

MARKING SCHEME
MID-TERM EXAMINATION (2026-27)
CLASS : XII
SUBJECT: MATHEMATICS (041)

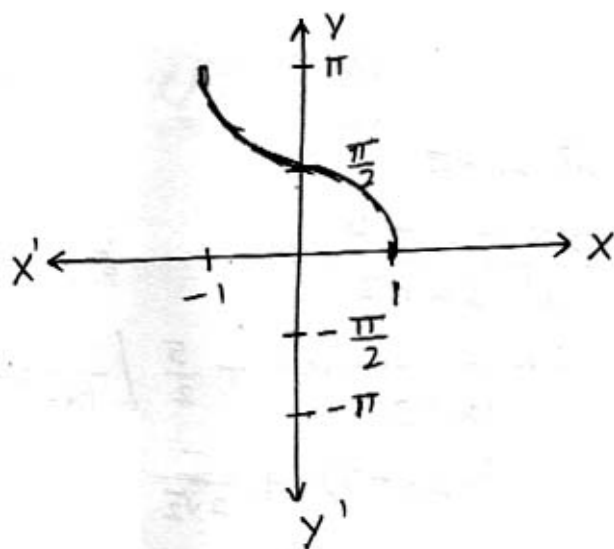
SECTION-A

- | | | | |
|-----|-----|----------------------------------|---|
| 1. | (c) | $\{(1, 1), (2, 2), (3, 3)\}$ | 1 |
| 2. | (b) | 0 | 1 |
| 3. | (a) | 1 | 1 |
| 4. | (d) | Neither injective nor surjective | 1 |
| 5. | (c) | $ A = B \Rightarrow A = B$ | 1 |
| 6. | (d) | $\frac{\pi}{3}$ | 1 |
| 7. | (c) | $\frac{3x^2}{1+x^6}$ | 1 |
| 8. | (c) | $(-\infty, 0)$ | 1 |
| 9. | (d) | $\mathbb{R}^+ \cup \{0\}$ | 1 |
| 10. | (d) | 3 | 1 |
| 11. | (a) | 0 | 1 |
| 12. | (c) | 2, 1 | 1 |

13. (a) $\frac{2}{1+x^2}$ 1
14. (c) $\frac{\sin^8 x}{8} + C$ 1
15. (d) 1, 5 1
16. (a) $8 \text{ cm}^2/\text{sec}$ 1
17. (a) $\frac{1}{5}$ 1
18. (c) $f'(x) = 0$ and $f'(x)$ does not change its sign from left to right of that point 1
19. (b) Both Assertion (A) and Reason (R) are true and the Reason (R) is not the correct explanation of the Assertion (A). 1
20. (a) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A). 1

SECTION-B

21. 2



For Visually Impaired Students only

$$\sin \left[\cot^{-1} \sqrt{2} \left\{ \cos \frac{\pi}{4} \right\} \right] \quad \frac{1}{2}$$

$$= \sin \left[\cot^{-1} \sqrt{2} \times \frac{1}{\sqrt{2}} \right] \quad \frac{1}{2}$$

$$= \sin \frac{\pi}{4} \quad \frac{1}{2}$$

$$= \frac{1}{\sqrt{2}} \quad \frac{1}{2}$$

22. $y = \sin^{-1} \frac{\sqrt{1+x} + \sqrt{1-x}}{2}$

Put $x = \cos \theta$

$$y = \sin^{-1} \frac{\sqrt{1+\cos \theta} + \sqrt{1-\cos \theta}}{2} \quad \frac{1}{2}$$

$$\text{Or, } y = \sin^{-1} \left(\frac{\sqrt{2} \cos \frac{\theta}{2} + \sqrt{2} \sin \frac{\theta}{2}}{2} \right)$$

$$\text{or, } y = \sin^{-1} \left(\frac{1}{\sqrt{2}} \cos \frac{\theta}{2} + \frac{1}{\sqrt{2}} \sin \frac{\theta}{2} \right)$$

$$\text{or, } y = \sin^{-1} \left(\sin \frac{\pi}{4} \cos \frac{\theta}{2} + \cos \frac{\pi}{4} \sin \frac{\theta}{2} \right)$$

