Directorate of Education, GNCT of Delhi Practice Paper

Class XI | Mathematics —Mid Term Answer Key

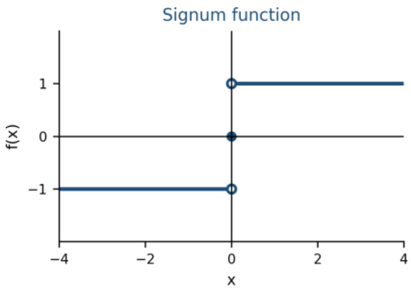
Full Syllabus • Sections A–E • 38 Questions • 80 Marks

Five Questions in the source paper (Q2, Q8, Q13, Q15 and Q30(a)) carry apparent print/OCR defects; each correction is flagged inline in red.

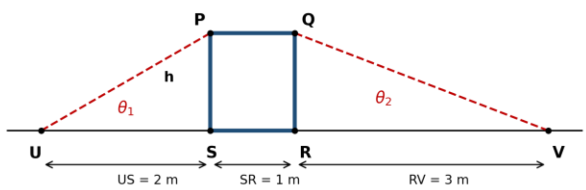
Q. No	Step-by-Step Solution	Marks per Step
SECTION – A (Q1–20, 1 mark each)		
1	$A \cap B = B$ means every element of B is also in A, i.e. $B \subseteq A$. **(b)** $B \subset A$	*1 mark for the correct option.*
2	$n(A) = 3,\ n(B) = 2 \Rightarrow n(A \times B) = 6$. Number of relations $= 2^6 = 64$; non-empty relations $= 64 - 1 = 63$. *Note: printed option (d) shows "632", a typo for 63.* **(d)** 63	*1 mark for the correct option.*
3	$\sin(45^\circ + \theta) = \frac{1}{\sqrt{2}}(\cos\theta + \sin\theta)$; $\cos(45^\circ - \theta) = \frac{1}{\sqrt{2}}(\cos\theta + \sin\theta)$. $\sin(45^\circ + \theta) - \cos(45^\circ - \theta) = 0$. **(d)** 0	*1 mark for the correct option.*
4	$(X \cup Y)' = X' \cap Y'$ (De Morgan's law). $X \cap (X \cup Y)' = X \cap (X' \cap Y') = (X \cap X') \cap Y' = \varphi \cap Y' = \varphi$. **(c)** φ	*1 mark for the correct option.*
5	$1 + i^2 + i^4 + \cdots + i^{2026} = \sum_{k=0}^{1013} i^{2k} = \sum_{k=0}^{1013} (-1)^k$ (1014 terms). An even number of alternating $+1, -1$ terms sums to 0. **(a)** 0	*1 mark for the correct option.*
6	$\lvert x - 2\rvert < 5 \Rightarrow -5 < x - 2 < 5 \Rightarrow -3 < x < 7$. **(d)** $-3 < x < 7$	*1 mark for the correct option.*
7	$f(x) = \sqrt{7 - x}$ needs $7 - x \geq 0 \Rightarrow x \leq 7$. **(c)** 7	*1 mark for the correct option.*
8	Diagonals $= \frac{n(n-3)}{2} = 44 \Rightarrow n^2 - 3n - 88 = 0$. $n = \frac{3 \pm \sqrt{9 + 352}}{2} = \frac{3 \pm 19}{2} \Rightarrow n = 11$. *Note: option (c) is unreadable in the source PDF; the correct value is 11 sides.* **(c)** 11	*1 mark for the correct option.*
9	$\cos 120^\circ = -\frac{1}{2}$; $\sin 210^\circ = -\frac{1}{2}$. $\cos 120^\circ + \sin 210^\circ = -1$. **(d)** -1	*1 mark for the correct option.*

