STD 9 Maths [Manjari]
World of numbers

*** Choose the right answer from the given options. [1 Marks Each]** **[34]**

1. $0.6 + 0.\overline{7} + 0.4\overline{7}$ is equal to:

   (A) $\frac{155}{90}$ (B) $\frac{147}{90}$ (C) $\frac{167}{90}$ (D) none of these

   Ans. : $\frac{167}{90}$

2. When written in decimal form, which of the following will be a non-terminating and non-repeating number?

   (A) $1^{1/9}$ (B) $2^{1/9}$ (C) 2^{-9} (D) $9^{1/2}$

   Ans. : $2^{1/9}$

3. Which of these can definitely be expressed as a product of primes?

   (i) $\sqrt{n}$
   (ii) n
   (iii) $\frac{\sqrt{n}}{2}$

   (A) only (ii)

   (B) only (i) and (ii)

   (C) all (i), (ii) and (iii)

   (D) cannot be determined without knowing n

   Ans. : only (ii)

4. Every rational number is:

   (A) a natural number (B) an integer

   (C) a real number (D) a whole number

   Ans. : a real number

5. Between two rational numbers:

   (A) there is no rational number

   (B) there is exactly one rational number

   (C) there are infinitely many rational numbers

   (D) there are only rational numbers and no irrational numbers

   Ans. : there are infinitely many rational numbers

6. $2\sqrt{3} + \sqrt{3} - 2\sqrt{3}$ is equal to:

   (A) $2\sqrt{6}$ (B) $\sqrt{3}$ (C) $4\sqrt{6}$ (D) 5

   Ans. : $\sqrt{3}$

7. $\sqrt{10} \times \sqrt{15}$ is equal to:

(A) $6\sqrt{5}$ (B) $5\sqrt{6}$ (C) $\sqrt{25}$ (D) $10\sqrt{5}$

Ans. : $5\sqrt{6}$

8. The value of $\frac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}}$ is equal to:

(A) $\sqrt{2}$ (B) 2 (C) 4 (D) 8

Ans. : 2

9. The simplest form of $1.\overline{6}$ is:

(A) $\frac{5}{3}$ (B) $\frac{7}{4}$ (C) $\frac{11}{9}$ (D) $\frac{15}{13}$

Ans. : $\frac{5}{3}$

10. For rationalising the denominator of the expression $\frac{x}{\sqrt{12}}$, we multiply by:

(A) $\sqrt{2}$ (B) $\frac{\sqrt{3}}{\sqrt{3}}$ (C) $\sqrt{3}$ (D) $\frac{\sqrt{2}}{\sqrt{2}}$

Ans. : $\frac{\sqrt{3}}{\sqrt{3}}$

11. If $x = 0.\overline{473}$, then $1000x$ is equal to:

(A) $473.\overline{473}$ (B) $0.473\ldots$ (C) $0.473473473\overline{473}$ (D) $0.4737\overline{473}$

Ans. : $473.\overline{473}$

12. If p is prime number, then $\sqrt{p}$ is a/an:

(A) irrational number (B) whole number

(C) real number (D) natural number

Ans. : irrational number

13. If $x = \sqrt{5} + 2$, then $x - \frac{1}{x}$ equals:

(A) 4 (B) 3 (C) 2 (D) 1

Ans. : 4

14. If $\sqrt{13 - a\sqrt{10}} = 2\sqrt{2} + \sqrt{5}$, then $a^2 + a^{-1}$ is:

(A) 64 (B) $\frac{-63}{4}$ (C) $\frac{1}{4}$ (D) 0

Ans. : $\frac{-63}{4}$

15. $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5} =$

(A) $\sqrt{3}$ (B) $\sqrt{2}$ (C) $\sqrt{5}$ (D) $2\sqrt{5}$

Ans. : $\sqrt{5}$

16. $\frac{6-4\sqrt{2}}{6+4\sqrt{2}} =$

(A) $15 + 12\sqrt{2}$ (B) $17 - 12\sqrt{2}$ (C) $19 + 2\sqrt{2}$ (D) $18 + 3\sqrt{2}$

Ans. : $17-12\sqrt{2}$

17. $\frac{p}{q}$ form of $0.000\overline{2}$ is:

(A) $\frac{1}{4500}$ (B) $\frac{1}{3200}$ (C) $\frac{1}{3600}$ (D) $\frac{1}{9000}$

Ans. : $\frac{1}{3200}$

18. $0.6+0.\overline{7}+0.4\overline{7}$ is equal to :

(A) $\frac{155}{90}$ (B) $\frac{147}{90}$ (C) $\frac{167}{90}$ (D) none of these

Ans. : (C) $\frac{167}{90}$

19. $2\sqrt{3}+\sqrt{3}-2\sqrt{3}$ is equal to :

(A) $2\sqrt{6}$ (B) $\sqrt{3}$ (C) $4\sqrt{6}$ (D) 5

Ans. : (B) $\sqrt{3}$

20. Between two rational numbers :

(A) there is no rational number

(B) there is exactly one rational number

(C) there are infinitely many rational numbers

(D) there are only rational numbers and no irrational numbers

Ans. : (C) there are infinitely many rational numbers

21. Every rational number is :

(A) a natural number (B) an integer

(C) a real number (D) a whole number

Ans. : (C) a real number

22. For rationalising the denominator of the expression $\frac{x}{\sqrt{12}}$, we multiply by:

(A) $\sqrt{2}$ (B) $\frac{\sqrt{3}}{\sqrt{3}}$ (C) $\sqrt{3}$ (D) $\frac{\sqrt{2}}{\sqrt{2}}$

Ans. : (B) $\frac{\sqrt{3}}{\sqrt{3}}$

23. $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$ =

(A) $15+12\sqrt{2}$ (B) $17-12\sqrt{2}$ (C) $19+2\sqrt{2}$ (D) $18+3\sqrt{2}$

Ans. : (B) $17-12\sqrt{2}$

24. $\frac{p}{q}$ form of $0.000\overline{2}$ is:

(A) $\frac{1}{4500}$ (B) $\frac{1}{3200}$ (C) $\frac{1}{3600}$ (D) $\frac{1}{9000}$

Ans. : (B) $\frac{1}{3200}$

25. If p is prime number, then $\sqrt{p}$ is a/an:

(A) irrational number (B) whole number

(C) real number

(D) natural number

Ans.: (A) irrational number

26. If $\sqrt{13 - a\sqrt{10}} = 2\sqrt{2} + \sqrt{5}$, then $a^2 + a^{-1}$ is :

(A) 64

(B) $\frac{-63}{4}$

(C) $\frac{1}{4}$

(D) 0

Ans. : (B) $\frac{-63}{4}$

27. If $x = 0.\overline{473}$, then $1000x$ is equal to :

(A) $473.\overline{473}$

(B) $0.473...$

(C) $0.473473473\overline{473}$

(D) $0.4737\overline{473}$

Ans.: (A) $473.\overline{473}$

28. If $x = \sqrt{5} + 2$, then $x - \frac{1}{x}$ equals :

(A) 4

(B) 3

(C) 2

(D) 1

Ans.: (A) 4

29. $\sqrt{10} \times \sqrt{15}$ is equal to :

(A) $6\sqrt{5}$

(B) $5\sqrt{6}$

(C) $\sqrt{25}$

(D) $10\sqrt{5}$

Ans. : (B) $5\sqrt{6}$

30. $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$ =

(A) $\sqrt{3}$

(B) $\sqrt{2}$

(C) $\sqrt{5}$

(D) $2\sqrt{5}$

Ans. : (C) $\sqrt{5}$

31. $\sqrt{n}$ is a natural number such that $n > 1$.
Which of these can definitely be expressed as a product of primes?

(i) $\sqrt{n}$

(ii) n

(iii) $\frac{\sqrt{n}}{2}$

(A) only (ii)

(B) only (i) and (ii)

(C) all (i), (ii) and (iii)

(D) cannot be determined without knowing n

Ans.: (A) only (ii)

32. The simplest form of $1.\overline{6}$ is :

(A) $\frac{5}{3}$

(B) $\frac{7}{4}$

(C) $\frac{11}{9}$

(D) $\frac{15}{13}$

Ans.: (A) $\frac{5}{3}$

33. The value of $\frac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}}$ is equal to :

(A) $\sqrt{2}$

(B) 2

(C) 4

(D) 8

Ans. : (B) 2

34. When written in decimal form, which of the following will be a non-terminating and non-repeating number?

(A) $1^{1/9}$ (B) $2^{1/9}$ (C) 2^{-9} (D) $9^{1/2}$

Ans. : (B) $2^{1/9}$

*** A statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option.** **[12]**

Directions: In each question below, there are two statements labelled as Assertion (A) and Reason (R). Choose the correct answer as per the options given below:
(a) Both (A) and (R) are true and (R) is the correct explanation of (A)
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A)
(c) (A) is true but (R) is false
(d) (A) is false but (R) is true

35. Assertion (A) : Sum of $0.\overline{4}$ and $\sqrt{2}$ is an irrational number.
Reason (R) : Sum of an irrational number with a rational number is always an irrational number.

Ans. : Both (A) and (R) are true and (R) is the correct explanation of (A)

36. Assertion (A) : $\frac{4}{\sqrt{7}-\sqrt{3}}$ is equal to $\sqrt{7}+\sqrt{3}$.
Reason (R) : Rationalization of denominator.

Ans. : Both (A) and (R) are true and (R) is the correct explanation of (A)

37. Assertion (A) : Sum of 2 and $\sqrt{3}$ is an irrational number.
Reason (R) : Sum of any two real numbers is always irrational number.

Ans. : (A) is true but (R) is false

38. Assertion (A) : Three rational number between $\frac{2}{5}$ and $\frac{3}{5}$ are $\frac{9}{20}, \frac{10}{20}$ and $\frac{11}{20}$.
Reason (R) : A rational number between two rational numbers p and q is $\frac{p+q}{2}$.

Ans. : Both (A) and (R) are true and (R) is the correct explanation of (A)

39. Assertion (A) : $\sqrt{p}$ is an irrational number, where p is a prime number.
Reason (R) : Square root of any prime number is an irrational number.

Ans. : Both (A) and (R) are true and (R) is the correct explanation of (A)

40. Assertion (A) : $\sqrt{5}$ is an irrational number.
Reason (R) : If m is an odd number greater than 1 , then $\sqrt{m}$ is irrational.

Ans. : (A) is true but (R) is false

41. Directions : In each question below, there are two statements labelled as Assertion (A) and Reason (R). Choose the correct answer as per the options

given below:

Assertion (A) : $\frac{4}{\sqrt{7}-\sqrt{3}}$ is equal to $\sqrt{7}+\sqrt{3}$.

Reason (R) : Rationalization of denominator.

Ans. : A

42. Directions : In each question below, there are two statements labelled as Assertion (A) and Reason (R). Choose the correct answer as per the options given below:

Assertion (A): $\sqrt{5}$ is an irrational number.

Reason (R) : If m is an odd number greater than 1, then $\sqrt{m}$ is irrational.

Ans. : C

43. Directions : In each question below, there are two statements labelled as Assertion (A) and Reason (R).Choose the correct answer as per the options given below:

Assertion (A): $\sqrt{p}$ is an irrational number, where p is a prime number.

Reason (R) : Square root of any prime number is an irrational number.

Ans. : A

44. Directions : In each question below, there are two statements labelled as Assertion (A) and Reason (R).Choose the correct answer as per the options given below:

Assertion (A): Sum of $0.\bar{4}$ and $\sqrt{2}$ is an irrational number.

Reason (R) : Sum of an irrational number with a rational number is always an irrational number.

Ans. : A

45. Directions : In each question below, there are two statements labelled as Assertion (A) and Reason (R). Choose the correct answer as per the options given below:

Assertion (A) : Sum of 2 and $\sqrt{3}$ is an irrational number.

Reason (R) : Sum of any two real numbers is always irrational number.

Ans. : C

46. Directions : In each question below, there are two statements labelled as Assertion (A) and Reason (R). Choose the correct answer as per the options given below:

Assertion (A) : Three rational number between $\frac{2}{5}$ and $\frac{3}{5}$ are $\frac{9}{20}$, $\frac{10}{20}$ and $\frac{11}{20}$.

Reason (R) : A rational number between two rational numbers p and q is $\frac{p+q}{2}$.

Ans. : A

*** Answer the following short questions. [2 Marks Each]** **[80]**

47. The temperature in the high-altitude desert of Ladakh is recorded as $4^{\circ}C$ at noon. By midnight, it drops by $15^{\circ}C$. What is the midnight temperature?

