Mind Curve Mid-Term Test Series 2026–27

Class XII | Test 06 – Answer Key (Step-by-Step Format)

Relations & Functions, Inverse Trigonometric Functions, Matrices, Determinants, Continuity & Differentiability, Application of Derivatives, Integrals, Application of Integrals, Differential Equations, Linear Programming

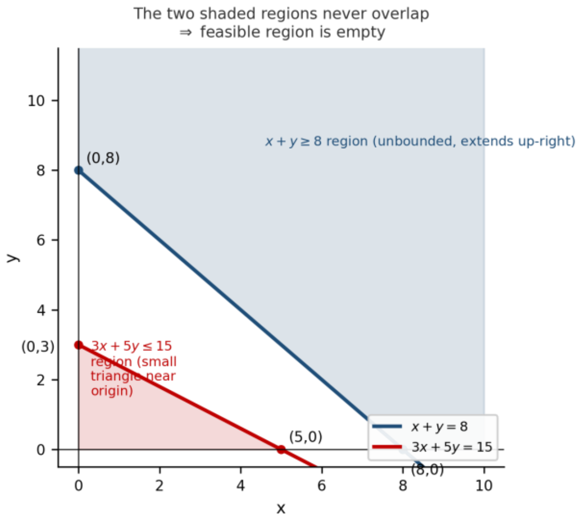
Q.No	Step-by-Step Solution	Marks per Step
	SECTION – A (Q1–20, 1 mark each)	
1	$\begin{pmatrix}2 & -8 & 3\end{pmatrix}$ is 1×3, the middle matrix is 3×3, and the column vector is 3×1. Matrix product dimensions: $(1\times 3)(3\times 3)(3\times 1) = 1\times 1$. So A is a single number (scalar) written as a 1×1 matrix – no need to compute its value. **Answer: (b)** 1×1	1 mark for the correct option.
2	For a set with n elements, the number of reflexive relations is 2^{n^2-n}. Here $n=3$, so the count is $2^{9-3} = 2^6 = 64$. **Answer: (b) 64**	1 mark for the correct option.
3	For a scalar c and an $n\times n$ matrix M: $\lvert cM\rvert = c^n\lvert M\rvert$. Here $\left\lvert \frac{A^{-1}}{2}\right\rvert = \left(\frac{1}{2}\right)^3 \lvert A^{-1}\rvert = \frac{1}{8}\cdot\frac{1}{\lvert A\rvert} = \frac{1}{8\lvert A\rvert}$ (using A is 3×3 and $\lvert A^{-1}\rvert = 1/\lvert A\rvert$). Comparing with $\frac{1}{k\lvert A\rvert}$ gives $k=8$. **Answer: (b) 8**	1 mark for the correct option.
4	$Z = px+qy$. For the minimum to occur at *both* $(3,0)$ and $(1,1)$, Z must take the same value at both corner points. $Z(3,0)=3p$ and $Z(1,1)=p+q$. Setting them equal: $3p=p+q \Rightarrow 2p=q$. Check: with $q=2p$, $Z(0,3)=3q=6p\geq 3p$, so the minimum genuinely occurs along the edge joining $(3,0)$ and $(1,1)$. **Answer: (c)** $2p=q$	1 mark for the correct option.
5	As printed, "A is a symmetric matrix" makes every option symmetric: $A+A^T=2A$, $AA^T=A^2$, $A-A^T=O$, and $A^T=A$ are all symmetric when $A^T=A$. This is the standard question "If A is a *square* matrix, which of the following is **not** symmetric?" – for a general square A, $A-A^T$ is skew-symmetric, not symmetric (it equals O only when A is itself symmetric). **Answer: (c)** $A-A^T$	1 mark for the correct option.
6	$x=\sin^3 t \Rightarrow \frac{dx}{dt} = 3\sin^2 t\cos t$. $y=\cos^3 t \Rightarrow \frac{dy}{dt} = -3\cos^2 t\sin t$. $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-3\cos^2 t\sin t}{3\sin^2 t\cos t} = -\frac{\cos t}{\sin t} = -\cot t$. **Answer: (d)** $-\cot t$	1 mark for the correct option.