Q. No	Step-by-Step Solution	Marks per Step
10	$\binom{n}{25} = \binom{n}{14} \Rightarrow n - 25 = 14 \Rightarrow n = 39.$ $\binom{n}{1} = n = 39.$ **(d)** 39	*1 mark for the correct option.*
11	$-3x + 17 < -13 \Rightarrow -3x < -30 \Rightarrow x > 10$ (dividing by -3 reverses the inequality). **(a)** $x \in (10, \infty)$	*1 mark for the correct option.*
12	$\sqrt{-9} = 3i,\ \sqrt{-16} = 4i.$ $2\sqrt{-9} \times \sqrt{-16} = 2 \times 3i \times 4i = 24i^2 = -24.$ **(b)** -24	*1 mark for the correct option.*
13	As printed, $5\cos x + 7$ has maximum 12 (at $\cos x = 1$) — not among the given options. *Note: the coefficient appears to be a print error for 2. Taking $2\cos x + 7$: since $-1 \le \cos x \le 1$, range is $[5, 9]$.* **(b)** 9	*1 mark for the correct option.*
14	$5 - x > 2 \Rightarrow x < 3.$ Natural numbers less than 3: $x \in \{1, 2\}$. **(d)** $\{1, 2\}$	*1 mark for the correct option.*
15	$(4 + 3i)^{-1} = \frac{4 - 3i}{4^2 + 3^2} = \frac{4 - 3i}{25} = \frac{4}{25} - \frac{3}{25}i.$ *Note: option (d) as printed appears OCR-garbled; this matches the calculation exactly.* **(d)** $\frac{4}{25} - \frac{3}{25}i$	*1 mark for the correct option.*
16	Let $t = [x]$. $t^2 - 5t + 6 = 0 \Rightarrow (t-2)(t-3) = 0 \Rightarrow t = 2$ or 3. $[x] = 2 \Rightarrow x \in [2, 3)$; $[x] = 3 \Rightarrow x \in [3, 4)$; union $= [2, 4)$. **(d)** $x \in [2, 4)$	*1 mark for the correct option.*
17	$A = \{3, 6, 9, 12\}$ needs a rule generating exactly these 4 values. Option (d): $x = 3(n-1),\ n \in N,\ 1 < n < 6 \Rightarrow n = 2, 3, 4, 5 \Rightarrow x = 3, 6, 9, 12.$ **(d)** $A = \{x : x = 3(n-1),\ n \in N,\ 1 < n < 6\}$	*1 mark for the correct option.*
18	Each toss has 2 outcomes; for 6 independent tosses, total outcomes $= 2^6 = 64$. **(b)** 64	*1 mark for the correct option.*
19	$f(x) = \sin 5x + \cos 5x = \sqrt{2}\sin\left(5x + \frac{\pi}{4}\right)$, amplitude $\sqrt{2}$. Since $-1 \le \sin\theta \le 1$ for all θ (Reason R, true), range of f is $[-\sqrt{2}, \sqrt{2}]$ (Assertion A, true). R is exactly the fact used to derive A, so R correctly explains A.	*1 mark for the correct option.*

Q. No	Step-by-Step Solution	Marks per Step
	(a) Both A and R are true and R is the correct explanation of A.	
20	$\{1, 2, 3, 4\}$ vs $\{3, 2, 5, 1\}$: the element 4 is not in the second set — sets are NOT equal. Assertion A is FALSE. Reason R (the definition of equal sets) is TRUE. (d) A is false but R is true.	*1 mark for the correct option.*
SECTION – B (Q21–25, 2 marks each)		
21	**Part (a)** $A - B = A \cap B'$, so $(A \cap B) \cup (A - B) = (A \cap B) \cup (A \cap B') = A \cap (B \cup B')$. $B \cup B' = U \Rightarrow A \cap (B \cup B') = A \cap U = A$. **Hence** $(A \cap B) \cup (A - B) = A$. **Part (b)** $A = \{x : x^2 + 7\|x\| + 6 = 0\}$. Let $t = \|x\| \geq 0$: $t^2 + 7t + 6 = 0 \Rightarrow (t + 1)(t + 6) = 0 \Rightarrow t = -1, -6$. Both roots negative — invalid for $\|x\|$ — so no real x exists. **Answer:** $A = \varphi$; its only subset is φ itself.	*Step 1 (a): 1 mark for the proof using $A - B = A \cap B'$. Step 1 (b): 1 mark for finding $A = \varphi$ and its only subset.*
22	$(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = \cos^2 x + \sin^2 x + \cos^2 y + \sin^2 y + 2(\cos x \cos y - \sin x \sin y)$. $= 2 + 2\cos(x + y) = 2[1 + \cos(x + y)]$. Using $1 + \cos\theta = 2\cos^2(\theta/2)$: $= 4\cos^2\left(\frac{x+y}{2}\right)$. **Hence proved.**	*Step 1: 1 mark for expanding and grouping using $\cos^2 + \sin^2 = 1$. Step 2: 1 mark for the half-angle step to the final form.*
23	**Part (a)** $x + iy = \frac{a + ib}{a - ib} \Rightarrow \|x + iy\| = \frac{\|a + ib\|}{\|a - ib\|} = 1$ (equal moduli). $\Rightarrow x^2 + y^2 = 1$. **Part (b)** $\left(\frac{1}{3} + 3i\right)^3 = (a^3 - 3ab^2) + i(3a^2b - b^3)$, with $a = \frac{1}{3}$, $b = 3$. Real part $= \frac{1}{27} - 9 = -\frac{242}{27}$; Imaginary part $= 1 - 27 = -26$. **Answer:** $-\frac{242}{27} - 26i$	*Step 1 (a): 1 mark for using $\|a + ib\| = \|a - ib\|$. Step 1 (b): 1 mark for the binomial expansion to $a + ib$ form.*
24	**(i) Repetition allowed** $5 \times 5 \times 5 = 125$. **(ii) Repetition not allowed** $5 \times 4 \times 3 = 60$.	*Step 1: 1 mark for 125 (repetition allowed). Step 2: 1 mark for 60 (repetition not allowed).*

