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MID-TERM EXAMINATION (2026-27)**CLASS : XII****SUBJECT: MATHEMATICS (041)****Time Allowed : 3 hours****Maximum Marks : 80****समय : 3 घंटे****अधिकतम अंक - 80****सामान्य निर्देश:**

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख्ती से पालन कीजिए।

1. इस प्रश्न पत्र में 38 प्रश्न हैं। सभी प्रश्न अनिवार्य हैं।
2. यह प्रश्न पत्र पाँच खंडों में विभाजित है - क, ख, ग, घ एवं ङ।
3. खंड-क में प्रश्न संख्या 1 से 18 तक बहुविकल्पीय तथा प्रश्न संख्या 19 एवं 20 अभिकथन एवं तर्क आधारित 1 अंक के प्रश्न हैं।
4. खंड-ख में प्रश्न संख्या 21 से 25 तक अति लघुउत्तरीय (VSA) प्रकार के 2 अंक के प्रश्न हैं।
5. खंड-ग में प्रश्न संख्या 26 से 31 तक अति लघुउत्तरीय (SA) प्रकार के 3 अंक के प्रश्न हैं।
6. खंड-घ में प्रश्न संख्या 32 से 35 तक दीर्घ-घुउत्तरीय (LA) प्रकार के 5 अंकों के प्रश्न हैं।
7. खंड-ङ में प्रश्न संख्या 36 से 38 तक प्रकरण अध्ययन आधारित 4 अंकों के प्रश्न हैं।
8. प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड-ख के 2 प्रश्नों में, खण्ड-ग के 3 प्रश्नों में, खण्ड-घ के 2 प्रश्नों में और खण्ड-ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।
9. कैल्कुलेटर का उपयोग वर्जित है।

GENERAL INSTRUCTIONS:

Read the following instructions very carefully and strictly follow them :

1. This question paper contains 38 questions. All questions are compulsory.
2. This question paper is divided into five sections – A, B, C, D and E.
3. In Section-A, question No. 1 to 18 are multiple choice questions (MCQs) type and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section-B, question No. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.
5. In Section-C, question No. 26 to 31 are Short answer (SA) type questions, carrying 3 marks each.
6. In Section-D, question No. 32 to 35 are long answer (LA) type questions, carrying 5 marks each.
7. In Section-E, question No. 36 to 38 are Case Study based questions carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section-B, 3 questions in Section-C, 2 questions in Section-D and 2 questions in Section-E.
9. Use of calculators is not allowed.

SECTION-A

**This Section comprises multiple choice questions (MCQ's) of 1 mark each.
(QUESTION 1-18)**

1. Let $f: \mathbb{R} \rightarrow \mathbb{R}$, be a function defined as $f(x) = x^3 + 2$, then f is:

(a) one-one onto function
(b) many-one function
(c) one-one but not onto function
(d) neither one-one nor onto function (1)

2. If $y = 3\cos^{-1} 2x$, then

(a) $0 \leq y \leq \pi$ (b) $0 \leq y \leq 2\pi$
(c) $0 \leq y \leq 3\pi$ (d) $0 \leq y \leq 6\pi$ (1)

3. The principal Value of $\sec^{-1}(2) + 3\operatorname{cosec}^{-1}(2) + \cot^{-1}\sqrt{3}$ is:

(a) $\frac{\pi}{2}$ (b) π
(c) $\frac{3\pi}{2}$ (d) $\frac{5\pi}{6}$ (1)

4. The Value of $\sin(2\sin^{-1}(0.6))$ is

(a) 0.6 (b) 1.2
(c) 0.48 (d) 0.96 (1)

5. The number of all possible matrices of order 3×3 with each entry 2026 or 2027 is:

(a) 256 (b) 512
(c) $(2026)^9$ (d) $(2027)^9$ (1)

6. If $P = \begin{bmatrix} 7 & a & b \\ -3 & 8 & c \\ 2 & 4 & 9 \end{bmatrix}$ is a symmetric matrix, then the Value of $(a + b + c)^{a+c}$ is:
- (a) 3 (b) 6
(c) 9 (d) 36 (1)
7. Let $P = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$ and $Q = \begin{bmatrix} 3 & 1 & 2 \end{bmatrix}$, then what will be the value of $|M|$, where $M = PQ$:
- (a) 0 (b) 6
(c) 4 (d) 8 (1)
8. If $A = [a_{ij}]$ is a 2×2 matrix whose elements are given by $a_{ij} = i^j - j$, then A' is:
- (a) $\begin{bmatrix} 0 & -2 \\ 2 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -2 \\ 1 & 2 \end{bmatrix}$
(c) $\begin{bmatrix} 0 & -1 \\ 3 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 3 \\ -1 & 2 \end{bmatrix}$ (1)
9. If a square matrix A is such that $A^2 = A$ and $(1 - A)^4 = pA + 1$, then $p =$
- (a) -1 (b) -2
(c) -3 (d) -4 (1)
10. If $x + 2y = 3xy$ then $\frac{dy}{dx}$ is:
- (a) $\frac{1-3y}{3x-2}$ (b) $\frac{3y-2}{3x-1}$
(c) $\frac{3y-1}{3x-2}$ (d) 1 (1)