$$\text{or, } y = \sin^{-1} \sin \left(\frac{\pi}{4} + \frac{\theta}{2} \right) \quad \frac{1}{2}$$

$$\text{or, } y = \frac{\pi}{4} + \frac{\theta}{2}$$

$$\text{or, } y = \frac{\pi}{4} + \frac{1}{2} \cos^{-1} x \quad \frac{1}{2}$$

Differentiate w.r.t. x

$$\frac{dy}{dx} = -\frac{1}{2\sqrt{1-x^2}}$$

$$2\sqrt{1-x^2} \frac{dy}{dx} + 1 = 0 \quad \frac{1}{2}$$

OR

$$\text{Let } y = (\sin x)^x$$

Taking log both the sides

$$\log y = x \log \sin x \quad \frac{1}{2}$$

Differentiate w.r.t. x

$$\frac{1}{y} \cdot \frac{dy}{dx} = x \frac{1}{\sin x} \cos x + \log \sin x \cdot 1 \quad \frac{1}{2}$$

$$\text{Or, } \frac{dy}{dx} = (\sin x)^x (x \cot x + \log \sin x) \quad 1$$

$$23. \int \frac{x+3}{x^2-2x+3} dx$$

$$= \frac{1}{2} \int \frac{2x-2+5}{x^2-2x+3} dx \quad \frac{1}{2}$$

$$= \frac{1}{2} \int \frac{2x-2}{x^2-2x+3} dx + \frac{5}{2} \int \frac{1}{(x-1)^2 + (\sqrt{2})^2} dx \quad 1$$

$$= \frac{1}{2} \log(x^2-2x+3) + \frac{5}{2\sqrt{2}} \tan^{-1} \frac{x-1}{\sqrt{2}} + C \quad \frac{1}{2}$$

OR

$$\int e^x \left[\tan^{-1} x + \frac{1}{1+x^2} \right] dx$$

$$\int e^x [f(x) + f'(x)] dx = e^x f(x) + C \quad 1$$

$$\text{Here } f(x) = \tan^{-1} x \quad f'(x) = \frac{1}{1+x^2} \quad \frac{1}{2}$$

$$= e^x \tan^{-1} x + C \quad \frac{1}{2}$$

$$24. \quad f(x) = \cos 3x, \quad x \in \left[0, \frac{\pi}{2} \right]$$

$$f'(x) = -3 \sin 3x$$

$$f'(x) = 0$$

$$\Rightarrow -3 \sin 3x = 0 \quad \frac{1}{2}$$

$$\Rightarrow 3x = 0, \pi, 2\pi, \dots$$

$$\Rightarrow x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \dots \quad \frac{1}{2}$$

$$0, \frac{\pi}{3} \in \left[0, \frac{\pi}{2}\right]$$

$$f'(x) < 0 \text{ if } x \in \left[0, \frac{\pi}{3}\right] \quad \text{Decreasing} \quad \frac{1}{2}$$

$$f'(x) > 0 \text{ if } x \in \left[\frac{\pi}{3}, \frac{\pi}{2}\right] \quad \text{Increasing} \quad \frac{1}{2}$$

$$25. \quad \begin{bmatrix} x & 2x & 3 \end{bmatrix} \begin{bmatrix} X \\ 1 \\ -1 \end{bmatrix} = 0$$

$$[x^2 + 2x - 3] = [0] \quad 1$$

$$\text{or, } x^2 + 2x - 3 = 0$$

$$\text{or, } (x + 3)(x - 1) = 0 \quad \frac{1}{2}$$

$$\text{or, } x = 1, -3 \quad \frac{1}{2}$$

SECTION-C

$$26. \quad \int \frac{x + \sin x}{1 + \cos x} dx$$

$$= \int \frac{x + \sin x}{2 \cos^2 \frac{x}{2}} dx$$

$$= \frac{1}{2} \int x \sec^2 \frac{x}{2} dx + \frac{1}{2} \int \frac{\cancel{2} \sin \frac{x}{2} \cancel{\cos \frac{x}{2}}}{\cancel{2} \cos \frac{x}{2}} dx \quad 1$$

