

CRPF PUBLIC SCHOOL, ROHINI, DELHI

MID TERM EXAMINATION (2026-27)

MATHEMATICS (SET- A)

CLASS - XII

Time Allowed: 3 hrs

Maximum Marks: 80

General Instructions:

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has **18 MCQ's** and **02 Assertion Reasoning based questions** of 1 mark each.
3. **Section B** has **5 Very Short Answer (VSA)-type questions** of 2 marks each.
4. **Section C** has **6 Short Answer (SA)-type questions** of 3 marks each.
5. **Section D** has **4 Long Answer (LA)-type questions** of 5 marks each.
6. **Section E** has **3 source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

SECTION – A (MCQ) 1 Mark Questions

Q1	Domain of $f(x) = \cos^{-1} x + \sin x$ is : (A) $\mathbb{R}$ (B) $(-1, 1)$ (C) $[-1, 1]$ (D) ϕ
Q2	If R be a relation defined as aRb iff $ a - b > 0$, $a, b \in \mathbb{R}$ then R is : (A) reflexive (B) symmetric (C) transitive (D) symmetric and transitive
Q3	The given graph illustrates : (A) $y = \sec^{-1} x$ (B) $y = \cot^{-1} x$ (C) $y = \tan^{-1} x$ (D) $y = \operatorname{cosec}^{-1} x$

Q4	The value of $\cos \left(\frac{\pi}{6} + \cot^{-1}(-\sqrt{3}) \right)$ is (A) -1 (B) $\frac{-\sqrt{3}}{2}$ (C) 0 (D) 1
Q5	If $\begin{bmatrix} x+y & 3y \\ 3x & x+3 \end{bmatrix} = \begin{bmatrix} 9 & 4x+y \\ x+6 & y \end{bmatrix}$, then $(x-y) = ?$ (A) -7 (B) -3 (C) 3 (D) 7
Q6	If A and B are two square matrices of the same order, then $(A+B)(A-B)$ is equal to : (A) $A^2 - AB + BA - B^2$ (B) $A^2 + AB - BA - B^2$ (C) $A^2 - AB - BA - B^2$ (D) $A^2 - B^2 + AB + BA$
Q7	If $A = \begin{bmatrix} 1 & 12 & 4y \\ 6x & 5 & 2x \\ 8x & 4 & 6 \end{bmatrix}$ is a symmetric matrix, then $(2x+y)$ is (A) -8 (B) 0 (C) 6 (D) 8
Q8	If $A = [a_{ij}]$ is a 3×3 diagonal matrix such that $a_{11} = 1$, $a_{22} = 5$ and $a_{33} = -2$, then A is : (A) 0 (B) -10 (C) 10 (D) 1
Q9	If $\begin{bmatrix} 4+x & x-1 \\ -2 & 3 \end{bmatrix}$ is a singular matrix, then the value of x is : (A) 0 (B) 1 (C) -2 (D) -4
Q10	If $f(x) = \begin{cases} 1, & \text{if } x \leq 3 \\ ax+b, & \text{if } 3 < x < 5 \\ 7, & \text{if } 5 \leq x \end{cases}$ is continuous in R, then the values of a and b are : (A) $a = 3, b = -8$ (B) $a = 3, b = 8$ (C) $a = -3, b = -8$ (D) $a = -3, b = 8$

Q11	If $f(x) = x + x - 1$, then which of the following is correct ? (A) $f(x)$ is both continuous and differentiable, at $x = 0$ and $x = 1$. (B) $f(x)$ is differentiable but not continuous, at $x = 0$ and $x = 1$. (C) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$. (D) $f(x)$ is neither continuous nor differentiable, at $x = 0$ and $x = 1$.
Q12	If $\tan^{-1}(x^2 - y^2) = a$, where 'a' is a constant, then $\frac{dy}{dx}$ is : (A) $\frac{x}{y}$ (B) $-\frac{x}{y}$ (C) $\frac{a}{x}$ (D) $\frac{a}{y}$
Q13	The derivative of $\tan^{-1}(x^2)$ w.r.t. x is : (A) $\frac{x}{1+x^4}$ (B) $\frac{2x}{1+x^4}$ (C) $-\frac{2x}{1+x^4}$ (D) $\frac{1}{1+x^4}$
Q14	$\int \sqrt{1+\sin x} \, dx$ is equal to : (A) $2\left(-\sin \frac{x}{2} + \cos \frac{x}{2}\right) + C$ (B) $2\left(\sin \frac{x}{2} - \cos \frac{x}{2}\right) + C$ (C) $-2\left(\sin \frac{x}{2} + \cos \frac{x}{2}\right) + C$ (D) $2\left(\sin \frac{x}{2} + \cos \frac{x}{2}\right) + C$
Q15	If $\int e^{-3 \log x} \, dx = f(x) + C$, then $f(x)$ is : (A) $e^{-3 \log x}$ (B) $e^{\log\left(\frac{1}{x^3}\right)}$ (C) $\frac{-1}{2x^2}$ (D) $\frac{-1}{4x^4}$
Q16	The value of $\int_{-1}^1 x \, dx$ is : (A) -2 (B) -1 (C) 1 (D) 2
Q17	The restrictions imposed on decision variables involved in an objective function of a linear programming problem are called : (A) feasible solutions (B) constraints (C) optimal solutions (D) infeasible solutions