Ans. : Initial temperature $= 4^{\circ}C$
Drop $= 15^{\circ}C$
Midnight temperature $= 4 - 15 = -11^{\circ}C$
Therefore, the midnight temperature is $-11^{\circ}C$.

48. Find three distinct rational numbers that lie strictly between -1/2 = 1/4.

Ans. : Given numbers: -1/2 and 1/4
LCM of 2 and 4 = 4
Converting -1/2 to common denominator:
-1/2 = -2/4
Now the given numbers: -2/4 and 1/4
So numbers between -2/4 and 1/4 are -1/4, 0, 1/8
Therefore, three rational numbers are -1/4, 0, 1/8.

49. Simplify the expression: (-1/4) + (5/12)

Ans. : LCM of 4 and 12 = 12
Converting -1/4 to common denominator:
-1/4 = -3/12
So, -3/12 + 5/12
= 2/12
= 1/6
Therefore, the result of (-1/4) + (5/12) is 1/6.

50. Rationalise the following :

$\frac{16}{\sqrt{41}-5}$

Ans. : $\frac{16}{\sqrt{41}-5} = \frac{16}{\sqrt{41}-5} \times \frac{\sqrt{41}+5}{\sqrt{41}+5} = \frac{16(\sqrt{41}+5)}{(\sqrt{41})^2-(5)^2} = \frac{16\sqrt{41}+80}{41-25} = \frac{16\sqrt{41}+80}{16} = \frac{16\sqrt{41}}{16} + \frac{80}{16} = \sqrt{41} + 5$

51. Rationalise the following :

$\frac{2+\sqrt{3}}{2-\sqrt{3}}$

Ans. : $\frac{2+\sqrt{3}}{2-\sqrt{3}} = \frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{(2+\sqrt{3})^2}{(2)^2-(\sqrt{3})^2} = \frac{2^2+(\sqrt{3})^2+2\times2\times\sqrt{3}}{4-3}$

$= \frac{4+3+4\sqrt{3}}{1} = 7 + 4\sqrt{3}$

52. Rationalise the following :

$\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$

Ans. : $\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}} = \frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \frac{\sqrt{6}(\sqrt{2}-\sqrt{3})}{(\sqrt{2})^2-(\sqrt{3})^2} = \frac{\sqrt{12}-\sqrt{18}}{2-3}$

$= \frac{\sqrt{2\times2\times3}-\sqrt{2\times3\times3}}{-1} = \frac{2\sqrt{3}-3\sqrt{2}}{-1} = -(2\sqrt{3} - 3\sqrt{2})$

$= 3\sqrt{2} - 2\sqrt{3}$

53. Classify the following number as rational or irrational with justification.

$\sqrt{196}$

Ans. : $\sqrt{196}=\sqrt{14\times 14}=14$, i.e., rational

[$\because 14$ can be expressed as $\frac{14}{1}=\frac{p}{q}$, where p,q are integers and $q\neq 0$]

54. Simplify:

$12\sqrt{18}+6\sqrt{20}-6\sqrt{147}+3\sqrt{50}$

Ans. : $51\sqrt{2}-42\sqrt{3}+12\sqrt{5}$

55. Locate $3-\sqrt{3}$ on the number line.

Ans. : 1.26

56. Locate $\sqrt{3}$ on the number line.

Ans. : $\sqrt{3}$

57. All irrational numbers are included in the category of rational numbers. State whether the given statement is true or false. Justify.

Ans. : False

Rational and irrational numbers are separate sets. An irrational number, by definition, is a real number that is not rational.

58. Write the following rational numbers as decimals:

$\frac{-6}{11}$

Ans. : $-0.5454\ldots(-0.\overline{54})$

59. Express the following in the form $\frac{p}{q}$, where p and q are integers and $q\neq 0$.

$0.2555...$

Ans. : $\frac{23}{90}$

60. Express the following in the form $\frac{p}{q}$, where p and q are integers and $q\neq 0$.

$0.404040...$

Ans. : $\frac{40}{99}$

61. Express the following as a fraction in simplest form.

$0.\overline{0001}$

Ans. : $\frac{1}{9999}$

62. Express the following in the form of $\frac{p}{q}$, where p and q are integers, $q\neq 0$.

$5.\overline{3}$

Ans. : $\frac{16}{3}$

63. Express $1.3\overline{2}+0.\overline{35}$ as a fraction in the simplest form.

Ans. : $\frac{553}{330}$

64. Write the following rational numbers in the decimal form:

$\frac{1}{3}$

Ans. : 0.333

65. Write the following rational numbers in the decimal form:

$\frac{17}{9}$

Ans. :

66. Find three rational numbers lying between $\frac{1}{3}$ and $\frac{1}{5}$

Ans. : $\frac{9}{30}, \frac{8}{30}, \frac{7}{30}$

67. Find a rational number between $\frac{1}{3}$ and $\frac{1}{4}$

Ans. : $\frac{7}{24}$

68. Find the additive inverse of :

-2

Ans. : 2

69. Find the additive inverse of :

$\frac{-4}{9}$

Ans. : $\frac{4}{9}$

70. Evaluate :

$$\left[\frac{35}{18} \times \left(\frac{-9}{5}\right)\right] \div \left[\frac{-49}{63} \times \left(\frac{-18}{21}\right)\right]$$

Ans. : $\frac{-21}{4}$

71. Evaluate :

$$\left(\frac{-7}{11} + \frac{6}{5}\right) \div \left(-4 + \frac{-2}{5}\right)$$

Ans. : $\frac{-31}{242}$

72. Evaluate :

$$\left(\frac{1}{3} + \frac{3}{4}\right) \times \left(\frac{6}{5} + \frac{2}{3}\right)$$

Ans. : $\frac{91}{45}$

73. Evaluate :

$$\left[\frac{22}{-9} \times \left(\frac{-6}{11}\right)\right] - \left[\frac{3}{22} \times \frac{8}{11}\right]$$

Ans. : $\frac{448}{363}$

74. Evaluate :

$$\left[\frac{6}{11} - \left(\frac{-2}{55}\right)\right] \times \left[\frac{7}{5} + \left(\frac{-3}{10}\right)\right]$$

Ans. : $\frac{16}{25}$

75. Simplify :

$\frac{27}{16} \times \left(\frac{-24}{9}\right)$

Ans. : $\frac{-9}{2}$

76. Evaluate :

$\frac{-5}{7} - \frac{15}{14} + \left(\frac{-9}{28}\right)$

Ans. : $\frac{-59}{28}$

77. Evaluate :

$\frac{-6}{13} + \frac{7}{13} + \frac{1}{26}$

Ans. : $\frac{1}{26}$

78. Subtract :

$\frac{-14}{5}$ from $\frac{-7}{20}$

Ans. : $\frac{49}{20}$

79. Subtract :

$\frac{3}{7}$ from $\frac{-5}{14}$

Ans. : $\frac{-11}{14}$

80. Subtract :

$\frac{-9}{16}$ from $\frac{-5}{24}$

Ans. : $\frac{17}{48}$

81. Represent the following on a separate number line :

$\frac{-3}{2}$

Ans. : -1 and -2

82. Represent the following on a separate number line :

$\frac{3}{8}$

Ans. : 0 and 1

83. Reduce to lowest terms (standard form) :

$\frac{25}{-35}$

Ans. : $\frac{5}{-7}$

84. Reduce to lowest terms (standard form) :

$\frac{-52}{-24}$

Ans. : $\frac{13}{6}$

85. Write three rational number equivalent to :

$\frac{-5}{8}$

Ans. : $\frac{-10}{16}, \frac{-15}{24}, \frac{-20}{32}$

86. Verify:

$\left|\frac{-7}{4}\right| + \left|\frac{-2}{3}\right| = \left|\frac{-7}{4} - \frac{2}{3}\right|$

Ans. : $\left|\frac{-7}{4}\right| = \frac{7}{4}$

$\left|\frac{-2}{3}\right| = \frac{2}{3}$

$\frac{7}{4} + \frac{2}{3} = \frac{7\times3}{12} + \frac{2\times4}{12} = \frac{21}{12} + \frac{8}{12} = \frac{29}{12}$

$\frac{-7}{4} - \frac{2}{3} = \frac{-21-8}{12} = \frac{-29}{12}$

$\left|\frac{-29}{12}\right| = \frac{29}{12}$

*** Answer the following questions. [3 Marks Each]** **[180]**

87. Ancient Indians used the joints of their fingers to count, a practice still seen today. Each finger has 3 joints, and the thumb is used to count them. How many can you count on one hand? How does this relate to the ancient base-12 counting systems?

Ans. : Each finger (except thumb) has 3 joints.
Number of fingers used = 4 (excluding thumb)
Total joints = 4 × 3 = 12
So, we can count up to 12 using one hand.
Relation to base-12 system:
Since counting reaches 12 on one hand, it naturally leads to a base-12 (duodecimal) counting system used in ancient times.
Therefore:
- Total count = 12
- This explains the origin of base-12 counting system.

88. A spice trader takes a loan (debt) of ₹850. The next day, he makes a profit (fortune) of ₹1,200. The following week, he incurs a loss of ₹450. Write this sequence as an equation using integers and calculate his final financial standing.

Ans. : Debt = − ₹850
Profit = + ₹1200
Loss = − ₹450
Equation: − ₹850 + ₹1200 − ₹450
Step-by-step calculation:
= ₹350 − ₹450
= − ₹100
Therefore, his final financial standing is −₹100 (a loss of ₹100).

89. Explain, using a real-world example of debt, why subtracting a negative number is the same as adding a positive number (e.g., 10 − (−5) = 15).

Ans. : Consider you have ₹10.
A negative number represents debt.
So, −₹5 means you owe ₹5.
Now, 10 − (−5) means removing a debt of ₹5.
If your debt is removed, your money increases by ₹5.
So, 10 − (−5) = 10 + 5 = 15
Thus, subtracting a negative number is the same as adding a positive number.

90. Find the product:
(i) 2/3 × 3/10
(ii) 7/11 × 5/8
(iii) -4/7 × 5/14

Ans. : (i) 2/3 × 3/10
= (2 × 3)/(3 × 10)
= 6/30
= 1/5 [After simplification]
Therefore, the product is 1/5.
(ii) 7/11 × 5/8
= (7 × 5)/(11 × 8)
= 35/88
Therefore, the product is 35/88.
(iii) -4/7 × 5/14
= (-4 × 5)/(7 × 14)
= -20/98
= -10/49 [After simplification]
Therefore, the product is -10/49.

91. Find the quotient:
(i) 2/3 ÷ 3/10
(ii) 7/11 ÷ 5/8
(iii) -4/7 ÷ 5/14

Ans. : (i) To divide fractions, multiply by the reciprocal.
2/3 ÷ 3/10
= 2/3 × 10/3
= (2 × 10)/(3 × 3)
= 20/9
Therefore, the quotient is 20/9.
(ii) 7/11 ÷ 5/8
= 7/11 × 8/5
= (7 × 8)/(11 × 5)

= 56/55

Therefore, the quotient is 56/55.

(iii) -4/7 ÷ 5/14

= -4/7 × 14/5

= (-4 × 14)/(7 × 5)

= -56/35

= -8/5 [After simplification]

Therefore, the quotient is -8/5.

92. Show that: (1/2 + 3/4) × 8/3 = 1/2 × 8/3 + 3/4 × 8/3

Ans. : LHS = (1/2 + 3/4) × 8/3

= (2/4 + 3/4) × 8/3 [First add inside the bracket: 1/2 = 2/4]

= (5/4) × 8/3 [Since 2/4 + 3/4 = 5/4]

= 5/4 × 8/3

= (5 × 8)/4 × 3)

= 40/12

= 10/3 [After simplification]

RHS = 1/2 × 8/3 + 3/4 × 8/3

= (1 × 8)/(2 × 3) + (3 × 8)/(4 × 3)

= 8/6 + 24/12

= 4/3 + 6/3 [After simplification: 8/6 = 4/3 and 24/12 = 6/3]

= 10/3

Since LHS = RHS,

Therefore, (1/2 + 3/4) × 8/3 = 1/2 × 8/3 + 3/4 × 8/3

Hence proved.

93. A tailor has $15\frac{3}{4}$ metres of fine silk. If making one kurta requires $2\frac{1}{4}$ metres of silk, exactly how many kurtas can he make?

