CBSE CLASS XII · MATHEMATICS (041)

Chapter 1
Relations and Functions

Chapter-wise Previous Year Questions with Marking Scheme

Board Examinations 2022 – 2026 (including Compartment / Supplementary)

Reviewed by **Deepika Bhati**
Maths Mentor Since 2009 | Maths Passionate..

Types of Relations & Equivalence Relations

Multiple Choice Questions (1 mark)

Q1. **2023** : 65/5/1, 65/5/2 **[1 Mark]**

Let A = {3, 5}. Then number of reflexive relations on A is

(a) 2 (b) 4

(c) 0 (d) 8

Ans. (b) 4 **1**

Q2. **2023** : 65/5/3 **[1 Mark]**

Let R be a relation in the set N given by R = {(a, b) : a = b − 2, b > 6}. Then

(a) (8, 7) ∈ R (b) (6, 8) ∈ R

(c) (3, 8) ∈ R (d) (2, 4) ∈ R

Ans. (b) (6, 8) ∈ R **1**

Q3. **2024** : 65/1/2 **[1 Mark]**

A relation R defined on set A = {x : x ∈ Z and 0 ≤ x ≤ 10} as R = {(x, y) : x = y} is given to be an equivalence relation. The number of equivalence classes is :

(A) 1 (B) 2

(C) 10 (D) 11

Ans. (D) 11 **1**

Q4. **2024** : 65/1/3 **[1 Mark]**

A relation R defined on a set of human beings as R = {(x, y) : x is 5 cm shorter than y} is :

(A) reflexive only

(B) reflexive and transitive

(C) symmetric and transitive

(D) neither transitive, nor symmetric, nor reflexive

Ans. (D) Neither transitive, nor symmetric, nor reflexive **1**

Q5. **2024** : 65/2/2 **[1 Mark]**

Which of the following statements is ***not*** true about equivalence classes A_i (i = 1, 2, ... n) formed by an equivalence relation R defined on a set A ?

(A) $\bigcup_{i=1}^{n} A_i = A$

(B) $A_i \cap A_j \neq \phi$, $i \neq j$

(C) $x \in A_i$ and $x \in A_j \Rightarrow A_i = A_j$

(D) All elements of A_i are related to each other, for all i

Ans. (B) $A_i \cap A_j \neq \phi$, $i \neq j$ **1**

Q6. **2025** : 65/7/2 **[1 Mark]**

If R be a relation defined as aRb iff $|a - b| > 0$, $a, b \in \mathbb{R}$, then R is :

(A) reflexive (B) symmetric

(C) transitive (D) symmetric and transitive

Ans. (B) symmetric **1**

Q7. **2026** : 65/2/1 **[1 Mark]**

A relation R on set A = {1, 2, 3} defined as R = {(1, 1), (2, 2), (1, 2)} is

(A) Reflexive only (B) Reflexive and Transitive

(C) Symmetric and Transitive (D) Transitive only

Ans. (D) Transitive only **1**

Q8. **2026** : 65/2/2 **[1 Mark]**

A relation R on set A = {1, 2, 3} defined as R = {(1, 2), (2, 1), (2, 2)} is

(A) Reflexive only (B) Reflexive and Transitive

(C) Symmetric and Transitive (D) Symmetric only

Ans. (D) Symmetric only **1**

Q9. **2026** : 65/2/3 **[1 Mark]**

A relation R on set A = {1, 2, 3} is defined as R = {(1, 3), (3, 3), (1, 1), (2, 2), (3, 1)} is

(A) only reflexive and symmetric (B) reflexive only

(C) only reflexive and transitive (D) reflexive, symmetric and transitive

Ans. (D) reflexive, symmetric and transitive **1**

AN EDUCATIONAL INSTITUTE

Assertion – Reason Questions (1 mark)

Q10. **2024** : 65/4/1, 65/4/2, 65/4/3 **[1 Mark]**

Assertion (A) : The relation R = {(x, y) : (x + y) is a prime number and x, y ∈ N} is not a reflexive relation.

Reason (R) : The number '2n' is composite for all natural numbers n.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Ans. (C) Assertion (A) is true, but Reason (R) is false. **1**

Q11. **2026** : 65/5/1, 65/5/2, 65/5/3 **[1 Mark]**

Assertion (A) : A relation R on the set {1, 2, 3} defined as R = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)} is an equivalence relation.

