

MARKING SCHEME**CRPF PUBLIC SCHOOL, ROHINI, DELHI****MID-TERM EXAMINATION (2026-27)****MATHEMATICS (SET- A)****CLASS - XII**

SECTION – A (MCQ) 1 Mark Questions	
Q1	(C) $[-1, 1]$
Q2	(B) symmetric
Q3	(A) $y = \sec^{-1} x$
Q4	(A) -1
Q5	(B) -3
Q6	(A) $A^2 - AB + BA - B^2$
Q7	(D) 8
Q8	(B) -10
Q9	(C) -2
Q10	(A) $a = 3, b = -8$
Q11	(C) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$.
Q12	(A) $\frac{x}{y}$
Q13	(B) $\frac{2x}{1+x^4}$
Q14	(B) $2\left(\sin \frac{x}{2} - \cos \frac{x}{2}\right) + C$
Q15	(C) $\frac{-1}{2x^2}$
Q16	(C) 1
Q17	(B) constraints
Q18	(C) It will have both maximum and minimum values.
Q19	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
Q20	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
SECTION – B (Very Short Answer (VSA)-type questions) 2 Marks Each	

Q21	$x = e^{\sin^{-1} t} \Rightarrow \frac{dx}{dt} = e^{\sin^{-1} t} \times \frac{1}{\sqrt{1-t^2}}$$y = e^{\cos^{-1} t} \Rightarrow \frac{dy}{dt} = e^{\cos^{-1} t} \times \frac{-1}{\sqrt{1-t^2}}$ Now, $\frac{dy}{dx} = -\frac{e^{\cos^{-1} t}}{e^{\sin^{-1} t}}$ or $-e^{\cos^{-1} t - \sin^{-1} t}$ $\left. \frac{dy}{dx} \right _{t=\frac{1}{\sqrt{2}}} = -1$ Note: Proportionate marks may be allotted if a student uses property $\sin^{-1} t + \cos^{-1} t = \frac{\pi}{2}$ to solve this question.	I II III	1 $\frac{1}{2}$ $\frac{1}{2}$
OR			
(b) Given $\sqrt{3}(x^2 + y^2) = 4xy$ On differentiating w.r.t. x, we get $\sqrt{3}\left(2x + 2y \frac{dy}{dx}\right) = 4\left(x \frac{dy}{dx} + y\right)$ $\Rightarrow x\sqrt{3} + y\sqrt{3} \frac{dy}{dx} = 2x \frac{dy}{dx} + 2y$ $\Rightarrow (y\sqrt{3} - 2x) \frac{dy}{dx} = 2y - x\sqrt{3}$ $\Rightarrow \frac{dy}{dx} = \frac{2y - x\sqrt{3}}{y\sqrt{3} - 2x}$ Hence, $\frac{dy}{dx}$ at $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) = \frac{2 \times \frac{\sqrt{3}}{2} - \sqrt{3} \times \frac{1}{2}}{\sqrt{3} \times \frac{\sqrt{3}}{2} - 2 \times \frac{1}{2}} = \frac{\sqrt{3} - \frac{\sqrt{3}}{2}}{\frac{3}{2} - 1} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$.			
Q22	(a) Let $y = \tan^{-1}\left(\frac{\cos 2x - \sin 2x}{\cos 2x + \sin 2x}\right)$, $0 < x < \frac{\pi}{4}$ $\Rightarrow y = \tan^{-1}\left(\frac{1 - \tan 2x}{1 + \tan 2x}\right) = \tan^{-1}\left(\frac{\tan \frac{\pi}{4} - \tan 2x}{1 + \tan \frac{\pi}{4} \cdot \tan 2x}\right) = \tan^{-1}\left[\tan\left(\frac{\pi}{4} - 2x\right)\right]$ $\therefore y = \left(\frac{\pi}{4} - 2x\right)$ $\because 0 < x < \frac{\pi}{4} \Rightarrow 0 < 2x < \frac{\pi}{2} \Rightarrow -\frac{\pi}{4} < 2x - \frac{\pi}{4} < \frac{\pi}{4} \Rightarrow -\frac{\pi}{4} < \frac{\pi}{4} - 2x < \frac{\pi}{4}$ Also $\tan^{-1} \tan x = x$, when $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.		