Ans. : Converting improper fractions:$15\frac{3}{4}$ = 63/4

$2\frac{1}{4}$= 9/4

Total amount of silk cloth = $15\frac{3}{4}$ metres

In $2\frac{1}{4}$ metres of silk, number of kurta = 1

⇒ In 1 metres of silk, number of kurta = $1/2\frac{1}{4}$

⇒ In $15\frac{3}{4}$ metres of silk, number of kurta = $1/2\frac{1}{4}\times 15\frac{3}{4}$

= 1/(9/4) × (63/4)

= 4/9 × 63/4

= (4 × 63)/(9 × 4)

= 63/9 = 7

Therefore, he can make exactly 7 kurtas from $15\frac{3}{4}$ metres of fine silk.

94. Can you think of other way(s) to find a rational number between any two rational numbers?

Ans. : Yes, there are methods:

1. Taking average:

If a and b are two rational numbers, then
(a + b)/2 lies between them.

2. By making common denominator:

Convert both numbers to same denominator and choose a number in between.

3. By decimal expansion:

Convert into decimals and pick numbers in between.

95. Classify the following numbers as rational or irrational:

(i) $\sqrt{81}$

(ii) $\sqrt{12}$

(iii) 0.33333 ...

Ans. : (i) $\sqrt{81} = 9$

Since 9 can be written as 9/1, it is a rational number.

Therefore, $\sqrt{81}$ is rational.

(ii) $\sqrt{12} = 2\sqrt{3}$

Since $\sqrt{3}$ is irrational, $\sqrt{12}$ is irrational.

Therefore, $\sqrt{12}$ is irrational.

(iii) 0.33333... = 1/3

This is a repeating decimal, so it is rational.

Therefore, 0.33333... is rational.

Explicit fraction: 1/3

96. The number $0.\overline{9}$ (which means $0.99999\ldots$) is a rational number. Using algebra (let $x = 0.\overline{9}$, multiply by 10 , and subtract), explain why $0.\overline{9}$ is exactly equal to 1.

Ans. : Let x = 0.99999...

Multiply both sides by 10:

10x = 9.99999...

Now subtract:

10x – x = 9.99999... – 0.99999...

9x = 9

x = 1

But x = 0.99999...

Therefore, 0.99999... = 1.

97. We have seen that the repeating block of $\frac{1}{7}$ is a cyclic number. Try to find more numbers (n) whose reciprocals $\left(\frac{1}{n}\right)$ produce decimals with repeating blocks that are cyclic.

Ans. : Some examples are:

1/7 = 0.142857142857142857...

1/17 = 0.0588235294117647058823529411764705882352941176 47...
1/19 = 0.052631578947368421052631578947368421052631578947368421...
These reciprocals produce repeating blocks that show cyclic behaviour.
Therefore, examples of such numbers are 7, 17, 19, ...
These are numbers whose reciprocals can give cyclic repeating decimals.

98. Convert the following decimal numbers in the form of $\frac{p}{q}$.

(i) 12.6

(ii) 0.0120

(iii) $3.0\overline{52}$

Ans. : (i) 12.6 = 126/10 = 63/5
Therefore, 12.6 = 63/5
(ii) 0.0120 = 120/10000 = 3/250
Therefore, 0.0120 = 3/250
(iii) Let x = 3.052222...
Then, 100x = 305.2222...
1000x = 3052.2222...
Subtract: 1000x – 100x
= 3052.2222... – 305.2222...
⇒ 900x = 2747
⇒ x = 2747/900
Therefore, $3.0\overline{52} = 2747/900$

99. Find 6 rational numbers between 3 and 4.

Ans. : We can write 3 and 4 with a common denominator:
3 = 21/7
4 = 28/7
So, six rational numbers between 3 and 4 are:
22/7, 23/7, 24/7, 25/7, 26/7, 27/7
Therefore, the required six rational numbers are:
22/7, 23/7, 24/7, 25/7, 26/7, 27/7

100. Find 5 rational numbers between $\frac{2}{5}$ and $\frac{3}{5}$.

Ans. : Write both fractions with a larger common denominator:
2/5 = 20/50
3/5 = 30/50
So, five rational numbers between 20/50 and 30/50 are:
21/50, 22/50, 23/50, 24/50, 25/50
Therefore, the required five rational numbers are:
21/50, 22/50, 23/50, 24/50, 25/50

101. Find 5 rational numbers between $\frac{1}{6}$ and $\frac{2}{5}$.

Ans. : First take a common denominator:
1/6 = 5/30
2/5 = 12/30
Now the fractions between 5/30 and 12/30 are:
6/30, 7/30, 8/30, 9/30, 10/30
Therefore, five rational numbers between 1/6 and 2/5 are:
6/30, 7/30, 8/30, 9/30, 10/30
These may also be written in simplest form as:
1/5, 7/30, 4/15, 3/10, 1/3

102. If $\frac{x}{3}+\frac{x}{5}=\frac{16}{15}$, find the rational number x.

Ans. : Given: x/3 + x/5 = 16/15
Taking x common, we have:
x(1/3 + 1/5) = 16/15
Now, 1/3 + 1/5 = 5/15 + 3/15 = 8/15
So, x × 8/15 = 16/15
Therefore, x = (16/15) ÷ (8/15)
= (16/15) × (15/8)
= 16/8
= 2
Therefore, the rational number x is 2.

103. Without performing division, determine whether the decimal expansion of $\frac{18}{125}$ is terminating or non-terminating. If it terminates, state the number of decimal places.

Ans. : Given: 18/125

The denominator is $125=5^3$
Since the denominator has only the prime factor 5, the decimal expansion is terminating.
To find the number of decimal places, make the denominator a power of 10: 125 × 8 = 1000
So, 18/125 = 144/1000 = 0.144
Thus, it terminates with 3 decimal places.
Therefore, 18/125 is a terminating decimal with 3 decimal places.

104. A rational number in its lowest form has denominator $2^3\times5$. How many decimal places will its decimal expansion have? Explain your answer.

Ans. : Given denominator:

$2^3\times5=8\times5=40$

A rational number with denominator of the form $2^{m\times5^n}$ has a terminating decimal expansion. The number of decimal places is equal to the greater of m and n.
Here: m = 3, n = 1

So, number of decimal places = 3

Example: 1/40 = 0.025

Therefore, the decimal expansion will have 3 decimal places.

105. Prove that $3+2\sqrt{5}$ is irrational.

Ans. : Let us assume to the contrary that $3+2\sqrt{5}$ is rational. Then, there exists positive integers a and b such that $3+2\sqrt{5}=\frac{a}{b},a,b$ are co-prime, $b\neq 0$

$\Rightarrow 2\sqrt{5}=\frac{a}{b}-3=\frac{a-3b}{b}\Rightarrow \sqrt{5}=\frac{a-3b}{2b}$

$\Rightarrow \sqrt{5}$ is rational. [$\because a,b,2$ and 3 are integers. So, $\frac{a-3b}{2b}$ is rational].

This contradicts the fact that $\sqrt{5}$ is irrational. So our assumption is wrong.

Hence, $3+2\sqrt{5}$ is irrational. Proved.

106. Prove that $3\sqrt{2}$ is irrational.

Ans. : Let us assume to the contrary that $3\sqrt{2}$ is rational. Then, there exists positive integers a and b such that $3\sqrt{2}=\frac{a}{b},a$ and b are co-prime, $b\neq 0$.

$\Rightarrow \sqrt{2}=\frac{a}{3b}\Rightarrow \sqrt{2}$ is rational. [$\because a,b$ and 3 are integers, so, $\frac{a}{3b}$ is rational]

This contradicts the fact that $\sqrt{2}$ is irrational. So, our assumption is wrong.

Thus, $3\sqrt{2}$ is not rational. Hence, it is irrational. Proved.

107. Find the value of a in each of the following :

$\frac{3-\sqrt{5}}{3+2\sqrt{5}}=a\sqrt{5}-\frac{19}{11}$

Ans. : $\frac{3-\sqrt{5}}{3+2\sqrt{5}}=\frac{3-\sqrt{5}}{3+2\sqrt{5}}\times\frac{3-2\sqrt{5}}{3-2\sqrt{5}}=\frac{3(3-2\sqrt{5})-\sqrt{5}(3-2\sqrt{5})}{(3)^2-(2\sqrt{5})^2}$

$=\frac{9-6\sqrt{5}-3\sqrt{5}+10}{9-20}=\frac{19-9\sqrt{5}}{-11}=\frac{-(19-9\sqrt{5})}{11}=\frac{9\sqrt{5}-19}{11}=\frac{9}{11}\sqrt{5}-\frac{19}{11}$

Now, $a\sqrt{5}-\frac{19}{11}=\frac{9}{11}\sqrt{5}-\frac{19}{11}\Rightarrow a=\frac{9}{11}$

108. Find the value of a in each of the following :

$\frac{5+2\sqrt{3}}{7+4\sqrt{3}}=a-6\sqrt{3}$

Ans. : $\frac{5+2\sqrt{3}}{7+4\sqrt{3}}=\frac{5+2\sqrt{3}}{7+4\sqrt{3}}\times\frac{7-4\sqrt{3}}{7-4\sqrt{3}}=\frac{5(7-4\sqrt{3})+2\sqrt{3}(7-4\sqrt{3})}{(7)^2-(4\sqrt{3})^2}=\frac{35-20\sqrt{3}+14\sqrt{3}-24}{49-48}=\frac{11-6\sqrt{3}}{1}=11-6\sqrt{3}$

Now, $a-6\sqrt{3}=11-6\sqrt{3}\Rightarrow a=11$.

109. Evaluate : $\left\{\sqrt{5-2\sqrt{6}}\right\}+\left\{\sqrt{3+2\sqrt{2}}\right\}$

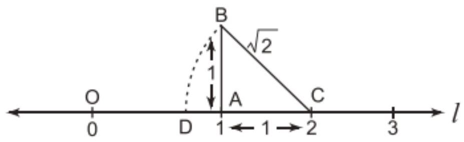
Ans. : $\sqrt{5-2\sqrt{6}}=\sqrt{(\sqrt{3})^2+(\sqrt{2})^2-2\times\sqrt{3}\times\sqrt{2}}=\sqrt{(\sqrt{3}-\sqrt{2})^2}=\sqrt{3}-\sqrt{2}$

Also, $\sqrt{3+2\sqrt{2}}=\sqrt{(\sqrt{2})^2+(\sqrt{1})^2+2\times\sqrt{2}\times\sqrt{1}}=\sqrt{(\sqrt{2}+\sqrt{1})^2}=\sqrt{2}$
$+\sqrt{1}$

Now, $\sqrt{5-2\sqrt{6}}+\sqrt{3+2\sqrt{2}}=\sqrt{3}-\sqrt{2}+\sqrt{2}+\sqrt{1}=\sqrt{3}+\sqrt{1}=\sqrt{3}+1$

110. Represent $2-\sqrt{2}$ on the number line.

Ans. :



On number line l, mark the points O , A and C such that $OA = AC = 1$ unit.

Draw $BA \perp l$ such that $BA = 1$ unit. Join BC .

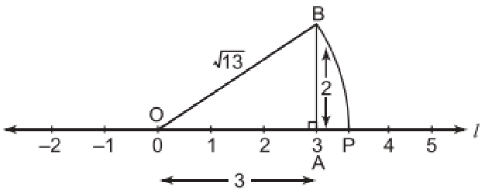
Here, $BC = \sqrt{1^2 + 1^2} = \sqrt{2}$ units

With C as centre and BC as radius draw an arc intersecting line l at D.

Here, point D represents $2 - \sqrt{2}$ on the number line.

111. Locate $\sqrt{13}$ on the number line.

Ans. :



Let $OA = 3$ units and let $BA \perp OA$ such that $AB = 2$ units. Join OB .

In $\triangle OAB, OB = \sqrt{OA^2 + AB^2}$

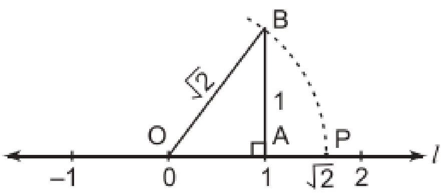
[Using Baudhāyana-Pythagoras theorem]

$\Rightarrow OB = \sqrt{3^2 + 2^2} = \sqrt{9 + 4} = \sqrt{13}$

With O as centre and OB as radius, draw an arc meeting the real line l at P , Then $OB = OP = \sqrt{13}$ units

112. Locate $\sqrt{2}$ on the number line.

Ans. :



Let l be a real number line and O be a point representing 0 (zero).

Take $OA = 1$ unit. Draw $AB \perp OA$ such that $AB = 1$ unit. Join OB.

Clearly, $OB = \sqrt{OA^2 + AB^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$ [Using Baudhayana-Pythagoras theorem]

With O as centre and OB as radius, draw an arc, meeting line l at P .

