5 common difference d = 3.

Formula: $t_n = a + (n-1)d$

$t_n = -5 + (n-1)3$

$= -5 + 3n - 3$

$= 3n - 8$

Now checking whether 52 is a term:

Let $t_n = 52$

$\Rightarrow 3n - 8 = 52$

$\Rightarrow 3n = 60$

$\Rightarrow n = 20$

Since n = 20 is a natural number, 52 is a term of the sequence. Therefore, 52 is the 20th term.

179. Let $T_1 = 1, T_2 = 2, T_3 = 4$, and $T_n = T_{n-1} + T_{n-2} + T_{n-3}$ for $n \geq 4$. Find T_4, T_5, T_6, T_7 , and $T_{8.}$

Ans. : Given:

$T_1 = 1$

$T_2 = 2$

$T_3 = 4$

$T_n = T_{n^{-1}} + T_{n^{-2}} + T_{n^{-3}}$ (for $n \geq 4$)

Now computing step by step:

$T_4 = T_3 + T_2 + T_1$ [Using $T_n = T_{n^{-1}} + T_{n^{-2}} + T_{n^{-3}}$ and putting $n = 4$]

= 4 + 2 + 1

= 7

$T_5 = T_4 + T_3 + T_2$ [Using $T_n = T_{n^{-1}} + T_{n^{-2}} + T_{n^{-3}}$ and putting $n = 5$]

= 7 + 4 + 2

= 13

$T_6 = T_5 + T_4 + T_3$ [Using $T_n = T_{n^{-1}} + T_{n^{-2}} + T_{n^{-3}}$ and putting $n = 6$]

= 13 + 7 + 4

= 24

$T_7 = T_6 + T_5 + T_4$ [Using $T_n = T_{n^{-1}} + T_{n^{-2}} + T_{n^{-3}}$ and putting $n = 7$]

= 24 + 13 + 7

= 44

$T_8 = T_7 + T_6 + T_5$ [Using $T_n = T_{n^{-1}} + T_{n^{-2}} + T_{n^{-3}}$ and putting $n = 8$]

= 44 + 24 + 13

= 81.

180. If $a_n = 3 - 4n$, show that $a_1, a_2, a_3, \ldots\ldots$ form an AP. Also, find S_{20}.

Ans. : We have, $a_n = 3 - 4n$

$\therefore a_1 = 3 - 4(1) = 3 - 4 = -1$

$a_2 = 3 - 4(2) = 3 - 8 = -5$

$a_3 = 3 - 4(3) = 3 - 12 = -9$

Here, $a_2 - a_1 = a_3 - a_2 = -4$

$\therefore a_1, a_2, a_3, \ldots$ form an AP with common difference $d = -4$.

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow$ $S_{20} = \frac{20}{2}[2(-1) + (20-1)(-4)] \Rightarrow = 10(-2 - 76) = 10 \times (-78) =$.
-780

181. Which term of the AP: $-2, -7, -12, \ldots.$ will be -77? Find the sum of this AP up to the term -77.

Ans. : Let ath term (a_n) of the AP be -77. Then a_{n} = - 77

Here, $a = -2$ and $d = -7 - (-2) = -7 + 2 = -5$

We have, $a_n = -77 \Rightarrow a + (n-1)d = -77$

$\Rightarrow$ $(-2) + (n-1)(-5) = -77$ $\Rightarrow$ $-2 - 5n + 5 = -77$

$\Rightarrow$ $-5n + 3 = -77$ $\Rightarrow$ $-5n = -80 \Rightarrow n = 16$

Hence, 16th term of the AP is -77.

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow$ $S_{16} = \frac{16}{2}[2(-2) + (16-1)(-5)]$

$\Rightarrow$ $S_{16} = 8(-4 - 75) = 8 \times (-79) = -632$.

182. Divya deposited ₹1000 at compound interest at the rate of 10% per annum. The amounts at the end of first year, second year, third year, form an AP. Justify your answer.

