

Marking Scheme
MID-TERM EXAMINATION (2026-27)
Subject: MATHEMATICS (041)
Class: XII

Time Allotted: 3 HOURS

Maximum Marks: 80

Instructions:-

- 1. Evaluation is to be made as per instructions provided in the marking scheme. Marking scheme should be strictly adhered to and religiously followed. However, while evaluating, answers which are innovative they may be assessed for their correctness otherwise and Marks to be awarded to them.**
- 2. If a student has attempted an extra question, answer of the question deserving more marks should be retained and other answer score crossed out.**
- 3. A full scale of marks (0-80) has to be used. Please do not hesitate to award full marks, if the answer deserves it.**

SECTION-A

1. (a) one-one onto function	1
2. (c) $0 \leq y \leq 3\pi$	1
3. (b) π	1
4. (d) 0.96	1
5. (b) 512	1
6. (a) 3	1
7. (a) 0	1
8. (d) $\begin{bmatrix} 0 & 3 \\ -1 & 2 \end{bmatrix}$	1
9. (a) -1	1
10. (a) $\frac{1-3y}{3x-2}$	1

11. (d) $2 \int_0^4 \sqrt{x} dx$	1
12. (d) $\frac{6x^2}{1+4x^6}$	1
13. (c) $-\tan^2 x$	1
14. (c) $10\sqrt{3}$	1
15. (c) $\frac{3}{8}$	1
16. (d) $\sqrt{3}$	1
17. (a) 2, 1	1
18. (d) $\cos y \cdot \cos x$	1
19. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).	1
20. (d) Assertion (A) is false but Reason (R) is true.	1

SECTION-B

21. (a) $\tan \left[\sin^{-1} \left(\frac{\sqrt{3}}{2} \right) - \cos^{-1} \left(\frac{-1}{\sqrt{2}} \right) \right] = \tan \left[\sin^{-1} \left(\frac{\sqrt{3}}{2} \right) - \pi + \cos^{-1} \left(\frac{1}{\sqrt{2}} \right) \right] = \tan \left(-\frac{5\pi}{12} \right)$	1½
$= -\tan \frac{5\pi}{12} = \frac{-(\sqrt{3}+1)}{\sqrt{3}-1} \text{ or } -2 - \sqrt{3}$	½
OR	
(b) $\sin^{-1} \left(\sin \frac{3\pi}{4} \right) = \frac{\pi}{4}, \cos^{-1} \left(\cos \frac{5\pi}{6} \right) = \frac{5\pi}{6}, \tan^{-1} \left(\tan \frac{7\pi}{5} \right) = \frac{2\pi}{5}$	1½
So, $\sin^{-1} \left(\sin \frac{3\pi}{4} \right) + \cos^{-1} \left(\cos \frac{5\pi}{6} \right) + \tan^{-1} \left(\tan \frac{2\pi}{5} \right) = \frac{89}{60}$	½
22. $A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$ or (Student can give any other correct answer)	1
$A^4 = A^2 \cdot A^2 = (-4I)(-4I) = 16I = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}$	1
23. (a) $f(x) = \frac{x-1-1}{x-1} = 1 - \frac{1}{x-1} \Rightarrow f'(x) = \frac{1}{(x-1)^2}, x \neq 1$	1

As $f'(x) > 0$ since $(x - 1)^2 > 0$, so $f(x)$ is strictly increasing on $\mathbb{R} - \{1\}$ and $f(x)$ is strictly decreasing on $\{1\}$.

