STD 9 Maths [Manjari]
I am up and down

*** Choose the right answer from the given options. [1 Marks Each]** **[15]**

1. A circle is the set of all points in a plane that are at a fixed distance from a:
   (A) Diameter (B) Chord (C) Centre (D) Arc

   Ans. : (C) Centre
   A circle is defined as the set of all points in a plane that are equidistant from a fixed point called the centre.

2. The longest chord of a circle is:
   (A) Radius (B) Diameter (C) Arc (D) Tangent

   Ans. : (B) Diameter
   The diameter passes through the centre of the circle and has the greatest possible length among all chords.

3. If two chords of a circle are equal, then they subtend:
   (A) Unequal angles at the centre
   (B) Equal angles at the centre
   (C) Right angles at the centre
   (D) Supplementary angles at the centre

   Ans. : (B) Equal angles at the centre
   Equal chords of a circle subtend equal angles at the centre according to the theorem on equal chords.

4. If two chords of a circle subtend equal angles at the centre, then the chords are:
   (A) Parallel (B) Equal (C) Perpendicular (D) Unequal

   Ans. : (B) Equal
   Chords of a circle that subtend equal angles at the centre are equal in length.

5. The perpendicular drawn from the centre of a circle to a chord:
   (A) Divides the circle into two equal parts
   (B) Bisects the chord
   (C) Is parallel to the chord
   (D) Forms a tangent

   Ans. : (B) Bisects the chord
   The perpendicular from the centre of a circle to a chord always bisects the chord.

6. If the radius of a circle is 13 cm and the perpendicular distance from the centre to a chord is 5 cm, then the length of the chord is:

(A) 10 cm (B) 12 cm (C) 24 cm (D) 26 cm

Ans. : (C) 24 cm

Half of the chord length $= \sqrt{13^2 - 5^2}$

$= \sqrt{169 - 25}$

$= \sqrt{144}$

$= 12$

Therefore, chord length = 2 × 12 = 24 cm.

7. The angle subtended by a diameter at any point on the circle is:

(A) 45° (B) 60° (C) 90° (D) 180°

Ans. : (C) 90°

The angle subtended by a diameter at the circumference of a circle is always a right angle.

8. If an arc subtends an angle of 100° at the centre, then the angle subtended by the same arc at a point on the remaining circle is:

(A) 25° (B) 50° (C) 100° (D) 200°

Ans. : (B) 50°

The angle subtended by an arc at the centre is double the angle subtended at any point on the remaining part of the circle.

$100^\circ \div 2 = 50^\circ$

9. A quadrilateral is called cyclic if:

(A) All its sides are equal

(B) Opposite sides are parallel

(C) Its vertices lie on the same circle

(D) Diagonals bisect each other

Ans. : (C) Its vertices lie on the same circle

A quadrilateral whose four vertices lie on the same circle is called a cyclic quadrilateral.

10. The sum of opposite angles of a cyclic quadrilateral is:

(A) 90° (B) 180° (C) 270° (D) 360°

Ans. : (B) 180°

Opposite angles of a cyclic quadrilateral are supplementary.

11. In a cyclic quadrilateral, if one angle measures 75°, then its opposite angle measures:

(A) 75° (B) 90° (C) 105° (D) 115°

Ans. : (C) 105°

Opposite angles of a cyclic quadrilateral add up to 180°.

$180^\circ - 75^\circ = 105^\circ$

12. If two chords of a circle are equidistant from the centre, then the chords are:
    (A) Equal (B) Parallel (C) Perpendicular (D) Diameters

    Ans. : (A) Equal
    Chords that are equidistant from the centre of a circle are equal in length.

13. If two distinct points are given on a plane, the number of circles passing through them is:
    (A) One (B) Two (C) Three (D) Infinitely many

    Ans. : (D) Infinitely many
    Every point on the perpendicular bisector of the line segment joining the two points can act as the centre of a circle passing through them.

14. Three non-collinear points determine:
    (A) No circle
    (B) Exactly one circle
    (C) Exactly two circles
    (D) Infinitely many circles

    Ans. : (B) Exactly one circle
    There is a unique circle passing through three non-collinear points.

15. If one chord of a circle is longer than another chord, then the longer chord is:
    (A) Farther from the centre
    (B) Closer to the centre
    (C) Equal distance from the centre
    (D) A diameter

    Ans. : (B) Closer to the centre
    Among unequal chords of a circle, the longer chord lies nearer to the centre.

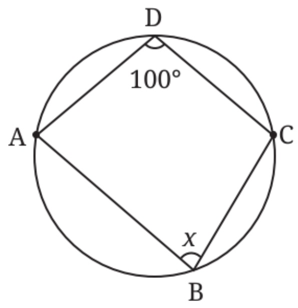
*** Answer the following short questions. [2 Marks Each]** **[2]**

16. What is the least possible radius of a circle through two points A and B?

    Ans. : For all circles passing through two fixed points A and B, the centre lies on the perpendicular bisector of AB.
    The smallest circle occurs when the centre is the midpoint of AB.
    In that case, AB becomes the diameter of the circle.
    Therefore, the least possible radius is:
    Radius = AB/2
    Hence, the least possible radius of a circle through A and B is half of AB.

*** Answer the following questions. [3 Marks Each]** **[18]**

17. Find x in following figure



Ans. : In the figure, A, D, C and B are points on the same circle.
So, quadrilateral ADCB is a cyclic quadrilateral.
Given: $\angle ADC = 100^\circ$
In a cyclic quadrilateral, opposite angles are supplementary.
Therefore, $\angle ABC + \angle ADC = 180^\circ$
$\Rightarrow x + 100^\circ = 180^\circ$
$\Rightarrow x = 80^\circ$
Hence, $x = 80^\circ$.

18. An arc of a circle subtends an angle of 70° at the centre. What is the measure of the angle subtended by the arc at a point on the circle?

Ans. : We know:
Angle subtended by an arc at the centre
= 2 × angle subtended by the same arc at a point on the circle
Given: Angle at the centre $= 70^\circ$
So, angle at the circle
$= 70^\circ/2$
$= 35^\circ$
Hence, the required angle is 35°.

19. A cyclic quadrilateral has sides $5, 5, 12, 12$ units. Find its area.

Ans. : Since the cyclic quadrilateral has sides 5, 5, 12, 12, its semiperimeter is
s = (5 + 5 + 12 + 12)/2
= 34/2
= 17
For a cyclic quadrilateral, area is given by Brahmagupta's formula:
Area $= \sqrt{[(s-a)(s-b)(s-c)(s-d)]}$
Substituting the vaues, we have:
Area $= \sqrt{[(17-5)(17-5)(17-12)(17-12)]}$
$= \sqrt{[(12)(12)(5)(5)]}$
$= \sqrt{3600}$
= 60
Hence, the area of the cyclic quadrilateral is 60 square units.

20. Consider a cyclic quadrilateral. Without drawing its circumcircle, how can we find out whether the centre of the circumcircle lies inside the quadrilateral or outside? What is the best way of finding out?

Ans. : In a cyclic quadrilateral, we know that opposite angles are supplementary (their sum is 180°).
To find whether the centre of the circumcircle (circumcentre) lies inside or outside, we can observe the angles of the quadrilateral:
If all angles are less than 90° (acute), then the circumcentre lies inside the quadrilateral.
If any one angle is greater than 90° (obtuse), then the circumcentre lies outside the quadrilateral.
Best way: Check the angles of the quadrilateral.
This method is easy and does not require drawing the circumcircle.
So, by just looking at whether the quadrilateral has an obtuse angle or not, we can decide the position of the circumcentre.

21. Consider all chords of a circle of a fixed length. What is the shape formed by the midpoints of all these chords?

Ans. : Let the radius of the circle be r.
Let each chord have fixed length x.
The perpendicular from the centre to a chord bisects the chord.
So, for every chord, half chord $= x/2$.
Let d be the distance of the midpoint of the chord from the centre.
Using Pythagoras theorem:
$r^2 = d^2 + (x/2)^2$
So, $d^2 = r^2 - (x/2)^2$
Since r and x are fixed, d is also fixed.
Therefore, every midpoint is at the same distance from the centre.
Hence, the midpoints form a circle with the same centre as the original circle.

22. In a circle, two chords CC’ and DD’ are drawn perpendicular to a diameter AB. Prove that the segment MM’ joining the midpoints of the chords CD and C’D’ is perpendicular to AB.