11. Which of the following expressions will give the area of the region bounded by the curve $y^2 = x$ and line $x = 4$?

(a) $\int_0^4 \sqrt{x} \, dx$

(b) $\int_0^4 x^2 \, dx$

(c) $2 \int_0^2 \sqrt{x} \, dx$

(d) $2 \int_0^4 \sqrt{x} \, dx$ (1)

12. The derivative of $\tan^{-1}(2x^3)$ w.r.t. x is:

(a) $\frac{6x}{1+2x^6}$

(b) $\frac{3x}{1+4x^6}$

(c) $\frac{3x^2}{1+4x^6}$

(d) $\frac{6x^2}{1+4x^6}$ (1)

13. Derivative of $\sin^4 x$ with respect to $\cos^4 x$ is:

(a) -1

(b) $-\tan x$

(c) $-\tan^2 x$

(d) $-\tan^3 x$ (1)

14. The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. The rate at which the area increases, when side is 10 cm is: (in cm^2/sec)

(a) 10

(b) $5\sqrt{3}$

(c) $10\sqrt{3}$

(d) $\frac{10}{3}$ (1)

15. If $\int \frac{3x}{5+4x^2} dx = A \log(5+4x^2) + c$, then the Value of A is:

(a) 8

(b) $\frac{8}{3}$

(c) $\frac{3}{8}$

(d) $\frac{3}{4}$ (1)

16. If $\int_0^a \frac{dx}{1+x^2} = \frac{\pi}{12}$, then the value of a is:
- (a) 2 (b) 3
- (c) $\frac{1}{3}$ (d) $\sqrt{3}$ (1)
17. The order and degree of the differential equation $\left[2 - \left(\frac{dy}{dx}\right)^3\right]^3 = 4\frac{d^2y}{dx^2}$ are respectively:
- (a) 2, 1 (b) 2, 3
- (c) 2, 9 (d) 2, 6 (1)
18. The differential equation $\frac{dy}{dx} = f(x, y)$ will not be a homogeneous differential equation, if $f(x, y)$ is:
- (a) $\frac{x+y}{x}$ (b) $\frac{x^2 + y^2}{x^2 - yx}$
- (c) $\frac{y}{x} + \sin\left(\frac{y}{x}\right)$ (d) $\cos y \cos x$ (1)

Assertion-Reason Based Questions

Direction: Question number 19 and 20 are Assertion (A) and Reason (R) based questions carrying 1 mark each. If two statements are given, one labelled Assertion (A) and other labelled Reason (R).

Select the correct answer from the codes (a), (b), (c) and (d) as given below.

- (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).
- (c) Assertion (A) is true but Reason (R) is false.
- (d) Assertion (A) is false but Reason (R) is true.

19. **Assertion (A):** A Relation R on the set $\{a, b, c\}$ defined as $R = \{(a, b), (b, b), (c, c)\}$ is an equivalence relation.

Reason (R): A relation that is reflexive, symmetric and transitive is an equivalence relation. (1)

20. **Assertion (A):** $f(x) = e^{7x} - 8$ is a decreasing function, where $x \in \mathbb{R}$.

Reason (R): If $f'(x) < 0$, then $f(x)$ is a decreasing function. (1)

SECTION-B

This section comprises of 5 very short answer (VSA) type questions of 2 marks each.

21. (a) Evaluate $\tan \left[\sin^{-1} \left(\frac{\sqrt{3}}{2} \right) - \cos^{-1} \left(\frac{-1}{\sqrt{2}} \right) \right]$. (3)

OR

- (b) Find the value of $\sin^{-1} \left(\sin \frac{3\pi}{4} \right) + \cos^{-1} \left(\cos \frac{3\pi}{6} \right) + \tan^{-1} \left(\tan \frac{7\pi}{5} \right)$

22. Construct a 2×2 matrix A such that $A^2 = -4I$. Hence find A^4 , where I is an identity matrix. (2)

23. (a) Determine the Values of x for which $f(x) = \frac{x-2}{x-1}$, $x \neq 1$ is an increasing or a decreasing function. (2)

OR

- (b) f and g are continuous functions on interval $[a, b]$. Given that $f(a-x) = f(x)$ and $g(x) + g(a-x) = a$, show that $\int_0^a f(x)g(x)dx = \frac{a}{2} \int_0^a f(x)dx$.