$$= \frac{1}{2} \int x \sec^2 \frac{x}{2} dx + \int \tan \frac{x}{2} dx$$

$$= \frac{1}{2} x \int \sec^2 \frac{x}{2} dx - \frac{1}{2} \int 1 \left[\sec^2 \frac{x}{2} dx \right] + \int \tan \frac{x}{2} dx + C \quad 1$$

$$= \frac{1}{2} x \cdot \cancel{\int \tan \frac{x}{2}} - \frac{1}{2} \cdot \cancel{\int \tan \frac{x}{2} dx} + \int \tan \frac{x}{2} dx + C$$

$$= x \tan \frac{x}{2} + C \quad 1$$

27. $\sin^{-1}(3-2x)$

$$-1 \leq 3-2x \leq 1 \quad 1$$

$$\text{or, } -1-3 \leq \cancel{3} - 2x - \cancel{3} \leq 1-3$$

$$\text{or, } -4 \leq -2x \leq -2 \quad 1$$

Divide by -2

$$\text{or } 2 \geq x \geq 1$$

$$\text{or } 1 \leq x \leq 2$$

$$\text{Hence domain of the function is } [1, 2] \quad 1$$

$$28. \quad f(x) = \begin{cases} \frac{e^{3x} - e^{-5x}}{x} & , \text{ if } x \neq 0 \\ a & , \text{ if } x = 0 \end{cases}$$

$\therefore$ The function is continuous

$$\therefore \lim_{x \rightarrow 0} f(x) = f(0)$$

1

$$\lim_{x \rightarrow 0} f(x)$$

$$= \lim_{x \rightarrow 0} \frac{e^{3x} - e^{-5x}}{x}$$

$$= \lim_{x \rightarrow 0} \frac{e^{3x} - \frac{1}{e^{5x}}}{x}$$

$$= \lim_{x \rightarrow 0} \frac{e^{8x} - 1}{x e^{5x}}$$

$$= \lim_{x \rightarrow 0} \frac{8(e^{8x} - 1)}{8x \cdot e^{5x}}$$

1

$$= 8, x \neq 0 \quad \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

$$\therefore a = 8$$

1

OR

$$f(x) = \begin{cases} x + 2 & , \text{ if } x < 1 \\ 0 & , \text{ if } x = 1 \\ x - 2 & , \text{ if } x > 1 \end{cases}$$

The linear function $x + 2$ & $x - 2$ both are polynomials and polynomials are continuous every where on their respective open intervals. 1

Let's check at $x = 1$

$$\text{L.H.L.} = \lim_{x \rightarrow 1-h} f(x)$$

$$= \lim_{h \rightarrow 0} f(1-h)$$

$$= \lim_{h \rightarrow 0} 1-h+2$$

$$= \lim_{h \rightarrow 0} 3-h$$

$$= 3, h \neq 0$$

$\frac{1}{2}$

$$\text{R.H.L.} = \lim_{x \rightarrow 1+h} f(x)$$

$$= \lim_{h \rightarrow 0} f(1+h)$$

$$= \lim_{h \rightarrow 0} 1+h-2 = -1$$

$\frac{1}{2}$

Since L.H.L. $\neq$ R.H.L. Therefore, the point of discontinuity is $x = 1$. 1

$$29. \quad A^2 = A.A. = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} \quad \frac{1}{2}$$

$$A^2 + \alpha A + \beta I = 0$$

$$\Rightarrow \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 3\alpha & 2\alpha \\ \alpha & \alpha \end{bmatrix} + \begin{bmatrix} \beta & 0 \\ 0 & \beta \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \frac{1}{2}$$

$$\Rightarrow \begin{bmatrix} 11+3\alpha+\beta & 8+2\alpha \\ 4+\alpha & 3+\alpha+\beta \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \frac{1}{2}$$