Ans. : Let AB be the diameter of the circle.
Since chords CC’ and DD’ are perpendicular to AB, the points C and C’ are symmetric about AB.
Similarly, D and D’ are also symmetric about AB.
Now, let M be the midpoint of CD and M’ be the midpoint of C’D’.
Since C corresponds to C’ and D corresponds to D’ by reflection in AB,
the midpoint of CD corresponds to the midpoint of C’D’.
Therefore, M and M’ are symmetric about AB.
The line joining two points that are symmetric about a line is perpendicular to that

line.

Hence, $MM' \perp AB$

Therefore, the segment joining the midpoints of CD and C’D’ is perpendicular to AB.

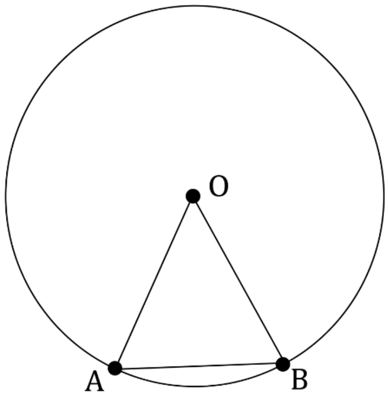
Hence proved.

*** Questions with calculation. [4 Marks Each]** **[24]**

23. Show that the triangle formed by a chord and the centre of the circle is isosceles.

Ans. : Let AB be a chord of a circle with centre O.

Join OA and OB.



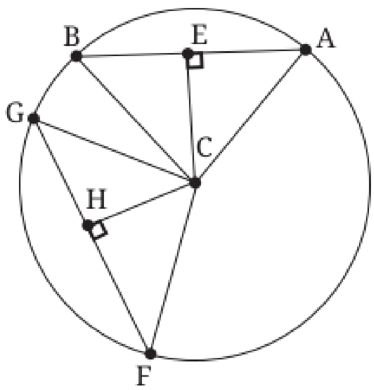
In triangle OAB

OA = OB [Radii of the same circle]

Therefore, triangle OAB is an isosceles triangle.

Hence proved.

24. Consider Fig If CE is perpendicular to AB, CH is perpendicular to GH , and CE $= CH$, show that $AB = GF$.



Ans. : Given: $CE \perp AB, CH \perp GF$ and $CE = CH$.

To Prove: AB = GF

Proof: Since CE is perpendicular to chord AB, the perpendicular from the centre to a chord bisects the chord.

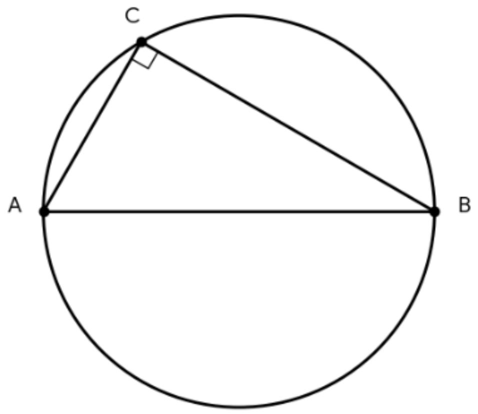
Therefore, AE = EB

Similarly, since CH is perpendicular to chord GF. So, GH = HF

In $\triangle CEA$ and $\triangle CHG$, we have
CA = CG [Radii of the same circle]
CE = CH [Given]
$\angle CEA = \angle CHG$ [Each 90°]
Therefore, by RHS congruence
$\triangle CEA \cong \triangle CHG$
So, AE = GH [CPCT]
⇒ 2AE = 2GH
⇒ AB = GF [AB = 2AE and GF = 2GH]
Hence proved.

25. The diameter of a circle is AB . Point C is on the circumference What is the measure of the $\angle ACB$? Explain your reasoning.

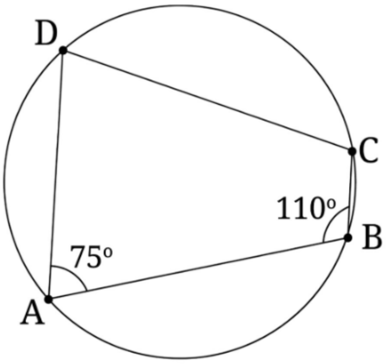
Ans. : AB is a diameter.
The angle subtended by a diameter at any point on the circle is 90°.
Therefore, $\angle ACB = 90^\circ$
Hence, the measure of $\angle ACB$ is 90°.



26. ABCD is a cyclic quadrilateral inscribed in a circle. If $\angle A$ measures 75°, what is the measure of $\angle C$? If $\angle B$ measures 110°, what is the measure of $\angle D$?

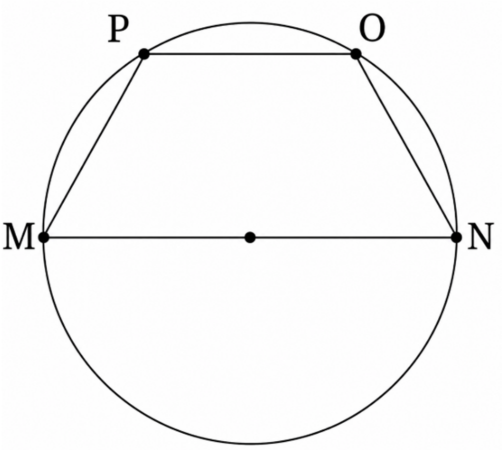
Ans. : In a cyclic quadrilateral, opposite angles are supplementary.
So, $\angle A + \angle C = 180^\circ$
$\Rightarrow 75^\circ + \angle C = 180^\circ$
$\Rightarrow \angle C = 105^\circ$
Also, $\angle B + \angle D = 180^\circ$
$\Rightarrow 110^\circ + \angle D = 180^\circ$
$\Rightarrow \angle D = 70^\circ$

Hence, $\angle C = 105^\circ$ and $\angle D = 70^\circ$.



27. A quadrilateral MNOP is inscribed in a circle. If MN is a diameter, what can you say about $\angle MOP$ and $\angle MNP$? Explain your reasoning.

Ans. : Since MNOP is inscribed in a circle, it is a cyclic quadrilateral.
So, opposite angles of a cyclic quadrilateral are supplementary.
Therefore,
$\angle MOP + \angle MNP = 180^\circ$
Hence, $\angle MOP$ and $\angle MNP$ are supplementary angles.



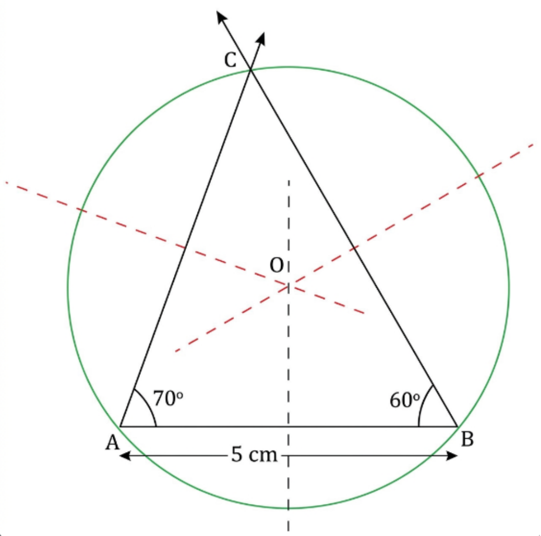
28. There is no chord of a circle that is longer than its diameter. How do you justify this statement?

Ans. : A chord is a line segment joining two points on a circle.
The longest chord passes through the centre of the circle.
Such a chord is called the diameter.
For any other chord, its perpendicular distance from the centre is greater than 0.
If radius = r and distance of chord from centre = d, then:
Chord length $= 2\sqrt{(r^2 - d^2)}$
Since $d > 0$ for any chord not passing through the centre,
$r^2 - d^2 < r^2$
So, $2\sqrt{(r^2 - d^2)} < 2r$
But diameter = 2r.
Therefore, every chord other than the diameter is shorter than the diameter.
Hence, no chord of a circle is longer than its diameter.

* **Answer the following questions. [5 Marks Each]**

29. Draw $\triangle ABC$ with $AB = 5cm, \angle A = 70^\circ$ and $\angle B = 60^\circ$. Draw the circumcircle of $\triangle ABC$. Is the centre inside or outside the triangle?

Ans. : 1. Draw the line segment AB = 5 cm.
2. At point A , construct $\angle CAB = 70^\circ$.
3. At point B , construct $\angle CBA = 60^\circ$.
4. Let the two rays meet at C. Then$\triangle ABC$ is formed.
5. Draw the perpendicular bisector of AB.
6. Draw the perpendicular bisectors of BC and AC.
7. Let the three perpendicular bisectors meet at O. This point O is the circumcentre.
8. With centre O and radius OA, draw a circle passing through A, B and C. This is the circumcircle of $\triangle ABC$.