Then, $OP = OB = \sqrt{2}$ units

Thus, the point P represents $\sqrt{2}$ on the real number line.

113. If $x = 1 - \sqrt{2}$, find $x^2 + \frac{1}{x^2}$.

Ans. : 6

114. If $x=\frac{\sqrt{2}-1}{\sqrt{2}+1}$, $y=\frac{\sqrt{2}+1}{\sqrt{2}-1}$, then find x^2+xy+y^2.

Ans. : 35

115. Rationalise the denominator :

$\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$

Ans. : $\frac{4\sqrt{30}+9}{21}$

116. Rationalise the denominator :

$\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$

Ans. : 11

117. Rationalise the denominator :

$\frac{\sqrt{3}+1}{\sqrt{3}-1}+\frac{\sqrt{3}-1}{\sqrt{3}+1}$

Ans. : 4

118. Rationalise the denominator :

$\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}+\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

Ans. : 8

119. Rationalise the denominator of the following.

$\frac{3+\sqrt{2}}{4-\sqrt{2}}$

Ans. : $\frac{2+\sqrt{2}}{2}$

120. Rationalise the denominator of the following.

$\frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}}$

Ans. : $2\sqrt{3}+\sqrt{6}$

121. Rationalise the denominator of the following.

$\frac{4+\sqrt{5}}{4-\sqrt{5}}$

Ans. : $\frac{21+8\sqrt{5}}{11}$

122. Rationalise the denominator of the following.

$\frac{\sqrt{7}-1}{\sqrt{7}+1}$

Ans. : $\frac{4-\sqrt{7}}{3}$

123. Rationalise the denominator of the following.

$\frac{1}{\sqrt{3}+\sqrt{2}}$

Ans. : $\sqrt{3}-\sqrt{2}$

124. Rationalise the denominator of the following.

$\frac{\sqrt{2}}{2+\sqrt{2}}$

Ans. : $\sqrt{2}-1$

125. Find the values of a and b in the following :

$\frac{2\sqrt{6}-\sqrt{5}}{\sqrt{45}-\sqrt{24}}=a+b\sqrt{30}$

Ans. : $a=\frac{3}{7}, b=\frac{4}{21}$

126. Find the values of a and b in the following :

$\frac{3+\sqrt{7}}{3-\sqrt{7}}=a+b\sqrt{7}$

Ans. : $a=8; b=3$

127. Find the values of a and b in the following :

$\frac{5+\sqrt{3}}{7-4\sqrt{3}}=a+\sqrt{3}b$

Ans. : $a=47, b=27$

128. Find the values of a and b in the following :

$\frac{5+\sqrt{6}}{5-\sqrt{6}}=a+b\sqrt{6}$

Ans. : $a=\frac{31}{19}, b=\frac{10}{19}$

129. Evaluate : $\left\{\sqrt{5+2\sqrt{6}}\right\}+\left\{\sqrt{8-2\sqrt{15}}\right\}$

Ans. : $\sqrt{2}+\sqrt{5}$

130. Show that $7.4\overline{78}$ can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q\neq 0$.

Ans. : The repeating decimal $7.4\overline{78}$ can be expressed in the form $\frac{p}{q}$ as $\frac{1234}{165}$, where $p=1234$ and $q=165$ are integers and $q\neq 0$.

131. Insert eight rational numbers between x and $|x|$, where $x=\frac{-5}{11}$

Ans. : $\frac{1}{11}, \frac{2}{11}, \frac{3}{11}, \frac{4}{11}, \frac{-1}{11}, \frac{-2}{11}, \frac{-3}{11}, \frac{-4}{11}$,

132. Find five rational numbers between :

$\frac{1}{4}$ and $\frac{7}{5}$

Ans. : $\frac{6}{20}, \frac{7}{20}, \frac{8}{20}, \frac{9}{20}, \frac{10}{20}$

133. Find five rational numbers between :

$\frac{-2}{7}$ and $\frac{1}{11}$

Ans. : $\frac{-21}{77}, \frac{-20}{77}, \frac{-19}{77}, \frac{-18}{77}, \frac{-17}{77}$

134. Find five rational numbers between :

-$\frac{3}{2}$ and $\frac{5}{3}$

Ans. : $\frac{-8}{6}, \frac{-7}{6}, \frac{-5}{6}, \frac{-4}{6}, \frac{-3}{6}$

135. Find ten rational numbers between -$\frac{5}{6}$ and $\frac{3}{8}$

Ans. : $\frac{-19}{24}, \frac{-18}{24}, \frac{-17}{24}, \frac{-16}{24}, \frac{-15}{24}, \frac{-14}{24}, \frac{-13}{24}, \frac{-12}{24}, \frac{-11}{24}, \frac{-10}{24}$

136. Insert three rational numbers lying between $-\frac{1}{2}$ and $\frac{-3}{4}$

Ans. : $\frac{-9}{16}, \frac{-5}{8}$ or $\frac{-10}{16}, \frac{-11}{16}$

137. Find four rational numbers between -6 and $\frac{4}{5}$

Ans. : $\frac{-13}{5}, \frac{-43}{10}, \frac{-9}{10}, \frac{-1}{20}$

138. Verify that $(x+y)+z = x+(y+z)$ by taking :

$x = \frac{2}{3}, y = \frac{3}{4}, z = \frac{4}{5}$

Ans. : $\frac{2}{3} + \frac{3}{4} = \frac{8+9}{12} = \frac{17}{12}$

$\frac{17}{12} + \frac{4}{5} = \frac{85+48}{60} = \frac{133}{60}$

$\frac{3}{4} + \frac{4}{5} = \frac{15+16}{20} = \frac{31}{20}$

$\frac{2}{3} + \frac{31}{20} = \frac{40+93}{60} = \frac{133}{60}$

139. Verify that $(x+y)+z = x+(y+z)$ by taking :

$x = \frac{-2}{5}, y = \frac{6}{11}, z = \frac{-3}{2}$

Ans. : $\frac{-2}{5} + \frac{6}{11} = \frac{-22+30}{55} = \frac{8}{55}$

$\frac{8}{55} + \left(\frac{-3}{2}\right) = \frac{16-165}{110} = \frac{-149}{110}$

$\frac{6}{11} + \left(\frac{-3}{2}\right) = \frac{12-33}{22} = \frac{-21}{22}$

$\frac{-2}{5} + \left(\frac{-21}{22}\right) = \frac{-44-105}{110} = \frac{-149}{110}$

140. Verify that $x+y = y+x$ by taking :

$x = \frac{11}{13}, y = \frac{12}{26}$

Ans. : $\frac{11}{13} + \frac{6}{13} = \frac{11+6}{13} = \frac{17}{13}$

$\frac{6}{13} + \frac{11}{13} = \frac{6+11}{13} = \frac{17}{13}$

141. Verify that $x+y = y+x$ by taking :

$x = \frac{2}{3}, y = \frac{5}{6}$

Ans. : $\frac{2}{3} + \frac{5}{6}$

$\frac{2\times 2}{6} + \frac{5}{6} = \frac{4+5}{6} = \frac{9}{6} = \frac{3}{2}$

$\frac{5}{6} + \frac{2}{3}$

$\frac{5}{6} + \frac{2\times 2}{6} = \frac{5+4}{6} = \frac{9}{6} = \frac{3}{2}$

142. Verify that $x+y = y+x$ by taking :

$x = \frac{-6}{5}, y = \frac{3}{7}$

Ans. : $\frac{-6}{5}+\frac{3}{7}$

$\frac{-6\times7}{35}+\frac{3\times5}{35}=\frac{-42+15}{35}=\frac{-27}{35}$

$\frac{3}{7}+\frac{-6}{5}$

$\frac{3\times5}{35}+\frac{-6\times7}{35}=\frac{15-42}{35}=\frac{-27}{35}$

143. A chef mixes $\frac{2}{3}kg$ flour with $\frac{1}{6}kg$ more. Is total still rational?

Ans. : Yes

144. A delivery van covers $\frac{7}{9}km$ of its route in morning. No extra distance is added due to diversion (0 km). What is total distance covered?

Ans. : $\frac{7}{9}km$

145. A student gains $\frac{9}{10}$ marks in bonus but loses the same due to penalty. Find net change.

Ans. : No change

146. One notebook costs $\frac{5}{4}$ rupees (discounted rate). A student buys $\frac{3}{2}$ dozens for class A and $\frac{1}{2}$ dozen for class B. Use distributive property to find total cost.

Ans. : ₹30

*** Questions with calculation. [4 Marks Each]** **[76]**

147. Calculate the following using Brahmagupta's laws:
(i) (−12) × 5
(ii) (−8) × (−7)
(iii) 0 − (−14)
(iv) (−20) ÷ 4

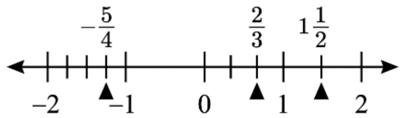
Ans. : (i) Negative × Positive = Negative [As Debt × Fortune = Debt]
Therefore, (−12) × 5 = −60
(ii) Negative × Negative = Positive [As Debt × Debt = Fortune]
Therefore, (−8) × (−7) = 56
(iii) As per Brahmagupta, zero minus debt is a fortune.
Subtracting a negative is same as adding:
Therefore, 0 − (−14) = 0 + 14 = 14
(iv) Negative ÷ Positive = Negative [As Debt ÷ Fortune = Debt]
Therefore, (−20) ÷ 4 = −5.

148. Represent the rational numbers $\frac{2}{3},-\frac{5}{4}$ and $1\frac{1}{2}$ on a single number line.

Ans. : Convert mixed number:

$1\frac{1}{2}$= 3/2
Now to compare the number first convert into decimal:
• -5/4 = -1.25

• 2/3 ≈ 0.67
• 3/2 = 1.5
On the number line:
• -5/4 lies between -2 and -1
• 2/3 lies between 0 and 1
• 3/2 lies between 1 and 2
So, order is:
-5/4 < 2/3 < 3/2
So, we can easily Mark these points accordingly on the number line.



149. Convert the following decimal numbers in the form of $\frac{p}{q}$.

(i) $1.2\overline{35}$

(ii) $0.\overline{23}$

(iii) $2.0\overline{5}$

Ans. : (i) Let x = 1.235235235...
Then, 1000x = 1235.235235...
Subtract: 1000x – x = 1235.235235... – 1.235235...
⇒ 999x = 1234
⇒ x = 1234/999
Therefore, $1.2\overline{35} = 1234/999$
(ii)Let x = 0.232323...
Then, 100x = 23.232323...
Subtract: 100x – x = 23.232323... – 0.232323...
⇒ 99x = 23
So, x = 23/99
Therefore, $0.\overline{23} = 23/99$
(iii) Let x = 2.05555...
Then, 10x = 20.5555...
100x = 205.5555...
Subtract: 100x – 10x = 205.5555... – 20.5555...
⇒ 90x = 185
So, x = 185/90 = 37/18
Therefore, $2.05 = 37/18$

150. Convert the following decimal numbers in the form of $\frac{p}{q}$.

(i) $2.12\overline{5}$

(ii) $3.12\overline{5}$

(iii) $2.\overline{1625}$

Ans. : (i) Let x = 2.125555...

Then, 100x = 212.5555...

1000x = 2125.5555...

Subtract: 1000x – 100x = 2125.5555... – 212.5555...

⇒ 900x = 1913

So, x = 1913/900

Therefore, $2.125 = 1913/900$

(ii) Let x = 3.125125125...

Then, 1000x = 3125.125125...

Subtract: 1000x – x = 3125.125125... – 3.125125...

⇒ 999x = 3122

So, x = 3122/999

Therefore, $3.12\overline{5} = 3122/999$

(iii) Let x = 2.162516251625...

Then, 10000x = 21625.16251625...

Subtract: 10000x – x = 21625.16251625... – 2.16251625...

⇒ 9999x = 21623

So, x = 21623/9999

Therefore, $2.\overline{1625} = 21623/9999$

151. Locate the following rational numbers on the number line.

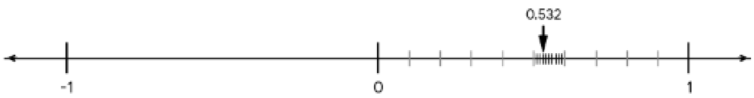
(i) 0.532

(ii) $1.1\overline{5}$

Ans. : (i) 0.532 lies between 0 and 1.

More precisely: 0.532 = 532/1000

So, we have to mark a point between 0.53 and 0.54, slightly after 0.53



(ii) $1.1\overline{5} = 1.15555\ldots$

It lies between 1 and 2.