Comparing the corresponding elements

$$11 + 3\alpha + \beta = 0 \quad \dots(i)$$

$$4 + \alpha = 0 \quad \dots(ii)$$

$$8 + 2\alpha = 0 \quad \dots(iii)$$

$$3 + \alpha + \beta = 0 \quad \dots(iv)$$

From equation (ii) & (iv)

$$\alpha = -4, \beta = 1$$

$\frac{1}{2}$

1

30. Let $I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x^9 - 3x^5 + 7x^3 - x + 1}{\cos^2 x} dx$

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (x^9 - 3x^5 + 7x^3 - x) \sec^2 x \, dx + \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x \, dx \quad \frac{1}{2}$$

$$I = I_1 + I_2$$

$$I_1 = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (x^9 - 3x^5 + 7x^3 - x) \sec^2 x \, dx \quad 1$$

$$\int_{-a}^a f(x) \, dx = 0 \quad \text{If } f(x) \text{ is an odd function}$$

$$= 2 \int_0^a f(x) \, dx \quad \text{If } f(x) \text{ is an even function}$$

$$f(x) = (x^9 - 3x^5 + 7x^3 - x) \sec^2 x$$

$$f(-x) = (-x^9 + 3x^5 - 7x^3 + x) \sec^2 x$$

$$= -f(x) \quad [\because f(x) \text{ is an odd function}]$$

$$\Rightarrow I_1 = 0 \quad \frac{1}{2}$$

$$I_2 = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x \, dx$$

$$= 2 \int_0^{\frac{\pi}{4}} \sec^2 x \, dx = 2 [\tan x]_0^{\frac{\pi}{4}} = 2 \quad \frac{1}{2}$$

$$\text{Hence } \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (x^9 - 3x^5 + 7x^3 - x) / \cos^2 x \, dx = 2 \quad \frac{1}{2}$$

OR

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \log_e \sin x \, dx \quad \dots(i)$$

$$= \int_0^{\frac{\pi}{2}} \log \sin\left(\frac{\pi}{2} - x\right) dx \quad \Bigg| \quad = \int_0^a f(x) dx = \int_0^a f(a-x) dx$$

$$I = \int_0^{\frac{\pi}{2}} \log_e \cos x \, dx \quad \dots(ii)$$

$\frac{1}{2}$

Add equation (i) and (ii)

$$2I = \int_0^{\frac{\pi}{2}} [\log_e \sin x + \log_e \cos x] dx$$

$$\text{or, } 2I = \int_0^{\frac{\pi}{2}} [\log \sin x \cos x + \log 2 - \log 2] dx$$

$$\text{or, } 2I = \int_0^{\frac{\pi}{2}} \log \sin 2x dx - \int_0^{\frac{\pi}{2}} \log 2 \, dx$$

$\frac{1}{2}$

Put $2x = t$

$$\text{or, } 2I = \frac{1}{2} \int_0^{\pi} \log \sin t \, dt - \log 2 \left[x \right]_0^{\frac{\pi}{2}}$$

$\frac{1}{2}$

$$\text{or, } 2I = \frac{1}{2} \int_0^{\pi} \log \sin x \, dx - \frac{\pi}{2} \log_e 2 \quad \int_a^b f(x) = \int_a^b f(t) dt$$

$\frac{1}{2}$

$$\text{or, } 2I = \frac{1}{2} \times \cancel{\int_0^{\pi} \log \sin x \, dx} - \frac{\pi}{2} \log_e 2 \quad \Bigg| \quad \int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx$$

If $f(2a - x) = f(x)$

$$\text{or, } 2I = I - \frac{\pi}{2} \log_e 2 \quad \frac{1}{2}$$

$$\text{or, } I = -\frac{\pi}{2} \log_e 2$$

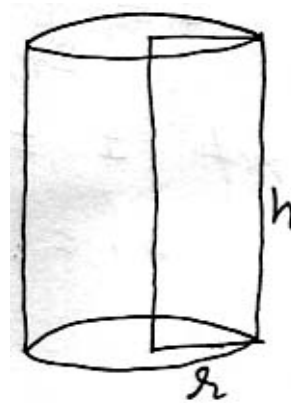
$$\text{Hence, } \int_0^{\frac{\pi}{4}} \log_e \sin x \, dx = -\frac{\pi}{2} \log_e 2 \quad \frac{1}{2}$$