Q.No	Step-by-Step Solution	Marks per Step
	SECTION – A (Q1–20, 1 mark each)	
7	Expand the outer derivative: $\frac{d}{dx}\left[\left(\frac{d^2y}{dx^2}\right)^3\right] = 3\left(\frac{d^2y}{dx^2}\right)^2\frac{d^3y}{dx^3} = 0.$ The highest derivative present is $\frac{d^3y}{dx^3}$, so order $= 3$. It appears to the power 1 (once cleared of the equation), so degree $= 1$. Sum of order and degree $= 3 + 1 = 4$. **Answer: (b) 4**	1 mark for the correct option.
8	**Answer: (d)** $\frac{x}{y} = c$	1 mark for the correct option.
9	$\frac{7\pi}{6}$ lies in the third quadrant; $\tan\frac{7\pi}{6} = \tan\left(\frac{7\pi}{6} - \pi\right) = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$. The principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, and $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$. **Answer: (c)** $\frac{\pi}{6}$	1 mark for the correct option.
10	$f(x) = a(x - \cos x) \Rightarrow f'(x) = a(1 + \sin x)$. Since $1 + \sin x \geq 0$ for all x (equal to 0 only at isolated points), f is strictly decreasing iff $f'(x) \leq 0$ everywhere with equality only at isolated points, i.e. iff $a < 0$. **Answer: (c)** $(-\infty, 0)$	1 mark for the correct option.
11	$y = 5e^{7x} + 6e^{-7x} \Rightarrow y' = 35e^{7x} - 42e^{-7x} \Rightarrow y'' = 245e^{7x} + 294e^{-7x}$. $y'' = 49\left(5e^{7x} + 6e^{-7x}\right) = 49y$, so $y'' - 49y = 0$, matching $y'' - ky = 0$ with $k = 49$. **Answer: (c) 49**	1 mark for the correct option.
12	For any $n \times n$ matrix, $A(\text{adj } A) = \|A\|\, I_n$, so $\|A(\text{adj } A)\| = \|A\|^n$. Here $n = 3$, $\|A\| = -2$, so $\|A(\text{adj } A)\| = (-2)^3 = -8$. **Answer: (c)** -8	1 mark for the correct option.
13	Let $u = \sqrt{x}$, so $du = \frac{1}{2\sqrt{x}}dx$, i.e. $dx = 2u\,du$. $\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx = \int \frac{\cos u}{u} \cdot 2u\,du = 2\int \cos u\,du = 2\sin u + C = 2\sin\sqrt{x} + C$. **Answer: (d)** $2\sin\sqrt{x} + C$	1 mark for the correct option.
14	A standard LPP theorem: if an LPP has two optimal solutions, then every point on the line segment joining them is also optimal, so it has infinitely many optimal solutions. **Answer: (d) If an LPP admits two optimal solutions, then it has infinitely many optimal solutions.**	1 mark for the correct option.
15	$\cot^{-1}\sqrt{3} = \frac{\pi}{6}$, so the equation becomes $\cos\left(\tan^{-1} x + \frac{\pi}{6}\right) = 0$. $\Rightarrow \tan^{-1} x + \frac{\pi}{6} = \frac{\pi}{2} \Rightarrow \tan^{-1} x = \frac{\pi}{3} \Rightarrow x = \tan\frac{\pi}{3} = \sqrt{3}$.	1 mark for the correct option.