Q. No	Step-by-Step Solution	Marks per Step
25	$l = r\theta \Rightarrow r_1\theta_1 = r_2\theta_2;\ \theta_1 = 75^\circ = \dfrac{5\pi}{12},\ \theta_2 = 90^\circ = \dfrac{\pi}{2}.$ $\dfrac{r_1}{r_2} = \dfrac{\theta_2}{\theta_1} = \dfrac{\pi/2}{5\pi/12} = \dfrac{6}{5}.$ **Answer:** radii in ratio $6:5$.	*Step 1: 1 mark for $r_1\theta_1 = r_2\theta_2$ with correct radian conversion. Step 2: 1 mark for the final ratio $6:5$.*
SECTION – C (Q26–31, 3 marks each)		
26	 $f(x) = 1$ if $x > 0$; $f(x) = 0$ if $x = 0$; $f(x) = -1$ if $x < 0$. **Domain:** $\mathbb{R}$. **Range:** $\{-1, 0, 1\}$.	*Step 1: 1 mark for the piecewise definition. Step 2: 1 mark for the graph. Step 3: 1 mark for domain and range.*
27	**Part (a)** $x + iy = (u + iv)^{1/3} \Rightarrow (x + iy)^3 = u + iv = (x^3 - 3xy^2) + i(3x^2y - y^3).$ $u = x(x^2 - 3y^2),\ v = y(3x^2 - y^2) \Rightarrow \dfrac{u}{x} = x^2 - 3y^2,\ \dfrac{v}{y} = 3x^2 - y^2.$ $\dfrac{u}{x} + \dfrac{v}{y} = 4x^2 - 4y^2 = 4(x^2 - y^2).$ **Part (a) — OR:** $z_1 = 2 - i,\ z_2 = 1 + i$ $z_1 + z_2 + 1 = 4;\quad z_1 - z_2 + i = 1 - i.$ $\dfrac{4}{1-i} = \dfrac{4(1+i)}{2} = 2 + 2i \Rightarrow \lvert 2 + 2i\rvert = \sqrt{8} = 2\sqrt{2}.$ **Answer:** $2\sqrt{2}$	*Step 1: 1 mark for cubing and expanding $(x + iy)^3$. Step 2: 1 mark for forming u/x, v/y. Step 3: 1 mark for the final simplification (OR: rationalising to $2\sqrt{2}$).*
28	**Part (a)** Acid quantity fixed $= 0.45 \times 1125 = 506.25$ L. Let x L water be added. $25 < \dfrac{50625}{1125 + x} < 30 \Rightarrow 562.5 < x < 900.$ **Answer:** between 562.5 L and 900 L. **Part (b) — OR** $5(2x - 7) - 3(2x + 3) \le 0 \Rightarrow x \le 11.$ $2x + 19 \le 6x + 47 \Rightarrow x \ge -7.$ **Answer:** $x \in [-7, 11]$	*Step 1: 1 mark for the fixed acid quantity and inequality setup. Step 2: 1 mark for solving both bounds. Step 3: 1 mark for the final interval.*