Ans. : Amount at the end of 1st year

$= P\left(1 + \frac{r}{100}\right)^n = ₹1000\left(1 + \frac{10}{100}\right)^1 = ₹1000 \times \frac{11}{10} = ₹1100$

Amount at the end of the 2 nd year $= ₹1000\left(1 + \frac{10}{100}\right)^2$

$= ₹1000\left(\frac{11}{10}\right)^2 = ₹1000 \times \frac{121}{100} = ₹1210$

Amount at the end of the 3rd year $= ₹1000\left(1 + \frac{10}{100}\right)^3$

$= ₹1000\left(\frac{11}{10}\right)^3 = ₹1000 \times \frac{1331}{1000} = ₹1331$

So, the amounts (in ₹) at the end of 1st year, 2nd year, 3rd year, are $1100, 1210, 1331$ respectively.

Here,

$a_2 - a_1 = 1210 - 1100 = 110, a_3 - a_2 = 1331 - 1210 = 121 a_2 - a_1 \neq a_3 - a_2$. It does not form an AP.

183. Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

(i) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$.

(ii) $0, -4, -8, -12$

(iii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \ldots$

Ans. : (i) Here, $a_1 = 3; a_2 = 3+\sqrt{2}, a_3 = 3+2\sqrt{2}, a_4 = 3+3\sqrt{2}, \ldots\ldots$

We have, $a_2 - a_1 = 3+\sqrt{2}-3 = \sqrt{2}$

$a_3 - a_2 = (3+2\sqrt{2}) - (3+\sqrt{2}) = 3+2\sqrt{2}-3-\sqrt{2} = \sqrt{2}$

$a_4 - a_3 = 3+3\sqrt{2} - (3+2\sqrt{2}) = 3+3\sqrt{2}-3-2\sqrt{3} = \sqrt{2}$.

Since $a_{n+1} - a_n$ is same every time. So, the given numbers form an AP with common difference $d = \sqrt{2}$.

Now, next three terms are:

$a_5 = 3+3\sqrt{2}+\sqrt{2} = 3+4\sqrt{2}, a_6 = 3+4\sqrt{2}+\sqrt{2} = 3+5\sqrt{2}, a_7 = 3 + 5\sqrt{2}+\sqrt{2} = 3+6\sqrt{2}$

(ii) Here, $a_1 = 0, a_2 = -4, a_3 = -8, a_4 = -12, \ldots\ldots$

We have, $a_2 - a_1 = -4 - 0 = -4$

$a_3 - a_2 = -8 - (-4) = -8 + 4 = -4$

$a_4 - a_3 = -12 - (-8) = -12 + 8 = -4$

Since $a_{n+1} - a_n$ is same every time. So, the given numbers form an AP with common difference $d = -4$.

Now, next three terms are:

$a_5 = -12 + (-4) = -12 - 4 = -16; a_6 = -16 + (-4) = -16 - 4 = -20$

$a_7 = -20 + (-4) = -20 - 4 = -24$

(iii) Here, $a_1 = -\frac{1}{2}; a_2 = -\frac{1}{2}; a_3 = -\frac{1}{2}; a_4 = -\frac{1}{2}, \ldots$.

We have, $a_2 - a_1 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$

$a_3 - a_2 = -\frac{1}{2} - \left(\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$

$a_4 - a_3 = -\frac{1}{2} - \left(\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$

Since $a_{n+1} - a_n$ is same every time. So, the given numbers form an AP with common difference $d = 0$.

Now, next three terms are : $a_5 = -\frac{1}{2} + 0 = -\frac{1}{2}, a_6 = -\frac{1}{2} + 0 = -\frac{1}{2}, a_7 = -\frac{1}{2} + 0 = -\frac{1}{2}$.

184. The sum of first n terms of an AP is represented by $S_n = 6n - n^2$ Find the common difference.

Ans. : -2

185. If $S_n = 3n^2 - 4n$, then find the nth terms.

Ans. : $6n - 7$

186. If $S_n = \frac{1}{2}(3n^2 + 7n)$, find nth term and 20th term.

Ans. : $3n + 2; 62$

187. In an AP, the sum of first in terms is $\left(\frac{3n^2}{2} + \frac{5n}{2}\right)$. Find its 25^{th} term.

Ans. : (i) $a_{25} = 76$

188. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees that each section of each class will plant, will be the same as the class in which they are studying, e.g., a section of class I will plant I tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. How many trees will be planted by the students?