Q.No	Step-by-Step Solution	Marks per Step
	SECTION – A (Q1–20, 1 mark each)	
	Answer: (d) $\sqrt{3}$	
16	$2\cos 2x - 1 = 2(1-2\sin^2 x) - 1 = 1 - 4\sin^2 x = (1-2\sin x)(1+2\sin x)$. So $\dfrac{2\cos 2x - 1}{1+2\sin x} = 1 - 2\sin x$ (for $1 + 2\sin x \neq 0$). $\int (1-2\sin x)\,dx = x + 2\cos x + C$. **Answer: (b)** $x + 2\cos x + C$	1 mark for the correct option.
17	$\cos^{-1}\left(\dfrac{1}{2}\right) = \dfrac{\pi}{3}$, so $\sin^{-1}(2x) = \dfrac{\pi}{2} - \dfrac{\pi}{3} = \dfrac{\pi}{6}$. $\Rightarrow 2x = \sin\dfrac{\pi}{6} = \dfrac{1}{2} \Rightarrow x = \dfrac{1}{4}$. **Answer: (d)** $\dfrac{1}{4}$	1 mark for the correct option.
18	$A = \begin{pmatrix} 3 & 5 \\ 7 & 9 \end{pmatrix} \Rightarrow A^T = \begin{pmatrix} 3 & 7 \\ 5 & 9 \end{pmatrix}$. $P = \dfrac{A + A^T}{2} = \dfrac{1}{2}\begin{pmatrix} 6 & 12 \\ 12 & 18 \end{pmatrix} = \begin{pmatrix} 3 & 6 \\ 6 & 9 \end{pmatrix}$. **Answer: (b)** $\begin{pmatrix} 3 & 6 \\ 6 & 9 \end{pmatrix}$	1 mark for the correct option.
19	$\sin^{-1} x + \cos^{-1} x = \dfrac{\pi}{2}$ for all $x \in [-1, 1]$, so $\sin^{-1} x + 2\cos^{-1} x = \dfrac{\pi}{2} + \cos^{-1} x$. As $\cos^{-1} x$ ranges over $[0, \pi]$, the expression ranges over $\left[\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right]$, **not** $[0, \pi]$ – so Assertion (A) is **false**. Reason (R), that the principal branch of $\sin^{-1} x$ has range $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$, is a correct standard fact – Reason is **true**. **Answer: A is false but R is true.**	1 mark for the correct option.
20	Let x = distance of the man from the pole, y = shadow length. By similar triangles, $\dfrac{x+y}{6} = \dfrac{y}{2} \Rightarrow 2(x+y) = 6y \Rightarrow y = \dfrac{x}{2}$. So $\dfrac{dy}{dt} = \dfrac{1}{2}\dfrac{dx}{dt} = \dfrac{1}{2}(10) = 5$ m/min – Assertion is **true**, and Reason correctly explains why the factor is exactly one-half. **Answer: (a) Both Assertion and Reason are correct and Reason is the correct explanation for Assertion.**	1 mark for the correct option.
	SECTION – B (Q21–25, 2 marks each)	
21	**(A)** $f(x) = \cos\left(2x + \dfrac{\pi}{4}\right) \Rightarrow f'(x) = -2\sin\left(2x + \dfrac{\pi}{4}\right)$. For $x \in \left(\dfrac{3\pi}{8}, \dfrac{5\pi}{8}\right)$: $2x + \dfrac{\pi}{4} \in \left(\pi, \dfrac{3\pi}{2}\right)$, the third quadrant, where $\sin(\cdot) < 0$.	1 mark for f'(x); 1 mark for the conclusion.

Q.No	Step-by-Step Solution	Marks per Step
SECTION – A (Q1–20, 1 mark each)		
	So $f'(x) = -2(\text{negative}) > 0$ throughout this interval – f is **strictly increasing** on $\left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$. **Answer: increasing on the given interval.** **(B)** $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = 4\pi r^2$. At $r = 10$ cm: $\frac{dV}{dr} = 4\pi(100) = 400\pi$ cm^3 per cm. **Answer:** 400π **cm^3/cm**	
22	$X + Y = \begin{pmatrix} 7 & 0 \\ 2 & 5 \end{pmatrix}$, $X - Y = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$. $X = \frac{(X+Y)+(X-Y)}{2} = \frac{1}{2}\begin{pmatrix} 10 & 0 \\ 2 & 8 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 1 & 4 \end{pmatrix}$. $Y = \frac{(X+Y)-(X-Y)}{2} = \frac{1}{2}\begin{pmatrix} 4 & 0 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$. **Answer:** $X = \begin{pmatrix} 5 & 0 \\ 1 & 4 \end{pmatrix}$, $Y = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$	1 mark for X; 1 mark for Y.
23	$1 + \sin 2x = (\sin x + \cos x)^2$, so $\sqrt{1 + \sin 2x} = \sin x + \cos x$ (taking the positive root). $1 + \cos 2x = 2\cos^2 x$. So the integrand becomes $e^x \cdot \frac{\sin x + \cos x}{2\cos^2 x} = \frac{1}{2}e^x(\sec x \tan x + \sec x)$. Using $\int e^x[f(x)+f'(x)]\,dx = e^x f(x) + C$ with $f(x) = \sec x$: the integral is $\frac{1}{2}e^x \sec x + C$. **Answer:** $\frac{1}{2}e^x \sec x + C$	1 mark for simplifying the integrand; 1 mark for applying $\int e^x[f + f']dx$.
24	$x = \cos t + \ln\tan\frac{t}{2}$: $\frac{dx}{dt} = -\sin t + \frac{\sec^2(t/2)}{2\tan(t/2)} = -\sin t + \frac{1}{\sin t} = \frac{\cos^2 t}{\sin t}$. $y = \sin t \Rightarrow \frac{dy}{dt} = \cos t$. $\frac{dy}{dx} = \frac{\cos t}{\cos^2 t/\sin t} = \frac{\sin t}{\cos t} = \tan t$. $\frac{d^2y}{dx^2} = \frac{d(\tan t)/dt}{dx/dt} = \frac{\sec^2 t}{\cos^2 t/\sin t} = \frac{\sin t}{\cos^4 t}$. At $t = \frac{\pi}{4}$: $\sin t = \cos t = \frac{1}{\sqrt{2}}$, so $\cos^4 t = \frac{1}{4}$, giving $\frac{d^2y}{dx^2} = \frac{1/\sqrt{2}}{1/4} = 2\sqrt{2}$. **Answer:** $2\sqrt{2}$	1 mark for dy/dx; 1 mark for the value of d²y/dx².
25	**(A)** Let $u = x + \sqrt{x^2 - 1}$, so $y = u^2$ and $\frac{du}{dx} = 1 + \frac{x}{\sqrt{x^2-1}} = \frac{u}{\sqrt{x^2-1}}$. $\frac{dy}{dx} = 2u\frac{du}{dx} = \frac{2u^2}{\sqrt{x^2-1}} = \frac{2y}{\sqrt{x^2-1}}$.	1 mark for dy/dx; 1 mark for the final identity.