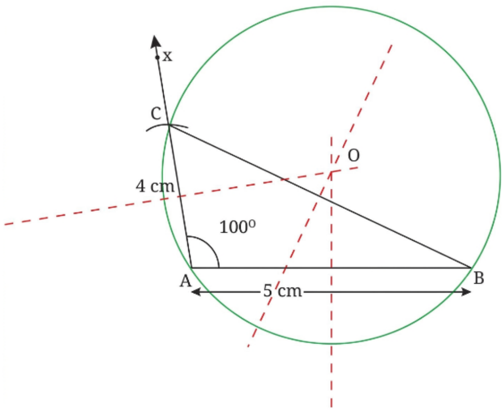


Now:
$\angle C = 180^\circ - (70^\circ + 60^\circ) = 50^\circ$
Since all the angles of the triangle are less than 90°, the triangle is acute-angled.
In an acute-angled triangle, the circumcentre lies inside the triangle.
Therefore, the centre is inside the triangle.

30. Draw $\triangle ABC$ with $AB = 5cm, \angle A = 100^\circ, AC = 4cm$. Draw the circumcircle of $\triangle ABC$. Is the centre inside or outside the triangle?

Ans. : 1. Draw the line segment AB = 5 cm.
2. At point A , construct $\angle BAC = 100^\circ$.
3. On the ray AX, mark point C such that AC = 4 cm.
4. Join C to B. Then ΔABC is formed.
5. Draw the perpendicular bisector of AB.
6. Draw the perpendicular bisector of AC and BC.
7. Let these bisectors meet at O. This point O is the circumcentre.
8. With centre O and radius OA, draw the circle through A, B and C. This is the

circumcircle.



Reasoning:

Since $\angle A = 100^\circ$, the triangle is obtuse-angled.

In an obtuse-angled triangle, the circumcentre lies outside the triangle.

Therefore, the centre is outside the triangle.

31. Draw $\triangle ABC$, with $AB = 6cm, BC = 7cm$ and $CA = 7cm$. Draw the circumcircle of $\triangle ABC$. Let the circumcentre be O . Measure OA , OB, OC.

Ans. : 1. Draw AB = 6 cm.

2. With centre A and radius 7 cm, draw an arc.

3. With centre B and radius 7 cm, draw another arc cutting the first arc at C.

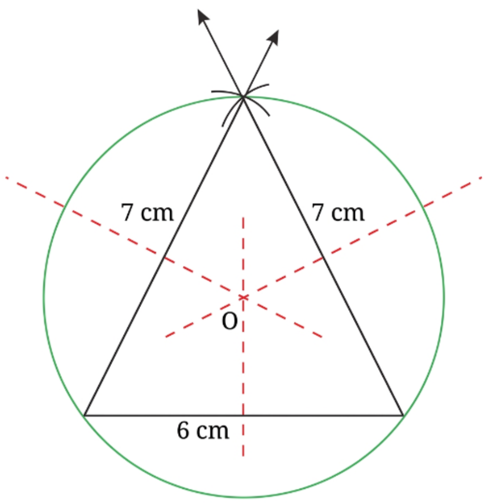
4. Join AC and BC . Then $\triangle ABC$ is formed.

5. Draw the perpendicular bisector of AB.

6. Draw the perpendicular bisectors of AC and BC).

7. Let they intersect at O. This point O is the circumcentre.

8. With centre O and radius OA, draw the circumcircle.



Observation:

Since O is the circumcentre, it is equidistant from all three vertices.

Therefore, OA = OB = OC

Approximate measurement:

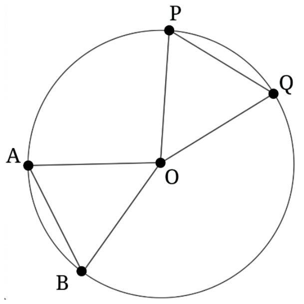
For the triangle with sides 7 cm, 7 cm, 6 cm, OA = OB = OC ≈ 4 cm

So, the three measurements are equal, each approximately 4 cm.

32. Show that if two such isosceles triangles (occurring in the previous question) have equal base length, they are congruent to each other.

Ans. : Let AB and PQ be two equal chords of the same circle with centre O.

Then triangles formed are $\triangle OAB$ and $\triangle OPQ$.



Now, in $\triangle OAB$ and $\triangle OPQ$

OA = OP [Radii of the same circle]

OB = OQ [Radii of the same circle]

AB = PQ [Given]

So, by SSS congruence criterion

$\triangle OAB \cong \triangle OPQ$

Hence, if two such isosceles triangles have equal base length, they are congruent to each other.

Hence proved.

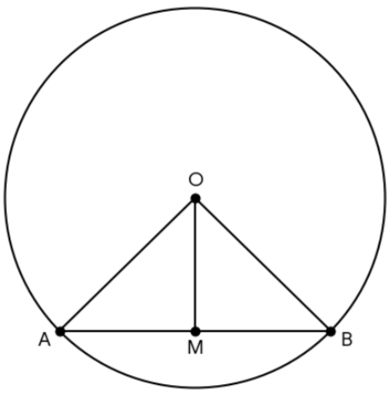
33. Can you explain why the converse to Theorem 4 is true, i.e., why does the perpendicular from the centre of a circle to a chord of the circle bisect the chord?

(Hint: Use Fig.You are told that $\angle CMA = \angle CMB = 90^\circ$. You need to show that $AM = BM$.)

Ans. : Converse of Theorem 4: The perpendicular from the centre of a circle to a chord of the circle bisects the chord.

Let AB be a chord of a circle with centre O , and let $OM \perp AB$.

To Prove: AM = BM

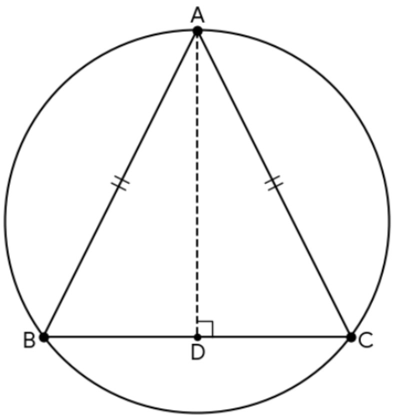


In $\triangle OMA$ and $\triangle OMB$,

OA = OB [Radii of the same circle]
OM = OM [Common to both triangles]
$\angle OMA = \angle OMB$ [Each 90°, as $OM \perp AB$]
Therefore, by RHS congruence
$\triangle OMA \cong \triangle OMB$
Hence, AM = BM [CPCT]
So, M is the midpoint of AB.
Therefore, the perpendicular from the centre of a circle to a chord bisects the chord.
Hence proved.

34. An isosceles triangle ABC is inscribed in a circle, with $AB = AC$. Show that the altitude from A to BC passes through the centre of the circle.

Ans. : Given: $\triangle ABC$ is inscribed in a circle and $AB = AC$.
Since AB = AC, triangle ABC is isosceles with base BC.
Let AD be the altitude from A to BC.
Then $AD \perp BC$



In $\triangle ABD$ and $\triangle ACD$,
AB = AC [Given]
AD = AD [Common to both triangles]
$\angle ADB = \angle ADC$ [Each 90°, as $AD \perp BC$]
Therefore, by RHS congruence
$\triangle ABD \cong \triangle ACD$
Hence, BD = DC [CPCT]
So, AD is bisector of BC.
Since AD is perpendicular to BC and also bisects BC, AD is the perpendicular bisector of chord BC.
Therefore, AD passes through the centre of the circle.
Hence, the altitude from A to BC passes through the centre of the circle.
Hence proved.

35. Two parallel chords of lengths 6 cm and 8 cm are on opposite sides of the centre of a circle. If the radius of the circle is 5 cm , find the distance between

the midpoints of the chords.

