

# Mind Curve Mid-Term Test Series 2025-26

Class X | Test 06 – Answer Key

**Chapters 1–9, 13, 14: Real Numbers, Polynomials, Pair of Linear Equations, Quadratic Equations, Arithmetic Progression, Similar Triangles, Coordinate Geometry, Introduction to Trigonometry, Applications of Trigonometry, Statistics & Probability**

| Q.No                                    | Step-by-Step Solution                                                                                                                                                                                                                                                                                                                               | Marks per Step                 |
|-----------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------|
| <b>SECTION – A (Q1–20, 1 mark each)</b> |                                                                                                                                                                                                                                                                                                                                                     |                                |
| <b>1</b>                                | <b>Q1</b><br>$y = 0$ and $y = -7$ represent two distinct horizontal lines (both parallel to the x-axis)<br>Distinct parallel lines never intersect, so the pair has no common solution<br><b>Answer: (d) no solution</b>                                                                                                                            | 1 mark for the correct option. |
| <b>2</b>                                | <b>Q2</b><br>Let the common difference be $d$ . 4th terms: $(-1 + 3d)$ and $(-8 + 3d)$<br>Difference = $(-1 + 3d) - (-8 + 3d) = -1 + 8 = 7$ (the $d$ terms cancel, so the difference of 4th terms equals the difference of first terms)<br><b>Answer: (c) 7</b>                                                                                     | 1 mark for the correct option. |
| <b>3</b>                                | <b>Q3</b><br>$\triangle ABC \sim \triangle DEF \Rightarrow \frac{AB}{DE} = \frac{BC}{EF}$<br>$\frac{3}{4} = \frac{6}{EF} \Rightarrow EF = \frac{6 \times 4}{3} = 8$<br><b>Answer: (a) 8 cm</b>                                                                                                                                                      | 1 mark for the correct option. |
| <b>4</b>                                | <b>Q4</b><br>In parallelogram PQRS, diagonals PR and QS bisect each other<br>Midpoint of PR = $\left(\frac{3-3}{2}, \frac{4-2}{2}\right) = (0, 1)$<br>Let $S = (x, y)$ . Midpoint of QS = $\left(\frac{-2+x}{2}, \frac{3+y}{2}\right) = (0, 1)$<br>$-2 + x = 0 \Rightarrow x = 2; \quad 3 + y = 2 \Rightarrow y = -1$<br><b>Answer: (c) (2, -1)</b> | 1 mark for the correct option. |
| <b>5</b>                                | <b>Q5</b><br>$\text{HCF}(a, b) = p^{\min(3,2)} q^{\min(4,3)} = p^2 q^3 \Rightarrow m = 2, n = 3$<br>$\text{LCM}(a, b) = p^{\max(3,2)} q^{\max(4,3)} = p^3 q^4 \Rightarrow r = 3, s = 4$<br>$(m + n)(r + s) = (2 + 3)(3 + 4) = 5 \times 7 = 35$<br><b>Answer: (c) 35</b>                                                                             | 1 mark for the correct option. |
| <b>6</b>                                | <b>Q6</b><br>Since $p$ is prime, its only factors are 1 and $p$ — these are the required roots<br>Sum of roots = $1 + p$ , product of roots = $1 \times p = p$<br>Equation: $x^2 - (\text{sum})x + (\text{product}) = 0 \Rightarrow x^2 - (p + 1)x + p = 0$<br><b>Answer: (b) <math>x^2 - (p + 1)x + p = 0</math></b>                               | 1 mark for the correct option. |
| <b>7</b>                                | <b>Q7</b><br>Squaring $\sin \theta + \cos \theta = \sqrt{2}$ : $1 + 2 \sin \theta \cos \theta = 2 \Rightarrow \sin \theta \cos \theta = \frac{1}{2}$                                                                                                                                                                                                | 1 mark for the correct option. |