Q.No	Step-by-Step Solution	Marks per Step
	SECTION – A (Q1–20, 1 mark each)	
	Squaring: $\left(\frac{dy}{dx}\right)^2 = \frac{4y^2}{x^2-1}$, so $(x^2-1)\left(\frac{dy}{dx}\right)^2 = 4y^2$. **Shown.** **(B)** $y = 5\cos x - 3\sin x \Rightarrow y' = -5\sin x - 3\cos x \Rightarrow y'' = -5\cos x + 3\sin x$. $y'' + y = (-5\cos x + 3\sin x) + (5\cos x - 3\sin x) = 0$. **Proved.**	
	SECTION – C (Q26–31, 3 marks each)	
26	For $x < 0$, we evaluate the limit as $x \to 0^-$: $$\text{LHL} = \lim_{x\to 0^-} \frac{x + \sin x}{\sin(a+1)x}$$ Divide both the numerator and the denominator by x: $$\text{LHL} = \lim_{x\to 0^-} \frac{1 + \frac{\sin x}{x}}{\frac{\sin(a+1)x}{x}}$$ Using the standard limit property $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$, we get: $\text{LHL} = \frac{1+1}{a+1} = \frac{2}{a+1}$ For the function to be continuous, the LHL must equal $f(0)$, which is given as 2: $$\frac{2}{a+1} = 2$$ $$a + 1 = 1$$ $$a = 0$$ $\text{RHL} = \lim_{x\to 0^+} \frac{2(e^{\sin bx} - 1)}{bx}$ Rewrite the limit by multiplying and dividing by $\sin bx$: $$\text{RHL} = 2 \cdot \lim_{x\to 0^+} \left(\frac{e^{\sin bx} - 1}{\sin bx} \cdot \frac{\sin bx}{bx} \right)$$ Using the standard limits $\lim_{u\to 0} \frac{e^u - 1}{u} = 1$ and $\lim_{v\to 0} \frac{\sin v}{v} = 1$: $$\text{RHL} = 2 \cdot 1 \cdot 1 = 2$$ Since $\text{RHL} = f(0) = 2$, the function is perfectly consistent for any non-zero value of b	1 mark for LHL; 1 mark for RHL; 1 mark for the value of a.