Ans. : Given: Radius of the circle OA = OC = 5 cm

Lengths of the two chords are AB = 6 cm and CD = 8 cm

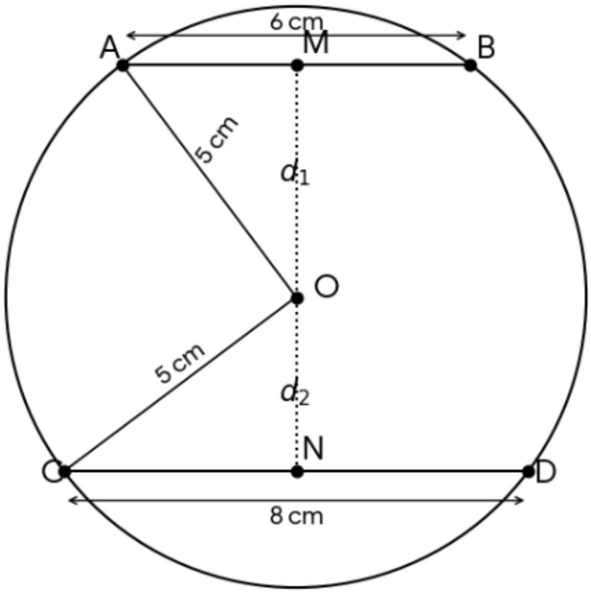
The perpendicular from the centre O to a chords AB and CD bisects the chords at M and N respectively.

So, half-lengths are:

For the 6 cm chord: AM = 3 cm

For the 8 cm chord: CN = 4 cm

Let the distances of the chords from the centre be d_1 and d_2.



In triangle OAM, using Pythagoras theorem:

$OM^2 + AM^2 = OA^2$

$\Rightarrow d_1^2 + 3^2 = 5^2$

$\Rightarrow d_1^2 + 9 = 25$

$\Rightarrow d_1^2 = 16$

$\Rightarrow d_1 = 4cm$

Similarly, in triangle OCN, using Pythagoras theorem:

$ON^2 + CN^2 = OC^2$

$\Rightarrow d_2^2 + 4^2 = 5^2$

$\Rightarrow d_2^2 + 16 = 25$

$\Rightarrow d_2^2 = 9$

$\Rightarrow d_2 = 3cm$

Since the chords are on opposite sides of the centre and their centers lies on same line.

So, the distance between their midpoints $= d_1 + d_2 = 4 + 3 = 7cm$

Therefore, the distance between the midpoints of the two chords is 7 cm.

36. Use the Baudhāyana-Pythagoras theorem to show why Theorem 6 must be true.

Ans. : Theorem 6: Chords of a circle having the same length are at the same distance from the centre.

Let AB and CD be two equal chords of a circle with centre O.

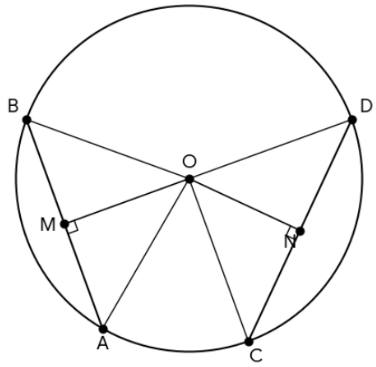
Let $OM \perp AB$ and $ON \perp CD$.

We know that the perpendicular from the centre to a chord bisects the chord.

So, AM = MB = AB/2 and CN = ND = CD/2

Since AB = CD, we get: AM = CN

Also, OA = OC = radius of the circle.



Now apply the Baudhāyana–Pythagoras theorem in right triangles OMA and ONC:

In $\triangle OMA : OA^2 = OM^2 + AM^2$

In $\triangle ONC : OC^2 = ON^2 + CN^2$

Since OA = OC and AM = CN,

So, we have: $OM^2 + AM^2 = ON^2 + CN^2$

Therefore, $OM^2 = ON^2$

⇒ OM = ON

Hence, the equal chords AB and CD are equidistant from the centre.

Thus, Theorem 6 is proved.

37. Consider Fig. If CE is perpendicular to AB, CH is perpendicular to GF, and CE = CH, show that AB = GF. Solve this using the Baudhāyana-Pythagoras theorem.

Ans. : Given: $CE \perp AB, CH \perp GF$ and $CE = CH$.

To Prove: AB = GF

Proof: Since CE is perpendicular to chord AB, it bisects AB.

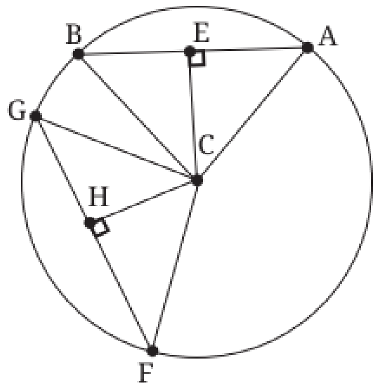
So, E be the midpoint of AB.

⇒ AE = AB/2

Similarly, since CH is perpendicular to chord GF, it bisects GF.

So, H be the midpoint of GF.

⇒ GH = GF/2



Now apply the Baudhāyana-Pythagoras theorem in right triangles $\triangle CEA$ and $\triangle CHG$.

In $\triangle CEA : CA^2 = CE^2 + AE^2$

In $\triangle CHG : CG^2 = CH^2 + GH^2$

But, CA = CG [Radii of same circle]

CE = CH [Given]

Therefore, $CE^2 + AE^2 = CH^2 + GH^2$

$\Rightarrow AE^2 = GH^2$[Since $CE = CH$]

Hence, AE = GH

So, AB/2 = GF/2

Therefore, AB = GF

Hence proved.

38. Find the length of the chord of a circle where the radius is 7 cm and perpendicular distance is 6 cm .

Ans. : Let the chord be AB and O be the centre of the circle.

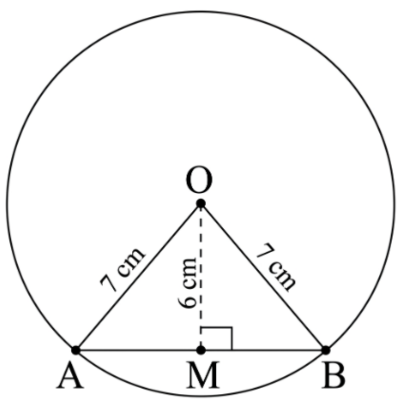
Let OM be the perpendicular from O to chord AB.

Given: Radius, OA = OB = 7 cm

Perpendicular distance, OM = 6 cm

We know that the perpendicular from the centre to a chord bisects the chord.

So, AM = MB



In right triangle OMA,

$OA^2 = OM^2 + AM^2$

$\Rightarrow 7^2 = 6^2 + AM^2$

$\Rightarrow 49 = 36 + AM^2$

$\Rightarrow AM^2 = 13$

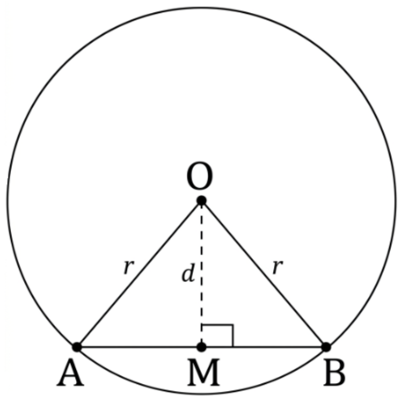
$\Rightarrow AM = \sqrt{13}$

Therefore, chord AB = 2 × AM $= 2\sqrt{13} cm$

Hence, the length of the chord is $2\sqrt{13} cm$.

39. Explain why the following statement is true: If the perpendicular distance of a chord from the centre is d and the radius is r, then the chord length is $2\sqrt{r^2 - d^2}$.

Ans. : Let AB be a chord of a circle with centre O.
Let OM be the perpendicular from O to AB.
Given: Radius = r
Perpendicular distance from centre to chord = d
Since the perpendicular from the centre to a chord bisects the chord, so
M is the midpoint of AB.
So, AM = MB = half of the chord length.



In right triangle OMA, By the Baudhāyana–Pythagoras theorem:

$OA^2 = OM^2 + AM^2$

$\Rightarrow r^2 = d^2 + AM^2$

$\Rightarrow AM^2 = r^2 - d^2$

$\Rightarrow AM = \sqrt{(r^2 - d^2)}$

But chord length AB = 2 × AM

So, $AB = 2\sqrt{(r^2 - d^2)}$

Hence, the chord length is $2\sqrt{(r^2 - d^2)}$.

40. In a circle, if the distance of chord AB from the centre is twice the distance of another chord CD from the centre, then can we conclude that CD = 2 AB ? Give reasons for your answer.