| Q.No | Step-by-Step Solution                                                                                                                                                                                                                                                                                                              | Marks per Step                 |
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|      | $\tan \theta + \cot \theta = \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} = \frac{1}{1/2} = 2$ <p><b>Answer: (b) 2</b></p>                                                                                                                                                                                        |                                |
| 8    | <p><b>Q8</b></p> $x \tan 60^\circ \cos 60^\circ = \sin 60^\circ \cot 60^\circ \Rightarrow x \times \sqrt{3} \times \frac{1}{2} = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}}$ $x \times \frac{\sqrt{3}}{2} = \frac{1}{2} \Rightarrow x = \frac{1}{\sqrt{3}} = \tan 30^\circ$ <p><b>Answer: (b) <math>\tan 30^\circ</math></b></p> | 1 mark for the correct option. |
| 9    | <p><b>Q9</b></p> <p>For <math>f(x) = px^2 - 2x + 3p</math>: <math>\alpha + \beta = \frac{2}{p}</math>, <math>\alpha\beta = \frac{3p}{p} = 3</math></p> <p>Given <math>\alpha + \beta = \alpha\beta</math>: <math>\frac{2}{p} = 3 \Rightarrow p = \frac{2}{3}</math></p> <p><b>Answer: (b) <math>\frac{2}{3}</math></b></p>         | 1 mark for the correct option. |
| 10   | <p><b>Q10</b></p> $\triangle ABC \sim \triangle DEF \Rightarrow \frac{AB}{DE} = \frac{4}{6} = \frac{2}{3}$ $BC = \frac{2}{3} \times EF = \frac{2}{3} \times 9 = 6; \quad CA = \frac{2}{3} \times FD = \frac{2}{3} \times 12 = 8$ <p>Perimeter = <math>AB + BC + CA = 4 + 6 + 8 = 18</math> cm</p> <p><b>Answer: (d) 18 cm</b></p>  | 1 mark for the correct option. |
| 11   | <p><b>Q11</b></p> <p>Since <math>DE \parallel BC</math>, <math>\triangle ADE \sim \triangle ABC</math>, giving <math>\frac{AE}{AC} = \frac{DE}{BC}</math></p> $\frac{a}{a+b} = \frac{x}{y} \Rightarrow x = \frac{ay}{a+b}$ <p><b>Answer: (c) <math>x = \frac{ay}{a+b}</math></b></p>                                               | 1 mark for the correct option. |
| 12   | <p><b>Q12</b></p> <p>Since <math>AD \parallel BC</math>, <math>\triangle AOD \sim \triangle COB</math> (AA, alternate angles), giving <math>\frac{AO}{OC} = \frac{AD}{BC}</math></p> $\frac{1}{2} = \frac{4}{BC} \Rightarrow BC = 8 \text{ cm}$ <p><b>Answer: (c) 8cm</b></p>                                                      | 1 mark for the correct option. |
| 13   | <p><b>Q13</b></p> <p>First 3-digit multiple of 7: <math>105 = 7 \times 15</math>. Last: <math>994 = 7 \times 142</math></p> <p>Count = <math>142 - 15 + 1 = 128</math></p> <p><b>Answer: (c) 128</b></p>                                                                                                                           | 1 mark for the correct option. |
| 14   | <p><b>Q14</b></p> <p>For <math>p(x) = 2x^2 - 7x + k</math>: product of zeroes = <math>\frac{k}{2}</math></p> <p>For reciprocal zeroes, product = 1: <math>\frac{k}{2} = 1 \Rightarrow k = 2</math></p> <p><b>Answer: (b) 2</b></p>                                                                                                 | 1 mark for the correct option. |

| Q.No                                      | Step-by-Step Solution                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | Marks per Step                 |
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| 15                                        | <p><b>Q15</b></p> $5 \tan \beta = 4 \Rightarrow \tan \beta = \frac{4}{5}$ <p>Dividing numerator and denominator of <math>\frac{5 \sin \beta - 2 \cos \beta}{5 \sin \beta + 2 \cos \beta}</math> by <math>\cos \beta</math>:</p> $\frac{5 \tan \beta - 2}{5 \tan \beta + 2} = \frac{4 - 2}{4 + 2} = \frac{2}{6} = \frac{1}{3}$ <p><b>Answer: (a) <math>\frac{1}{3}</math></b></p>                                                                                                                                   | 1 mark for the correct option. |
| 16                                        | <p><b>Q16</b></p> <p>A.P.: 3, 8, 13, 18, ... has <math>a = 3</math>, <math>d = 5</math></p> $a_n = 3 + (n - 1)5 = 78 \Rightarrow (n - 1)5 = 75 \Rightarrow n = 16$ <p><b>Answer: (d) 16th</b></p>                                                                                                                                                                                                                                                                                                                  | 1 mark for the correct option. |
| 17                                        | <p><b>Q17</b></p> <p>Midpoint of P(-6,5) and Q(-2,3) has <math>x</math>-coordinate <math>\frac{-6 - 2}{2} = -4</math></p> $\frac{p}{3} = -4 \Rightarrow p = -12$ <p><b>Answer: (b) -12</b></p>                                                                                                                                                                                                                                                                                                                     | 1 mark for the correct option. |
| 18                                        | <p><b>Q18</b></p> <p>Total frequency = <math>13 + 10 + 15 + 8 + 11 = 57</math>; <math>\frac{n}{2} = 28.5</math></p> <p>Since these are inclusive (gap-of-1) class intervals, the true (continuous) boundaries are obtained by subtracting/adding 0.5: e.g. 12–17 becomes 11.5–17.5</p> <p>Cumulative frequencies: 13, 23, 38, ... — the class reaching 28.5 is 11.5–17.5, so the upper limit of the median class is 17.5</p> <p><b>Answer: (d) 17.5</b></p>                                                        | 1 mark for the correct option. |
| 19                                        | <p><b>Q19</b></p> <p>By the Midpoint theorem, <math>DE = \frac{1}{2}BC</math>, i.e. <math>BC = 2 \times DE</math></p> $DE = \sqrt{(3 - (-3))^2 + (5 - (-3))^2} = \sqrt{36 + 64} = \sqrt{100} = 10$ $BC = 2 \times 10 = 20 \text{ units} \text{ — Assertion (A) is TRUE}$ <p>Reason (R) correctly states the Midpoint theorem, and is exactly why (A) holds</p> <p><b>Answer: (a) Both Assertion and Reason are correct, and Reason is the correct explanation for Assertion.</b></p>                               | 1 mark for the correct option. |
| 20                                        | <p><b>Q20</b></p> $\text{LCM} = \frac{\text{product of the numbers}}{\text{HCF}} = \frac{5780}{17} = 340 \text{ — Assertion (A) is TRUE}$ <p>Reason (R), "HCF is always a factor of LCM", is a TRUE general fact, but it is not what directly explains why <math>\text{LCM} = 340</math> here — that follows from the product–HCF–LCM relationship, not merely from HCF dividing LCM</p> <p><b>Answer: (b) Both Assertion and Reason are correct, but Reason is not the correct explanation for Assertion.</b></p> | 1 mark for the correct option. |
| <b>SECTION – B (Q21–25, 2 marks each)</b> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |                                |
| 21                                        | <p><b>Q21</b></p> <p>Total cards = 20</p> <p>Primes from 1–20: 2, 3, 5, 7, 11, 13, 17, 19 — 8 numbers. <math>P(\text{prime}) = \frac{8}{20} = \frac{2}{5}</math></p>                                                                                                                                                                                                                                                                                                                                               | 1 mark for each probability.   |