OR

(b) Let $I = \int_0^a f(x)g(x)dx = \int_0^a f(a-x)g(a-x)dx$

$$I = \int_0^a f(x) \cdot [a - g(x)]dx = a \int_0^a f(x)dx - I$$

$$2I = a \int_0^a f(x)dx \Rightarrow I = \frac{a}{2} \int_0^a f(x)dx$$

24. Let $I = \int \frac{dx}{\sqrt{x^2+4x+5}} = \int \frac{dx}{\sqrt{(x+2)^2+1^2}}$

$$I = \log|x+2+\sqrt{x^2+4x+5}|+c$$

25. Given differential eq can be written as $\frac{dy}{y} = \cot 4x dx$

$$\Rightarrow \int \frac{1}{y} dy = \int \cot 4x dx$$

$$\Rightarrow \log y = \frac{1}{4} \log |\sin 4x| + c \Rightarrow y = (\sin 4x)^{1/4} \cdot c$$

when $x = \frac{\pi}{8}, y = 1 \Rightarrow c = 1$

$\therefore y = (\sin 4x)^{1/4}$ is the required particular solution of given differential eq.

SECTION – C

26. (a) R is not reflexive, since $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \notin R$

Let $(x, y) \in R \Rightarrow x + y - \sqrt{2}$ is an irrational number

$\Rightarrow y + x - 2$ is an irrational number, ($\because x + y = y + x$)

So, R is symmetric.

Now as $\left(\frac{1}{\sqrt{2}}, \sqrt{2}\right) \in R \Rightarrow \frac{1}{\sqrt{2}} + \sqrt{2} - \sqrt{2} = \frac{1}{\sqrt{2}}$ is an irrational number.

and $\left(\sqrt{2}, \frac{1}{\sqrt{2}}\right) \in R \Rightarrow \sqrt{2} + \frac{1}{\sqrt{2}} - \sqrt{2} = \frac{1}{\sqrt{2}}$ is an irrational number.

but $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \notin R, \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - \sqrt{2} = 0$ is not an irrational number.

So, R not transitive.

1

OR

(b) Let $(x, x) \in R$ for any $x \in A$

$\Rightarrow x + x = 2x$, which is divisible by 2 for any integer $x \in A$.

Thus, R is Reflexive.

$\frac{1}{2}$

Let $(x, y) \in R \Rightarrow x + y = 2k, k \in \mathbb{Z} \Rightarrow y + x = 2k \quad (\because x + y = y + x)$

So, $(y, x) \in R$, R is symmetric.

1

Let $(x, y) \in R$ and $(y, z) \in R \Rightarrow x + y = 2k$ and $y + z = 2n$

$\Rightarrow x + z = 2k + 2n - 2y = 2(k + n - y)$, is divisible by 2.

Thus $(x, z) \in R$, R is transitive.

1

Since R is reflexive, symmetric and transitive, so, R is an equivalence relation.

$\frac{1}{2}$

27. $AB = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow (AB)^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

1

$B^{-1} \cdot A^{-1} = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (1 \text{ mark for finding } A^{-1} \text{ and } B^{-1})$

1+ $\frac{1}{2}$

So, $(AB)^{-1} = B^{-1}A^{-1}$

$\frac{1}{2}$

Alternately,

we observe $A = B^{-1}$ and $B = A^{-1}$,

1

$AB = I \Rightarrow (AB)^{-1} = I^{-1} = I,$

1

$B^{-1}A^{-1} = AB = I$

$\frac{1}{2}$

$\therefore (AB)^{-1} = B^{-1}A^{-1}.$

$\frac{1}{2}$

28. (a) $\frac{dx}{d\theta} = e^{\theta}(\cos \theta + \sin \theta - \sin \theta + \cos \theta) = 2e^{\theta} \cos \theta$

1

$\frac{dy}{d\theta} = e^{\theta}(\sin \theta - \cos \theta + \cos \theta + \sin \theta) = 2e^{\theta} \sin \theta$

1

$\frac{dy}{dx} = \frac{2e^{\theta} \sin \theta}{2e^{\theta} \cos \theta} = \tan \theta \text{ (Hence Proved)}$

1

OR

(b) $\log y = x[\log(x+1) - \log(x-1)]$

$$\frac{1}{y} \frac{dy}{dx} = x \left[\frac{1}{x+1} - \frac{1}{x-1} \right] + \log(x+1) - \log(x-1)$$

$$\frac{dy}{dx} = y \left[\log \left(\frac{x+1}{x-1} \right) - \frac{2x}{x^2-1} \right]$$