Reason (R) : A relation that is reflexive, symmetric and transitive is an equivalence relation.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Ans. (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). **1**

Q12. **2026 Supp** : 65/7/1, 65/7/2, 65/7/3 **[1 Mark]**

Assertion (A) : Let R be a relation on set A of all books in a school library given by R = {(x, y) : x, y ∈ A, x and y have the same number of pages} and S is a relation on set A from the same library such that S = {(p, q) : p and q are Mathematics books}. Also, R ∩ S ≠ ϕ. Then, R ∩ S is an equivalence relation.
Reason (R) : The intersection of two equivalence relations on a set is an equivalence relation on the set.
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Ans. (D) Assertion (A) is false, but Reason (R) is true. **1**

Very Short Answer Questions (2 marks)

Q13. **2026** : 65/3/1, 65/3/2, 65/3/3 **[2 Marks]**

A relation R on A = {1, 2, 3} is defined as R = {(1, 1), (3, 3), (1, 2)}. Is R a symmetric relation ? Justify. Write the smallest relation set R_1 such that R ∪ R_1 becomes an equivalence relation on the set {1, 2, 3}.

Answer (as per CBSE Marking Scheme)	Marks
As (1, 2) ∈ R, but (2, 1) ∉ R ⇒ R is not symmetric relation.	**1**
R_1 = {(2, 2), (2, 1)}	**1**

Q14. **2026 Supp** : 65/7/1, 65/7/2, 65/7/3 **[2 Marks]**

Students are trying to fix thumb pins on a graph sheet on the class notice board. A relation R is defined on the set A of thumb pins as R = {(P, Q) : P, Q ∈ A and distance between P and Q is less than 3 cm}. Determine if R is an equivalence relation.

Answer (as per CBSE Marking Scheme)	Marks
Let d(P, Q) denotes distance between any point P and Q. For any point P on set A, the distance from P to itself is zero. d(P, P) = 0 < 3, (P, P) ∈ R. Therefore, R is **reflexive**.	**½**
Let (P, Q) ∈ R, then d(P, Q) < 3. As, d(P, Q) = d(Q, P). Thus, d(Q, P) < 3, which means (Q, P) ∈ R. Therefore, R is **symmetric**.	**½**
Let (P, Q) ∈ R, so d(P, Q) < 3 and (Q, S) ∈ R so d(Q, S) < 3, then (P, S) ∉ R, for example: Let P, Q and R be any three different points on the same line such that d(P, Q) = 2 cm and d(Q, R) = 2 cm so d(P, R) = 4 cm > 3. Therefore, R is **not transitive**. Thus, **R is not an equivalence relation.**	**1**

Short Answer Questions (3 marks)

Q15. **2024** : 65/1/1, 65/1/2, 65/1/3 **[3 Marks]**

(a) A relation R on set A = {1, 2, 3, 4, 5} is defined as R = {(x, y) : $|x^2 - y^2| < 8$}. Check whether the relation R is reflexive, symmetric and transitive.

OR

(b) A function f is defined from $\mathbb{R} \to \mathbb{R}$ as f(x) = ax + b, such that f(1) = 1 and f(2) = 3. Find function f(x). Hence, check whether function f(x) is one-one and onto or not.

Answer (as per CBSE Marking Scheme)	Marks
(a) **Reflexive:** $\because |x^2 - x^2| < 8 \ \forall\ x \in A \Rightarrow (x, x) \in R \therefore$ R is reflexive.	½
Symmetric: Let (x, y) ∈ R for some x, y ∈ A $\therefore |x^2 - y^2| < 8 \Rightarrow |y^2 - x^2| < 8 \Rightarrow (y, x) \in R$. Hence R is symmetric.	1
Transitive: (1, 2), (2, 3) ∈ R as $|1^2 - 2^2| < 8$, $|2^2 - 3^2| < 8$ respectively. But $|1^2 - 3^2| \not< 8 \Rightarrow (1, 3) \notin R$. Hence R is not transitive.	1½
OR	
(b) f(x) = ax + b. Solving a + b = 1 and 2a + b = 3 to get a = 2, b = −1. f(x) = 2x − 1	1
Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in \mathbb{R}$: $2x_1 - 1 = 2x_2 - 1 \Rightarrow x_1 = x_2$. Hence f is one-one.	1
Let y = 2x − 1, y ∈ ℝ (Codomain) $\Rightarrow x = \frac{y+1}{2} \in \mathbb{R}$ (domain). Also, $f(x) = f(\frac{y+1}{2}) = y \therefore$ f is onto.	1

Q16. **2025** : 65/2/1 **[3 Marks]**

Let R be a relation defined over N, where N is set of natural numbers, defined as "mRn if and only if m is a multiple of n, m, n ∈ N." Find whether R is reflexive, symmetric and transitive or not.