Q.No	Step-by-Step Solution	Marks per Step
	SECTION – A (Q1–20, 1 mark each)	
27	 Constraints: $x + y \geq 8,\ 3x + 5y \leq 15,\ x, y \geq 0$. Since $y \geq 0$, $5y \geq 3y$, so on the region $x+y \geq 8$: $3x+5y \geq 3x+3y = 3(x+y) \geq 3(8) = 24$. But the second constraint requires $3x + 5y \leq 15$. Since $24 > 15$, no point with $x, y \geq 0$ can satisfy both constraints at once. **Answer: The feasible region is empty – this LPP has no feasible solution.**	1 mark for identifying the bound $3x + 5y \geq 24$; 2 marks for concluding infeasibility.
28	Split $[1, 5]$ at the corner points $x = 1, 2, 4$: on $[1, 2]$ the sum is $(x - 1) + (2 - x) + (4 - x) = 5 - x$; on $[2, 4]$ it is $(x - 1) + (x - 2) + (4 - x) = x + 1$; on $[4, 5]$ it is $(x - 1) + (x - 2) + (x - 4) = 3x - 7$. $\int_1^2 (5 - x)\, dx = \left[5x - \frac{x^2}{2}\right]_1^2 = 8 - 4.5 = 3.5.$ $\int_2^4 (x + 1)\, dx = \left[\frac{x^2}{2} + x\right]_2^4 = 12 - 4 = 8.$ $\int_4^5 (3x - 7)\, dx = \left[\frac{3x^2}{2} - 7x\right]_4^5 = 2.5 - (-4) = 6.5.$ Total $= 3.5 + 8 + 6.5 = 18$. **Answer: 18**	1 mark for splitting the interval; 1 mark for each piece's integral; 1 mark for the total.
29	**(A)** $x\frac{dy}{dx} + y - x + xy \cot x = 0 \Rightarrow \frac{dy}{dx} + y\left(\frac{1}{x} + \cot x\right) = 1$ (dividing by x) – linear in y. Integrating factor $= e^{\int (1/x + \cot x)\, dx} = e^{\ln x + \ln \sin x} = x \sin x$. $y \cdot x \sin x = \int x \sin x\, dx = -x \cos x + \sin x + C$ (by parts). **Answer:** $xy \sin x = -x \cos x + \sin x + C$ **(B)** Separate variables: $\frac{dy}{1 + y^2} = \frac{-e^x\, dx}{1 + e^{2x}}$. Integrate: $\tan^{-1} y = -\tan^{-1}(e^x) + C$ (substitute $u = e^x$ on the right).	1 mark for the linear form; 1 mark for the integrating factor; 1 mark for the solution.

Q.No	Step-by-Step Solution	Marks per Step
	SECTION – A (Q1–20, 1 mark each)	
	At $x=0, y=1$: $\frac{\pi}{4} = -\frac{\pi}{4} + C \Rightarrow C = \frac{\pi}{2}$, so $\tan^{-1} y + \tan^{-1}(e^x) = \frac{\pi}{2}$, which simplifies to $y = e^{-x}$. **Answer:** $y = e^{-x}$	
30	**(A)** $f(1) = \frac{1+1}{2} = 1$ (1 is odd) and $f(2) = \frac{2}{2} = 1$ (2 is even) – so $f(1) = f(2) = 1$ with $1 \neq 2$. f is **not injective**, so f is **not bijective**. **Answer: f is not bijective (fails injectivity, e.g. f(1)=f(2)=1).** **(B)** **Reflexive:** $(a,b)R(a,b)$ needs $ab(b+a) = ba(a+b)$, true by commutativity for all (a,b). **Symmetric:** $ad(b+c) = bc(a+d)$ is unchanged (just relabelled) when $(a,b) \leftrightarrow (c,d)$ are swapped – so $(a,b)R(c,d) \Rightarrow (c,d)R(a,b)$. **Key simplification:** expanding $ad(b+c) = bc(a+d)$ and rearranging gives $bd(a-c) = ac(b-d)$, i.e. $\frac{1}{c} - \frac{1}{a} = \frac{1}{d} - \frac{1}{b}$, i.e. $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$. **Transitive:** the relation is exactly "$g(a,b) = g(c,d)$" where $g(a,b) = \frac{1}{a} - \frac{1}{b}$. Equality of real numbers is always reflexive, symmetric and transitive, so R is an **equivalence relation**.	1 mark for the counterexample; 1 mark for the conclusion.
31	$A' = A^{-1}$ means $A^T A = I$. With $A = \begin{pmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{pmatrix}$, compute $A^T A$ column-by-column. Diagonal entries: $(A^T A)_{11} = 0^2 + x^2 + x^2 = 2x^2$; $(A^T A)_{22} = (2y)^2 + y^2 + (-y)^2 = 6y^2$; $(A^T A)_{33} = z^2 + (-z)^2 + z^2 = 3z^2$. Off-diagonal entries all vanish by direct computation (each is a sum that telescopes to 0), confirming $A^T A = \mathrm{diag}(2x^2, 6y^2, 3z^2)$. Setting $A^T A = I$: $2x^2 = 1,\ 6y^2 = 1,\ 3z^2 = 1 \Rightarrow x = \frac{1}{\sqrt{2}},\ y = \frac{1}{\sqrt{6}},\ z = \frac{1}{\sqrt{3}}$. **Answer:** $x = \frac{1}{\sqrt{2}},\ y = \frac{1}{\sqrt{6}},\ z = \frac{1}{\sqrt{3}}$	1 mark for setting up $A^T A = I$; 2 marks for the three values (combined).
	SECTION – D (Q32–35, 5 marks each)	
32	**(A)** $\sqrt{\tan x} + \sqrt{\cot x} = \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}}$. Let $t = \sin x - \cos x$, so $dt = (\cos x + \sin x)dx$ and $t^2 = 1 - 2\sin x \cos x$, giving $\sin x \cos x = \frac{1-t^2}{2}$. Integral becomes $\int \frac{dt}{\sqrt{(1-t^2)/2}} = \sqrt{2} \int \frac{dt}{\sqrt{1-t^2}} = \sqrt{2} \sin^{-1} t + C$. At $x = 0$: $t = -1$; at $x = \pi/4$: $t = 0$. Definite value $= \sqrt{2}\left[\sin^{-1}(0) - \sin^{-1}(-1)\right] = \sqrt{2}\left(0 + \frac{\pi}{2}\right) = \frac{\pi}{\sqrt{2}}$.	1 mark for the substitution; 2 marks for the antiderivative; 2 marks for evaluating the limits.