31. Given $\frac{dV}{dt} = -20 \text{ cm}^3 / \text{sec}$

$$\ell = 30 \text{ cm}$$

$$h = 1.2 \text{ m} = 120 \text{ cm}$$

To calculate $\left[\frac{dh}{dt} \right]_{h=80}$



$$\text{Volume of cylinder } v = \pi r^2 h$$

$$\text{or } v = 900 \pi h \quad \dots(i) \quad \frac{1}{2}$$

Difference w.r.t. t

$$\frac{dv}{dt} = 900\pi \frac{dh}{dt} \quad \frac{1}{2}$$

$$-20 = 900\pi \frac{dh}{dt}$$

$$\text{or, } \frac{dh}{dt} = -\frac{20}{900\pi} = -\frac{1}{45\pi} \quad 1$$

$$\text{Hence, the level of the diesel is decreasing at the rate of } \frac{1}{45\pi} \text{ cm/sec} \quad \frac{1}{2}$$

OR

Let the monthly charges is increased by ₹ x.

Hence, x subscriber will discontinue the service.

New revenue $r = (300 + x) (500 - x)$

$$r = 150000 + 200x - x^2 \quad \dots(i) \quad 1$$

Differentiate w.r.t. x

$$\frac{dr}{dx} = 200 - 2x \quad \dots(ii) \quad \frac{1}{2}$$

For maxima or minima

$$\frac{dr}{dx} = 0$$

$$\Rightarrow x = 100 \quad \frac{1}{2}$$

Differentiate equation number (ii) w.r.t. x

$$\frac{d^2r}{dx^2} = -2 \text{ (Negative)}$$

This is the condition of maxima. $\frac{1}{2}$

Hence increase of ₹ 100 in monthly charges will bring maximum revenue. $\frac{1}{2}$

SECTION-D

32. $A = \begin{bmatrix} 2 & 3 & 1 \\ -3 & 2 & 1 \\ 5 & -4 & -2 \end{bmatrix}$

$$|A| = 2(-4 + 4) - 3(6 - 5) + 1(12 - 10)$$

$$= 0 - 3 + 2$$

$$= -1$$

$\frac{1}{2}$

Cofactor matrix of $A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$

1

Adj A = Transpose of cofactor matrix of A

$$= \begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}$$

$\frac{1}{2}$

$$A^{-1} = \frac{\text{Adj } A}{|A|} = \begin{bmatrix} 0 & -2 & -1 \\ 1 & 9 & 5 \\ -2 & -23 & -13 \end{bmatrix}$$

$\frac{1}{2}$

$$2x - 3y + 5z = 11$$

$$3x + 2y - 4z = -5$$

$$x + y - 2z = -3$$

System of equations in matrix form may be written as

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$B \quad X \quad = \quad C \quad \Rightarrow X = B^{-1}C \quad \frac{1}{2}$$

$$B = A' \text{ (By observation)}$$

$$B^{-1} = [A']^{-1}$$

$$= [A^{-1}]' \quad \frac{1}{2}$$

$$= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

$$X = B^{-1}C$$

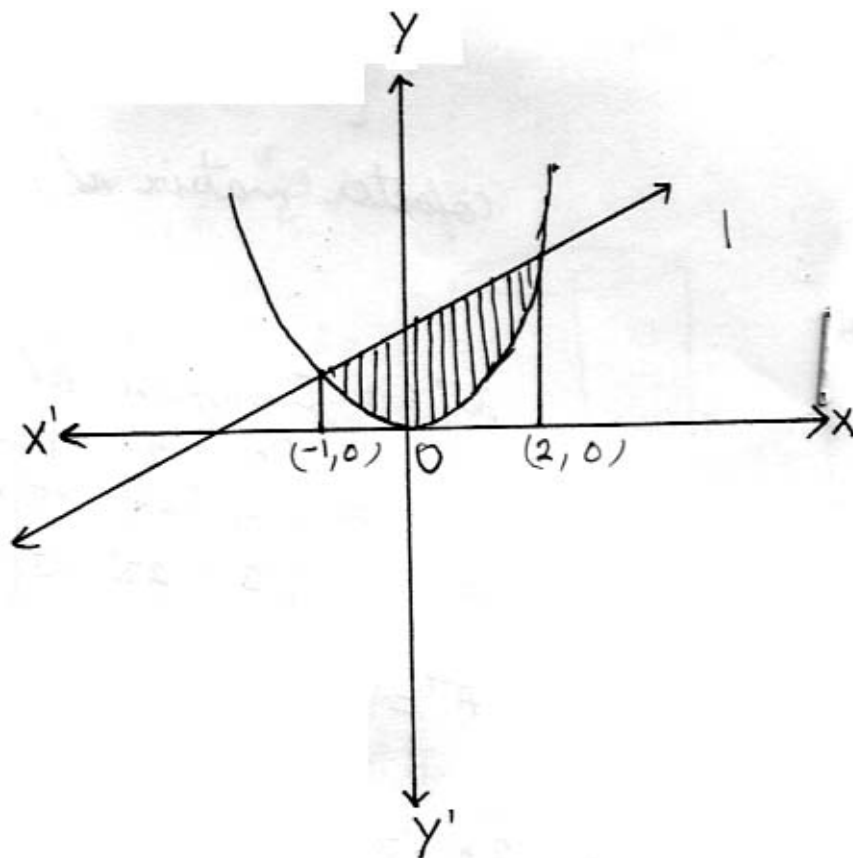
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 + 6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\text{Hence } x = 1, y = 2, z = 3 \quad 1$$