Answer (as per CBSE Marking Scheme)	Marks
Let x ∈ N. Then we know that x is a multiple of itself. ⇒ xRx. Hence, R is reflexive.	1
We have 2, 8 ∈ N such that 8 is a multiple of 2 ⇒ 8R2. But, 2 is not a multiple of 8. Hence, 2 is not R-related to 8. Therefore, R is not symmetric.	1
Let x, y, z ∈ N such that xRy, yRz. Then x = my, y = nz for some m, n ∈ N ⇒ x = mnz ⇒ x = pz, where p = mn ∈ N. Hence, xRz. Therefore, R is transitive.	1

Q17. **2025** : 65/2/3 **[3 Marks]**

Let R be a relation on set of real numbers ℝ defined as {(x, y) : $x - y + \sqrt{3}$ is an irrational number, x, y ∈ ℝ}. Verify R for reflexivity, symmetry and transitivity.

Answer (as per CBSE Marking Scheme)	Marks
Let x ∈ ℝ. Then we know that $x - x + \sqrt{3} = \sqrt{3}$, which is an irrational number. ⇒ (x, x) ∈ R. Hence, R is reflexive.	1
We have $\sqrt{3}$, 2 ∈ ℝ such that $\sqrt{3} - 2 + \sqrt{3} = 2(\sqrt{3} - 1)$, which is an irrational number ⇒ $(\sqrt{3}, 2) \in R$. But, $2 - \sqrt{3} + \sqrt{3} = 2$, which is a rational number. Hence, $(2, \sqrt{3}) \notin R$. Therefore, R is not symmetric.	1
Let $-\sqrt{3}, \sqrt{3}, 2 \in \mathbb{R}$ such that $(-\sqrt{3}, \sqrt{3}), (\sqrt{3}, 2) \in R$. But, $(-\sqrt{3}, 2) \notin R$. Therefore, R is not transitive.	1

Q18. **2025** : 65/6/1, 65/6/2, 65/6/3 **[3 Marks]**

(a) A student wants to pair up natural numbers in such a way that they satisfy the equation $2x + y = 41$, $x, y \in N$. Find the domain and range of the relation. Check if the relation thus formed is reflexive, symmetric and transitive. Hence, state whether it is an equivalence relation or not.

OR

(b) Show that the function $f : N \to N$, where N is a set of natural numbers, given by $f(n) = \begin{cases} n - 1, & \text{if n is even} \\ n + 1, & \text{if n is odd} \end{cases}$ is a bijection.

Answer (as per CBSE Marking Scheme)	Marks
(a) R = {(1, 39), (2, 37), ..., (20, 1)}. Domain = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}	½
Range = {1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39}	½
(1, 1) does not belong to R hence not reflexive	½
(1, 39) belongs to R but (39, 1) does not belong to R hence not symmetric	½
(11, 19) and (19, 3) belong to R but (11, 3) does not belong to R hence not transitive. Hence R is not an equivalence relation.	1
OR	
(b) Let f(x) = f(y). Let x and y are both odd or both even. Then either $x + 1 = y + 1$ or $x - 1 = y - 1$ gives $x = y$	½
x odd and y even is rejected as $x + 1 = y - 1$ gives $x - y = -2$ not possible as odd number and even number cannot differ by 2	½
Hence f is one-one	½
For onto: Let f(x) = y gives $x = y + 1$ or $x = y - 1$. If y is odd, x is even and if y is even, x is odd.	1
Range = N = co-domain, hence onto. As f is both one-one and onto hence bijective	½

Q19. **2026 Supp** : 65/7/1, 65/7/2, 65/7/3 **[3 Marks]**

(a) If R is a relation on set of natural numbers N such that $R = \{(x, y) : x, y \in N, x + 2y = 25\}$, then find the Domain and Range of the relation R. Also, determine if it is an equivalence relation.