| Q.No | Step-by-Step Solution                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Marks per Step                                                                                                                   |
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|      | <p>Multiples of 6 from 1-20: 6, 12, 18 — 3 numbers. <math>P(\text{multiple of 6}) = \frac{3}{20}</math></p> <p><b>Answer:</b> <math>P(\text{prime}) = \frac{2}{5}</math>; <math>P(\text{multiple of 6}) = \frac{3}{20}</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |                                                                                                                                  |
| 22   | <p><b>Q22</b></p> <p>From <math>x + y = 2m</math>: <math>y = 2m - x</math></p> <p>Substitute into <math>mx - ny = m^2 + n^2</math>: <math>mx - n(2m - x) = m^2 + n^2</math></p> <p><math>mx - 2mn + nx = m^2 + n^2 \Rightarrow x(m + n) = m^2 + 2mn + n^2 = (m + n)^2</math></p> <p><math>x = m + n</math> (for <math>m + n \neq 0</math>); then <math>y = 2m - (m + n) = m - n</math></p> <p><b>Answer:</b> <math>x = m + n</math>, <math>y = m - n</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                   | <p>1 mark for finding <math>x</math>;<br/>1 mark for finding <math>y</math>.</p>                                                 |
| 23   | <p><b>Q23</b></p> <p><b>(A)</b></p> <p>Let the x-axis divide A(1,-5), B(-4,5) in ratio <math>k : 1</math></p> <p>y-coordinate of division point <math>= \frac{5k - 5}{k + 1} = 0 \Rightarrow 5k = 5 \Rightarrow k = 1</math></p> <p>Ratio <math>= 1 : 1</math>; <math>x = \frac{1(-4) + 1(1)}{1 + 1} = \frac{-3}{2}</math></p> <p><b>Answer:</b> Ratio <math>= 1 : 1</math>; point of division <math>= \left(-\frac{3}{2}, 0\right)</math></p> <p><b>(B)</b></p> <p>Let the line <math>x - 3y = 7</math> divide A(2,-3), B(3,7) in ratio <math>k : 1</math></p> <p>Point <math>= \left(\frac{3k + 2}{k + 1}, \frac{7k - 3}{k + 1}\right)</math></p> <p>Substitute: <math>\frac{3k + 2}{k + 1} - \frac{3(7k - 3)}{k + 1} = 7 \Rightarrow -18k + 11 = 7(k + 1)</math></p> <p><math>-18k + 11 = 7k + 7 \Rightarrow 4 = 25k \Rightarrow k = \frac{4}{25}</math></p> <p><b>Answer:</b> Ratio <math>= 4 : 25</math></p> | <p>1 mark for the section-formula setup; 1 mark for the final ratio/coordinates.</p>                                             |
| 24   | <p><b>Q24</b></p> <p>In <math>\triangle ABD</math>: <math>\angle BAD = \angle 1</math> and <math>\angle BDA = \angle 2</math>; since <math>\angle 1 = \angle 2</math> (given), <math>\triangle ABD</math> is isosceles with <math>AB = BD</math></p> <p>Given <math>\frac{AD}{AE} = \frac{AC}{BD}</math>; substituting <math>BD = AB</math>: <math>\frac{AD}{AE} = \frac{AC}{AB} \Rightarrow \frac{AB}{AC} = \frac{AE}{AD}</math></p> <p>In <math>\triangle BAE</math> and <math>\triangle CAD</math>: <math>\frac{AB}{AC} = \frac{AE}{AD}</math> (shown) and <math>\angle BAE = \angle CAD</math> (the common angle at A, <math>= \angle 1 = \angle 2</math>)</p> <p>By SAS similarity, <math>\triangle BAE \sim \triangle CAD</math></p> <p><b>Answer:</b> <math>\triangle BAE \sim \triangle CAD</math> (shown).</p>                                                                                           | <p>1 mark for showing <math>\triangle ABD</math> isosceles; 1 mark for the ratio <math>\frac{AB}{AC} = \frac{AE}{AD}</math>.</p> |
| 25   | <p><b>Q25</b></p> <p><b>(A)</b></p> <p><math>\sin(A + B) = 1 \Rightarrow A + B = 90^\circ</math> (the only value in the given range)</p> <p><math>2 \cos(A - B) = \sqrt{3} \Rightarrow \cos(A - B) = \frac{\sqrt{3}}{2} \Rightarrow A - B = 30^\circ</math></p> <p>Adding: <math>2A = 120^\circ \Rightarrow A = 60^\circ</math>; then <math>B = 90^\circ - 60^\circ = 30^\circ</math></p> <p><b>Answer:</b> <math>A = 60^\circ</math>, <math>B = 30^\circ</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                              | <p>1 mark for setting up the equation(s); 1 mark for the final angle(s).</p>                                                     |

