

Half Yearly Examination

Mathematics.

Class XII-A

Time allowed 3 Hrs .

Maximum marks :- 80

General Instructions :-

Read the following instructions carefully and follow them .

1. This question paper contains 38 Questions . All questions are compulsory
2. This Question paper is divided into 5 sections A,B,C,D and E
3. In Section A question no 1 to 18 are Multiple choice questions and Question no 19 and 20 are Assertion and reason questions of 1 mark
4. In section B question no 21 to 25 are Very short answer type questions carrying 2 marks .
5. In section C , question no 26 to 31 are short answer type questions carrying 3 marks .
6. In section D question no 32 to 35 are long answer type questions carrying 5 marks
7. In section E , question no 36 to 38 are case study based integrated questions carrying 4 marks .
8. Draw neat figures wherever required. Use $\pi = 22/7$ wherever required if not stated.

Section A

Q1. Let $R = \{(x,y) \in \mathbb{R} \times \mathbb{R} : x^2 - 4xy + 3y^2 = 0\}$. The relation R is

- (A) Symmetric but not transitive (B) reflexive but not Symmetric
(C) An equivalence relation. (D) transitive but not reflexive

Q2. Which one of the following functions is one one ?

1. $f : \mathbb{R} \rightarrow \mathbb{R}$ such that $f(x) = x^2$
2. $f : \mathbb{R} \rightarrow \mathbb{R}$ such that $f(x) = 3x + 2$
3. $f : \mathbb{N} \rightarrow \mathbb{N}$ such that $f(x) = x^2$
4. $f : \mathbb{R} \rightarrow \mathbb{R}$ such that $f(x) = [x]$, greatest integer $\leq x$

- (A) 1 and 3 only. (B) 2 and 3 only. (C) 3 and 4 only. (D) 1,2 and 3 only.

- Q3. The principal value $\sin^{-1}(\cos(33\pi/5))$ is
 (A) $3\pi/5$ (B) $-\pi/10$ (C) $\pi/10$ (D) $-3\pi/5$
- Q4. If A is a non singular matrix of order 3 such that $|\text{adj}A| = 64$, then $|A^T|$ is
 (A) ± 8 . (B) ± 4 . (C) 64 (D) 8
- Q5. If $f(x) = |x-1| + |x-2|$, then the number of points where $f(x)$ fails to be differentiable is |
 (A) 1 (B) 2 (C) 0 (D) infinite
- Q6. The function $f(x) = [x]^2 - [x]^2$ (where $[.]$ represents the greatest integer function) is discontinuous at
 (A) All integer points (B) All integers except 0
 (C) All integers except 1 (D) All integers except 0 and 1
- Q7. If $f(x) = \log_x(\log x)$, then the value of $f'(x)$ at $x = e$ is
 (A) $1/e$ (B) e (C) 1 (D) 0
- Q8. The side of an equilateral triangle is increasing at the rate of 2cm/sec. The rate at which area is increasing when side is 10 cm is
 (A) $10\sqrt{3} \text{ cm}^2/\text{sec}$ (B) $20\sqrt{3} \text{ cm}^2/\text{sec}$ (C) $10 \text{ cm}^2/\text{sec}$ (D) $5\sqrt{3} \text{ cm}^2/\text{sec}$
- Q9. For what value of k the function $f(x) = \sin x - kx$ is strictly decreasing for all real x
 (A) $k > 1$. (B) $k < 1$. (C) $k \geq 1$. (D) $k \leq -1$
- Q10. The minimum value of $x \log x$ is equal to
 (A) e (B) $1/e$ (C) $-1/e$ (D) $2e$
- Q11. The integral $\int \sqrt{1+\sin 2x} \, dx$ for $x \in (0, \pi/4)$ equals
 (A) $\sin x - \cos x + c$ (B) $\cos x - \sin x + c$
 (C) $-\sin x - \cos x + c$ (D) $\sin x + \cos x + c$
- Q12 The value of $\sec^2(\tan^{-1}2) + \text{cosec}^2(\cot^{-1}3)$ is
 (A) 15 (B) 13. (C) 11. (D) 14

Q13. Let R be a relation on the set of natural numbers N defined by nRm if n divides m. Then R is

1. Reflexive relation.
2. Symmetric relation
3. Transitive relation.
4. Identity relation

Choose the correct option from the following :-

- (A) 1 and 3. (B) 1. and 2 (C) 1 and 4 (D) 2 and 3

Q14. A function $f: \mathbb{R} \rightarrow \mathbb{A}$ such that $f(x) = x^2 + 1$ is onto if A

- (A) $(-\infty, \infty)$ (B) $(1, \infty)$ (C) $[1, \infty)$ (D) $[-1, \infty)$

Q15. If the following matrix is skew symmetric then the value of $5x - y$ is

$$\begin{bmatrix} 0 & -1 & 3x \\ 1 & y & -5 \\ -6 & 5 & 0 \end{bmatrix}$$

- (A) 12 (B) 15. (C) 10 (D) 14

Q16. If $e^y = x^x$, then which of the following is true

- (A) $y \frac{d^2y}{dx^2} = 1$ (B) $\frac{d^2y}{dx^2} - y = 0$
 (C) $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$ (D) $y \frac{d^2y}{dx^2} - \frac{dy}{dx} + 1 = 0$

Q17. Which of the following statements are correct for $f(x) = (x-5)^2(x+8)^3$?

1. $x = 8$ is a point of local minima
2. $x = -\frac{1}{5}$ is a point of local maxima
3. $x = 5$ is a point of local minima .
4. $x = -8$ is a point of inflection.