29. Let V, r, l, h, α be the volume of water in the conical vessel, slant height, radius, height and semi-vertical angle of the cone.

$$\sin \alpha = \frac{r}{l} \Rightarrow r = l \sin \alpha = \frac{l}{2} \cos \alpha = \frac{h}{l} \Rightarrow h = l \cos \alpha = \frac{\sqrt{3}l}{2}$$

$$\text{So, } V = \frac{1}{3} \pi r^2 h = \frac{\sqrt{3}\pi}{24} l^3$$

$$\text{Now, } \frac{dV}{dt} = \frac{\sqrt{3}\pi}{8} l^2 \frac{dl}{dt}$$

$$\text{Thus, } -1 = \frac{\sqrt{3}\pi}{8} (16) \frac{dl}{dt} \Rightarrow \frac{dl}{dt} = -\frac{1}{2\sqrt{3}\pi} \text{ cm/sec}$$

30. (a) Let $\log x = p \Rightarrow x = e^p \Rightarrow dx = e^p dp$

$$\text{So, } I = \int \left(\log(\log x) + \frac{1}{(\log x)^2} \right) dx = \int \left(\log p + \frac{1}{p^2} \right) e^p dp$$

$$I = \int e^p \left(\log p + \frac{1}{p} - \frac{1}{p} + \frac{1}{p^2} \right) dp$$

$$\text{Let } f(p) = \log p - \frac{1}{p} \Rightarrow f'(p) = \frac{1}{p} + \frac{1}{p^2}$$

So, using formula $\int e^x (f(x) + f'(x)) dx = e^x f(x) + c$

$$I = e^p \left(\log p - \frac{1}{p} \right) + c = x \left(\log(\log x) - \frac{1}{\log x} \right) + c$$

OR

(b) Let $I = \int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx = \int_0^\pi \frac{(\pi-x) \sin x}{1 + \cos^2 x} dx$

$$2I = \pi \int_0^\pi \frac{\sin x}{1 + \cos^2 x} dx$$

Put $\cos x = p \Rightarrow \sin x dx = -dp$, When $x = \pi$, then $p = -1$, $x = 0$, then $p = 1$

$$2I = 2\pi \int_{-1}^1 \frac{dp}{1+p^2} = 2\pi \int_0^1 \frac{dp}{1+p^2}$$

$$I = (\tan^{-1} p)_0^1 \cdot 2\pi = \frac{\pi}{4} \cdot 2\pi = \frac{\pi^2}{4}$$

31. Given differential equation can be written as $\frac{dx}{dy} + \frac{1}{1+y^2} \cdot x = \frac{\tan^{-1} y}{1+y^2}$

$$\text{Integrating Factor} = e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1} y}$$

$$\text{Solution: } x \cdot e^{\tan^{-1} y} = \int e^{\tan^{-1} y} \frac{\tan^{-1} y}{1+y^2} dy$$

$$\text{put } \tan^{-1} y = p \Rightarrow \frac{1}{1+y^2} dy = dp$$

$$x \cdot e^{\tan^{-1} y} = \int e^p p \cdot dp = e^p (p - 1) + c = e^{\tan^{-1} y} (\tan^{-1} y - 1) + c$$

$$\therefore x = (\tan^{-1} y - 1) + c \cdot e^{-\tan^{-1} y}$$

SECTION-D

$$32. A_1 = \int_0^1 y dx = \int_0^1 2\sqrt{x} dx = \frac{4}{3} (x^{3/2})_0^1 = \frac{4}{3} \text{ sq. units}$$

$$A_2 = 2 \int_0^4 2\sqrt{x} dx = \frac{8}{3} (x^{3/2})_0^4 = \frac{64}{3} \text{ Sq. units}$$

$$\text{So, } A_1 : A_2 = \frac{4}{3} : \frac{64}{3} = 1 : 16$$

For Visually impaired Students:

$$\text{Required Area, } A = \int_{-1}^1 y dx$$

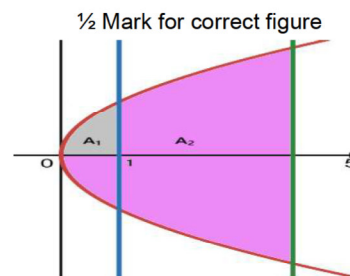
$$A = \int_{-1}^1 x^2 dx = 2 \int_0^1 x^2 dx$$

$$A = 2 \left(\frac{x^3}{3} \right)_0^1 = \frac{2}{3} \text{ Sq. units}$$

33. (a) Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in \mathbb{R} - \left\{ \frac{3}{5} \right\}$

$$\frac{3x_1 + 2}{5x_1 - 3} = \frac{3x_2 + 2}{5x_2 - 3} \Rightarrow (3x_1 + 2)(5x_2 - 3) = (3x_2 + 2)(5x_1 - 3) \Rightarrow x_1 = x_2$$

So f is one-one function.



$$\text{Let } y = f(x) = \frac{3x+2}{5x-3} \Rightarrow x = \frac{3y+2}{5y-3}$$

$$\text{As, } y \in \mathbb{R} - \left\{\frac{3}{5}\right\}, \text{ Range} = \mathbb{R} - \left\{\frac{3}{5}\right\} = \text{Co-domain}$$

Thus, f is onto function.

OR

$$\text{(b) } p(x) \text{ is defined when } -1 \leq 1 - 4x^2 \leq 1 \Rightarrow -2 \leq -4x^2 \leq 0 \Rightarrow 0 \leq x^2 \leq \frac{1}{2} \quad 1\frac{1}{2}$$

$$\text{Domain of } p(x) = \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right] \quad \frac{1}{2}$$

$$\text{Now, when } p(x) = \frac{\pi}{6} \Rightarrow \sin^{-1}(1 - 4x^2) = \frac{\pi}{6} \Rightarrow 1 - 4x^2 = \frac{1}{2} \Rightarrow x^2 = \frac{1}{8}$$

$$\text{so, } x = \pm \frac{1}{2\sqrt{2}} \quad 1$$

$$\text{As, } \sin^{-1} \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], \text{ so } -\frac{3\pi}{2} \leq 3p(x) \leq \frac{3\pi}{2} \Rightarrow -\pi \leq 3p(x) + \frac{\pi}{2} \leq 2\pi$$

$$\text{Range of } 3p(x) + \frac{\pi}{2} = [-\pi, 2\pi] \quad 2$$

$$\text{34. Let } D = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \frac{1}{2}$$

$$AB = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix} \quad 1$$

$$CD = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2a + 5c & 2b + 5d \\ 3a + 8c & 3b + 8d \end{bmatrix} \quad 1$$

$$\text{So, } \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix} = \begin{bmatrix} 2a + 5c & 2b + 5d \\ 3a + 8c & 3b + 8d \end{bmatrix}$$

$$\text{On solving, we get } a = -191, b = -110, c = 77, d = 44 \quad 2$$

$$D = \begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix} \quad \frac{1}{2}$$

NOTE: Alternatively student can find D using inverse too, so allot marks accordingly.

$$\text{35. (a) Let } I = \int_0^{\pi/2} \log(\sin x) dx = \int_0^{\pi/2} \log(\cos x) dx \quad 1$$

$$2I = \int_0^{\pi/2} \log(\sin 2x) dx - \int_0^{\pi/2} \log 2 dx \quad 1$$

$$2I = I_1 - \frac{\pi}{2} \log 2$$

1/2

$$\text{Now, } I_1 = \int_0^{\pi/2} \log(\sin 2x) dx, \text{ Put } 2x = p \Rightarrow dx = \frac{1}{2} dp$$