More precisely: 1.15 < 1.15555... < 1.16

So, we have to mark a point between 1.15 and 1.16, slightly after 1.15.

152. A rational number has a terminating decimal expansion whose last non-zero digit occurs in the 4th decimal place. Show that such a number can be written in the form $\frac{p}{10^4}$, where p is an integer not divisible by 10. Is it necessary that the denominator of this rational number, when written in the lowest form, is divisible by 2^4 or 5^4 ? Give reasons.

Ans. : If the last non-zero digit occurs in the 4th decimal place, then the number has exactly 4 decimal places.

So it can be written as: $p/10^4$, where p is an integer.
Also, since the last non-zero digit is in the 4th decimal place, p is not divisible by 10.
Otherwise, the decimal would end earlier.

Hence, the number can be written in the form: $p/10^4$, where p is an integer not divisible by 10 .

Now, $10^4 = 2^4 \times 5^4$
When the fraction is reduced to lowest form, some common factors between p and 10^4 may cancel.
Therefore, it is NOT necessary that the denominator in lowest form is divisible by 2^4 or 5^4.
Example: 0.1250 = 1250/10000 = 1/8

Here, in lowest form the denominator is $8 = 2^3$, which is not divisible by 2^4 or 5^4.
Therefore, it is not necessary.

153. Three rational numbers x, y, z satisfy $x + y + z = 0$ and $xy + yz + zx = 0$. Show that all the rational numbers x, y, z must be simultaneously zero.

Ans. : Given:
x + y + z = 0 ...(1)
xy + yz + zx = 0 ...(2)
We use the identity:
$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$
From (1), $(x + y + z)^2 = 0^2 = 0$
From (2), xy + yz + zx = 0
So, $0 = x^2 + y^2 + z^2 + 2(0)$
Therefore, $x^2 + y^2 + z^2 = 0$
Now, x^2, y^2 and z^2 are all non-negative rational numbers.
The sum of three non-negative numbers is 0 only when each one is 0.
Hence, $x^2 = 0, y^2 = 0, z^2 = 0$
Therefore, x = 0, y = 0, z = 0
So, all the rational numbers x, y and z are simultaneously zero.

154. Prove that $\sqrt{3} + \frac{5}{\sqrt{3}}$ is irrational.

Ans. : Let us assume that $\sqrt{3}+\frac{5}{\sqrt{3}}$ is rational.

Then, $\sqrt{3}+\frac{5}{\sqrt{3}}=\frac{a}{b}$ $[a, b$ are co-prime and $b \neq 0]$

$\Rightarrow \frac{3+5}{\sqrt{3}}=\frac{a}{b} \Rightarrow \frac{8}{\sqrt{3}}=\frac{a}{b} \Rightarrow \frac{1}{\sqrt{3}}=\frac{a}{8b} \Rightarrow \frac{1}{\sqrt{3}}$ is rational

[$\because a, b$ and 8 are integers. So, $\frac{a}{8b}$ is rational]

⇒ This contradicts the fact $\frac{1}{\sqrt{3}}$ is irrational. So, our assumption is wrong.

Thus, $\sqrt{3}+\frac{5}{\sqrt{3}}$ is irrational. Proved.

155. If $x=\frac{3-\sqrt{2}}{3+\sqrt{2}}$ and $y=\frac{3+\sqrt{2}}{3-\sqrt{2}}$ find the value of x^2+xy+y^2.

Ans. : We have, $x^2+y^2+xy=(x+y)^2-2xy+xy=(x+y)^2-xy$

Now, $x+y=\frac{3-\sqrt{2}}{3+\sqrt{2}}+\frac{3+\sqrt{2}}{3-\sqrt{2}}=\frac{(3-\sqrt{2})^2+(3+\sqrt{2})^2}{3^2-(\sqrt{2})^2}$

$\frac{3^2+(\sqrt{2})^2-2\times3\times\sqrt{2}+3^2+(\sqrt{2})^2+2\times3\times\sqrt{2}}{9-2}$

$=\frac{9+2-6\sqrt{2}+9+2+6\sqrt{2}}{7}=\frac{22}{7}$

And, $xy=\frac{3-\sqrt{2}}{3+\sqrt{2}}\times\frac{3+\sqrt{2}}{3-\sqrt{2}}=1$

Now, $x^2+xy+y^2=(x+y)^2-xy$ $\left[\because x^2+y^2=(x+y)^2-2xy\right]$

$=\left(\frac{22}{7}\right)^2-1=\frac{484}{49}-1=\frac{484-49}{49}=\frac{435}{49}$

156. If $x=7-4\sqrt{3}$, then find the value of $\sqrt{x}+\frac{1}{\sqrt{x}}$.

Ans. : We have, $x=7-4\sqrt{3}$. Then, $\frac{1}{x}=\frac{1}{7-4\sqrt{3}}$

$\Rightarrow \frac{1}{x}=\frac{1}{7-4\sqrt{3}}\times\frac{7+4\sqrt{3}}{7+4\sqrt{3}}=\frac{7+4\sqrt{3}}{(7)^2-(4\sqrt{3})^2}=\frac{7+4\sqrt{3}}{49-48}=7+4\sqrt{3}$

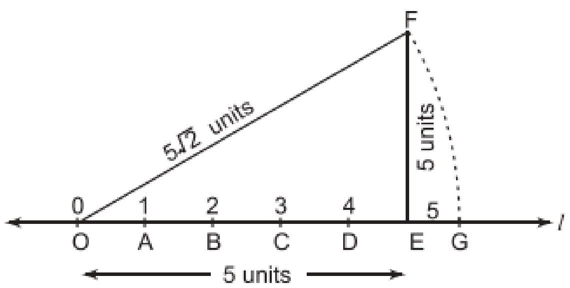
Now, $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2=(\sqrt{x})^2+\left(\frac{1}{\sqrt{x}}\right)^2+2\times\sqrt{x}\times\frac{1}{\sqrt{x}}$

$\Rightarrow \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2=x+\frac{1}{x}+2 \Rightarrow \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2=7-4\sqrt{3}+7+4\sqrt{3}+2$

$\Rightarrow \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2=16 \Rightarrow \sqrt{x}+\frac{1}{\sqrt{x}}=\pm4$

157. Represent $5\sqrt{2}$ on the number line.

Ans. :



On the number line mark O , A , B , C , D and E such that OA = AB = BC = CD = DE =1 unit.

At E draw $FE \perp l$ such that $FE = 5$ units.
Join OF. Here,

$OF = \sqrt{OE^2 + EF^2} = \sqrt{5^2 + 5^2}$

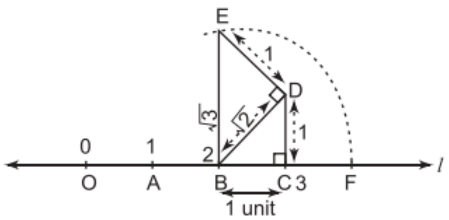
$= \sqrt{25+25} = \sqrt{50} = \sqrt{2 \times 25} = 5\sqrt{2}$ units.

With O as centre and OF as radius draw an arc intersecting line l at G.

Here, point G represents $5\sqrt{2}$ on the number line.

158. Represent $2+\sqrt{3}$ on number line.

Ans. :



On the number line l, mark the points O, A, B and C such that $OA = AB = BC = 1$ unit.
Draw $CD \perp l$ such that $CD = 1$ unit. Join BD.

In $\triangle BCD$, $\quad BD = \sqrt{1^2 + 1^2} = \sqrt{2}$ units
At D , draw $ED \perp BD$, such that $DE = 1$ unit.
Join BE.

In $\triangle BED, BE = \sqrt{(\sqrt{2})^2 + 1^2} = \sqrt{3}$ units

Taking B as centre and BE as radius, draw an arc intersecting line l at F .

$\therefore BE = BF = \sqrt{3}$ units

Here, $OB = 2$ units, $BF = \sqrt{3}$ units

$\therefore OF = OB + BF = (2+\sqrt{3})$ units

159. If $a = 2+\sqrt{3}+\sqrt{5}$ and $b = 3+\sqrt{3}-\sqrt{5}$, find $(a-2)^2 + (b-3)^2$.

Ans. : 16

160. If $x = \frac{-2}{3}$, $y = \frac{1}{4}$, $z = \frac{-5}{8}$ verify that :
$x(y+z) = xy + xz$

Ans. : $\frac{-2}{3} \times \left(\frac{1}{4} + \frac{-5}{8}\right) = \frac{-2}{3} \times \left(\frac{2-5}{8}\right) = \frac{-2}{3} \times \frac{-3}{8} = \frac{6}{24} = \frac{1}{4}$

$\left(\frac{-2}{3} \times \frac{1}{4}\right) + \left(\frac{-2}{3} \times \frac{-5}{8}\right) = \frac{-2}{12} + \frac{10}{24} = \frac{-4+10}{24} = \frac{6}{24} = \frac{1}{4}$

161. Verify the following : $\frac{3}{4} \times \left(\frac{2}{7} \times \frac{1}{5}\right) = \left(\frac{3}{4} \times \frac{2}{7}\right) \times \frac{1}{5}$

Ans. : $\frac{3}{4} \times \left(\frac{2 \times 1}{7 \times 5}\right) = \frac{3}{4} \times \frac{2}{35} = \frac{6}{140} = \frac{3}{70}$
$\left(\frac{3 \times 2}{4 \times 7}\right) \times \frac{1}{5} = \frac{6}{28} \times \frac{1}{5} = \frac{6}{140} = \frac{3}{70}$

162. Verify that : $\left(\frac{-9}{20}\right) \times \left(\frac{56}{-36}\right) = \left(\frac{56}{-36}\right) \times \left(\frac{-9}{20}\right)$

Ans. : $\frac{-9}{20} \times \frac{56}{-36}$

$\frac{56}{-36} \times \frac{-9}{20} = \frac{56 \times -9}{-36 \times 20} = \frac{-504}{-720} = \frac{7}{10}$

163. A shopkeeper sold rice in three transactions: Morning: $\frac{1}{4}$ quintal, Afternoon: $\frac{2}{5}$ quintal, Evening: $\frac{3}{20}$ quintal. Check whether grouping changes total sale.

Ans. : $\left(\frac{1}{4} + \frac{2}{5}\right) + \frac{3}{20} = \left(\frac{5+8}{20}\right) + \frac{3}{20} = \frac{13}{20} + \frac{3}{20} = \frac{16}{20} = \frac{4}{5}$ quintal

$\frac{1}{4} + \left(\frac{2}{5} + \frac{3}{20}\right) = \frac{1}{4} + \left(\frac{8+3}{20}\right) = \frac{1}{4} + \frac{11}{20} = \frac{5+11}{20} = \frac{16}{20} = \frac{4}{5}$ quintal

164. A student runs $\frac{3}{5} km$ in one round of the park. If she runs $\frac{4}{3}$ rounds in warm-up stretching equivalent distance, show that order of multiplication does not affect total distance.

Ans. : We need to show that $\frac{3}{5} \times \frac{4}{3} = \frac{4}{3} \times \frac{3}{5}$.

$\frac{3}{5} \times \frac{4}{3} = \frac{3 \times 4}{5 \times 3} = \frac{12}{15} = \frac{4}{5} km$

$\frac{4}{3} \times \frac{3}{5} = \frac{4 \times 3}{3 \times 5} = \frac{12}{15} = \frac{4}{5} km$

165. A water tank is filled by two pipes. Pipe A fills $\frac{3}{8}$ of the tank and Pipe B fills $\frac{5}{8}$. Show that total water filled is same whether Pipe A works first or Pipe B.

Ans. : We need to show that the total water filled is the same regardless of the order: $\frac{3}{8} + \frac{5}{8} = \frac{5}{8} + \frac{3}{8}$.

(A then B): $\frac{3}{8} + \frac{5}{8} = \frac{3+5}{8} = \frac{8}{8} = 1$

(B then A): $\frac{5}{8} + \frac{3}{8} = \frac{5+3}{8} = \frac{8}{8} = 1$

*** Answer the following questions. [5 Marks Each]** **[135]**

166. Prove that the following rational numbers are equal:
(i) 2/3 and 4/6
(ii) 5/4 and 10/8
(iii) -3/5 and -6/10
(iv) 9/3 and 3

Ans. : (i) To prove that two rational numbers are equal, we simplify them or compare their cross-products.
First Fraction: 2/3 = 2/3
Second Fraction: 4/6 = 2/3 (dividing numerator and denominator by 2)
Therefore, 2/3 and 4/6 are equal.
(ii) First Fraction: 5/4 = 5/4
Second Fraction: 10/8 = 5/4 (dividing numerator and denominator by 2)
Therefore, 5/4 and 10/8 are equal.
(iii) First Fraction: -3/5 = -3/5
Second Fraction: -6/10 = -3/5 (dividing numerator and denominator by 2)
Therefore, -3/5 and -6/10 are equal.