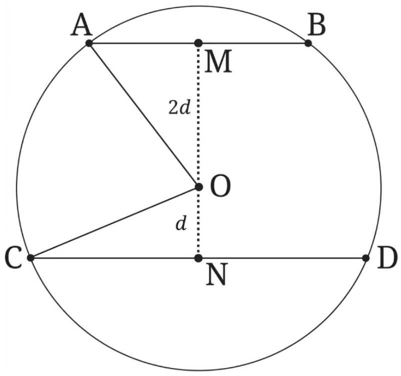
Ans. : No, we cannot conclude that CD = 2AB.
Reason: The length of a chord depends on the formula:
Chord length $= 2\sqrt{(r^2 - d^2)}$
This relation is not directly proportional to the distance from the centre.
Let the distance of chord CD from the centre be d.

Then the distance of chord AB from the centre is 2d.

So, $AB = 2\sqrt{\left(r^2 - (2d)^2\right)} = 2\sqrt{\left(r^2 - 4d^2\right)}$

$CD = 2\sqrt{\left(r^2 - d^2\right)}$

Clearly, CD is not equal to 2AB in general.



For example:

Let radius r = 5 cm

and d = 2 cm

Then distance of AB from centre = 4 cm

Now, $CD = 2\sqrt{\left(5^2 - 2^2\right)}$

$= 2\sqrt{(25 - 4)}$

$= 2\sqrt{21}$

$AB = 2\sqrt{\left(5^2 - 4^2\right)}$

$= 2\sqrt{(25 - 16)}$

$= 2\sqrt{9}$

= 6

Now, $2AB = 12$ but $CD = 2\sqrt{21}$, which is not equal to 12

Therefore, the statement is false.

Hence, we cannot conclude that CD = 2AB.

41. In a circle with centre O , the central angle AOB is 60°. If the radius of the circle is 12 cm , what is the length of the chord AB ?

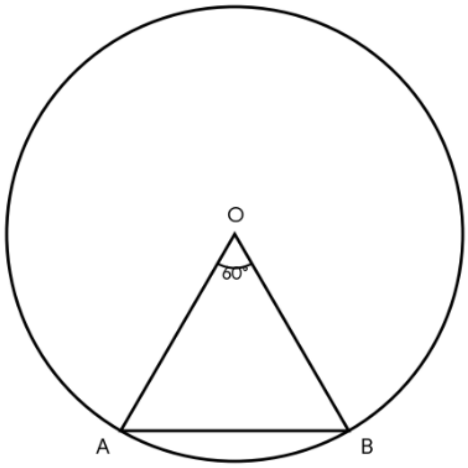
Ans. : Given: Radius OA = OB = 12 cm

$\angle AOB = 60^\circ$

In triangle AOB: OA = OB

So, $\angle OAB = \angle OBA$ [Angles opposite to equal sides of triangle]

Let $\angle OAB = \angle OBA = x$



In triangle, OAB,

$\angle AOB + \angle OAB + \angle OBA = 180^\circ$ [Angles sum property of the triangle]

$\Rightarrow 60^\circ + x + x = 180^\circ$ [Since $\angle AOB = 60^\circ$]

$\Rightarrow 2x = 180^\circ - 60^\circ = 120^\circ$

$\Rightarrow x = 120^\circ/2 = 60^\circ$

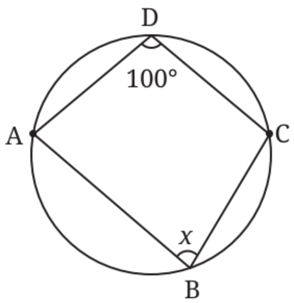
$\Rightarrow \angle OAB = \angle OBA = 60^\circ$

Thus, triangle AOB is equilateral.

Therefore, AB = OA = 12 cm

Hence, the length of chord AB is 12 cm.

42. Let A and B be two points on a circle with centre O .



(i) Are there points X, Y on the circle, on the same side of AB, such that $\angle AXB$ is different from $\angle AYB$?

(ii) Is it true that if $\angle AXB = \angle AYB$, then X and Y lie on the same side of the circle?

(iii) If $\angle AXB = \angle AYB$, and X and Y do not lie on the circle, does the circle through A, B and X also pass through Y ?

Ans. : (i) No.

Reason:

If X and Y lie on the same side of chord AB, then they lie on the same arc AB.

Angles subtended by the same chord in the same segment of a circle are equal.

Therefore, $\angle AXB = \angle AYB$

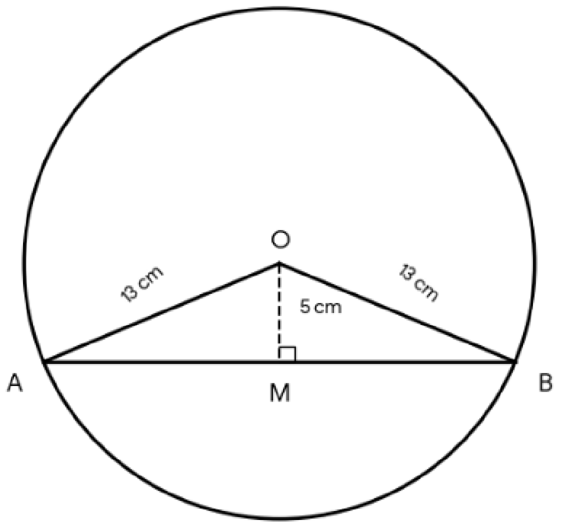
So, there are no such points X and Y on the same side of AB for which the angles are different.

(ii) No, this is not always true.

Reason:
Equal angles can also be subtended by the same chord AB at points on opposite arcs.
So, even if $\angle AXB = \angle AYB, X$ and Y need not lie on the same side of AB.
Therefore, the statement is false.
(iii) Yes.
If $\angle AXB = \angle AYB$, then the line segment AB subtends equal angles at X and Y.
By the converse theorem of concyclicity:
If a line segment subtends equal angles at two points on the same side, then the four points are concyclic.
Hence, A, B, X and Y lie on the same circle.
Therefore, the circle through A, B and X also passes through Y.

43. In a circle, a chord is 5 cm away from the centre. If the radius of the circle is 13 cm , what is the length of the chord?

Ans. : Let AB be the chord and O be the centre.
Let OM be the perpendicular from O to AB.
Given:
OM = 5 cm
OA = 13 cm
Since the perpendicular from the centre to a chord bisects the chord, so, AM = MB.



In right triangle OMA,
$OA^2 = OM^2 + AM^2$
$\Rightarrow 13^2 = 5^2 + AM^2$
$\Rightarrow 169 = 25 + AM^2$
$\Rightarrow AM^2 = 144$
⇒ AM = 12 cm
Therefore, AB = 2 × AM = 2 × 12 = 24 cm
Hence, the length of the chord is 24 cm.

44. The diameter of a circle is 26 cm . A chord of length 24 cm is drawn in the circle. Find the distance from the centre of the circle to the chord.

Ans. : Diameter = 26 cm

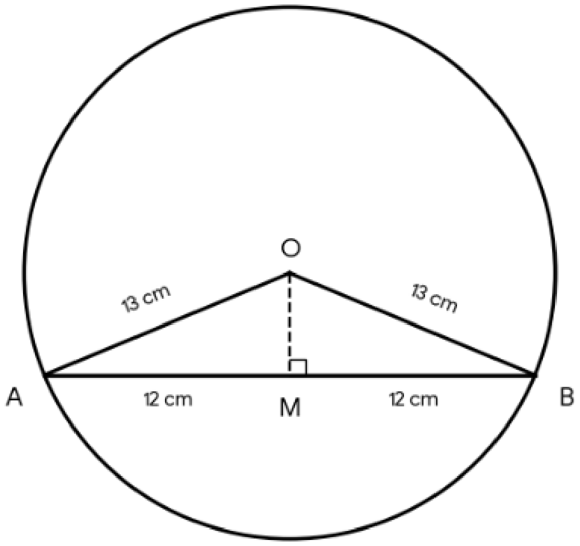
So, radius = 13 cm

Chord AB length = 24 cm

Half of chord AM = 12 cm

Let OM be the perpendicular distance from the centre O to the chord AB.

Then M is the midpoint of the chord.



In right triangle OMA,

$OA^2 = OM^2 + AM^2$

$\Rightarrow 13^2 = OM^2 + 12^2$

$\Rightarrow 169 = OM^2 + 144$

$\Rightarrow OM^2 = 25$

⇒ OM = 5 cm

Hence, the distance from the centre to the chord is 5 cm.

45. A circle has a radius of 15 cm . A chord is drawn. The distance from the centre of the circle to the chord is 9 cm . What is the length of the chord?

Ans. : Let AB be the chord and O be the centre.

Let OM be the perpendicular from O to AB.