Q18	If the feasible region of a linear programming problem with objective function $Z = ax + by$, is bounded, then which of the following is correct ? (A) It will only have a maximum value. (B) It will only have a minimum value. (C) It will have both maximum and minimum values. (D) It will have neither maximum nor minimum value.
Assertion Reasoning Based Questions Given below are two statements: one is labelled as Assertion A and other is labelled as Reason R. In the light of the above statements, choose the most appropriate answer from the options given below (A) Both A and R are correct and R is the correct explanation of A (B) Both A and R are correct but R is NOT the correct explanation of A (C) A is correct but R is not correct (D) A is not correct but R is correct	
Q19	<i>Assertion (A) :</i> Let $A = \{x \in \mathbb{R} : -1 \leq x \leq 1\}$. If $f : A \rightarrow A$ be defined as $f(x) = x^2$, then f is not an onto function. <i>Reason (R) :</i> If $y = -1 \in A$, then $x = \pm \sqrt{-1} \notin A$.
Q20	<i>Assertion (A) :</i> $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous at $x = 0$. <i>Reason (R) :</i> When $x \rightarrow 0$, $\sin \frac{1}{x}$ is a finite value between -1 and 1.
SECTION – B (Very Short Answer (VSA)-type questions) 2 Marks Each	
Q21	If $x = e^{\sin^{-1} t}$, $y = e^{\cos^{-1} t}$, find $\frac{dy}{dx}$ at $t = \frac{1}{\sqrt{2}}$ OR If $\sqrt{3}(x^2 + y^2) = 4xy$, then find $\frac{dy}{dx}$ at $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.
Q22	Simplify : $\tan^{-1} \left(\frac{\cos 2x - \sin 2x}{\cos 2x + \sin 2x} \right)$, $0 < x < \frac{\pi}{4}$.
Q23	If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then compute $A^2 - 5A + 4I$.
Q24	Find the interval in which the function $f(x) = x^4 - 4x^3 + 10$ is strictly decreasing. OR Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ has neither maxima nor minima.
Q25	If $\frac{d}{dx} (F(x)) = \frac{1}{e^x + 1}$, then find $F(x)$ given that $F(0) = \log \frac{1}{2}$.

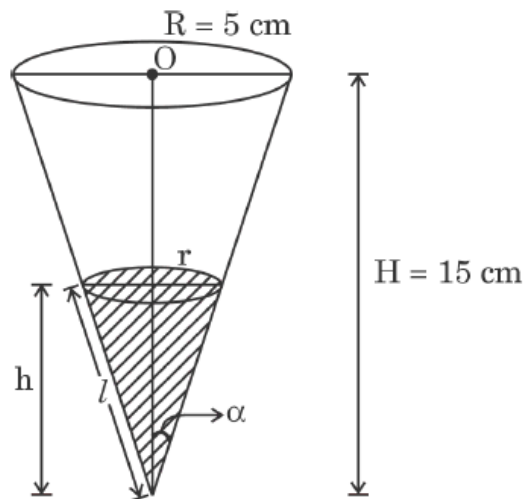
SECTION – C (Short Answer (SA)-type questions) 3 Marks Each	
Q26	Determine whether the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 2x^3 - 7$, $\forall x \in \mathbb{R}$ (Real numbers) is bijective or not. OR Students are trying to fix thumb pins on a graph sheet on the class notice board. A relation R is defined on the set A of thumb pins as $R = \{(P, Q) : P, Q \in A \text{ and distance between P and Q is less than 3 cm}\}$ Determine if R is an equivalence relation.
Q27	Find the values of 'a' and 'b', if $y = \frac{ax - b}{(x - 1)(x - 4)}$ has a critical point (3, 1).
Q28	If $y\sqrt{x^2 + 1} = \log[\sqrt{x^2 + 1} - x]$, show that $(x^2 + 1)\frac{dy}{dx} + xy + 1 = 0$.
Q29	Evaluate : $\int_0^{\pi} \frac{\sin^{2026} x}{\sin^{2026} x + \cos^{2026} x} dx$ OR Evaluate : $\int_0^1 x \tan^{-1} x dx$.
Q30	Find $\int e^{\cot^{-1} x} \left(\frac{1 - x + x^2}{1 + x^2} \right) dx$.
Q31	Solve the following linear programming problem graphically : Maximize $Z = 8000x + 12000y$ Subject to constraints $3x + 4y \leq 60$ $x + 3y \leq 30$ $x \geq 0, y \geq 0$
SECTION – D (Long Answer (LA)-type questions) 5 Marks Each	
Q32	Obtain the value of $\Delta = \begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$ in terms of x, y and z. Further, if $\Delta = 0$ and x, y, z are non-zero real numbers, prove that $x^{-1} + y^{-1} + z^{-1} = -1$. OR