(iv) First Fraction: 9/3 = 3
Hence, 9/3 and 3 are equal.

167. Find the sum:
(i) 2/5 + 3/10
(ii) 7/12 + 5/8
(iii) – 4/7 + 3/14

Ans. : (i) LCM of 5 and 10 = 10
Simplifying the first number: 2/5 = 4/10 [Making the same denominator]
Now, the sum
= 2/5 + 3/10
= 4/10 + 3/10
= 7/10
Therefore, the sum of 2/5 + 3/10 is 7/10.
(ii) LCM of 12 and 8 = 24
Simplifying the first number: 7/12 = 14/24 [Making the same denominator]
Simplifying the second number: 5/8 = 15/24 [Making the same denominator]
Now, the sum
= 7/12 + 5/8
= 14/24 + 15/24
= 29/24
Therefore, the sum of 7/12 + 5/8 is 29/24.
(iii) LCM of 7 and 14 = 14
Simplifying the first number: -4/7 = -8/14 [Making the same denominator]
So, the sum
= – 4/7 + 3/14
= – 8/14 + 3/14
= – 5/14
Therefore, the sum of – 4/7 + 3/14 is -5/14.

168. Find the difference:
(i) 5/6 – 1/4
(ii) 11/8 – 3/4
(iii) -7/9 – (-2/3)

Ans. : (i) LCM of 6 and 4 = 12
Simplifying the first number: 5/6 = 10/12 [Making the same denominator]
Simplifying the Second number: 1/4 = 3/12 [Making the same denominator]
So, the difference
= 5/6 – 1/4
= 10/12 – 3/12
= 7/12
Therefore, the difference is 7/12.

(ii) LCM of 8 and 4 = 8
Simplifying the Second number: 3/4 = 6/8 [Making the same denominator]
So, the difference
= 11/8 – 3/4
= 11/8 – 6/8
= 5/8
Therefore, the difference is 5/8.
(iii) -7/9 – (-2/3) = -7/9 + 2/3
LCM of 9 and 3 = 9
Simplifying the Second number: 2/3 = 6/9 [Making the same denominator]
So, the difference
= – 7/9 – (-2/3)
= – 7/9 + 6/9
= – 1/9
Therefore, the difference is -1/9.

169. Without performing long division, determine which of the following rational numbers will have terminating decimals and which will be repeating: $\frac{7}{20}, \frac{4}{15}$ and $\frac{13}{250}$. Then check your answers by explicitly performing the long divisions and expressing these rational numbers as decimals.

Ans. : A rational number p/q in lowest form has a terminating decimal:
- If the prime factors of q are only 2 and/or 5.
- If q has any prime factor other than 2 or 5, then the decimal is repeating.

(i) 7/20
Prime factorisation of 20:
20 = $2^2 \times 5$
Since the denominator has only 2 and 5 as prime factors, 7/20 has a terminating decimal.
Now convert into decimal:
7/20 = 0.35
Therefore, 7/20 is a terminating decimal.
(ii) 4/15
Prime factorisation of 15:
15 = 3 × 5
Since the denominator contains 3, which is neither 2 nor 5, 4/15 has a repeating decimal.
Now convert into decimal:
4/15 = 0.2666...
Therefore, 4/15 is a repeating decimal.
(iii) 13/250
Prime factorisation of 250:

250 = 2×5^3

Since the denominator has only 2 and 5 as prime factors, 13/250 has a terminating decimal.

Now convert into decimal:

13/250 = 0.052

Therefore, 13/250 is a terminating decimal.

170. Perform the long division for $\frac{1}{13}$. Identify the repeating block of digits. Does it show cyclic properties if you evaluate $\frac{2}{13}$? Now compute $\frac{3}{13}, \frac{4}{13}$, etc. What do you notice?

Ans. : Let us first write the decimal expansion of 1/13:

1/13 = 0.076923076923...

So, the repeating block of digits is 076923.

Now evaluate:

2/13 = 0.153846153846...

Repeating block: 153846

3/13 = 0.230769230769...

Repeating block: 230769

4/13 = 0.307692307692...

Repeating block: 307692

5/13 = 0.384615384615...

Repeating block: 384615

6/13 = 0.461538461538...

Repeating block: 461538

Here we notice that:

Each decimal is repeating.

The repeating blocks are cyclic shifts of one another.

So yes, the decimal expansion shows cyclic properties.

171. Classify the following numbers as rational or irrational:

(i) 0.123451234512345 ...

(ii) 1.01001000100001 ... (Notice the pattern: Is it repeating a single block?)

(iii) 23.5601856122239874790120

Find the explicit fractions in case they are rational.

Ans. : (i) The block 12345 repeats continuously.

A repeating decimal is rational.

Let x = 0.123451234512345...

Then, 100000x = 12345.0.123451234512345...

Subtract:

100000x – x = 12345

99999x = 12345

x = 12345/99999

x = 4115/33333
Therefore, 0.123451234512345... is rational.
Explicit fraction: 4115/33333
(ii) This decimal does not terminate and does not repeat a fixed block.
The number of zeros between 1s keeps changing.
So, it is non-terminating and non-repeating.
Therefore, 1.01001000100001... is irrational.
(iii) This is a terminating decimal.
Every terminating decimal is rational.
Convert into fraction:
23.560185612239874790120
= 23560185612239874790120 / 1000000000000000000000
Therefore, 23.560185612239874790120 is rational.
Explicit fraction:
23560185612239874790120 / 1000000000000000000000

172. Prove that $\sqrt{5}$ is an irrational number.

Ans. : We will use proof by contradiction.
Assume that $\sqrt{5}$ is rational. Then it can be written in the form of p/q :
Let $\sqrt{5} = p/q$, where p and q are integers, $q \neq 0$, and p/q is in lowest terms (that is, p and q have no common factor except 1).
Squaring both sides, we get:
$5 = p^2/q^2$
So, $p^2 = 5q^2$
This means p^2 is divisible by 5.
Therefore, p is also divisible by 5.
So let p = 5k, for some integer k.
Substitute into $p^2 = 5q^2$, we get:
$(5k)^2 = 5q^2$
$\Rightarrow 25k^2 = 5q^2$
$\Rightarrow 5k^2 = q^2$
This shows that q^2 is divisible by 5.
Therefore, q is also divisible by 5.
So both p and q are divisible by 5.
But this contradicts our assumption that p/q was in lowest terms.
Hence, our assumption is wrong.
Therefore, √5 is an irrational number.

173. Let $a = \frac{7}{12}$ and $b = \frac{5}{6}$. Express both a and b in the form $\frac{k_1}{m}$ and $\frac{k_2}{m}$ where k_1, k_2 and m are integers and $k_2 - k_1 > 6$. Using the same denominator m, write exactly five distinct rational numbers lying between a and b keeping an integer numerator.

Explain why the condition $k_2 - k_1 > n+1$ is necessary to find n such rational numbers between the two rational numbers a and b using this method.

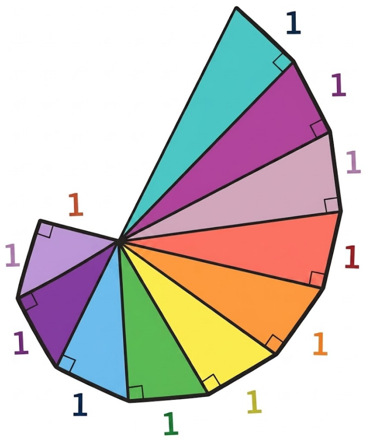
Ans. : Given:
a = 7/12
b = 5/6
First express both with the same denominator.
Since, 5/6 = 10/12,
we have: a = 7/12 and b = 10/12
Here, $k_2 - k_1$ = 10 − 7 = 3, which is not greater than 6.
So we multiply both fractions by 3:
a = 7/12 = 21/36
b = 10/12 = 30/36
Now, $k_1 = 21, k_2 = 30, m = 36$
and $k_2 - k_1 = 30 - 21 = 9 > 6$
So, five distinct rational numbers between a and b are:
22/36, 23/36, 24/36, 25/36, 26/36
These all lie strictly between 21/36 and 30/36.
Therefore, exactly five rational numbers between a and b are:
22/36, 23/36, 24/36, 25/36, 26/36
Why is the condition $k_2 - k_1 > n+1$ necessary?
If we want n rational numbers between k_1/m and k_2/m, then we need at least n integers strictly between k_1 and k_2.
The integers strictly between k_1 and k_2 are $k_1+1, k_1+2, \ldots, k_2-1$
Their number is $(k_2 - k_1 - 1)$
To get n rational numbers between them, we must have $k_2 - k_1 - 1 \geq n$
So, $k_2 - k_1 \geq n+1$
For safety in this method, the condition is taken as $k_2 - k_1 > n+1$
Thus, this ensures that enough integer numerators are available between k_1 and k_2 to form n distinct rational numbers.

174. Show that the rational number $\frac{(a+b)}{2}$ lies between the rational numbers a and b. numbers a and b.

Ans. : Let us assume that a < b.
We have to show that:
a < (a + b) / 2 < b
First, compare a and (a + b) / 2:
Since a < b,
adding a to both sides gives:
a + a < a + b
2a < a + b
Dividing both sides by 2: a < (a + b) / 2

Now, compare (a + b) / 2 and b:
Since a < b,
adding b to both sides gives:
a + b < b + b
a + b < 2b
Dividing both sides by 2:
(a + b) / 2 < b
Thus, a < (a + b) / 2 < b
Therefore, the rational number (a + b) / 2 lies between a and b.
Note: If b < a, then similarly we get:
b < (a + b) / 2 < a
So in either case, (a + b) / 2 lies between a and b.

175. Find the lengths of the hypotenuses of all the right triangles in Fig. which is referred to as the square root spiral.



Ans. : In the square root spiral, each new right triangle is formed by taking:
one leg = 1 unit
the other leg = hypotenuse of the previous triangle
Using Pythagoras theorem:
First triangle: Sides = 1, 1
Hypotenuse $= \sqrt{}(1^2 + 1^2) = \sqrt{}2$
Second triangle:
Sides $= \sqrt{}2, 1$
Hypotenuse $= \sqrt{}\left[(\sqrt{}2)^2 + 1^2\right] = \sqrt{}3$
Third triangle:
Sides $= \sqrt{3}, 1$
Hypotenuse $= \sqrt{}\left[(\sqrt{}3)^2 + 1^2\right] = \sqrt{}4 = 2$
Fourth triangle:
Sides = 2, 1
Hypotenuse $= \sqrt{}(2^2 + 1^2) = \sqrt{}5$
Fifth triangle:

Sides $= \sqrt{5}, 1$

Hypotenuse $= \sqrt{6}$

Sixth triangle:

Hypotenuse $= \sqrt{7}$

Seventh triangle:

Hypotenuse $= \sqrt{8} = 2\sqrt{2}$

Eighth triangle:

Hypotenuse $= \sqrt{9} = 3$

Ninth triangle:

Hypotenuse $= \sqrt{10}$

Tenth triangle:

Hypotenuse $= \sqrt{11}$

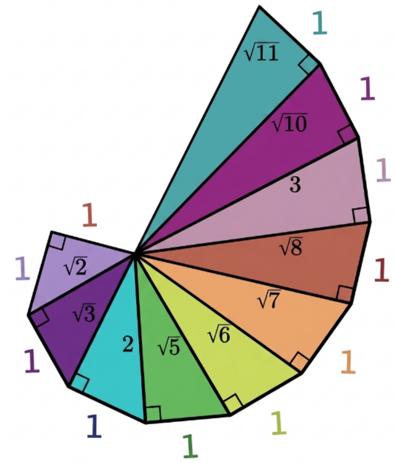
Thus, the lengths of the hypotenuses of the triangles in the square root spiral are:

$\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, \sqrt{9}, \sqrt{10}, \sqrt{11}, \ldots$

That is, $\sqrt{2}, \sqrt{3}, 2, \sqrt{5}, \sqrt{6}, \sqrt{7}, 2\sqrt{2}, 3, \sqrt{10}, \sqrt{11}, \ldots$

Hence, in general, the hypotenuse lengths follow the pattern:

$\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \ldots$ up to as many triangles as drawn in the figure.



176. Prove that $\frac{1}{\sqrt{2}}$ is irrational.

Ans. : Let us assume that $\frac{1}{\sqrt{2}}$ is rational.