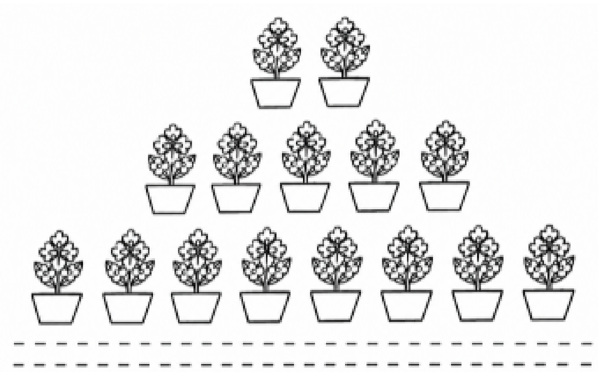
Ans. : 234

189. The sequence 2, 9, 16, is given.
(i) Identify if the given sequence is an AP.
(ii) Find the 20th term of the sequence.
(iii) Find the difference between the sum of its first 22 and 25 terms.
(iv) Is the term 102 belong to this sequence?
(v) If k is added to each of the above terms, will the new sequence be in AP?

Ans. : (i) yes
(ii) 135
(iii) 489
(iv) no
(v) yes

*** Case study based questions.** **[16]**

190.

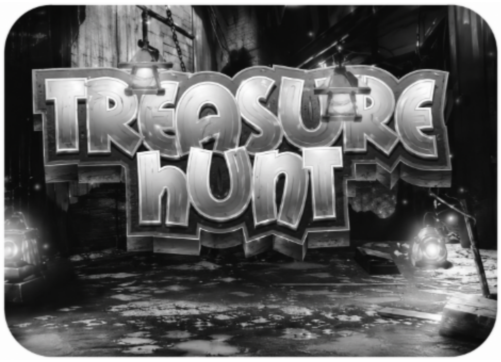


Roshni being a plant lover decides to start a nursery. She bought few plants with pots. She placed the pots in such a way that the number of pots in the first row is 2, in the second is 5, in the third row is 8 and so on. Based on the above

information, answer the following questions:
(i) How many pots are placed in the 7th row?
(ii) If Roshni wants to place 100 pots in total, then find the total number of rows formed in the arrangement.
(iii) How many pots are placed in the last row?
(iv) If Roshni has sufficient space for 12 rows, then how many total number of pots are placed by her with the same arrangement?

Ans. : (i) 20
(ii) 8
(iii) 23
(iv) 222

191.



Treasure Hunt is an exciting and adventurous game where participants follow a series of clues/numbers/ maps to discover hidden treasures. Players engage in a thrilling quest, solving puzzles and riddles to unveil the location of the coveted prize. While playing a treasure hunt game, some clues (numbers) are hidden in various spots collectively forming an A.P. If the number on the nth spot is $20+4n$, then answer the following questions to help the players in spotting the clues:
(i) Which number is on first spot?
(ii) Which spot is numbered as 112?
(iii) What is the sum of all the numbers on the first 10 spots?
(iv) Which number is on the $(n-2)^{th}$ spot?

Ans. : (i) 24
(ii) 23rd spot
(iii) 420
(iv) $(12+4n)$

192.

The marathon is a long-distance foot race with a distance of 42.195 km, usually run as a road race, but the distance can be covered on trail routes. The marathon can be completed by running or with a run/walk strategy. The marathon was one of the original modern Olympic events in 1896.

Neha, a student of class IX, wishes to participate in a marathon. She decided to begin her practice by gradually increasing her running distance. In the first week, she decided to run 3 km each day and increase the distance by 2 km each week, i.e., in the second week she would run 5 km each day, in the third week she would run 7 km each day and so on.

Based on the above information, answer the following questions:

(i) What distance will Neha cover each day of the 8^{th} week of her practice?

(ii) In which week would she be able to run for 45 km each day?

(iii) What is the total distance covered by Neha after 11 weeks, if she practiced for 5 days in each week?

(iv) Had she increased the distance by 3 km each week instead of 2 km each week, in how many weeks would she have trained herself to run for 42 km per day?

Ans. : (i) 17 km

(ii) 22nd week

(iii) 715 km

(iv) 14th week

193.

India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year.

(i) In which year, the production is 29,200 sets?

(ii) Find the production in the 8th year.

(iii) Find the production in first 3 years.

(iv) Find the difference of the production in 7th year and 4th year.

Ans. : (i) 12th year
(ii) 20400
(iii) 21600
(iv) 6600

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