Q. No	Step-by-Step Solution	Marks per Step
29	$\frac{2x-1}{3} \geq \frac{3x-2}{4} - \frac{2-x}{5}$. Multiply by LCM $= 60$: $20(2x-1) \geq 15(3x-2) - 12(2-x) \Rightarrow 40x - 20 \geq 57x - 54$. $34 \geq 17x \Rightarrow x \leq 2$. **Answer:** $x \in (-\infty, 2]$	*Step 1: 1 mark for clearing denominators. Step 2: 1 mark for expanding and simplifying. Step 3: 1 mark for the final answer.*
30	**Part (a)** *Note: printed denominator has a typo — coefficient of $\tan^4 x$ should be 1, not 4:* $\frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x} = \tan 4x$. $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$; $\tan 4x = \frac{2\tan 2x}{1-\tan^2 2x}$. Substituting and simplifying: $\tan 4x = \frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$. **Part (b) — OR: find** $\tan(\pi/8)$ $\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$, $\theta = 45°$: $\tan\frac{\pi}{8} = \frac{1-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \sqrt{2}-1$. **Answer:** $\sqrt{2}-1$	*Step 1: 1 mark for applying $\tan 2\theta$ twice. Step 2: 1 mark for expanding $(1-\tan^2 x)^2 - 4\tan^2 x$. Step 3: 1 mark for the final identity (OR: value $\sqrt{2}-1$).*
31	$U = \{1, ..., 9\}$, $A = \{2,4,6,8\}$, $B = \{2,3,5,7\}$. $A \cup B = \{2,3,4,5,6,7,8\} \Rightarrow (A \cup B)' = \{1,9\} = A' \cap B'$. **(i) verified.** $A \cap B = \{2\} \Rightarrow (A \cap B)' = \{1,3,4,5,6,7,8,9\} = A' \cup B'$. **(ii) verified.** **Both De Morgan's laws verified.**	*Step 1: 1 mark for verifying (i). Step 2: 1 mark for verifying (ii). Step 3: 1 mark for the concluding statement.*
	SECTION – D (Q32–35, 5 marks each)	
32	$(1+\cos\frac{5\pi}{8}) = 1-\cos\frac{3\pi}{8}$; $(1+\cos\frac{7\pi}{8}) = 1-\cos\frac{\pi}{8}$ (supplementary angles). Product $= \sin^2\frac{\pi}{8}\sin^2\frac{3\pi}{8} = \sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8} = \left(\frac{1}{2}\sin\frac{\pi}{4}\right)^2$. $= \left(\frac{\sqrt{2}}{4}\right)^2 = \frac{1}{8}$. **OR: Show** $2\sin^2\beta + 4\cos(\alpha+\beta)\sin\alpha\sin\beta + \cos 2(\alpha+\beta) = \cos 2\alpha$ Let $S = \alpha+\beta$. $2\sin^2\beta = 1-\cos 2\beta$. $2\sin\alpha\sin\beta = \cos(\alpha-\beta) - \cos S \Rightarrow 4\cos S\sin\alpha\sin\beta = 2\cos S\cos(\alpha-\beta) - 2\cos^2 S$. $2\cos S\cos(\alpha-\beta) = \cos 2\alpha + \cos 2\beta$ (sum-to-product, since $S \pm (\alpha-\beta) = 2\alpha, 2\beta$). Sum $= 1+\cos 2\alpha - 2\cos^2 S + \cos 2S$; using $2\cos^2 S = \cos 2S + 1$: $= \cos 2\alpha$.	*Step 1: 1 mark for pairing supplementary terms. Step 2: 2 marks for reducing to $\sin^2(\pi/8)\cos^2(\pi/8)$. Step 3: 2 marks for the final evaluation 1/8 (OR: 2+2+1 marks for the identity proof).*
33	$f(x) = \sqrt{2026-x^2}$ needs $2026 - x^2 \geq 0 \Rightarrow x^2 \leq 2026$. **Domain:** $x \in [-\sqrt{2026}, \sqrt{2026}]$	*Step 1: 2 marks for the domain. Step 2: 3 marks for the range.*