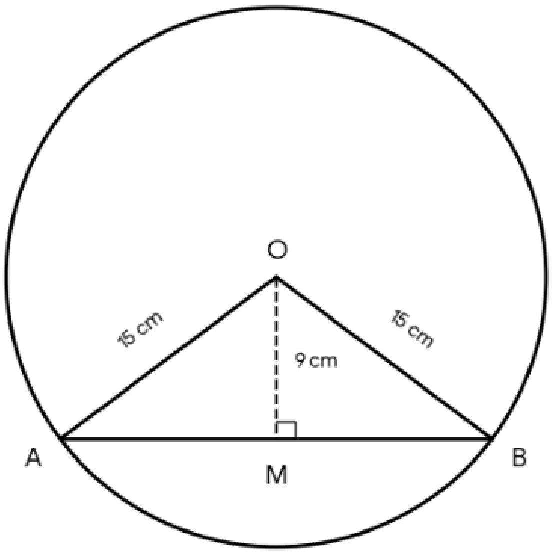
Given:

OA = 15 cm

OM = 9 cm

Since the perpendicular from the centre to a chord bisects the chord,

AM = MB



In right triangle OMA,

$OA^2 = OM^2 + AM^2$

$\Rightarrow 15^2 = 9^2 + AM^2$

$\Rightarrow 225 = 81 + AM^2$

$\Rightarrow AM^2 = 144$

⇒ AM = 12 cm

Therefore, AB = 2 × AM = 24 cm

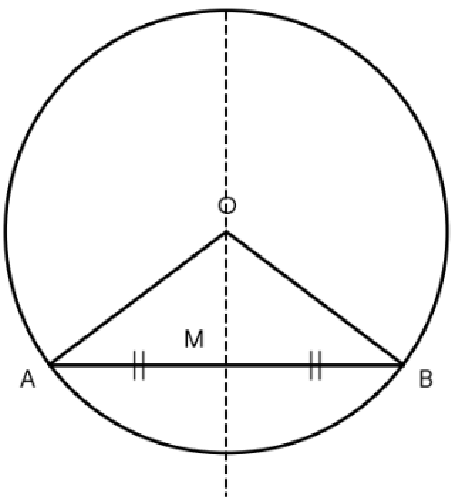
Hence, the length of the chord is 24 cm.

46. Prove that the perpendicular bisector of a chord passes through the centre of the circle.

Ans. : Let AB be a chord of a circle with centre O.

Let M be the midpoint of AB.

We have to show that OM is perpendicular to AB, which means the perpendicular bisector of AB passes through O.



So, in triangles OMA and OMB:

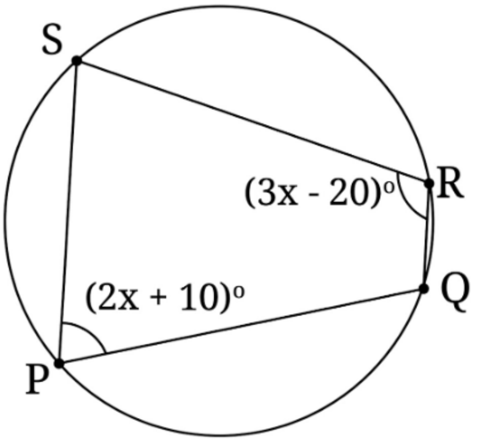
OA = OB [Radii of the same circle]

AM = MB [M is the midpoint of AB]

OM = OM [Common]
Therefore, by SSS congruence
$\triangle OMA \cong \triangle OMB$
Hence, $\angle OMA = \angle OMB$ [CPCT]
But these two angles form a linear pair, so $\angle OMA + \angle OMB = 180^\circ$
Since they are equal, $\angle OMA = \angle OMB = 90^\circ$
Therefore, $OM \perp AB$
So, the line through O and M is the perpendicular bisector of chord AB.
Hence proved.

47. Quadrilateral PQRS is inscribed in a circle. If $\angle P = (2x+10)^\circ$ and $\angle R = (3x-20)^\circ$, find the value of x and the measures of $\angle P$ and $\angle R$.

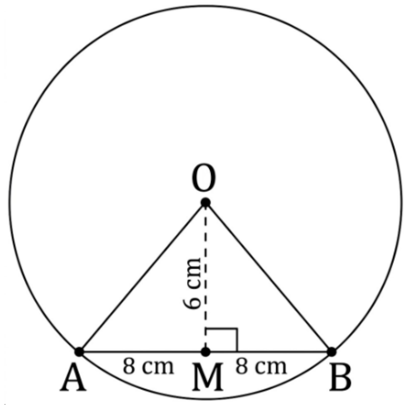
Ans. : Since PQRS is a cyclic quadrilateral, opposite angles are supplementary.
So, $\angle P + \angle R = 180^\circ$
⇒ (2x + 10) + (3x − 20) = 180
⇒ 5x − 10 = 180
⇒ 5x = 190
⇒ x = 38



Now,
$\angle P = 2x + 10 = 2(38) + 10 = 86^\circ$
$\angle R = 3x - 20 = 3(38) - 20 = 94^\circ$
Hence, $x = 38, \angle P = 86^\circ$ and $\angle R = 94^\circ$.

48. The distance of a chord of length 16 cm from the centre of a circle is 6 cm . Find the radius of the circle.

Ans. : Chord length AB = 16 cm
Half chord AM = MB = 8 cm
Distance from centre to chord OM = 6 cm
Let r be the radius.



In the right triangle OAM,

$OA^2 = AM^2 + OM^2$

$\Rightarrow r^2 = 8^2 + 6^2$

$\Rightarrow r^2 = 64 + 36$

$\Rightarrow r^2 = 100$

⇒ r = 10 cm

Hence, the radius of the circle is 10 cm.

49. When two chords intersect, each of them is divided into two line segments. Show that if the intersecting chords are of equal length, then the line segments of one chord are equal to the corresponding line segments of the other chord.

Ans. : Let chords AB and CD intersect at point P inside the circle, and O be the centre.

From the figure, $ON \perp CD$ and $OM \perp AB$.

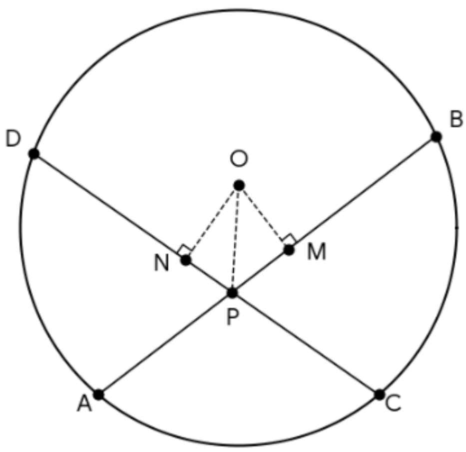
Equal chords ⇒ equal distances from centre

Given: AB = CD ...(1)

To Prove: PB = PD and AP = CP.

So, the perpendicular distances from the centre are equal.

So, OM = ON



In $\triangle OPM$ and $\triangle OPN$, we have

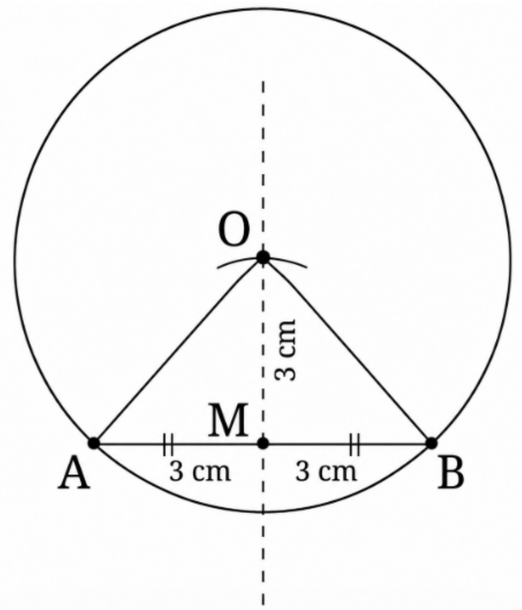
OP = OP [common side]

OM = ON [Proved above]

$\angle OMP = \angle ONP = 90^{\circ}[$ Perpendiculars $]$
So, by RHS congruency
$\triangle OPM \cong \triangle OPN$
So, PM = PN [CPCT]...(2)
Now, $OM \perp AB$, so $AM = MB$ [Perpendicular from the centre bisect the chord]
Similarly, $ON \perp CD$, so $CN = ND$
Since, AB = CD [From (1)]
Therefore, $BM = DN[BM = 1/2AB$ and $DN = 1/2CD] \ldots$ (3)
Adding (2) and (3), we get
PM + BM = PN + DN
⇒ PB = PD ...(4)
Subtracting (4) from (1), we get
AB – PB = CD – PD
⇒ AP = CP
Hence proved.