33. Correct figure 1 mark



$$x^2 = 4y \quad \dots(i)$$

$$x = 4y - 2 \quad \dots(ii)$$

From equation (i) & (ii)

$$x^2 = x + 2$$

$$\text{or, } x^2 - x - 2 = 0$$

$$\text{or, } (x - 2)(x + 1) = 0$$

$$\text{or, } x = -1, 2$$

1

Area of shaded region

$$= \int_{-1}^2 \frac{x+2}{4} dx - \int_{-1}^2 \frac{x^2}{4} dx \quad 1$$

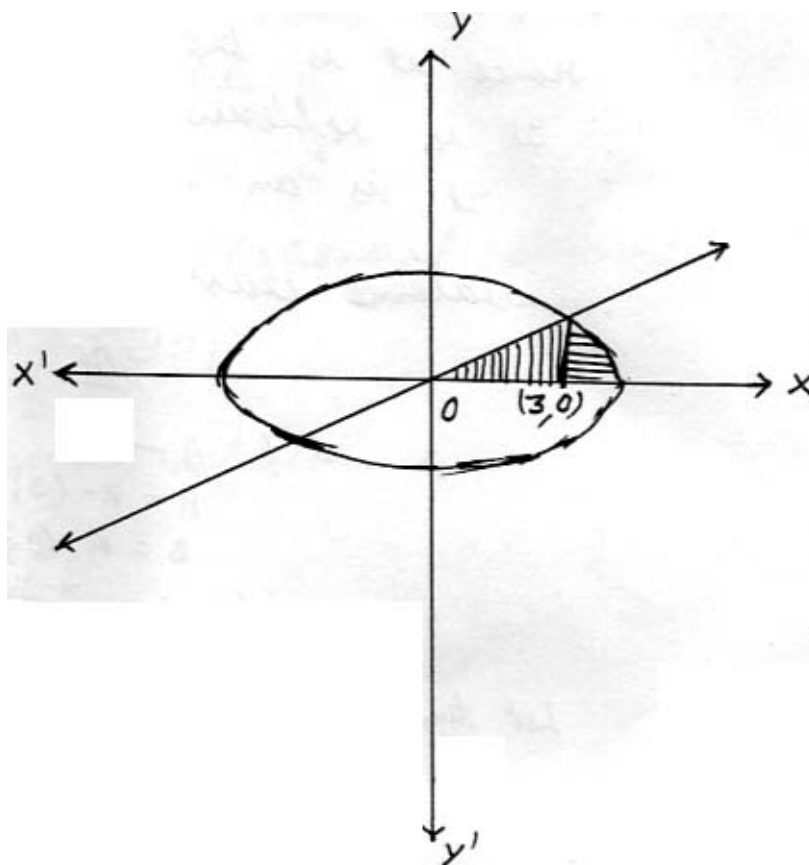
$$= \frac{1}{4} \left[\frac{x^2}{2} + 2x - \frac{x^3}{3} \right]_{-1}^2 \quad 1$$

$$= \frac{1}{4} \left[\frac{4^2}{2} + 4 - \frac{8}{3} - \frac{1}{2} + 2 - \frac{1}{3} \right] \quad \frac{1}{2}$$