1/2

$$I_1 = \frac{1}{2} \int_0^{\pi} \log(\sin p) dp = \int_0^{\pi/2} \log(\sin p) dp = I$$

1

$$\text{Thus, } 2I = I - \frac{\pi}{2} \log 2 \Rightarrow I = -\frac{\pi}{2} \log 2$$

$$\therefore \int_0^{\pi/2} \log(\sin x) dx = -\frac{\pi}{2} \log 2$$

1

OR

$$\text{(b) Let } I = \int \frac{x^2+x+1}{(x+2)(x^2+1)} dx$$

$$\text{Let } \frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$

1

$$\text{On solving, we get } A = \frac{3}{5}, B = \frac{2}{5}, C = \frac{1}{5}$$

1 1/2

$$\text{Thus, } I = \frac{3}{5} \int \frac{1}{x+2} dx + \frac{2}{5} \int \frac{x}{x^2+1} dx + \frac{1}{5} \int \frac{1}{x^2+1} dx$$

1

$$\therefore I = \frac{3}{5} \log |x+2| + \frac{1}{5} \log |x^2+1| + \frac{1}{5} \tan^{-1} x + c$$

1 1/2

SECTION-E

$$\text{36. (i) } f'(x) = 3x^2 - 3 \text{ (for } 0 < x < 2)$$

1

$$\text{(ii) For } x \geq 2, f(x) = 2x^2 - 8x + 12 \Rightarrow f'(x) = 4x - 8$$

1/2

$$\Rightarrow f'(3) = 4(3) - 8 = 4$$

1/2

$$\text{(iii) (a) LHL: } \lim_{x \rightarrow 2^-} f(x) = 4$$

1

$$\text{RHL: } \lim_{x \rightarrow 2^+} f(x) = 4$$

1/2

$$\text{Now, } f(2) = 4. \text{ Since LHL} = \text{RHL} = f(2),$$

$$\text{so } f(x) \text{ is continuous at } x = 2.$$

1/2

OR

$$\text{(iii) (b) For getting: LHD} = 9$$

1

For getting: $RHD = 0$

$\frac{1}{2}$

Since $LHD \neq RHD$, so function is not differentiable at $x = 2$.

$\frac{1}{2}$

37. (i) $A(x) = 80x - 2x^2 \Rightarrow \frac{dA}{dx} = 80 - 4x$

1

(ii) Put $A'(x) = 0 \Rightarrow 80 - 4x = 0 \Rightarrow x = 20$ m (critical point)

1

(iii) (a) $A''(x) = -4$, Since $A''(x) < 0$, so Area is maximum at $x = 20$.

1 + 1

OR

(iii) (b) As, $\lim_{h \rightarrow 0} f'(2 - h) > 0$, $\lim_{h \rightarrow 0} f'(2 + h) < 0$,

1

So, Area is maximum at $x = 20$

Max Area $A(20) = 1600 - 800 = 800 \text{ m}^2$

1

38. (a) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$A \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix} \Rightarrow \begin{bmatrix} a + 2b \\ c + 2d \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix} \text{ and } A \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \end{bmatrix} \Rightarrow \begin{bmatrix} 2a + b \\ 2c + d \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \end{bmatrix}$$

1

On solving, we get $A = \begin{bmatrix} 3 & 2 \\ \frac{14}{3} & \frac{5}{3} \end{bmatrix}$

1

(b) $A^{-1} = \frac{3}{-13} \begin{bmatrix} \frac{5}{3} & -2 \\ -\frac{14}{3} & 3 \end{bmatrix} = \begin{bmatrix} -\frac{5}{13} & \frac{6}{13} \\ \frac{14}{13} & -\frac{9}{13} \end{bmatrix}$

1

Decoded message $= A^{-1} \begin{bmatrix} 17 \\ 25 \end{bmatrix} = \begin{bmatrix} -\frac{5}{13} & \frac{6}{13} \\ \frac{14}{13} & -\frac{9}{13} \end{bmatrix} \begin{bmatrix} 17 \\ 25 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$

1