Then let $\frac{1}{\sqrt{2}} = \frac{a}{b}$, where a, b are co-prime, $b \neq 0, \Rightarrow \sqrt{2}a = b$

Squaring both sides, $(\sqrt{2}a)^2 = b^2 \Rightarrow 2a^2 = b^2$ ...(i)

$\Rightarrow a^2 = \frac{b^2}{2} \Rightarrow 2$ divides $b^2 \Rightarrow 2$ divides $b \Rightarrow 2$ is a factor of b

Let $b = 2c$ for some integer c.

Then, from $(i), 2a^2 = (2c)^2 \Rightarrow 2a^2 = 4c^2 \Rightarrow a^2 = 2c^2 \Rightarrow c^2 = \frac{a^2}{2}$

$\Rightarrow 2$ divides $a^2 \Rightarrow 2$ divides $a \Rightarrow 2$ is a factor of a

It means 2 is a common factor of a and b. This contradicts the assumption as there is no common factor of a and b other than 1 .

Thus, $\frac{1}{\sqrt{2}}$ is not rational. Hence, it is irrational. Proved.

177. Prove that $\sqrt{2}+\sqrt{5}$ is irrational.

Ans. : Let us assume to the contrary that $\sqrt{2}+\sqrt{5}$ is rational. Then, there exists positive integers a and b such that $\sqrt{2}+\sqrt{5}=\frac{a}{b}, a$ and b are co-prime, $b \neq 0$

$\Rightarrow \frac{a}{b}-\sqrt{2}=\sqrt{5} \Rightarrow\left(\frac{a}{b}-\sqrt{2}\right)^2=(\sqrt{5})^2$

$\Rightarrow \frac{a^2}{b^2}+2-2 \cdot \frac{a}{b} \cdot \sqrt{2}=5 \Rightarrow \frac{a^2}{b^2}+2-\frac{2 \sqrt{2} a}{b}=5$

$\Rightarrow \frac{a^2}{b^2}-\frac{2 \sqrt{2} a}{b}=3 \Rightarrow \frac{a^2-2 \sqrt{2} a b}{b^2}=3 \Rightarrow a^2-2 \sqrt{2} a b=3 b^2$

$\Rightarrow 2 \sqrt{2} a b=a^2-3 b^2 \Rightarrow \sqrt{2}=\frac{a^2-3 b^2}{2 a b}$

$\Rightarrow \sqrt{2}$ is a rational number. $[\because a, b, 2$ and 3 are integers. So, $\frac{a^2-3 b^2}{2 a b}$ is rational. $]$

This contradicts the fact that $\sqrt{2}$ is irrational. So, our assumption is wrong.

Hence, $\sqrt{2}+\sqrt{5}$ is irrational. Proved.

178. Prove that $\sqrt{3}$ is an irrational number.

Ans. : Let us assume to the contrary that $\sqrt{3}$ is a rational number. Then, there exists positive integers a and b such that $\sqrt{3}=\frac{a}{b}$, where a and b are integers having no common factor other than 1 (i.e., a and b are co-prime) and $b \neq 0$.

$\Rightarrow 3=\frac{a^2}{b^2}$ [Squaring both sides]

$\Rightarrow a^2=3 b^2$

$\Rightarrow 3$ divides $a^2 \Rightarrow 3$ divides $a \Rightarrow 3$ is a factor of a

Let $a=3 c$, for some integer c. Then from (i), we get $9 c^2=3 b^2 \Rightarrow(3 c)^2=b^2$

$\Rightarrow 3$ divides $b^2 \Rightarrow 3$ divides $b \Rightarrow 3$ is a factor of b.

It means 3 is a common factor of a and b. This contradicts the assumption as there is no common factor of a and b other than 1 .

Thus, $\sqrt{3}$ is not rational. Hence, $\sqrt{3}$ is irrational. Proved.

179. Prove that $\sqrt{2}$ is an irrational number.

Ans. : Let us assume to the contrary that $\sqrt{2}$ is a rational number. Then, there exists positive integers a and b, such that $\sqrt{2}=\frac{a}{b}$, where a and b are integers having no common factor other than 1 (i.e., a and b are co-prime) and $b \neq 0$.

$\Rightarrow 2=\frac{a^2}{b^2}$ [Squaring both sides]

$\Rightarrow a^2=2 b^2$

$\Rightarrow 2$ divides $a^2 \Rightarrow 2$ divides $a \Rightarrow 2$ is a factor of a

Let $a=2 c$ for some integer c. Then from (i), we get $4 c^2=2 b^2 \Rightarrow(2 c)^2=b^2$

$\Rightarrow 2$ divides $b^2 \Rightarrow 2$ divides $b \Rightarrow 2$ is a factor of b.

It means 2 is a common factor of a and b. This contradicts the assumption as there is no common factor of a and b other than 1.

Thus, $\sqrt{2}$ is not rational. Hence, $\sqrt{2}$ is irrational. Proved.

180. If $a=\frac{3+\sqrt{5}}{2}$, then find the value of $a^2+\frac{1}{a^2}$.

Ans. : We have, $a=\frac{3+\sqrt{5}}{2}$. Then,

$\frac{1}{a}=\frac{2}{3+\sqrt{5}}=\frac{2}{3+\sqrt{5}}\times\frac{3-\sqrt{5}}{3-\sqrt{5}}=\frac{2(3-\sqrt{5})}{(3)^2-(\sqrt{5})^2}=\frac{6-2\sqrt{5}}{9-5}$

$=\frac{6-2\sqrt{5}}{4}=\frac{2(3-\sqrt{5})}{4}=\frac{3-\sqrt{5}}{2}$.

Now, $a+\frac{1}{a}=\frac{3+\sqrt{5}}{2}+\frac{3-\sqrt{5}}{2}=\frac{3+\sqrt{5}+3-\sqrt{5}}{2}=\frac{6}{2}=3$

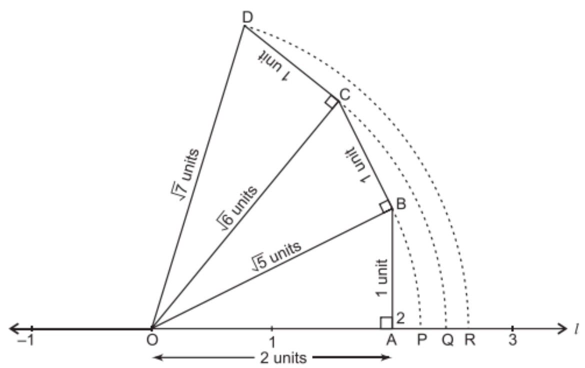
Squaring both sides, we get

$\left(a+\frac{1}{a}\right)^2=3^2 \Rightarrow a^2+\frac{1}{a^2}+2\times a\times\frac{1}{a}=9$

$\Rightarrow a^2+\frac{1}{a^2}+2=9 \quad \Rightarrow a^2+\frac{1}{a^2}=7$

181. Express $\sqrt{5}, \sqrt{6}$ and $\sqrt{7}$ on a real number line.

Ans. :



(i) Let $OA=2$ units and let $AB\perp OA$ such that $AB=1$ unit.

Join OB . In $\triangle OAB, OB=\sqrt{OA^2+AB^2}=\sqrt{2^2+1^2}=\sqrt{5}$

[Using Baudhāyana-Pythagoras theorem]

With O as centre and OB as radius, draw an arc meeting the real line l at P .

Then, $OP=OB=\sqrt{5}$ units

(ii) To represent $\sqrt{6}$ on the real line, draw $BC\perp OB$ such that $BC=1$ unit.

Join OC . In $\triangle OBC, OC=\sqrt{OB^2+BC^2}=\sqrt{(\sqrt{5})^2+1^2}=\sqrt{6}$ units

[Using Baudhāyana-Pythagoras theorem]

With O as centre and OC as radius, draw an arc meeting the real line l at Q .

Then, $OQ=OC=\sqrt{6}$ units

(iii) To represent $\sqrt{7}$ on the real line, draw $CD\perp OC$ such that $CD=1$ unit.

Join OD . In $\triangle OCD, OD=\sqrt{OC^2+CD^2}=\sqrt{(\sqrt{6})^2+1^2}=\sqrt{7}$

[Using Baudhāyana-Pythagoras theorem]

With O as centre and OD as radius, draw an arc meeting the real line l at R .

Then, $OR=OD=\sqrt{7}$ units

182. Prove that $\frac{2}{\sqrt{7}}$ is an irrational number.

Ans. : Assume that $\frac{2}{\sqrt{7}}$ is a rational number. If it is rational, it can be expressed as a fraction of two integers a and b :

$\frac{2}{\sqrt{7}}=\frac{a}{b} \quad$ (where $a, b \in Z, b \neq 0)$

We need to rearrange the equation to solve for $\sqrt{7}$. First, we can take the reciprocal of both sides:

$\frac{\sqrt{7}}{2}=\frac{b}{a}$

Now, multiply both sides by 2 :

$\sqrt{7}=\frac{2 b}{a}$

Let's look at the properties of the right side of our equation:

Since b is an integer, $2 b$ is also an integer.

Since $\frac{2}{\sqrt{7}}$ is a non-zero number, a must be a non-zero integer.

The expression $\frac{2 b}{a}$ is the ratio of two integers, which by definition makes it a rational number.

Our equation now states:

$\sqrt{7}=$ Rational Number

However, it is a known mathematical fact that $\sqrt{7}$ is irrational. (Since 7 is a prime number and not a perfect square, its square root cannot be expressed as a ratio of integers).

We have reached a contradiction: an irrational number cannot be equal to a rational number.

Because our assumption that $\frac{2}{\sqrt{7}}$ is rational led to a logical impossibility, the assumption must be false.

Therefore, $\frac{2}{\sqrt{7}}$ is an irrational number.

183. Prove that $\frac{1}{\sqrt{3}}$ is an irrational number.

Ans. : Assume that $\frac{1}{\sqrt{3}}$ is a rational number. This means it can be written as a fraction:

$\frac{1}{\sqrt{3}}=\frac{a}{b}$

where a and b are integers, $b \neq 0$, and we assume the fraction is in its simplest form (meaning a and b have no common factors other than 1).

Squaring both sides of the equation to remove the square root:

$\left(\frac{1}{\sqrt{3}}\right)^{2}=\left(\frac{a}{b}\right)^{2}$

$\frac{1}{3}=\frac{a^{2}}{b^{2}}$

Cross-multiply to solve for b^{2} :

$b^{2}=3 a^{2} \quad \ldots($ Equation 1)

From Equation 1 , we see that b^{2} is a multiple of 3 . Therefore, b must also be a multiple of 3 .

Since b is a multiple of 3 , we can write $b=3k$ for some integer k.
Substitute $b=3k$ back into Equation 1:

$(3k)^2=3a^2$

$9k^2=3a^2$

Divide both sides by 3:

$3k^2=a^2$

This new equation shows that a^2 is also a multiple of 3 , which means a must be a multiple of 3.

Now we have found that:

b is a multiple of 3 .

a is a multiple of 3.

This contradicts our initial assumption that the fraction $\frac{a}{b}$ was in its simplest form (having no common factors).

Since our assumption led to a contradiction, the assumption that $\frac{1}{\sqrt{3}}$ is rational must be false. Therefore, $\frac{1}{\sqrt{3}}$ is an irrational number.

184. Prove that $7-6\sqrt{5}$ is an irrational number.

Ans. : Assume that $7-6\sqrt{5}$ is a rational number. If it is rational, it can be written as a fraction of two integers a and b :

$7-6\sqrt{5}=\frac{a}{b} \quad$ (where $a,b\in Z$ and $b\neq 0)$

We need to rearrange the equation to solve for $\sqrt{5}$. First, subtract 7 from both sides:

$-6\sqrt{5}=\frac{a}{b}-7$

$-6\sqrt{5}=\frac{a-7b}{b}$

Now, divide both sides by -6 :

$\sqrt{5}=\frac{a-7b}{-6b}$

To make it cleaner, we can multiply the numerator and denominator by -1 :

$\sqrt{5}=\frac{7b-a}{6b}$

Since a and b are integers, ($7b-a$) is an integer.

Since b is a non-zero integer, $6b$ is also a non-zero integer.

The right side of the equation, $\frac{7b-a}{6b}$, is a ratio of two integers and is therefore a rational number.

$\sqrt{5}=$ Rational Number

However, it is a known mathematical fact that $\sqrt{5}$ is irrational (as 5 is a prime number and not a perfect square

Since an irrational number cannot equal a rational number, we have reached a contradiction.