OR

(b) Determine whether the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x^3 - 7$, $\forall x \in \mathbb{R}$ (Real numbers) is bijective or not.

Answer (as per CBSE Marking Scheme)	Marks
(a) R = {(23, 1), (21, 2), (19, 3), (17, 4), (15, 5), (13, 6), (11, 7), (9, 8), (7, 9), (5, 10), (3, 11), (1, 12)}	1
Domain of relation R = {23, 21, 19, 17, 15, 13, 11, 9, 7, 5, 3, 1}	½
Range of relation R = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}	½
Since, $1 \in N$, but $(1, 1) \notin R$, so R is not reflexive. Therefore, R is not an equivalence relation.	1
OR	
(b) Let f(a) = f(b), for some $a, b \in \mathbb{R} \Rightarrow 2a^3 - 7 = 2b^3 - 7$	1
$\Rightarrow a^3 = b^3 \Rightarrow a = b$. Thus, f is one-one function.	½
Now, let $y = f(x) = 2x^3 - 7 \Rightarrow x = (\frac{y + 7}{2})^{1/3}$. For any real number y, the expression $(\frac{y + 7}{2})^{1/3}$ is a real number. Therefore, Range = $\mathbb{R}$ = Co-domain, so f is onto function.	1

Since f is both one-one and onto, so **it is a bijective function.**	**½**

Long Answer Questions (5 marks)

Q20. **2023** : 65/1/1, 65/1/2, 65/1/3 **[5 Marks]**

A relation R is defined on a set of real numbers $\mathbb{R}$ as R = {(x, y) : x . y is an irrational number}. Check whether R is reflexive, symmetric and transitive or not.

Answer (as per CBSE Marking Scheme)	Marks
For reflexive: (1, 1) $\notin$ R as 1^2 is rational (or any other counter example). $\therefore$ R is not reflexive.	**1½**
For symmetric: Let (x, y) $\in$ R $\therefore$ x.y is an irrational number $\therefore$ (y.x) is an irrational number $\therefore$ (y, x) $\in$ R. $\therefore$ R is symmetric.	**1½**
For transitive: (1, $\sqrt{2}$) $\in$ R, ($\sqrt{2}$, 2) $\in$ R (or any other counter example), but (1, 2) $\notin$ R. $\therefore$ R is not transitive.	**2**

Q21. **2023** : 65/4/1, 65/4/2, 65/4/3 **[5 Marks]**

(a) If N denotes the set of all natural numbers and R is the relation on N × N defined by (a, b) R (c, d), if ad(b + c) = bc(a + d). Show that R is an equivalence relation.

OR

(b) Let $f : \mathbb{R} - \{-\frac{4}{3}\} \to \mathbb{R}$ be a function defined as $f(x) = \frac{4x}{3x + 4}$. Show that f is a one-one function. Also, check whether f is an onto function or not.

Answer (as per CBSE Marking Scheme)	Marks
(a) **Reflexive:** Here, (a, b) R (a, b) $\forall$ (a, b) $\in$ N × N since ab(b + a) = ba(a + b) is always true.	**1**
Symmetric: Let (a, b) R (c, d) $\forall$ (a, b), (c, d) $\in$ N × N. Then, ad(b + c) = bc(a + d) $\Rightarrow$ bc(a + d) = ad(b + c) $\Rightarrow$ (c, d) R (a, b).	**1½**
Transitive: Let (a, b) R (c, d) and (c, d) R (e, f) $\forall$ (a, b), (c, d), (e, f) $\in$ N × N. Then ad(b + c) = bc(a + d) and cf(d + e) = de(c + f) $\Rightarrow \frac{b + c}{bc} = \frac{a + d}{ad}$ and $\frac{d + e}{de} = \frac{c + f}{cf} \Rightarrow \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}$ and $\frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c}$. Adding, we get $\frac{1}{b} + \frac{1}{e} = \frac{1}{a} + \frac{1}{f} \Rightarrow \frac{e + b}{be} = \frac{f + a}{af} \Rightarrow$ af(b + e) = be(a + f) $\Rightarrow$ (a, b) R (e, f).	**2**
Hence, R is an equivalence relation.	**½**
OR	
(b) **For one-one:** Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in \mathbb{R} - \{-\frac{4}{3}\} \Rightarrow \frac{4x_1}{3x_1 + 4} = \frac{4x_2}{3x_2 + 4} \Rightarrow 12x_1x_2 + 16x_1 = 12x_1x_2 + 16x_2$	**1**
$\Rightarrow 16x_1 = 16x_2 \Rightarrow x_1 = x_2 \Rightarrow$ f is one-one.	**1**
For onto: Let y $\in$ $\mathbb{R}$, and for some x, $y = \frac{4x}{3x + 4} \Rightarrow 3xy + 4y = 4x \Rightarrow x(3y - 4) = -4y \Rightarrow x = \frac{4y}{4 - 3y}$	**1**
x is real if $y \neq \frac{4}{3}$. So $R_f = \mathbb{R} - \{\frac{4}{3}\} \neq$ Codomain (f)	**1**
So, f is not onto.	**1**