Q. No	Step-by-Step Solution	Marks per Step
	$2026-x^2$ ranges over $[0,2026]$ as x^2 ranges over $[0,2026]$; taking the square root: **Range:** $f(x)\in[0,\sqrt{2026}]$	
34	**(i) No girl** $\binom{7}{5}=21.$ **(ii) At least one boy and one girl** $\binom{11}{5}-\binom{7}{5}-0=462-21=441.$ **(iii) At least 3 girls** $\binom{4}{3}\binom{7}{2}+\binom{4}{4}\binom{7}{1}=84+7=91.$ **OR: PERMUTATIONS (12 letters, T repeated twice)** (i) Starts with P, ends with S: $\dfrac{10!}{2!}=1{,}814{,}400.$ (ii) Vowels together: $\dfrac{8!}{2!}\times 5!=20160\times 120=2{,}419{,}200.$ (iii) Exactly 4 letters between P, S: 7 position-pairs $\times 2\times\dfrac{10!}{2!}=14\times 1{,}814{,}400=25{,}401{,}600.$	*Step 1: (i) 1, (ii) 2, (iii) 2 marks — total 5. (OR:) Step 1: 1 mark (i); Step 2: 2 marks (ii); Step 3: 2 marks (iii).*
35	**Part 1 — Prove** $\tan 5x-\tan 4x-\tan x=\tan 5x\tan 4x\tan x$ $\tan 5x=\tan(4x+x)=\dfrac{\tan 4x+\tan x}{1-\tan 4x\tan x}.$ $\Rightarrow \tan 5x-\tan 5x\tan 4x\tan x=\tan 4x+\tan x\Rightarrow \tan 5x-\tan 4x-\tan x=\tan 5x\tan 4x\tan x.$ **Part 2 — Evaluate** $\dfrac{\tan 50^\circ}{2\tan 10^\circ+\tan 40^\circ}$ With $x=10^\circ$: $\tan 50^\circ-\tan 40^\circ-\tan 10^\circ=\tan 50^\circ\tan 40^\circ\tan 10^\circ.$ $\tan 40^\circ=\cot 50^\circ\Rightarrow\tan 50^\circ\tan 40^\circ=1\Rightarrow \text{RHS}=\tan 10^\circ.$ $\Rightarrow \tan 50^\circ=\tan 40^\circ+2\tan 10^\circ\Rightarrow\dfrac{\tan 50^\circ}{2\tan 10^\circ+\tan 40^\circ}=1.$	*Step 1: 2 marks for the identity proof. Step 2: 2 marks for using* $\tan 40^\circ=\cot 50^\circ$. *Step 3: 1 mark for the final value* 1.
SECTION – E (Q36–38, Case-Study Based, 4 marks each)		
36	Side elevation of the platform (not to scale)  $PQRS$ square, side 1 m $\Rightarrow$ height $h=1$ m (all sides of a square are equal). $\tan\theta_1=\dfrac{h}{US}=\dfrac{1}{2}$; $\tan\theta_2=\dfrac{h}{RV}=\dfrac{1}{3}.$	*Step 1: 1 mark for* $h=1$ *m and* $\tan\theta_1,\tan\theta_2$. *Step 2 (i): 1 mark. Step 3 (ii): 1 mark. Step 4 (iii): 2 marks for* $\tan(\theta_1+\theta_2)=1$ *(OR: 2 marks for* $\pi/4$*).*

Q. No	Step-by-Step Solution	Marks per Step
	(i) $\tan\theta_1 + \tan\theta_2 = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$. **(ii)** $\tan\theta_1 \cdot \tan\theta_2 = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$. **(iii)** $\tan(\theta_1+\theta_2) = \frac{5/6}{1-1/6} = \frac{5/6}{5/6} = 1$. **OR (iii): radian measure of $\theta_1+\theta_2$** Both $\theta_1, \theta_2 < 45°$, so $\theta_1+\theta_2 \in (0°, 90°)$; $\tan(\theta_1+\theta_2) = 1 \Rightarrow \theta_1+\theta_2 = \frac{\pi}{4}$.	
37	**(i)** $\binom{10}{2} \times \binom{20}{3} = 45 \times 1140 = 51{,}300$. **(ii)** $\binom{9}{1} \times \binom{20}{3} = 9 \times 1140 = 10{,}260$. **(iii)** $\binom{10}{2} \times \binom{19}{3} = 45 \times 969 = 43{,}605$. **OR (iii): a particular professor excluded** $\binom{9}{2} \times \binom{20}{3} = 36 \times 1140 = 41{,}040$.	*Step 1 (i): 1 mark for 51,300. Step 2 (ii): 1 mark for 10,260. Step 3 (iii): 2 marks for 43,605 (OR: 2 marks for 41,040).*
38	**(i)** $2^{10} = 1024$ subsets; smallest $= \varphi$; largest $= S = \{1, ..., 10\}$. **(ii)** For x, y, z in AP: $x, x+d, x+2d$ with $x \geq 1,\ x+2d \leq 10 \Rightarrow d \leq 4$. Counting: $d{=}1{:}8,\ d{=}2{:}6,\ d{=}3{:}4,\ d{=}4{:}2 \Rightarrow$ total $= 20$.	*Step 1 (i): 2 marks for 1024 with smallest/largest subsets. Step 2 (ii): 2 marks for counting all 20 three-term APs.*