| Q.No                                      | Step-by-Step Solution                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | Marks per Step                                                                                                                 |
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|                                           | <p><b>(B)</b></p> <p>Dividing numerator and denominator of the LHS by <math>\cos \theta</math>: <math>\frac{1 - \tan \theta}{1 + \tan \theta} = \frac{1 - \sqrt{3}}{1 + \sqrt{3}}</math></p> <p>Cross-multiplying: <math>(1 - \tan \theta)(1 + \sqrt{3}) = (1 + \tan \theta)(1 - \sqrt{3})</math></p> <p><math>1 + \sqrt{3} - \tan \theta - \sqrt{3} \tan \theta = 1 - \sqrt{3} + \tan \theta - \sqrt{3} \tan \theta</math></p> <p><math>2\sqrt{3} = 2 \tan \theta \Rightarrow \tan \theta = \sqrt{3} \Rightarrow \theta = 60^\circ</math></p> <p><b>Answer: <math>\theta = 60^\circ</math></b></p>                            |                                                                                                                                |
| <b>SECTION – C (Q26–31, 3 marks each)</b> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                |
| <b>26</b>                                 | <p><b>Q26</b></p> <p>Total students = 30: <math>2 + p + 7 + 6 + 6 + 6 = 30 \Rightarrow p = 3</math></p> <p>Class marks (midpoints): 17.5, 32.5, 47.5, 62.5, 77.5, 92.5</p> <p><math>\Sigma f_i x_i = 2(17.5) + 3(32.5) + 7(47.5) + 6(62.5) + 6(77.5) + 6(92.5)</math></p> <p><math>= 35 + 97.5 + 332.5 + 375 + 465 + 555 = 1860</math></p> <p>Mean = <math>\frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{1860}{30} = 62</math></p> <p><b>Answer: <math>p = 3</math>; Mean marks = 62</b></p>                                                                                                                                       | <p>1 mark for finding <math>p</math>;<br/>1 mark for the class marks/setup; 1 mark for the mean.</p>                           |
| <b>27</b>                                 | <p><b>Q27</b></p> <p>Total outcomes = 52</p> <p>(i) Black cards = 26, queens = 4, black queens = 2 (overlap).<br/><math>P = \frac{26 + 4 - 2}{52} = \frac{28}{52} = \frac{7}{13}</math></p> <p>(ii) Red cards = 26, kings = 4, red kings = 2 (overlap). Red or king<br/><math>= 26 + 4 - 2 = 28</math>; neither = <math>52 - 28 = 24</math>. <math>P = \frac{24}{52} = \frac{6}{13}</math></p> <p>(iii) Black kings: king of spades and king of clubs = 2. <math>P = \frac{2}{52} = \frac{1}{26}</math></p> <p><b>Answer: (i) <math>\frac{7}{13}</math> (ii) <math>\frac{6}{13}</math> (iii) <math>\frac{1}{26}</math></b></p> | <p>1 mark for each of the three probabilities.</p>                                                                             |
| <b>28</b>                                 | <p><b>Q28</b></p> <p>Suppose, to the contrary, that <math>5 + 2\sqrt{3}</math> is rational, say <math>\frac{p}{q}</math> (<math>q \neq 0</math>)</p> <p>Then <math>2\sqrt{3} = \frac{p}{q} - 5 = \frac{p - 5q}{q}</math>, so <math>\sqrt{3} = \frac{p - 5q}{2q}</math>, which is rational</p> <p>This contradicts the given fact that <math>\sqrt{3}</math> is irrational</p> <p>Hence our assumption is wrong, and <math>5 + 2\sqrt{3}</math> is irrational</p> <p><b>Answer: Proved by contradiction.</b></p>                                                                                                                | <p>1 mark for the assumption/setup; 1 mark for expressing <math>\sqrt{3}</math> as rational; 1 mark for the contradiction.</p> |
| <b>29</b>                                 | <p><b>Q29</b></p> <p>Zeroes of <math>2x^2 - 5x - 3</math>: <math>2x^2 - 6x + x - 3 = 2x(x - 3) + 1(x - 3) = (2x + 1)(x - 3)</math></p> <p>Zeroes: <math>x = 3</math> and <math>x = -\frac{1}{2}</math></p> <p>Double these: 6 and <math>-1</math> — these are the zeroes of <math>x^2 + px + q</math></p> <p>Sum = <math>6 + (-1) = 5 = -p \Rightarrow p = -5</math>; Product = <math>6 \times (-1) = -6 = q</math></p> <p><b>Answer: <math>p = -5</math>, <math>q = -6</math></b></p>                                                                                                                                         | <p>1 mark for the zeroes of the given polynomial; 1 mark for doubling them; 1 mark for <math>p</math> and <math>q</math>.</p>  |
| <b>30</b>                                 | <p><b>Q30</b></p> <p><math>S_6 = \frac{6}{2}[2a + 5d] = 36 \Rightarrow 2a + 5d = 12</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | <p>1 mark for each equation; 1 mark for <math>a</math>, <math>d</math>; 1 mark for <math>S_{10}</math>.</p>                    |