24. Find: $\int \frac{dx}{\sqrt{x^2 + 4x + 5}}$ (2)
25. Find the particular solution of the differential equation $\frac{dy}{dx} = y \cot 4x$, given that $y\left(\frac{\pi}{8}\right) = 1$. (2)

SECTION-C

This section comprises of 6 Short answer (SA) type questions of 3 marks each.

26. (a) Let R be a relation on set of real numbers $\mathbb{R}$ defined as

$$R = \{(x, y) : x + y - \sqrt{2} \text{ is an irrational number } x, y \in \mathbb{R}\}$$

Check the reflexivity, symmetricity and transitivity of R . (3)

OR

- (b) A Relation R on set $A = \{-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$ be defined as $R = \{(x, y) : x + y, \text{ is an integer divisible by } 2\}$. Show that R is an equivalence relation.

27. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$, then show that $(AB)^{-1} = B^{-1} A^{-1}$. (3)

28. (a) If $x = e^{\theta}(\cos \theta + \sin \theta)$, $y = e^{\theta}(\sin \theta - \cos \theta)$ then prove that $\frac{dy}{dx} = \tan \theta$. (3)

OR

(b) If $y = \left(\frac{x+1}{x-1}\right)^x$, $x > 1$, then prove that $\frac{dy}{dx} = y \left[\log_e \left(\frac{x+1}{x-1} - \frac{2x}{x^2-1} \right) \right]$. (3)

29. Water is dripping out at a steady rate of $1 \text{ cm}^3/\text{sec}$ through a tiny hole at the vertex of the conical vessel, whose axis is vertical. When the slant height of water in the vessel is 4 cm, find the rate of decrease of slant height, where the vertical angle of the conical vessel is $\frac{\pi}{6}$. (3)

30. (a) Find: $\int \left[\log(\log x) + \frac{1}{(\log x)^2} \right] dx$ (3)

OR

(b) Evaluate: $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$.

31. Solve the differential equation: $(\tan^{-1} y - x)dy = (1 + y^2)dx$. (3)

SECTION-D

This section comprises of 4 Long answer (LA) type questions of 5 marks each.

32. If A_1 denotes the Area of region bounded by $y^2 = 4x$, $x = 1$ and x-axis in the first quadrant and A_2 denotes the Area of region bounded by $y^2 = 4x$, $x = 4$, then find $A_1 : A_2$. (5)

For Visually impaired students:

Using integration, find the Area of the region bounded by the curve $y = x^2$, $x = -1$, $x = 1$ and x-axis. (5)

33. (a) A function $f: \mathbb{R} - \left\{\frac{3}{5}\right\} \rightarrow \mathbb{R} - \left\{\frac{3}{5}\right\}$ is defined as $f(x) = \frac{3x+2}{5x-3}$. Show that f is one-one and onto. (5)

OR

- (b) Find the domain of $p(x) = \sin^{-1}(1 - 4x^2)$. Hence find the value of x for which $p(x) = \frac{\pi}{6}$. Also, write the range of $3p(x) + \frac{\pi}{2}$.

34. Given that, $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$, find a matrix D such that $(AB - CD)$ is a null matrix. (5)

35. (a) Evaluate: $\int_0^{\pi/2} \log(\sin x) dx$. (5)

OR

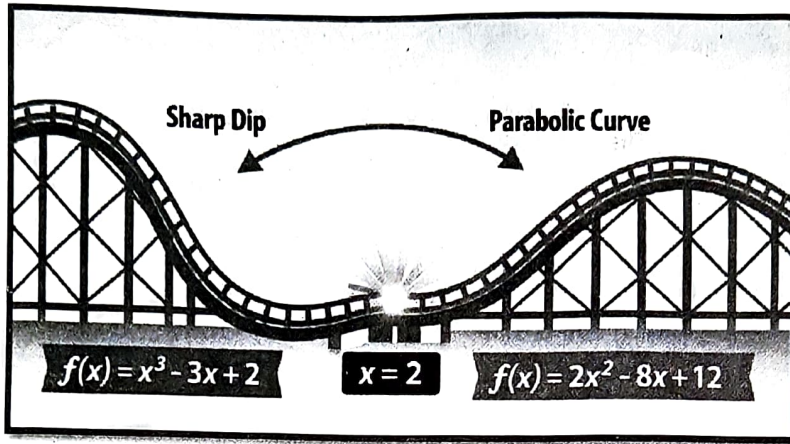
- (b) Find: $\int \frac{x^2 + x + 1}{(x+2)(x^2+1)} dx$. (5)

SECTION-E

This section comprises of 3 Case Study/Passage-based questions of 4 marks each. First two case-study questions have three sub-parts (i), (ii) and (iii) of marks 1, 1 and 2 respectively. The third case study question has two sub-parts (i) and (ii) of marks 2 each.