	If $A = \begin{bmatrix} -1 & a & 2 \\ 1 & 2 & x \\ 3 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ b & y & 3 \end{bmatrix}$, find the value of $(a + x) - (b + y)$.
Q33	(a) Differentiate $\tan^{-1} \left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right)$ with respect to $\cos^{-1} x^2$. (b) If $y = P \cos ux + Q \sin ux$, show that $\frac{d^2y}{dx^2} + u^2y = 0$.
Q34	A window is of the form of a semi-circle with a rectangle on its diameter. The total perimeter of the window is 10 m. Find the dimension of the window to admit maximum light through the whole opening.
Q35	Find : $\int \frac{x}{(x-1)(x^2+4)} dx$ OR $\int \frac{2 \sin 2\phi - \cos \phi}{6 - \cos^2 \phi - 4 \sin \phi} d\phi$
SECTION – E (Case Study questions) 4 Marks Each	
Q36	A school wants the students of class XII to do a project on ‘Sustainability’ keeping the world environment in mind. They select the student participants on the basis of an essay writing competition. 7 students out of 80 are selected for the project and are categorized into two sets such that : Girl students belong to Set A = {G₁, G₂, G₃, G₄}, Boy students belong to Set B = {B₁, B₂, B₃}. Based on the above information, answer the following questions : (i) How many relations are possible from Set A → Set B ?

	(ii) Let R be a relation from $A \rightarrow B$ such that $R = \{(G_1, B_1), (G_2, B_2), (G_3, B_2), (G_4, B_3), (G_1, B_2)\}$. Is R an injective function ? Justify your answer. 1 (iii) (a) Let the relation R from $A \rightarrow A$ be such that $R = \{(x, y), x, y \in A, x \text{ and } y \text{ are students from the same colony in the city}\}$ Verify if R is an equivalence relation. 2 OR (iii) (b) Verify if any function $f : B \rightarrow A$ is bijective. Give reason to support your answer. 2
Q37	A professional typist having his shop in a busy market charges ₹ 200 for typing 8 English and 4 Hindi pages, while he charges ₹ 275 for typing 5 English and 10 Hindi pages. Based on the above information, answer the following questions : (i) If he charges ₹ x for one page of English and ₹ y for one page of Hindi, express the above as a pair of linear equations. (ii) Express the information in terms of matrix equation $AX = B$. (iii) (a) Find A. OR (iii) (b) Find $(\text{adj } A)$.

Q38

At a birthday party, children are being served orange juice in conical cups, as shown in the figure.



Each cup is 15 cm deep and has a radius 5 cm. The juice is being poured into this cup at a rate of $0.1 \text{ cm}^3/\text{s}$.

On the basis of the above information, answer the following questions :

- (i) Establish a relation between the height h of the juice in the cup and radius r of the surface of the juice in the cup, if the semi-vertical angle of the cone is α . 1
- (ii) At what rate is the juice level in the cup rising when the juice is 6 cm deep ? 1
- (iii) (a) When the juice is 6 cm deep, then find at what rate is the upper surface area of juice increasing ? 2

OR

- (iii) (b) When the juice is 6 cm deep, then find the rate at which the wetted surface area of the cup is increasing. 2