| Q.No                                      | Step-by-Step Solution                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | Marks per Step                                                                                        |
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|                                           | $S_{16} = \frac{16}{2}[2a + 15d] = 256 \Rightarrow 2a + 15d = 32$ <p>Subtracting: <math>10d = 20 \Rightarrow d = 2</math>; then <math>2a + 10 = 12 \Rightarrow a = 1</math></p> $S_{10} = \frac{10}{2}[2a + 9d] = 5[2 + 18] = 5 \times 20 = 100$ <p><b>Answer: 100</b></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                                                                                                       |
| 31                                        | <p><b>Q31</b><br/><b>Prove the following identity:</b></p> $1 + \cot A + \tan A = 1 + \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A} = \frac{\sin A \cos A + 1}{\sin A \cos A}$ $\sec^3 A - \operatorname{cosec}^3 A = \frac{1}{\cos^3 A} - \frac{1}{\sin^3 A} = \frac{\sin^3 A - \cos^3 A}{\sin^3 A \cos^3 A}$ $\text{LHS} = \frac{(\sin A \cos A + 1)}{\sin A \cos A} \times (\sin A - \cos A) \times \frac{\sin^3 A \cos^3 A}{\sin^3 A - \cos^3 A}$ <p>Using <math>\sin^3 A - \cos^3 A = (\sin A - \cos A)(1 + \sin A \cos A)</math>, the factors <math>(\sin A - \cos A)</math> and <math>(1 + \sin A \cos A)</math> cancel</p> $\text{LHS} = \frac{\sin^3 A \cos^3 A}{\sin A \cos A} = \sin^2 A \cos^2 A = \text{RHS}$ <p><b>Answer: Identity proved.</b></p> <p><b>OR — Prove that:</b><br/>Multiply numerator and denominator of the LHS by <math>(\cos A + \sin A + 1)</math><br/>Numerator: <math>(\cos A + 1)^2 - \sin^2 A = 2 \cos^2 A + 2 \cos A = 2 \cos A(\cos A + 1)</math><br/>Denominator: <math>(\cos A + \sin A)^2 - 1 = 2 \sin A \cos A</math><br/>So <math>\text{LHS} = \frac{2 \cos A(\cos A + 1)}{2 \sin A \cos A} = \frac{\cos A + 1}{\sin A} = \cot A + \operatorname{cosec} A</math> — this proves the first equality</p> <p>For the second equality: <math>\operatorname{cosec} A + \cot A = \frac{1 + \cos A}{\sin A}</math>; multiply numerator and denominator by <math>(1 - \cos A)</math></p> $= \frac{1 - \cos^2 A}{\sin A(1 - \cos A)} = \frac{\sin^2 A}{\sin A(1 - \cos A)} = \frac{\sin A}{1 - \cos A}$ <p><b>Answer:</b> <math>\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A = \frac{\sin A}{1 - \cos A}</math> (both equalities proved).</p> | <p>3 marks for a correctly-justified, step-wise proof of either identity.</p>                         |
| <b>SECTION – D (Q32–35, 5 marks each)</b> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                       |
| 32                                        | <p><b>Q32</b><br/><b>(A) Basic Proportionality Theorem</b></p> <p>Given: <math>\triangle ABC</math> with <math>DE \parallel BC</math>, D on AB, E on AC. To prove: <math>\frac{AD}{DB} = \frac{AE}{EC}</math></p> <p>Construction: Join BE and CD; draw <math>EM \perp AB</math> and <math>DN \perp AC</math></p> $\text{ar}(\triangle ADE) = \frac{1}{2} \times AD \times EM, \quad \text{ar}(\triangle DBE) = \frac{1}{2} \times DB \times EM$ $\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle DBE)} = \frac{AD}{DB}; \text{ similarly } \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle DEC)} = \frac{AE}{EC}$ <p>Since <math>\triangle DBE</math> and <math>\triangle DEC</math> stand on the same base DE and lie between the same parallels DE and BC, <math>\text{ar}(\triangle DBE) = \text{ar}(\triangle DEC)</math></p> <p>So <math>\frac{AD}{DB} = \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle DBE)} = \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle DEC)} = \frac{AE}{EC}</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | <p>Up to 5 marks for a complete, correctly-justified proof with construction and all steps shown.</p> |