50. Draw a circle in which a chord of 6 cm length stands at a distance of 3 cm from the centre.
(Hint: Is it a circumcircle of a suitable triangle?)

Ans. : Given:
Chord length = 6 cm
Distance from centre to chord = 3 cm
Steps of Construction:
1. Draw a line segment AB = 6 cm.
2. Find the midpoint M of AB.
3. Draw a perpendicular line to AB at M.
4. On this perpendicular, mark a point O such that OM = 3 cm.
5. Join OA or OB.
6. With centre O and radius OA, draw a circle.



Since OM is perpendicular to AB and passes through its midpoint, AB is a chord of the circle.

Also, AM = 3 cm and OM = 3 cm

So, $OA^2 = OM^2 + AM^2$

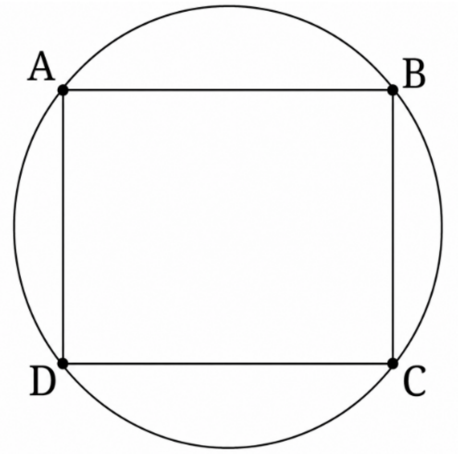
$\Rightarrow OA^2 = 3^2 + 3^2 = 18$

$\Rightarrow OA = 3\sqrt{2}cm$

Therefore, the required circle has centre O and radius $3\sqrt{2}cm$.

51. Show that rectangle is the only parallelogram that can be inscribed in a circle.

Ans. : Let ABCD be a parallelogram inscribed in a circle.



Since it is a cyclic quadrilateral:

$\angle A + \angle C = 180^\circ$

But in a parallelogram, opposite angles are equal: $\angle A = \angle C$

Therefore, $\angle A + \angle A = 180^\circ$

$\Rightarrow 2\angle A = 180^\circ$

$\Rightarrow \angle A = 90^\circ$

Similarly, all angles are 90°.

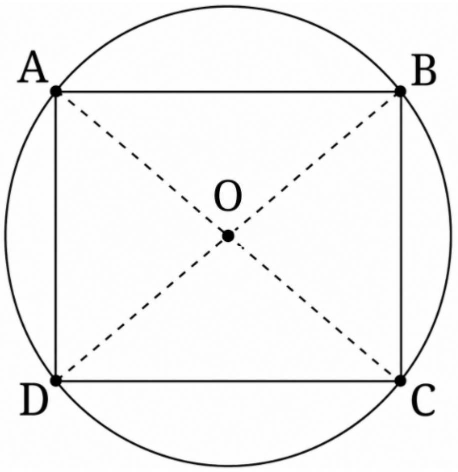
Hence, the parallelogram is a rectangle.

Therefore, a rectangle is the only parallelogram that can be inscribed in a circle.

52. Show that if a rectangle is inscribed in a circle, then the point of intersection of its diagonals must lie at the centre of the circle.

Ans. : Let ABCD be a rectangle inscribed in a circle.

Let its diagonals AC and BD intersect at O.



In a rectangle, diagonals are equal and bisect each other.

So, OA = OC and OB = OD

Also, AC = BD

Therefore, OA = OB = OC = OD

This means O is equidistant from all four vertices A, B, C and D.

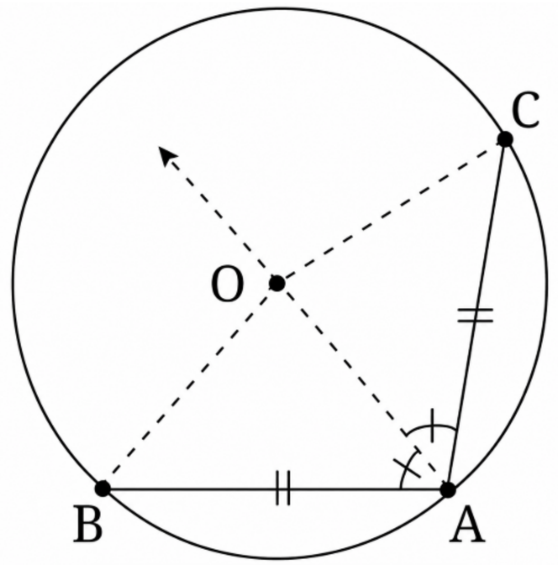
The point equidistant from all points on a circle is the centre.

Hence, the point of intersection of the diagonals is the centre of the circle.

53. In a circle with centre O, chords AB and AC are congruent. Explain why this statement is true: “The centre of the circle lies on the angle bisector of $\angle BAC$”.

Ans. : Given: AB = AC

Join OA, OB and OC.



In $\triangle AOB$ and $\triangle AOC$, we have

Now, OB = OC [Both are radii]

OA = OA [Common]

AB = AC [Given]

Therefore, by SSS congruence

$\triangle AOB \cong \triangle AOC$

So, $\angle BAO = \angle OAC$

Hence, AO bisects $\angle BAC$.

Therefore, the centre O lies on the angle bisector of $\angle BAC$.

54. Two parallel chords of lengths 10 cm and 24 cm are on the same side of the centre of a circle. The distance between the chords is 7 cm. Find the radius of the circle.

Ans. : Let the radius be r. So, OA = OC = r.

For chord AB of length 10 cm:

Half of chord AB = 5 cm

Let its distance from centre OM $= d_1$

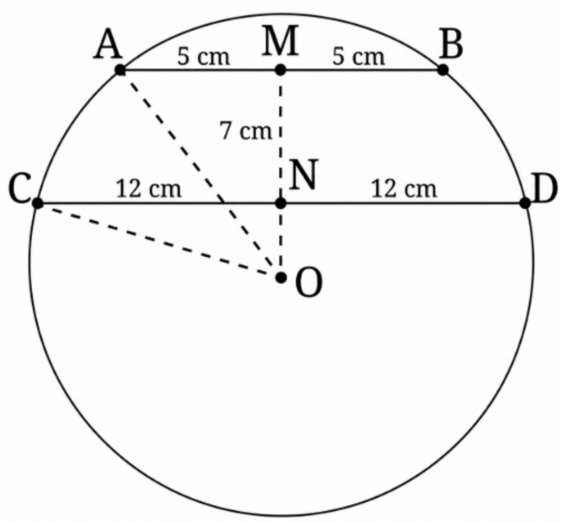
For chord CD of length 24 cm:

Half of chord CD = 12 cm

Let its distance from centre ON $= d_2$

Since the longer chord is closer to the centre:

$d_1 - d_2 = 7$ ...(1)



In triangle OAM, using Pythagoras theorem, we have

$OA^2 = OM^2 + AM^2$

$\Rightarrow r^2 = d_1^2 + 5^2$

$\Rightarrow r^2 = d_1^2 + 25$

In triangle OCN, using Pythagoras theorem, we have

$OC^2 = ON^2 + CN^2$

$\Rightarrow r^2 = d_2^2 + 12^2$

$\Rightarrow r^2 = d_2^2 + 144$

So, $d_1^2 + 25 = d_2^2 + 144$

$\Rightarrow d_1^2 - d_2^2 = 119$

$\Rightarrow (d_1 - d_2)(d_1 + d_2) = 119$,

$\Rightarrow 7(d_1 + d_2) = 119$ [Since $d_1 - d_2 = 7$]

$\Rightarrow d_1 + d_2 = 17 \ldots (2)$

Now solving equations (1) and (2), we have

$2d_1 = 24$

$\Rightarrow d_1 = 12$

Then $d_2 = 5$

Now, $r^2 = d_1^2 + 5^2$

$= 12^2 + 5^2$

$= 144 + 25$

$= 169$

$\Rightarrow r = 13cm$

Therefore, the radius of the circle is 13 cm.

55. A regular hexagon is inscribed in a circle of radius r. Find the length of the sides of the hexagon and the distance of each side from the centre of the circle.

Ans. : A regular hexagon divides the circle into 6 equal central angles.