Our original assumption that $7-6\sqrt{5}$ is rational must be incorrect. Therefore, $7-6\sqrt{5}$ is an irrational number.

185. Write whether $\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}}$ on simplification gives an irrational or a rational number.

Ans. : We can break down the square roots in the numerator into their prime factors to find perfect squares:

For $\sqrt{45}$: $\sqrt{45}=\sqrt{9\times 5}=3\sqrt{5}$

For $\sqrt{20}$: $\sqrt{20}=\sqrt{4\times 5}=2\sqrt{5}$

Now, substitute these simplified forms back into the numerator:

$2\sqrt{45}+3\sqrt{20}$

$=2(3\sqrt{5})+3(2\sqrt{5})$

$=6\sqrt{5}+6\sqrt{5}$

$=12\sqrt{5}$

Now, place the simplified numerator over the original denominator:

$\frac{12\sqrt{5}}{2\sqrt{5}}$

Since $\sqrt{5}$ appears in both the numerator and the denominator, they cancel out:

$\frac{12}{2}=6$

The simplified value is 6 .

Since 6 is an integer, it can be written as the fraction $\frac{6}{1}$. Therefore, the expression simplifies to a rational number.

186. Given that $\sqrt{2}$ is irrational, prove that $(5+3\sqrt{2})$ is an irrational number.

Ans. : Assume, for the sake of contradiction, that $5+3\sqrt{2}$ is a rational number. By definition, a rational number can be expressed as a fraction $\frac{a}{b}$ where a and b are integers and $b\neq 0$.

$5+3\sqrt{2}=\frac{a}{b}$

We will now use algebraic operations to isolate $\sqrt{2}$ on one side of the equation. First, subtract 5 from both sides:

$3\sqrt{2}=\frac{a}{b}-5$

Common denominator on the right side:

$3\sqrt{2}=\frac{a-5b}{b}$

Now, divide both sides by 3:

$\sqrt{2}=\frac{a-5b}{3b}$

Since a and b are integers, $(a-5b)$ must also be an integer (closure property of integers under subtraction and multiplication).

Since b is a non-zero integer, $3b$ is also a non-zero integer.

Therefore, the expression $\frac{a-5b}{3b}$ is a ratio of two integers, which makes it a rational number.

Irrational = Rational

This is a contradiction because we are given that $\sqrt{2}$ is irrational. A value cannot be both irrational and rational at the same time.

187. Prove that $3+\sqrt{2}$ is an irrational number.

Ans. : Assume that $3+\sqrt{2}$ is a rational number.
By definition, a rational number can be expressed in the form $\frac{a}{b}$, where a and b are integers, $b \neq 0$, and a and b are co-prime (they share no common factors other than 1).
$3+\sqrt{2}=\frac{a}{b}$
We need to isolate the radical term $(\sqrt{2})$ on one side:
$\sqrt{2}=\frac{a}{b}-3$
Now, simplify the right side by finding a common denominator:
$\sqrt{2}=\frac{a-3b}{b}$
The Right Side $\left(\frac{a-3b}{b}\right)$: Since a, b, and 3 are all integers, $(a-3b)$ is also an integer.Therefore, the right side is a ratio of two integers, which makes it a rational number.
The Left Side $(\sqrt{2})$: It is a known mathematical fact that $\sqrt{2}$ is an irrational number.
We have reached a logical impossibility:
Irrational Number = Rational Number
Since an irrational number cannot equal a rational number, our initial assumption that $3+\sqrt{2}$ is rational must be incorrect.
Therefore, $3+\sqrt{2}$ is an irrational number.

188. Prove that $\sqrt{5}$ is an irrational number.

Ans. : Assume that $\sqrt{5}$ is a rational number.
This means it can be written as a fraction:
$\sqrt{5}=\frac{a}{b}$
where a and b are integers, $b \neq 0$, and the fraction is in its simplest form (meaning a and b are co-prime and have no common factors other than 1).
Squaring both sides of the equation gives:
$5=\frac{a^2}{b^2}$
Multiplying both sides by b^2 :
$a^2=5b^2$
From the equation $a^2=5b^2$, we can see that a^2 is a multiple of 5 .
According to the fundamental theorem of arithmetic, if a^2 is divisible by a prime number (5), then a must also be divisible by 5.
$a=5k \quad$ (where k is some integer)
Substitute $a=5k$ back into the equation $a^2=5b^2$:
$(5k)^2=5b^2$

$25k^2 = 5b^2$

$5k^2 = b^2$

This shows that b^2 is also a multiple of 5 , which means b must also be divisible by 5 .

We have found that both a and b have 5 as a common factor.

However, this contradicts our initial assumption that a and b are co-prime (have no common factors).

Since our assumption leads to a contradiction, the assumption must be false.

Therefore, $\sqrt{5}$ is an irrational number.

189. Prove that $3-\sqrt{5}$ is an irrational number.

Ans. : Assume that $3-\sqrt{5}$ is a rational number.

By definition, it can be expressed in the form $\frac{a}{b}$, where a and b are integers, $b \neq 0$, and a and b are co-prime (no common factors other than 1).

$3-\sqrt{5}=\frac{a}{b}$

Isolate the irrational part $(\sqrt{5})$ by rearranging the equation:

$3-\frac{a}{b}=\sqrt{5}$

Combine the terms on the left side using a common denominator:

$\frac{3b-a}{b}=\sqrt{5}$

Now, let's analyze both sides of the equation:

Left Side $\left(\frac{3b-a}{b}\right)$: Since a, b, and 3 are integers, $(3b-a)$ is also an integer. Therefore, the fraction is a ratio of two integers, making it a rational number.

Right Side $(\sqrt{5})$: It is a well-known mathematical fact that $\sqrt{5}$ is an irrational number.

We have an equation stating that a rational number is equal to an irrational number:

Rational = Irrational

This is a contradiction. Our original assumption that $3-\sqrt{5}$ is rational must be false.

Therefore, $3-\sqrt{5}$ is an irrational number.

190. Prove that $\sqrt{5}+\frac{2}{\sqrt{5}}$ is irrational.

Ans. : Before starting the proof, it is helpful to combine the terms into a single fraction:

$\sqrt{5}+\frac{2}{\sqrt{5}}$

To add these, find a common denominator $(\sqrt{5})$:

$\frac{\sqrt{5} \cdot \sqrt{5}}{\sqrt{5}}+\frac{2}{\sqrt{5}}=\frac{5+2}{\sqrt{5}}=\frac{7}{\sqrt{5}}$

So, proving that $\sqrt{5}+\frac{2}{\sqrt{5}}$ is irrational is the same as proving that $\frac{7}{\sqrt{5}}$ is irrational.

Assume that $\frac{7}{\sqrt{5}}$ is a rational number. Therefore, it can be written as:

$\frac{7}{\sqrt{5}}=\frac{a}{b}$

where a and b are integers and $b\neq 0$.

Rearrange the equation to solve for $\sqrt{5}$. First, cross-multiply:

$7b=a\sqrt{5}$

Now, divide both sides by a :

$\sqrt{5}=\frac{7b}{a}$

Since b is an integer, $7b$ is also an integer.

Since the original expression $\sqrt{5}+\frac{2}{\sqrt{5}}$ is clearly not zero, a must be a non-zero integer.

The right side, $\frac{7b}{a}$, is a ratio of two integers, which makes it a rational number.

Our equation now claims:

$\sqrt{5}=$ Rational Number

However, we know that $\sqrt{5}$ is irrational (as 5 is a prime number and not a perfect square). A rational number cannot equal an irrational number.

The contradiction arises from our assumption that the expression is rational. Therefore, the assumption is false, and $\sqrt{5}+\frac{2}{\sqrt{5}}$ is an irrational number.

191. Prove that $5+\sqrt{2}$ is an irrational number.

Ans. : Assume that $5+\sqrt{2}$ is a rational number.

By definition, a rational number can be written in the form $\frac{a}{b}$, where a and b are integers, $b\neq 0$, and a and b are co-prime (they have no common factors other than 1).

$5+\sqrt{2}=\frac{a}{b}$

Isolate the radical $(\sqrt{2})$ on one side of the equation:

$\sqrt{2}=\frac{a}{b}-5$

Now, find a common denominator for the right side:

$\sqrt{2}=\frac{a-5b}{b}$

The Right Side: Since a,5, and b are all integers, the expression $\frac{a-5b}{b}$ is a ratio of two integers.

Therefore, the right side is a rational number.

The Left Side: We know that $\sqrt{2}$ is an irrational number.

We have reached a contradiction:

Irrational Number = Rational Number

This is impossible. Our initial assumption that $5+\sqrt{2}$ is rational must be wrong.

Therefore, $5+\sqrt{2}$ is an irrational number.

192. If $x=\frac{-2}{3}$, $y=\frac{1}{4}$, $z=\frac{-5}{8}$ verify that :

$y(x-z)=yx-yz$

Ans. : $\frac{1}{4}\times\left(\frac{-5}{8}-\frac{-2}{3}\right)=\frac{1}{4}\times\left(\frac{-5}{8}+\frac{2}{3}\right)=\frac{1}{4}\times\left(\frac{-15+16}{24}\right)=\frac{1}{4}\times\frac{1}{24}=\frac{1}{96}$

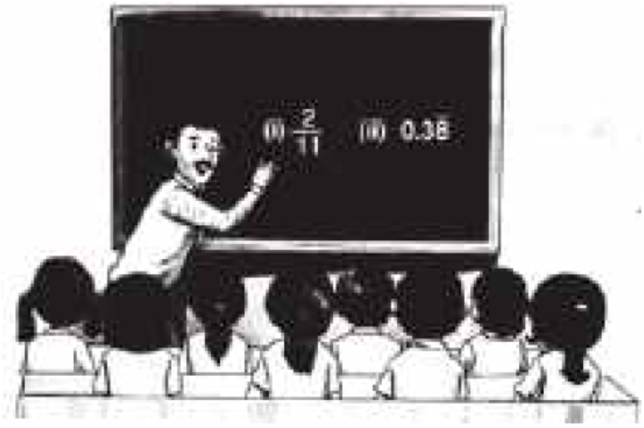
$\left(\frac{1}{4}\times\frac{-5}{8}\right)-\left(\frac{1}{4}\times\frac{-2}{3}\right)=\frac{-5}{32}-\frac{-2}{12}=\frac{-5}{32}+\frac{1}{6}=\frac{-15+16}{96}=\frac{1}{96}$

(iii) $\frac{1}{4}\times\left(\frac{-2}{3}-\frac{-5}{8}\right)=\frac{1}{4}\times\left(\frac{-16+15}{24}\right)=\frac{1}{4}\times\frac{-1}{24}=\frac{-1}{96}$

$\left(\frac{1}{4}\times\frac{-2}{3}\right)-\left(\frac{1}{4}\times\frac{-5}{8}\right)=\frac{-2}{12}-\frac{-5}{32}=\frac{-1}{6}+\frac{5}{32}=\frac{-16+15}{96}=\frac{-1}{96}$

*** Case study based questions.** **[4]**

To judge the preparation of students of class IX on topic 'Number System', Mathematics teacher writes two numbers on blackboard (as shown in figure), and asks some questions about the two numbers. Can you answer the questions?



193. Write the decimal form of $\frac{2}{11}$.

194. Write the $\frac{p}{q}$ form of $0.3\overline{8}$.

195. If $\frac{p}{q}$ form of $0.3\overline{8}$ is $\frac{m}{n}$, then find the value of $(m+n)$.

196. The decimal number $0.3\overline{8}$ is a terminating decimal. Is it true?

Ans. : 1. To find the decimal form, divide 2 by 11 :

- $2\div 11=0.181818$...

- Decimal form: $0.\overline{18}$

2. Let $x=0.3888\ldots$

i. Multiply by 10 to move the non-repeating digit: $10x$ = 3.888... (Equation 1)

ii. Multiply by 100 to move one repeating digit: $100x$ = 38.888... (Equation 2)

iii. Subtract Eq 1 from Eq 2 :

$$100x-10x=38.888\ldots-3.888\ldots$$
$$90x=35$$
$$x=\frac{35}{90}$$

iv. Simplify the fraction: $\frac{35\div 5}{90\div 5}=\frac{7}{18}$

- $\frac{p}{q}$ form: $\frac{7}{18}$

3. Value of $(m+n)$

From the previous step, the $\frac{p}{q}$ form is $\frac{7}{18}$.

- Here, $m = 7$ and $n = 18$.
- $m + n = 7 + 18 = 25$
- Value : 25

4. - No, it is False.

- The bar over the 8 indicates that the digit 8 repeats infinitely. Therefore, it is a non-terminating repeating (recurring) decimal.

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