$$= \frac{9}{8} = 1\frac{1}{8} \text{ square units} \quad \frac{1}{2}$$

OR

Correct figure 1 mark



$$2x^2 + 9y^2 = 36$$

$$\text{or, } \frac{x^2}{18} + \frac{y^2}{4} = 1 \quad \dots(\text{i})$$

1

$$y = \frac{\sqrt{2}}{3}x \quad \dots(\text{ii})$$

From equation (i) & (ii)

$$\frac{x^2}{18} + \frac{2}{36}x^2 = 1$$

$$2x^2 = 18$$

$$\text{or, } x = \pm 3$$

1

$$\text{Required area} = \int_0^{3\sqrt{2}} \frac{\sqrt{2}}{3}x \, dx + \frac{\sqrt{2}}{3} \int_0^{3\sqrt{2}} \sqrt{18 - x^2} \, dx$$

1

$$= \frac{\sqrt{2}}{3} \left\{ \left[\frac{x^2}{2} \right]_0^3 + \left[\frac{x}{2} \sqrt{18 - x^2} + \frac{18}{2} \sin^{-1} \frac{x}{3\sqrt{2}} \right]_3^{3\sqrt{2}} \right\}$$

$$= \frac{\sqrt{2}}{3} \left\{ \frac{9}{2} + \frac{3\sqrt{2}}{2} \times 0 + 9 \sin^{-1} 1 - \frac{3}{2} \times 3 - 9 \sin^{-1} \frac{1}{\sqrt{2}} \right\}$$

$$= \frac{\sqrt{2}}{3} \left\{ \cancel{\frac{9}{2}} + 9 \cdot \frac{\pi}{2} - \cancel{\frac{9}{2}} - 9 \frac{\pi}{4} \right\}$$

$\frac{1}{2}$

$$= \frac{\cancel{3} \sqrt{2}}{\cancel{3}} \times \frac{\pi}{4}$$

$$= \frac{3\sqrt{2}\pi}{4} \text{ square units}$$

$\frac{1}{2}$

$$34. \quad R = \{(a, b) : 3 \text{ divides } (a - b)\}$$

$$\therefore aRb \Rightarrow 3 \text{ divides } (a - b)$$

$$\therefore \forall a \in Z, aRa \Rightarrow a - a = 0 \text{ which is divisible by } 3$$

1

Hence it is reflexive relation.

$$\therefore aRb \Rightarrow 3 \text{ divides } (a - b)$$

$$\text{Let } a - b = 3m$$

$$\Rightarrow b - a = -3m$$

$$\Rightarrow b - a \text{ is divisible by } 3.$$

$$\Rightarrow \text{If } (a, b) \in R \Rightarrow (b, a) \in R$$

1

Hence, it is symmetric relation

$$\text{If } (a, b) \& (b, c) \in R$$

$$aRb \Rightarrow (a - b) \text{ is divisible by } 3.$$

$$bRc \Rightarrow (b - c) \text{ is divisible by } 3.$$

$$\text{Let } a - b = 3m \quad \dots(i)$$

$$b - c = 3n \quad \dots(ii)$$

$$a - c = 3(m + n)$$

$$\Rightarrow a - c \text{ is divisible by } 3$$

$$\Rightarrow \text{If } (a, b) \& (b, c) \in R \text{ then } (a, c) \in R$$

Hence, it is transitive relation.

$\therefore$ It is reflexive, symmetric & transitive relation.

$\therefore$ It is an equivalence relation.

1

Equivalence class $[1] = [\dots -5, -2, 1, 4, 7 \dots]$

1

OR

$$f: A \rightarrow B$$

$$A = \mathbb{R} - \{3\}$$

$$B = \mathbb{R} - \{1\}$$

$$f(x) = \frac{x-2}{x-3}$$

Let two distinct elements $x_1, x_2 \in A$ such that

$\frac{1}{2}$

$$f(x_1) = f(x_2)$$

$$\Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$$

$$\Rightarrow \cancel{x_1}x_2 - 2x_2 - 3x_1 + \cancel{6} = \cancel{x_1}x_2 - 3x_2 - 2x_1 + \cancel{6}$$