Q.No	Step-by-Step Solution	Marks per Step
	SECTION – A (Q1–20, 1 mark each)	
	Answer: $\dfrac{\pi}{\sqrt{2}}$ **(B)** Let $f(x) = \dfrac{x^2}{1+e^x}$. Then $f(-x) = \dfrac{x^2}{1+e^{-x}} = \dfrac{x^2 e^x}{e^x+1}$, so $f(x) + f(-x) = x^2$. By the property $\int_{-a}^{a} f(x)\,dx = \int_0^a \big[f(x) + f(-x)\big]\,dx$: the integral equals $\int_0^a x^2\,dx = \dfrac{a^3}{3}$. **Answer:** $\dfrac{a^3}{3}$	
33	**(A)** System: $x + 2y + 3z = 6,\ 2x - y + z = 2,\ 3x + 2y - 2z = 3$; matrix form $AX = B$ with $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 2 & -2 \end{pmatrix}$. $\lvert A\rvert = 1(2-2) - 2(-4-3) + 3(4+3) = 0 + 14 + 21 = 35 \neq 0$, so a unique solution exists. By Cramer's rule: $D_x = 35,\ D_y = 35,\ D_z = 35$, so $x = \dfrac{35}{35} = 1,\ y = 1,\ z = 1$. Check: $1 + 2 + 3 = 6$; $2 - 1 + 1 = 2$; $3 + 2 - 2 = 3$ – all satisfied. **Answer:** $x = 1,\ y = 1,\ z = 1$ **(B)** $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} \Rightarrow A^2 = \begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix}$ (direct multiplication). $A^3 = A^2 \cdot A = \begin{pmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{pmatrix}$. $A^3 - 6A^2 + 7A + 2I = \begin{pmatrix} 21-30+7+2 & 0 & 34-48+14+0 \\ 12-12+0+0 & 8-24+14+2 & 23-30+7+0 \\ 34-48+14+0 & 0 & 55-78+21+2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = O$. **Shown.**	1 mark for $\lvert A\rvert$; 3 marks for D_x, D_y, D_z; 1 mark for the solution.
34	**(A)**	1 mark for the intersection points; 3 marks for the integral; 1 mark for the final area.

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<tr><th>Q.No</th><th>Step-by-Step Solution</th><th>Marks per Step</th></tr>
<tr><td colspan="3">SECTION – A (Q1–20, 1 mark each)</td></tr>
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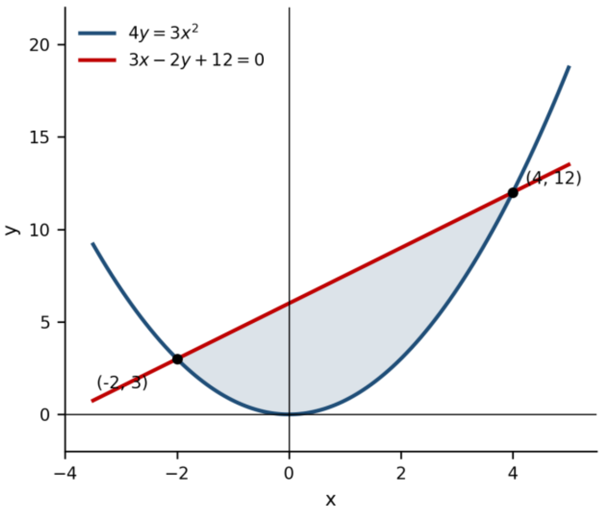

Parabola: $y = \frac{3}{4}x^2$. Line: $3x - 2y + 12 = 0 \Rightarrow y = \frac{3}{2}x + 6$.