Q23	$A^2 = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$ $A^2 - 5A + 4I = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ $A^2 - 5A + 4I = \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$	I	1½ (½ for any three correct elements)
		II	1
		III	½
Q24	$f(x) = x^4 - 4x^3 + 10$ $\Rightarrow f'(x) = 4x^3 - 12x^2 = 4x^2(x - 3)$ For $f'(x) = 4x^2(x - 3) = 0 \Rightarrow x = 0, 3$ Note that, in $x \in (-\infty, 0) \cup (0, 3)$, $f'(x) < 0$. So, the function $f(x)$ is strictly decreasing in $x \in (-\infty, 0) \cup (0, 3)$. OR $f(x) = 4x^3 - 18x^2 + 27x - 7$ $\Rightarrow f'(x) = 12x^2 - 36x + 27 = 3(4x^2 - 12x + 9) = 3(2x - 3)^2$ For $f'(x) = 0$, $3(2x - 3)^2 = 0 \Rightarrow x = \frac{3}{2}$ As the sign of $f'(x)$ does not change as x passes through $x = \frac{3}{2}$. That is, the sign of $f'(x)$ remains positive in the left and right neighborhood of $x = \frac{3}{2}$. So, the function f has neither maxima nor minima.		
Q25	$\frac{d}{dx}(F(x)) = \frac{1}{e^x + 1}$ $\Rightarrow F(x) = \int \frac{1}{e^x + 1} dx$ $= \int \frac{e^{-x}}{1 + e^{-x}} dx$ $= -\log(1 + e^{-x}) + C$ Now, $F(0) = -\log 2 + C \Rightarrow \log \frac{1}{2} + \log 2 = C \Rightarrow C = 0$ $\therefore F(x) = -\log(1 + e^{-x})$ Note: The above expression is equivalent to $F(x) = x - \log(e^x + 1)$	I	1
		II	1
		III	½
		IV	½
SECTION – C (Short Answer (SA)-type questions) 3 Marks Each			
Q26	One-One: $f(x_1) = f(x_2) \Rightarrow 2x_1^3 - 7 = 2x_2^3 - 7 \Rightarrow x_1^3 = x_2^3 \Rightarrow x_1 = x_2$. (Injective) Onto: Let $y = 2x^3 - 7 \Rightarrow x = \sqrt[3]{\frac{y+7}{2}} \in R$ for all $y \in R$. (Surjective). Thus, bijective.		

	OR Reflexive: $d(P, P) = 0 < 3 \Rightarrow (P, P) \in R$. Symmetric: If $d(P, Q) < 3$, then $d(Q, P) < 3 \Rightarrow (Q, P) \in R$. Transitive: If $d(P, Q) = 2$ and $d(Q, S) = 2$, $d(P, S)$ could be 4 (not < 3). Not transitive. Hence, not an equivalence relation.		
Q27	Solution: Curve passes through (3, 1): $1 = \frac{3a-b}{(2)(-1)} \Rightarrow 3a - b = -2$ (Eq 1). $\frac{dy}{dx} = \frac{a(x^2-5x+4)-(ax-b)(2x-5)}{(x^2-5x+4)^2}$. At $x = 3$, $\frac{dy}{dx} = 0 \Rightarrow a(-2) - (3a-b)(1) = 0 \Rightarrow -5a + b = 0 \Rightarrow b = 5a$ (Eq 2). Solving: $3a - 5a = -2 \Rightarrow a = 1$, and $b = 5$.		
Q28	(a) $y\sqrt{x^2+1} = \log[\sqrt{x^2+1}-x]$ $\Rightarrow y \times \frac{1}{2\sqrt{x^2+1}} \times 2x + \sqrt{x^2+1} \times \frac{dy}{dx} = \frac{1}{\sqrt{x^2+1}-x} \times \left(\frac{1}{2\sqrt{x^2+1}} \times 2x - 1 \right)$ $\Rightarrow \frac{yx}{\sqrt{x^2+1}} + \sqrt{x^2+1} \times \frac{dy}{dx} = \frac{1}{\sqrt{x^2+1}-x} \times \left(\frac{x}{\sqrt{x^2+1}} - 1 \right)$ $\Rightarrow \frac{yx}{\sqrt{x^2+1}} + \sqrt{x^2+1} \times \frac{dy}{dx} = \frac{1}{-\left[x - \sqrt{x^2+1} \right]} \times \left(\frac{x - \sqrt{x^2+1}}{\sqrt{x^2+1}} \right)$ $\Rightarrow \frac{yx}{\sqrt{x^2+1}} + \sqrt{x^2+1} \times \frac{dy}{dx} = -\frac{1}{\sqrt{x^2+1}}$ $\Rightarrow yx + (x^2+1) \times \frac{dy}{dx} = -1$ Hence, $(x^2+1) \frac{dy}{dx} + xy + 1 = 0$.		
Q29	Let $I = \int_0^{\pi} \frac{\sin^{2026}x}{\sin^{2026}x + \cos^{2026}x} dx = 2 \int_0^{\frac{\pi}{2}} \frac{\sin^{2026}x}{\sin^{2026}x + \cos^{2026}x} dx$ Getting $I = 2 \int_0^{\frac{\pi}{2}} \frac{\cos^{2026}x}{\sin^{2026}x + \cos^{2026}x} dx$ On adding, we get $2I = 2 \int_0^{\frac{\pi}{2}} 1 dx \Rightarrow I = \frac{\pi}{2}$ OR	I II III	1 1 1