Q22. **2023 Supp** : 65/C/1, 65/C/2, 65/C/3 | **2024 Comptt** : 65/S/1 **[5 Marks]**

(a) Show that the relation S in set $\mathbb{R}$ of real numbers defined by $S = \{(a, b) : a \leq b^3, a \in \mathbb{R}, b \in \mathbb{R}\}$ is neither reflexive, nor symmetric, nor transitive.

OR

(b) Let R be the relation defined in the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ by $R = \{(a, b) :$ both a and b are either odd or even$\}$. Show that R is an equivalence relation. Hence, find the elements of equivalence class [1].

Answer (as per CBSE Marking Scheme)	Marks
(a) We have $S = \{(a, b) : a \leq b^3\}$ where $a, b \in \mathbb{R}$. (i) **Reflexive:** we observe that $\frac{1}{2} \leq (\frac{1}{2})^3$ is not true. $\therefore (\frac{1}{2}, \frac{1}{2}) \notin S$. So, S is not reflexive.	**1½**
(ii) **Symmetric:** We observe that $1 \leq 3^3$ but $3 \not\leq 1^3$ i.e., $(1, 3) \in S$ but $(3, 1) \notin S$. So, S is not symmetric.	**1½**
(iii) **Transitive:** We observe that $10 \leq 3^3$ and $3 \leq 2^3$ but $10 \not\leq 2^3$, i.e., $(10, 3) \in S$ and $(3, 2) \in S$ but $(10, 2) \notin S$. So, S is not transitive. $\therefore$ S is neither reflexive, nor symmetric, nor transitive.	**2**
OR	
(b) $R = \{(a, b) :$ both a and b are either odd or even$\}$. **For reflexive:** Let $a \in A$; clearly both a and a are either odd or even $\therefore (a, a) \in R \Rightarrow$ R is reflexive.	**1**
For symmetric: Let $a, b \in A$. Let $(a, b) \in R \Rightarrow$ both a and b are either odd or even $\Rightarrow$ both b and a are either odd or even. So, $(a, b) \in R \Rightarrow (b, a) \in R \Rightarrow$ R is symmetric.	**1**
For transitive: Let $a, b, c \in A$. Let $(a, b) \in R$, $(b, c) \in R \Rightarrow$ both a and b are either odd or even & both b and c are either odd or even $\Rightarrow$ both a and c are either odd or even. So, $(a, b) \in R$, $(b, c) \in R \Rightarrow (a, c) \in R \Rightarrow$ R is transitive.	**2**
Equivalence class of $[1] = \{1, 3, 5, 7\}$	**1**

Q23. **2024** : 65/3/1 **[5 Marks]**

A relation R on set $A = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$ be defined as $R = \{(x, y) : x + y$ is an integer divisible by $2\}$. Show that R is an equivalence relation. Also, write the equivalence class [2].

Answer (as per CBSE Marking Scheme)	Marks
For reflexive: clearly $x + x$ i.e. $2x$ is integer divisible by $2 \Rightarrow (x, x) \in R \Rightarrow$ R is reflexive.	**1**
For symmetric: $(x, y) \in R \Rightarrow x + y$ is integer divisible by $2 \Rightarrow y + x$ is integer divisible by $2 \Rightarrow (y, x) \in R$	**1**
For transitive: $(x, y) \in R \Rightarrow x + y$ is integer divisible by 2 and $(y, z) \in R \Rightarrow y + z$ is integer divisible by 2. So, $(x + z) + 2y$ is integer divisible by $2 \Rightarrow x + z$ is integer divisible by $2 \Rightarrow (x, z) \in R$	**2**
Equivalence class $[2] = \{-4, -2, 0, 2, 4\}$	**1**

Q24. **2024** : 65/3/2 **[5 Marks]**

A relation R on set $A = \{x : -10 \leq x \leq 10, x \in Z\}$ is defined as $R = \{(x, y) : (x - y)$ is divisible by $5\}$. Show that R is an equivalence relation. Also, write the equivalence class [5].