Choose the correct option .

- (A) 1 and 2 (B) 1 and 3 (C) 2,3 and 4 (D) 1,2 and 4

Q18. In the following find the value of $(a+c)/b$, where d is a constant of integration.

$$\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} dx = ax + b \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c \tan^{-1}\left(\frac{x}{2}\right) + d$$

- (A) $-\sqrt{3}$ (B) $\sqrt{3}$ (C) $2\sqrt{3}$ (D) -2

In Question no 19 and 20 , a statement of assertion (A) is followed by a statement of reason (R) . Choose the correct option

- (a) Both assertion and reason are true and reason is the correct explanation of assertion
- (b) Both assertion and reason are true and reason is NOT the correct explanation of assertion .
- (c) Assertion is true but the reason is false .
- (d) Assertion is false but reason is true

Q19. Assertion :- $f(x) = [x]$ is not differentiable at $x=2$. where $[.]$ Is greatest integer function

Reason :- A function cannot be differentiable at a point where it is discontinuous.

Q20. Assertion :-The function $f(x) = \tan^{-1}x - x$ always decreases on $\mathbb{R}$.

Reason :- The derivative $f'(x) = -x^2/(x^2+1)$ is always strictly less than 0 for all $x \neq 0$.

Section B

Q21 . Sand is pouring from a pipe at the rate of $12 \text{ cm}^3/\text{sec}$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when height is 4 cm .

Q22. If A and B are symmetric matrices of the same order . Prove that matrix $AB - BA$ is a skew symmetric matrix.

Q23. If $\cos y = x \cos(a+y)$, prove that $dy/dx = (\cos^2(a+y))/\sin a$

Q24. Evaluate the following integral .

$$\int \frac{1}{x(x^5 + 1)} dx$$

Q25. Find the absolute maximum and absolute minimum value of $f(x) = \sin x + \cos x$ in the interval $[0, \pi]$

Section C

Q26. Let $f : \mathbb{R} \Rightarrow \mathbb{R}$ be a function defined by $f(x) = (x^2 - 3x + 2) / (x^2 + x + 1)$. Describe the complete range of $f(x)$ and analyze whether $f(x)$ is surjective function.

Q27. If A is an invertible square matrix of order 3 such that $|A| = 4$. Evaluate the precise scalar value of $|\text{adj}(\text{adj}(2A))|$.

Q28. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = 3\pi/2$, evaluate the precise value of

$$x^{100} + y^{100} + z^{100} - \frac{9}{x^{101} + y^{101} + z^{101}}$$

Q29. Discuss the continuity and differentiability of the function $f(x) = [x] + |x-1|$ within the closed interval $[0,2]$, where $[x]$ represents the greatest integer function.

Q30. Prove that the function $f(x) = (\log x)/x$ is strictly decreasing in the interval (e, ∞) and strictly increasing in $(0,e)$.

Q31. Find dy/dx for the following function.

$$y = x^{x^x} + (\sin x)^{\tan x}$$

Section D

Q32. Let $A = \mathbb{N} \times \mathbb{N}$ be set of all ordered pairs of natural numbers. Let R be a relation defined on A such that $(a,b), (c,d) \in A$.

$$(a,b) R (c,d) \Leftrightarrow ad(b+c) = bc(a+d)$$

Show that R is an equivalence relation. Provide explicit proofs for the reflexive, symmetric and transitive properties.

Q33 . Solve the following system of equations using matrices

$$x + 2y + z = 4$$

$$-x + y + z = 0$$

$$x - 3y + z = 2$$

Q34. A curve is represented parametrically by the equations :

$x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$. Find the exact value of second derivative d^2y/dx^2 at $\theta = \pi/4$

Q35 Evaluate the following integral .

$$\int \frac{x^2 + 1}{x^4 + x^2 + 1} dx$$

Section E

Q36. An algorithmic financial index tracks structural market shifts using an exponential-polynomial blend function over a time horizon t (in years). The asset valuation function $W(t)$ is defined by:

$$W(t) = t^2 \cdot e^{\frac{-t}{2}}, \text{ for } t \geq 0$$

1. Determine the rate of change of the index.
2. Find the exact intervals in which the asset index value is strictly increasing.
3. Find the exact intervals in which the index is strictly decreasing, and determine its long-term limiting behavior as t tends to ∞

Q37. A reconnaissance fighter jet is cruising along a low-altitude parabolic trajectory defined by the curve $y = x^2 + 7$. A stationary surface-to-air missile (SAM) battery is deployed at the fixed ground coordinate base $(3, 7)$.

- (i) Write down the distance-squared function, $S(x)$, between the jet's variable position (x, y) and the SAM battery base. (1 mark)
- (ii) Find the critical points of $S(x)$ by setting $S'(x) = 0$. (1 mark)
- (iii) Use the second derivative test to find the value of x that minimizes the distance between the jet and the base. (1 mark)
- (iv) Calculate the exact closest distance (minimum clearance) achieved by the fighter jet relative to the battery. (1 mark)

Q38. In theoretical physics quantum barrier simulation , the localised wave function $f(x)$ of a subatomic particle at the origin transition zone is modeled as

$$f(x) = \begin{cases} \frac{1-\cos(4x)}{x^2} & \text{if } x < 0 \\ p & \text{if } x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & \text{if } x > 0 \end{cases}$$

1. Evaluate the left hand limit .(1.5 marks)
2. Evaluate the right hand limit .(1.5 marks)
3. Determine the exact value of the parameter p that makes the quantum wave function continuous at $x = 0$ (1 mark)