| Q.No | Step-by-Step Solution                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | Marks per Step                                                                                                             |
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|      | <p><b>Answer:</b> <math>\frac{AD}{DB} = \frac{AE}{EC}</math> (proved).</p> <p><b>(B) Application to a trapezium</b><br/> Let ABCD be a trapezium with <math>AB \parallel CD</math>; diagonals meet at O. Let the line through O parallel to AB meet AD at E and BC at F</p> <p>Since <math>AB \parallel CD</math>, <math>\triangle AOB \sim \triangle COD</math> (AA), giving <math>\frac{AO}{OC} = \frac{BO}{OD} = \frac{AB}{CD}</math></p> <p>In <math>\triangle DAB</math>, <math>EO \parallel AB</math> (E on DA, O on DB), so by the theorem proved above:<br/> <math>\frac{DE}{EA} = \frac{DO}{OB} = \frac{CD}{AB}</math></p> <p>In <math>\triangle CAB</math>, <math>OF \parallel AB</math> (O on CA, F on CB), so similarly: <math>\frac{CF}{FB} = \frac{CO}{OA} = \frac{CD}{AB}</math></p> <p>So <math>\frac{DE}{EA} = \frac{CF}{FB}</math>, i.e. <math>\frac{AE}{ED} = \frac{BF}{FC}</math> — the non-parallel sides AD and BC are divided in the same ratio</p> <p><b>Answer:</b> <math>\frac{AE}{ED} = \frac{BF}{FC}</math> (shown).</p> |                                                                                                                            |
| 33   | <p><b>Q33</b></p> <p>Let the larger pipe alone take <math>x</math> hours to fill the pool; the smaller takes <math>x + 10</math> hours</p> <p>In 4 hours (larger) + 9 hours (smaller), half the pool is filled: <math>\frac{4}{x} + \frac{9}{x+10} = \frac{1}{2}</math></p> <p><math>8(x+10) + 18x = x(x+10) \Rightarrow 8x + 80 + 18x = x^2 + 10x</math></p> <p><math>x^2 - 16x - 80 = 0</math></p> <p><math>x = \frac{16 \pm \sqrt{256 + 320}}{2} = \frac{16 \pm 24}{2} \Rightarrow x = 20</math> (rejecting <math>x = -4</math>)</p> <p><b>Answer: Larger pipe: 20 hours; smaller pipe: 30 hours</b></p>                                                                                                                                                                                                                                                                                                                                                                                                                                          | <p>1 mark for the equation; 2 marks for simplifying to the quadratic; 1 mark for solving; 1 mark for both final times.</p> |
| 34   | <p><b>Q34</b></p> <p><b>(A)</b></p> <p>Let the usual speed be <math>x</math> km/hr; usual time = <math>\frac{600}{x}</math> hours</p> <p>Reduced speed = <math>x - 200</math>; new time = <math>\frac{600}{x-200}</math></p> <p><math>\frac{600}{x-200} - \frac{600}{x} = \frac{1}{2}</math></p> <p><math>600x - 600(x-200) = \frac{1}{2}x(x-200) \Rightarrow 120000 = \frac{x^2 - 200x}{2}</math></p> <p><math>x^2 - 200x - 240000 = 0 \Rightarrow x = \frac{200 \pm \sqrt{40000 + 960000}}{2} = \frac{200 \pm 1000}{2}</math></p> <p><math>x = 600</math> (rejecting the negative root)</p> <p><b>Answer: Scheduled duration = <math>\frac{600}{600} = 1</math> hour</b></p>                                                                                                                                                                                                                                                                                                                                                                       | <p>1 mark for the equation; 2 marks for simplifying to the quadratic; 1 mark for solving; 1 mark for the final answer.</p> |
| 35   | <p><b>Q35</b></p> <p>Since the "Below" classes have width 10 and start from "Below 15", the true class intervals are 5–15, 15–25, 25–35, 35–45, 45–55, 55–65</p> <p>Class frequencies (from consecutive differences of cumulative frequencies):<br/> 6, 11, 21, 23, 14, 5 (total 80)</p> <p>For the median: <math>\frac{n}{2} = 40</math>. Cumulative frequencies 6, 17, 38, 61, 75, 80 — the class reaching 40 is 35–45 (median class), with <math>cf = 38</math>, <math>f = 23</math>, <math>l = 35</math>, <math>h = 10</math></p> <p>Median = <math>l + \frac{\frac{n}{2} - cf}{f} \times h = 35 + \frac{40 - 38}{23} \times 10 = 35 + \frac{20}{23} \approx 35.87</math></p>                                                                                                                                                                                                                                                                                                                                                                    | <p>1 mark for the class frequencies; 2 marks for the median; 2 marks for the mode.</p>                                     |