Each central angle $= 360°/6 = 60°$

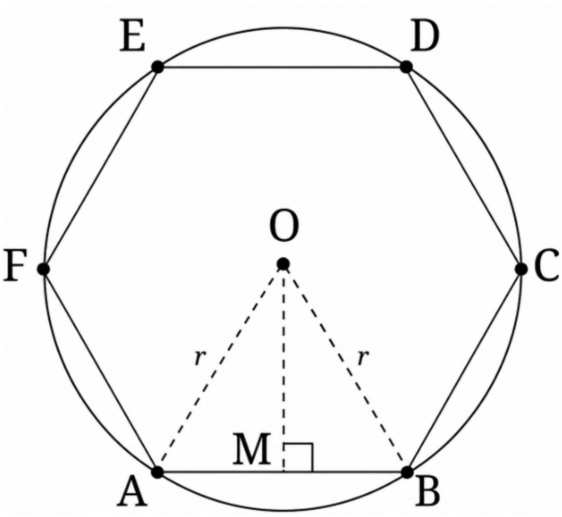
Let AB be one side of the hexagon and O be the centre.

Then $OA = OB = r$ and $\angle AOB = 60°$

So, $\triangle AOB$ is an equilateral triangle.

Therefore, AB = r

Thus, each side of the regular hexagon is r.



Now find the distance of each side from the centre.

Let OM be perpendicular to AB.

Since $\triangle AOB$ is equilateral: $AM = AB/2 = r/2$

In right triangle OMA:

$OA^2 = OM^2 + AM^2$

$\Rightarrow r^2 = OM^2 + (r/2)^2$

$\Rightarrow OM^2 = r^2 - r^2/4$

$\Rightarrow OM^2 = 3r^2/4$

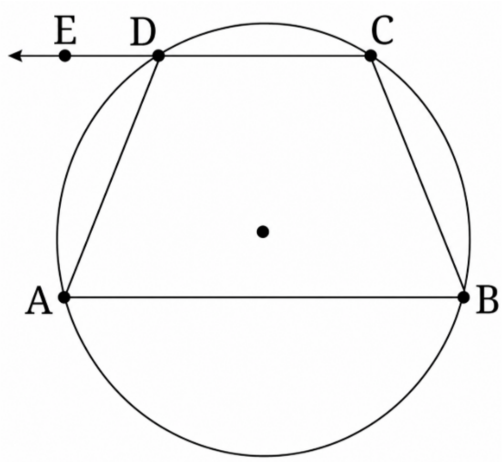
$\Rightarrow OM = (\sqrt{3}/2)r$

Therefore, Side length of hexagon = r

Distance of each side from centre = $= (\sqrt{3}/2)r$.

56. Let ABCD be a cyclic quadrilateral. Explain why the exterior angle at any vertex is equal to the interior opposite angle (e.g., $\angle CDE = \angle ABC$, where E is a point on the extension of side CD).

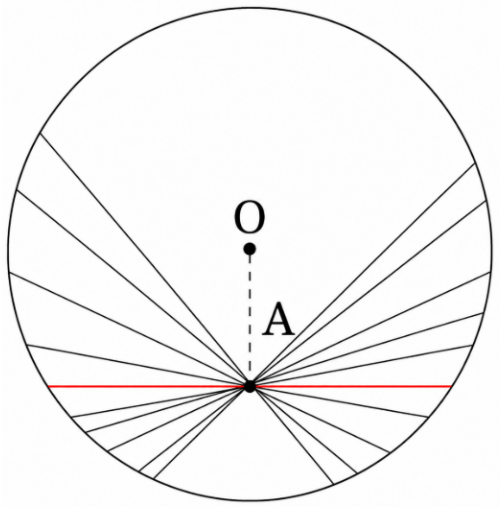
Ans. : In cyclic quadrilateral ABCD:
∠ABC + ∠ADC = 180° ...(1)
Since E lies on the extension of CD,
$\angle ADC$ and $\angle CDE$ form a linear pair.
So, $\angle ADC + \angle CDE = 180^\circ \ldots (2)$



From (1) and (2), we have
$\angle ABC + \angle ADC = \angle ADC + \angle CDE$
Subtracting ∠ADC from both sides, we get
$\angle ABC = \angle CDE$
Therefore, the exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.
Hence proved.

57. Let A be any point within a given circle with centre O. Show that the shortest chord of the circle that passes through point A is the one that is perpendicular to OA.

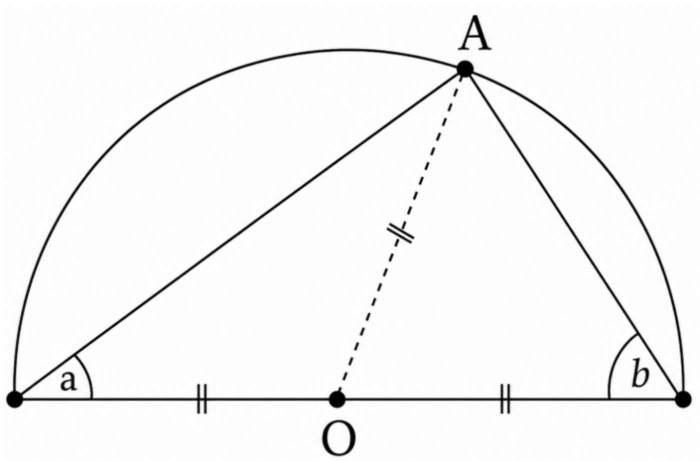
Ans. : Let the circle have centre O and let A be a point inside it.
Any chord passing through A will have some perpendicular distance from the centre O.
We know that the farther a chord is from the centre, the shorter it is.
So, the shortest chord through A will be the chord whose distance from O is maximum.
Among all lines passing through A, the perpendicular distance from O to the line is greatest when the line is perpendicular to OA.



Therefore, the chord through A that is perpendicular to OA is farthest from the centre.
Hence, it is the shortest chord passing through A.
Therefore, the shortest chord through A is the one perpendicular to OA.
Hence proved.

58. How would you use the following figure to justify the statement that the angle in a semicircle is 90°?

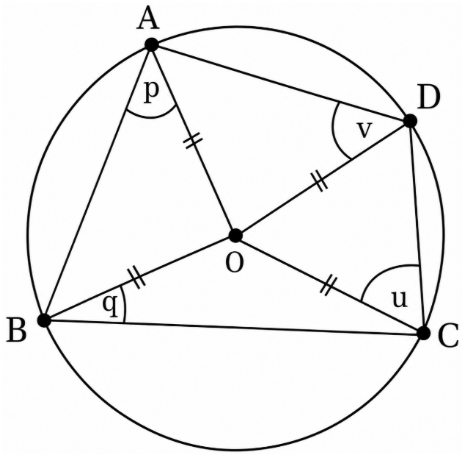
Ans. :



Let the diameter be BC and O be the centre of the circle.
Point A lies on the semicircle.
Join OA.
Since O is the centre:
OA = OB = OC
because all are radii of the same circle.
So, $\triangle AOB$ is isosceles and $\triangle AOC$ is isosceles.
Let: $\angle ABO = a$ and $\angle ACO = b$
Then: $\angle BAO = a$ and $\angle OAC = b$[Since $\angle ABO = \angle BAO$ and $\angle ACO = \angle OAC$]
Therefore, $\angle BAC = a + b$
Now, in triangle ABC:
$\angle ABC + \angle BAC + \angle ACB = 180^{\circ}$

So, $a + (a + b) + b = 180^\circ$
$\Rightarrow 2a + 2b = 180^\circ$
$\Rightarrow 2(a + b) = 180^\circ$
$\Rightarrow a + b = 90^\circ$
But, $\angle BAC = a + b$
Therefore, $\angle BAC = 90^\circ$
Hence, the angle in a semicircle is 90°.

59. How would you use the following figure to justify the statement that the sum of the opposite angles of a cyclic quadrilateral is 180°?



Ans. : Let ABCD be a cyclic quadrilateral with centre O.
Join OA, OB, OC and OD.
Since OA, OB, OC and OD are radii of the same circle:
OA = OB = OC = OD
So the triangles formed with the centre are isosceles triangles.
From the figure:
Let $\angle OAB = p$ and $\angle OBA = q$.
Let $\angle ODC = v$ and $\angle OCD = u$.
Using the angle-sum property in the triangles around the centre, the full angle around O is: 360°
The angle at the centre corresponding to the arcs gives twice the angle at the circumference.
Thus: $\angle A + \angle C = 180^\circ$
Similarly: $\angle B + \angle D = 180^\circ$
Therefore, the sum of opposite angles of a cyclic quadrilateral is 180°.
Hence proved.

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