$$\Rightarrow 3x_2 - 2x_2 = 3x_1 - 2x_1$$

$$\Rightarrow x_2 = x_1$$

1

$$\text{If } f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

Hence $f(x)$ is one-one function

$\frac{1}{2}$

Let $y \in B$ (co-domain)

$$y = \frac{x-2}{x-3}$$

$$\Rightarrow xy - 3y = x - 2$$

$$\Rightarrow x = \frac{3y-2}{y-1} = g(y) \text{ Let } \quad \frac{1}{2}$$

$$\text{Range} = R - \{1\} = B$$

$$\therefore \text{Range} = B \text{ (co-domain)} \quad 1$$

Hence it is an onto function.

$$\therefore \text{It is one-one and onto function.} \quad \frac{1}{2}$$

$\therefore f$ is invertible.

$$f^{-1}(x) = g(x) = \frac{3x-2}{x-1} \quad 1$$

35. $ydx - (x + 2y^2)dy = 0, y = 1 \text{ when } x = 0$

$$y \frac{dx}{dy} - x = 2y^2$$

$$\text{or, } \frac{dx}{dy} - \frac{x}{y} = 2y \quad 1$$

It is a linear differential equation in x . $\frac{1}{2}$

$$\text{I.F.} = e^{\int P dy} = e^{-\int \frac{1}{y} dy} = e^{-\log y} = \frac{1}{y} \quad \frac{1}{2}$$

Hence the solution is

$$\frac{x}{y} = \int \frac{2y}{y} dy + C \quad 1$$

$$\text{or, } \frac{x}{y} = 2y + C$$

$$\text{or, } x = 2y^2 + Cy \quad (\text{general solution}) \quad 1$$

$$\text{Put } x = 0, y = 1$$

$$\Rightarrow C = -2$$

Hence, the particular solution is

$$x = 2y^2 - 2y \quad 1$$

SECTION-E

$$36. \quad (i) \quad A = \begin{bmatrix} 18 & 12 \\ 15 & 10 \end{bmatrix}$$

$$A' = \begin{bmatrix} 18 & 15 \\ 12 & 10 \end{bmatrix} \quad 1$$

$$(ii) \quad B = \begin{bmatrix} 4 & 2 \\ 3 & 5 \end{bmatrix}$$

$$|B| = \begin{vmatrix} 4 & 2 \\ 3 & 5 \end{vmatrix}$$

$$= 20 - 6 = 14 \quad 1$$

(iii) Cofactor matrix of $B = \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}$

$$\text{adj } B = \begin{bmatrix} 5 & -2 \\ -3 & 4 \end{bmatrix} \quad 1$$

$$B^{-1} = \frac{\text{adj } B}{|B|} = \frac{1}{14} \begin{bmatrix} 5 & -2 \\ -3 & 4 \end{bmatrix} \quad 1$$

OR

Total prize amount awarded to Class XII-B

$$= 15 \times 500 + 10 \times 1000 \quad 1$$

$$= 7500 + 10000$$

$$= 17500 \quad 1$$

37. (i) By differentiating the above expression 1

(ii) Velocity function $v = \frac{ds}{dt} = 3t^2 - 12t + 9 \quad \dots(i) \quad 1$

(iii) If particle is at rest.

$$\Rightarrow 3t^2 - 12t + 9 = 0$$

$$\Rightarrow t^2 - 4t + 3 = 0$$

$$\Rightarrow (t-1)(t-3) = 0$$

Hence, particle will be on rest, when $t = 1$ sec & $t = 3$ sec. 1

Differentiate equation (i) w.r.t. t

$$a = \frac{dv}{dt} = \frac{d^2S}{dt^2} = 6t - 12 \quad 1$$

$$\left[\frac{dv}{dt} \right]_{t=3} = 18 - 12 = 6 \text{ m/sec}^2 \quad 1$$

38. (i) $\frac{dT}{dt} \propto (T - 25)$ 1

or $\frac{dT}{dt} = -K(T - 25)$ 1

(ii) $\frac{dT}{T - 25} = -Kdt$

or, $\log(T - 25) = -Kt + C$ where C is an arbitrary constant. 1

or, $T - 25 = e^{-Kt + C}$

or, $T = e^{-Kt + C} + 25$ 1