Intersection: $\frac{3}{4}x^2 = \frac{3}{2}x + 6 \Rightarrow 3x^2 - 6x - 24 = 0 \Rightarrow x^2 - 2x - 8 = 0 \Rightarrow (x-4)(x+2) = 0$, so $x = -2,\ 4$.

Area $= \int_{-2}^{4} \left[\left(\frac{3}{2}x + 6\right) - \frac{3}{4}x^2\right] dx = \left[\frac{3}{4}x^2 + 6x - \frac{1}{4}x^3\right]_{-2}^{4} = 20 - (-7) = 27.$

Answer: 27 square units

(B)

Intersection of $y^2 = 16ax$ and $y = 4mx$: $16m^2x^2 = 16ax \Rightarrow x = 0$ or $x = \frac{a}{m^2}$.

Area $= \int_0^{a/m^2} [4\sqrt{ax} - 4mx]\, dx = \frac{8\sqrt{a}}{3}\left(\frac{a}{m^2}\right)^{3/2} - 2m\left(\frac{a}{m^2}\right)^2 = \frac{8a^2}{3m^3} - \frac{2a^2}{m^3} = \frac{2a^2}{3m^3}$.

Setting $\frac{2a^2}{3m^3} = \frac{a^2}{12}$: $\frac{2}{3m^3} = \frac{1}{12} \Rightarrow m^3 = 8 \Rightarrow m = 2$.

Answer: $m = 2$

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<tr><td>35</td><td>

(A)

Let x = wire length used for the square, so side $= \frac{x}{4}$, area $= \frac{x^2}{16}$; remaining $(28 - x)$ forms the circle's circumference, so $r = \frac{28 - x}{2\pi}$, area $= \frac{(28-x)^2}{4\pi}$.

Total area $A(x) = \frac{x^2}{16} + \frac{(28-x)^2}{4\pi}$. $\frac{dA}{dx} = \frac{x}{8} - \frac{28-x}{2\pi}$; setting $= 0$: $2\pi x = 8(28 - x) \Rightarrow x = \frac{224}{2\pi + 8} = \frac{112}{\pi + 4}$.

$\frac{d^2A}{dx^2} = \frac{1}{8} + \frac{1}{2\pi} > 0$, confirming a minimum. Remaining length for the circle: $28 - x = \frac{28\pi}{\pi + 4}$.

Answer: square piece $= \frac{112}{\pi+4}$ **m, circle piece** $= \frac{28\pi}{\pi+4}$ **m (minimises combined area).**

</td><td>1 mark for setting up A(x); 2 marks for dA/dx=0; 1 mark for the second-derivative check; 1 mark for both lengths.</td></tr>
</table>