| Q.No | Step-by-Step Solution                                                                                                                                                                                                                                                                                                                                                                                                                                     | Marks per Step |
|------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------|
|      | <p>For the mode: the highest frequency (23) is in class 35–45 (modal class), with <math>f_1 = 23</math>, <math>f_0 = 21</math>, <math>f_2 = 14</math>, <math>l = 35</math>, <math>h = 10</math></p> $\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h = 35 + \frac{23 - 21}{46 - 21 - 14} \times 10 = 35 + \frac{20}{11} \approx 36.82$ <p><b>Answer: Median <math>\approx 35.87</math> years; Mode <math>\approx 36.82</math> years</b></p> |                |

### SECTION – E (Q36–38, 4 marks each)

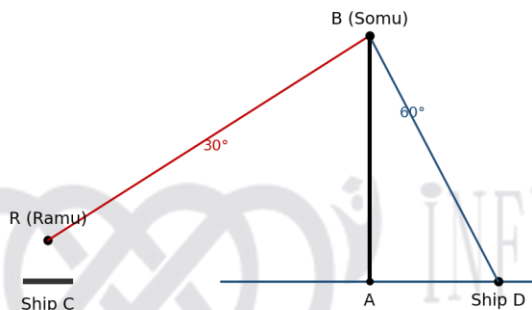
36

#### Q36

Let A be the base of the lighthouse, B its top, R the point on ship C's deck (10 m above water) where Ramu stands, and D the point where ship D meets the water

##### (i) Diagram

Ships C and D lie on opposite sides of the lighthouse AB, all in the same straight line. Ramu (R) is 10 m above the water on ship C; the line of sight RB makes a  $30^\circ$  angle of elevation, and  $RB = 100$  m. Somu (B), at the top of the lighthouse, observes ship D with a  $60^\circ$  angle of depression.



**Answer: See figure.**

##### (ii) Height of the light house

In the right triangle at R:  $\sin 30^\circ = \frac{\text{height of } B - 10}{RB} = \frac{h - 10}{100}$

$$h - 10 = 100 \times \frac{1}{2} = 50 \Rightarrow h = 60 \text{ m}$$

**Answer: 60 m**

##### (iii)(a) Distance between the ships

Horizontal distance from ship C to the lighthouse:

$$\cos 30^\circ = \frac{d_1}{100} \Rightarrow d_1 = 100 \times \frac{\sqrt{3}}{2} = 50\sqrt{3} = 50 \times 1.73 = 86.5 \text{ m}$$

Horizontal distance from the lighthouse to ship D:

$$\tan 60^\circ = \frac{60}{d_2} \Rightarrow d_2 = \frac{60}{\sqrt{3}} = 20\sqrt{3} = 20 \times 1.73 = 34.6 \text{ m}$$

$$\text{Distance between the ships} = d_1 + d_2 = 86.5 + 34.6 = 121.1 \text{ m}$$

**Answer: 121.1 m**

##### (iii)(b) [OR] Distance between Somu and ship D

In the right triangle at the lighthouse base:

$$\sin 60^\circ = \frac{60}{BD} \Rightarrow BD = \frac{60}{\sqrt{3}/2} = \frac{120}{\sqrt{3}} = 40\sqrt{3} = 40 \times 1.73 = 69.2 \text{ m}$$