36. A theme park designs a roller coaster track using two different mathematical models. To ensure the ride is smooth and safe, engineers check whether the track is continuous and differentiable at the point where the two parts join. The height of the track (in meters) is represented by the function:

$$f(x) = \begin{cases} x^3 - 3x + 2 & , \quad 0 < x < 2 \\ 2x^2 - 8x + 12 & , \quad x \geq 2 \end{cases}$$



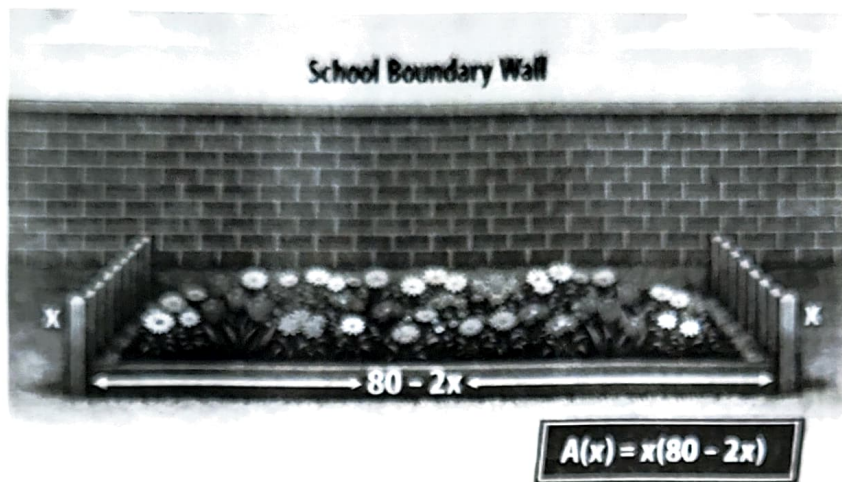
Based on the above information, answer the following questions:

- (i) Find $f'(x)$ for $0 < x < 2$. (1)
- (ii) Find $f'(3)$. (1)
- (iii) (a) Test whether the function is continuous at $x = 2$. (2)

OR

- (b) Test whether the function is differentiable at $x = 2$ (2)

37. A school plans to develop a rectangular flower garden along one side of an existing boundary wall. Since the wall acts as one side of the garden, fencing is required only for the remaining three sides. The school has 80 metres of fencing available. The objective is to design the garden so that it encloses the maximum possible Area. Let the width of the garden be x metres. The Area of the garden is given by $A(x) = x(80 - 2x)$, $0 < x < 40$



Based on the above information, answer the following questions:

- (i) Find $\frac{dA}{dx}$ (1)
- (ii) Find the critical points of $A(x)$ (1)
- (iii) (a) Using the second derivative Test, determine whether the critical points gives a maximum or a minimum value of the Area. (2)

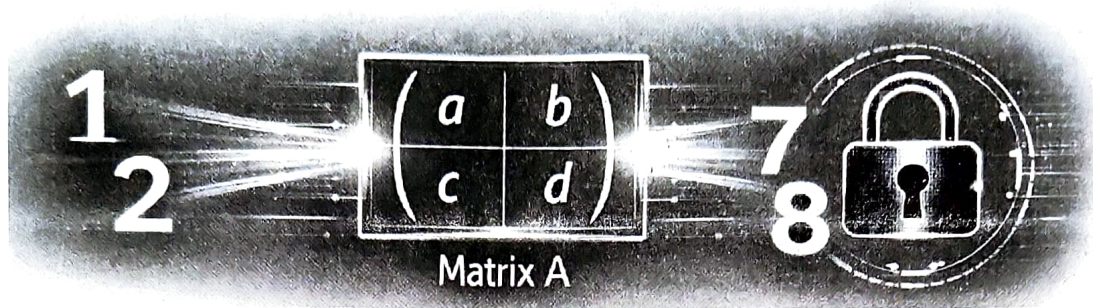
OR

- (b) Find the maximum area enclosed by the garden. (2)

38. A technology company has developed a secure digital lock system in which every message is encrypted using a 2×2 encoding matrix A . For security reasons the company does not disclose the encoding matrix. Instead, only the following information is available:

$$A \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}, \quad A \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \end{bmatrix}$$

To retrieve the original message, the system uses the inverse of the encoding matrix.



Answer the following questions based on the above information:

(i) Determine the encoding Matrix A. (2)

(ii) Without solving any system of Linear equations directly, find A^{-1} .

Hence, decode the encrypted message $\begin{bmatrix} 17 \\ 25 \end{bmatrix}$ (2)