Q.No	Step-by-Step Solution	Marks per Step
	SECTION – A (Q1–20, 1 mark each)	
	(B) $f(x) = (x-2)^2(x-1) = x^3 - 5x^2 + 8x - 4$. Rate of motion along the curve $= f'(x) = 3x^2 - 10x + 8$. Critical points: $3x^2 - 10x + 8 = 0 \Rightarrow x = \dfrac{10 \pm 2}{6} \Rightarrow x = 2$ or $x = \dfrac{4}{3}$. $f''(x) = 6x - 10$: at $x = \dfrac{4}{3}$, $f'' < 0$ (local max); at $x = 2$, $f'' > 0$ (local min). Since $f'(x) = 3(x-2)\left(x - \dfrac{4}{3}\right)$ is an upward parabola in x: $f' > 0$ (increasing) on $\left(-\infty, \dfrac{4}{3}\right) \cup (2, \infty)$, and $f' < 0$ (decreasing) on $\left(\dfrac{4}{3}, 2\right)$. **Answer: critical points $x = \frac{4}{3}, 2$; increasing on $(-\infty, \frac{4}{3}) \cup (2, \infty)$, decreasing on $(\frac{4}{3}, 2)$.**	
	SECTION – E (Q36–38, 4 marks each)	
36	**(i)** Length $x = 3k$ ("3 times as long as wide"). Volume $= x \cdot k \cdot h = 3k^2 h = 1\text{ ft}^3 \Rightarrow h = \dfrac{1}{3k^2}$. **(ii)** $S = 2(xk + xh + kh) = 2(3k^2 + 4kh) = 6k^2 + 8kh$. Substituting $h = \dfrac{1}{3k^2}$: $S(k) = 6k^2 + \dfrac{8}{3k}$. **(iii)(a)** $\dfrac{dS}{dk} = 12k - \dfrac{8}{3k^2}$. Setting $= 0$: $36k^3 = 8 \Rightarrow k^3 = \dfrac{2}{9} \Rightarrow k = \left(\dfrac{2}{9}\right)^{1/3}$. **(iii)(b)** $\dfrac{d^2S}{dk^2} = 12 + \dfrac{16}{3k^3}$, which is positive for every $k > 0$ – so it is positive at this k, confirming S is minimised there, i.e. this k gives the least surface area (material) for the required volume. **Answer:** $h = \dfrac{1}{3k^2}$; $S(k) = 6k^2 + \dfrac{8}{3k}$; $k = \left(\dfrac{2}{9}\right)^{1/3}$ **minimises S.**	1 mark each for (i) and (ii); 2 marks for (iii)(a)/(iii)(b) combined.
37	**(i)** Rows P, Q, R; columns notebooks, pens, pencils (in dozens): $X = \begin{pmatrix} 12 & 5 & 6 \\ 10 & 6 & 7 \\ 11 & 13 & 8 \end{pmatrix}$. **(ii)** P buys 12 dozen $= 144$ notebooks, 5 dozen$= 60$ pens, 6 dozen$= 72$ pencils. Amount $= 144(40) + 60(12) + 72(3) = 5760 + 720 + 216 = 6696$. **(iii)(a)** With $Y = \begin{pmatrix} 480 \\ 144 \\ 36 \end{pmatrix}$ (cost per dozen: 12×40, 12×12, 12×3), XY gives each shopkeeper's total bill: $XY = \begin{pmatrix} 12(480) + 5(144) + 6(36) \\ 10(480) + 6(144) + 7(36) \\ 11(480) + 13(144) + 8(36) \end{pmatrix} = \begin{pmatrix} 6696 \\ 5916 \\ 7440 \end{pmatrix}$. **Answer: P pays Rs. 6696;** $XY = (6696, 5916, 7440)^T$ **(P, Q, R's bills).** **OR (iii)(b)** $\det(X) = 12(48 - 91) - 5(80 - 77) + 6(130 - 66) = 12(-43) - 5(3) + 6(64) = -516 - 15 + 384 = -147$. **Answer:** $\det(X) = -147$	1 mark for (i); 1 mark for (ii); 2 marks for (iii)(a).
38	**(i)** $R = \{(x, y) : y = x + 5,\ x < 4\}$ on $\mathbb{N}$ gives the pairs $(1, 6), (2, 7), (3, 8)$.	1 mark for (i); 3 marks for (ii).

Q.No	Step-by-Step Solution	Marks per Step
	SECTION – A (Q1–20, 1 mark each)	
	Reflexive? Would need $x = x+5$ for some pair (x, x) – impossible, so R is **not reflexive**. *Symmetric?* $(1, 6) \in R$ but $(6, 1) \in R$ would need $1 = 6 + 5 = 11$ – false, so R is **not symmetric**. *Transitive?* Would need some $(x, y), (y, z) \in R$ with $y < 4$ – but every second element $(6, 7, 8)$ is ≥ 4, so no such chain exists; the condition holds **vacuously**, so R **is transitive**. **(ii)** $R = \{(a, b) : 2 \mid (a - b)\}$ on $\mathbb{Z}$. Reflexive: $a - a = 0$, and $2 \mid 0$. Symmetric: $2 \mid (a - b) \Rightarrow a - b = 2k \Rightarrow b - a = 2(-k) \Rightarrow 2 \mid (b - a)$. Transitive: $a - b = 2k$, $b - c = 2m \Rightarrow a - c = 2(k + m) \Rightarrow 2 \mid (a - c)$. All three properties hold, so R **is an equivalence relation** (it partitions $\mathbb{Z}$ into evens and odds). **Answer: (i) not reflexive, not symmetric, transitive (vacuously). (ii) R is an equivalence relation.**	

Prepared by Deepika Bhati | Infinity Think Beyond