**Answer: 69.2 m**

(i) 1 mark for a correctly labelled diagram. (ii) 1 mark. (iii) 2 marks for either part (1 mark for the horizontal distance(s); 1 mark for the final length).

| Q.No | Step-by-Step Solution                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | Marks per Step                                                                                                                                           |
|------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------|
| 37   | <p><b>Q37</b></p> <p>From the figure: inner octagon vertices <math>A(-2, 2)</math>, <math>B(2, 2)</math>, <math>C(4, 4)</math>, <math>D(4, 7)</math>, <math>E(2, 9)</math>, <math>F(-2, 9)</math>, <math>G(-4, 7)</math>, <math>H(-4, 4)</math>; outer points <math>W(-6, 2)</math>, <math>X(-4, 0)</math>, <math>O(6, 9)</math>, <math>P(4, 11)</math></p> <p><b>(i) Length of segment BF</b></p> <p><math>B = (2, 2)</math>, <math>F = (-2, 9)</math></p> $BF = \sqrt{(2 - (-2))^2 + (2 - 9)^2} = \sqrt{16 + 49} = \sqrt{65}$ <p><b>Answer:</b> <math>\sqrt{65} \approx 8.06</math> units</p> <p><b>(ii) Coordinates of Z (intersection of diagonals of WXOP)</b></p> <p>The diagonals of quadrilateral WXOP are WO and XP</p> <p>Midpoint of WO = <math>\left(\frac{-6 + 6}{2}, \frac{2 + 9}{2}\right) = (0, 5.5)</math></p> <p>Midpoint of XP = <math>\left(\frac{-4 + 4}{2}, \frac{0 + 11}{2}\right) = (0, 5.5)</math></p> <p>Both diagonals share this midpoint, so WXOP is a parallelogram and its diagonals bisect each other here</p> <p><b>Answer:</b> <math>Z = (0, 5.5)</math></p> <p><b>(iii)(a) Point on the y-axis equidistant from A and G</b></p> <p>Let the point be <math>(0, k)</math>. <math>A = (-2, 2)</math>, <math>G = (-4, 7)</math></p> $(0 - (-2))^2 + (k - 2)^2 = (0 - (-4))^2 + (k - 7)^2$ $4 + (k - 2)^2 = 16 + (k - 7)^2 \Rightarrow 8 - 4k = 65 - 14k \Rightarrow 10k = 57$ $k = 5.7$ <p><b>Answer:</b> <math>(0, 5.7)</math></p> <p><b>(iii)(b) [OR] Area of trapezium AFGH</b></p> <p><math>A(-2, 2)</math>, <math>F(-2, 9)</math>, <math>G(-4, 7)</math>, <math>H(-4, 4)</math>; sides AF and GH are both vertical, so they are parallel</p> $AF = 9 - 2 = 7; GH = 7 - 4 = 3; \text{distance between them} =  -2 - (-4)  = 2$ $\text{Area} = \frac{1}{2}(AF + GH) \times \text{height} = \frac{1}{2}(7 + 3) \times 2 = 10$ <p><b>Answer:</b> 10 square units</p> | <p>(i) 1 mark. (ii) 1 mark. (iii) 2 marks for either part (1 mark for the setup; 1 mark for the final value).</p>                                        |
| 38   | <p><b>Q38</b></p> <p>Production forms an A.P.; let first-year production be <math>a</math> and the yearly increase be <math>d</math></p> <p><b>(i) Production in the first year</b></p> $a_6 = a + 5d = 800; a_9 = a + 8d = 1130$ <p>Subtracting: <math>3d = 330 \Rightarrow d = 110</math>; then <math>a = 800 - 5(110) = 250</math></p> <p><b>Answer:</b> 250 rollers</p> <p><b>(ii) Yearly increase</b></p> <p>From above, <math>d = 110</math></p> <p><b>Answer:</b> 110 rollers</p> <p><b>(iii)(a) Year in which production was 1350 rollers</b></p> $a + (n - 1)d = 1350 \Rightarrow 250 + (n - 1)(110) = 1350$ $(n - 1)(110) = 1100 \Rightarrow n - 1 = 10 \Rightarrow n = 11$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | <p>(i) 1 mark for <math>a</math>, <math>d</math>. (ii) 1 mark. (iii) 2 marks for either part (1 mark for the equation; 1 mark for the final answer).</p> |

| Q.No | Step-by-Step Solution                                                                                                                                                      | Marks per Step |
|------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------|
|      | <p><b>Answer: 11th year</b></p> <p><b>(iii)(b) [OR] Production in the 8th year</b></p> $a_8 = a + 7d = 250 + 7(110) = 250 + 770 = 1020$ <p><b>Answer: 1020 rollers</b></p> |                |

Prepared by Deepika Bhati | Infinity Think Beyond