Q32	$\Delta = \begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$ $= (1+x)[(1+y)(1+z)-1] - 1[(1+z)-1] + 1[1-(1+y)]$ $= (1+x)(1+z+y+yz-1) - z - y$ $= xy + yz + zx + xyz$ $\Delta = 0 \Rightarrow xy + yz + zx + xyz = 0$ $\Rightarrow yz + zx + xy = -xyz$ Dividing both sides by xyz, $x^{-1} + y^{-1} + z^{-1} = -1$ Note: Proportionate marks may be allotted if a student uses properties of determinants to solve this question.	I II III IV	2 1 1 1
Q33	OR Since $AA^{-1} = I$ so, $\begin{bmatrix} -1 & a & 2 \\ 1 & 2 & x \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ b & y & 3 \end{bmatrix} = I$ $\Rightarrow \begin{bmatrix} -1-8a+2b & 1+7a+2y & 5-5a \\ -15+bx & 13+xy & -9+3x \\ -5+b & 4+y & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ By equality of matrices, we get $5-5a=0 \Rightarrow a=1$; $b-5=0 \Rightarrow b=5$; $4+y=0 \Rightarrow y=-4$; $-9+3x=0 \Rightarrow x=3$ Hence, $(a+x)-(b+y) = (1+3)-(5-4) = 4-1 = 3$.	I II III	$\frac{1}{2}$ $1\frac{1}{2}$ 1

(b)

$$(b) y = P \cos ux + Q \sin ux \dots (i)$$

$$\Rightarrow \frac{dy}{dx} = -u P \sin ux + u Q \cos ux$$

$$\Rightarrow \frac{d^2y}{dx^2} = -u^2 P \cos ux - u^2 Q \sin ux = -u^2 (P \cos ux + Q \sin ux)$$

$$\text{By (i), } \frac{d^2y}{dx^2} = -u^2 y$$

$$\therefore \frac{d^2y}{dx^2} + u^2 y = 0.$$

Q34

Let $ABCD$ be a rectangle and let the semi-circle is described on the side AB as its diameter. Let $AB = 2x$ and $AD = 2y$. Let $P = 10$ m be the given perimeter of window.

Therefore, $10 = 2x + 4y + \pi x$

$$\Rightarrow 4y = 10 - 2x - \pi x \dots (i)$$

Area of the window,

$$A = (2x)(2y) + \frac{1}{2} \pi x^2$$

$$\Rightarrow A = 4xy + \frac{1}{2} \pi x^2$$

$$\Rightarrow A = 10x - 2x^2 - \pi x^2 + \frac{1}{2} \pi x^2 \quad [\text{using (i)}]$$

$$\Rightarrow A = 10x - 2x^2 - \frac{1}{2} \pi x^2$$

$$\text{On diff. w.r.t. } x \text{ both sides, } \frac{dA}{dx} = 10 - 4x - \pi x$$

$$\text{Again diff. w.r.t. } x \text{ both sides, } \frac{d^2A}{dx^2} = -(4 + \pi)$$

For maxima or minima,

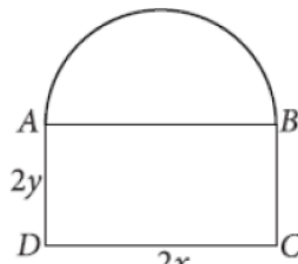
$$\frac{dA}{dx} = 0 \Rightarrow 10 - 4x - \pi x = 0 \Rightarrow x = \frac{10}{4 + \pi}$$

$$\left[\frac{d^2A}{dx^2} \right]_{x=\frac{10}{4+\pi}} = -(4 + \pi) < 0$$

So, A is maximum at $x = \left(\frac{10}{4 + \pi} \right)$ m.

Now, length of the window is $2x = \left(\frac{20}{4 + \pi} \right)$ m and

width is $2y = \left(\frac{10}{4 + \pi} \right)$ m.



Q35	Let $\frac{x}{(x-1)(x^2+4)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+4}$ on solving we get, $A = \frac{1}{5}, B = \frac{-1}{5}, C = \frac{4}{5}$ so, $\int \frac{x}{(x-1)(x^2+4)} dx = \frac{1}{5} \int \frac{dx}{x-1} - \frac{1}{10} \int \frac{2xdx}{x^2+4} + \frac{4}{5} \int \frac{dx}{x^2+4}$ $\int \frac{x}{(x-1)(x^2+4)} dx = \frac{1}{5} \log x-1 - \frac{1}{10} \log(x^2+4) + \frac{2}{5} \tan^{-1}\left(\frac{x}{2}\right) + c$	I	1
	OR	II	1½
	Let $I = \int \frac{2 \sin 2\phi - \cos \phi}{6 - \cos^2 \phi - 4 \sin \phi} d\phi$. Then, $I = \int \frac{(4 \sin \phi - 1) \cos \phi}{6 - (1 - \sin^2 \phi) - 4 \sin \phi} d\phi = \int \frac{(4 \sin \phi - 1) \cos \phi}{\sin^2 \phi - 4 \sin \phi + 5} d\phi$ Putting $\sin \phi = t$ and $\cos \phi d\phi = dt$, we get $I = \int \frac{4t-1}{t^2-4t+5} dt$ Let $(4t-1) = \lambda \frac{d}{dt}(t^2-4t+5) + \mu \Rightarrow (4t-1) = \lambda(2t-4) + \mu$. Comparing coefficients of like powers of t, we get: $2\lambda = 4, -4\lambda + \mu = -1 \Rightarrow \lambda = 2, \mu = 7$ $\therefore I = \int \frac{\lambda(2t-4) + \mu}{t^2-4t+5} dt = 2 \int \frac{2t-4}{t^2-4t+5} dt + 7 \int \frac{1}{t^2-4t+5} dt$ $\Rightarrow I = 2 \log t^2-4t+5 + 7 \int \frac{1}{(t-2)^2+1^2} dt = 2 \log t^2-4t+5 + 7 \tan^{-1}(t-2) + C$ $\Rightarrow I = 2 \log \sin^2 \phi - 4 \sin \phi + 5 + 7 \tan^{-1}(\sin \phi - 2) + C$	III	2½
SECTION – E (Case Study questions) 4 Marks Each			

Q36	(i)	Number of relations from A to B is 2^{12} or 4096	I	1
	(ii)	No R is not injective	I	$\frac{1}{2}$
		It is not a function as G_1 has two images B_1 and B_2	II	$\frac{1}{2}$
	(iii)	(a) R is reflexive as $(x, x) \in R \quad \forall x \in A$ because x and x are students from same colony	I	$\frac{1}{2}$
		Let $(x, y) \in R$ so x and y are students from same colony		
		Hence y and x are students from same colony		
		Thus, $(y, x) \in R$	II	$\frac{1}{2}$
		$\therefore$ R is symmetric		
		Let $(x, y) \in R$ and $(y, z) \in R$ so x and y are students from same colony, y and z are students from same colony		
		Thus, x and z are students from same colony	III	$\frac{1}{2}$
Q37		$\therefore (x, z) \in R$		
		Thus, R is transitive		
		As R is reflexive, symmetric and transitive, hence R is an equivalence relation	IV	$\frac{1}{2}$
		OR		
	(iii) (b)	Any function f from B to A will not be bijective	I	1
		As f from B to A cannot be surjective because $n(B) < n(A)$	II	1
		or		
		A and B are finite sets and $n(A) \neq n(B)$		
	(i)	$8x + 4y = 200 ; 5x + 10y = 275$		
	(ii)	$\begin{bmatrix} 8 & 4 \\ 5 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 200 \\ 275 \end{bmatrix}$		
Q37	(iii) (a)	$ A = 60$		
		Or		
	(iii) (b)	$\text{adj}A = \begin{bmatrix} 10 & -4 \\ -5 & 8 \end{bmatrix}$		

Q38	(i) $\tan \alpha = \frac{r}{h} = \frac{5}{15} \Rightarrow r = \frac{h}{3}$	I	1
	(ii) Volume, $V = \frac{1}{27} \pi h^3 \Rightarrow \frac{dV}{dt} = \frac{1}{9} \pi h^2 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{40\pi} \text{ cm/sec}$	I	1
	(iii) (a) Upper surface area of juice, $A = \pi r^2 = \frac{\pi h^2}{9}$	I	1
	$\frac{dA}{dt} = \frac{2\pi h}{9} \frac{dh}{dt} = \frac{1}{30} \text{ cm}^2/\text{sec}$	II	1
	OR		
	(iii) (b) Wetted surface area, $S = \pi r l = \frac{\sqrt{10}\pi h^2}{9}$	I	1
	$\frac{dS}{dt} = \frac{2\sqrt{10}\pi h}{9} \frac{dh}{dt} = \frac{\sqrt{10}}{30} \text{ cm}^2/\text{sec}$	II	1