Answer (as per CBSE Marking Scheme)	Marks
For reflexive relation: To prove $(x, x) \in R$, $x - x = 0$ which is divisible by 5 $\therefore (x, x) \in R \Rightarrow$ R is reflexive	**1**

For symmetric relation: Let $(x, y) \in R \Rightarrow x - y$ is divisible by $5 \Rightarrow x - y = 5m \Rightarrow y - x = 5(-m) \Rightarrow y - x$ is divisible by $5 \Rightarrow (y, x) \in R \therefore R$ is symmetric	**1**
For transitive relation: Let $(x, y) \in R$ and $(y, z) \in R$. $x - y$ is divisible by $5 \Rightarrow x - y = 5m$; $y - z$ is divisible by $5 \Rightarrow y - z = 5n \Rightarrow x - y + y - z = 5(m + n) \Rightarrow x - z = 5(m + n) \therefore x - z$ is divisible by $5 \Rightarrow (x, z) \in R \therefore R$ is transitive. R is an equivalence relation.	**2**
$[5] = \{-10, -5, 0, 5, 10\}$	**1**

Q25. **2024 Comptt** : 65/S/2 **[5 Marks]**

Check whether the relation S in the set $\mathbb{R}$ of real numbers, defined as $S = \{(a, b) : a \leq b^2\}$ is reflexive, symmetric or transitive. Also, determine all $x \in \mathbb{R}$ such that $(x, x) \in S$.

Answer (as per CBSE Marking Scheme)	Marks
Reflexive: Let $\frac{1}{2} \not\leq (\frac{1}{2})^2$, $\therefore$ S is not reflexive.	**1**
Symmetric: $1 \leq 2^2 \Rightarrow (1, 2) \in S$ but $2 \not\leq 1^2 \Rightarrow (2, 1) \notin S$. $\therefore$ S is not symmetric.	**1½**
Transitive: $10 \leq 6^2 \Rightarrow (10, 6) \in S$ & $6 \leq 3^2 \Rightarrow (6, 3) \in S$ but $10 \not\leq 3^2 \Rightarrow (10, 3) \notin S \therefore$ S is not transitive.	**1½**
Let $(x, x) \in S \Leftrightarrow x \leq x^2 \Leftrightarrow x \in \mathbb{R} - (0, 1)$	**1**

Types of Functions – One-one, Onto & Bijective

Multiple Choice Questions (1 mark)

Q26. **2024** : 65/1/1 **[1 Mark]**

A function $f : \mathbb{R}_+ \to \mathbb{R}$ (where $\mathbb{R}_+$ is the set of all non-negative real numbers) defined by $f(x) = 4x + 3$ is :

(A) one-one but not onto (B) onto but not one-one
(C) both one-one and onto (D) neither one-one nor onto

Ans. (A) one-one but not onto **1**

Q27. **2024** : 65/2/1, 65/2/3 **[1 Mark]**

Let $f : \mathbb{R}_+ \to [-5, \infty)$ be defined as $f(x) = 9x^2 + 6x - 5$, where $\mathbb{R}_+$ is the set of all non-negative real numbers. Then, f is :

(A) one-one (B) onto
(C) bijective (D) neither one-one nor onto

Ans. (C) Bijective **1**

Q28. **2024** : 65/3/1 **[1 Mark]**

Let $\mathbb{R}_+$ denote the set of all non-negative real numbers. Then the function $f : \mathbb{R}_+ \to \mathbb{R}_+$ defined as $f(x) = x^2 + 1$ is :

(A) one-one but not onto (B) onto but not one-one
(C) both one-one and onto (D) neither one-one nor onto

Ans. (A) one-one but not onto **1**

Q29. **2024** : 65/3/2 **[1 Mark]**

A function $f : \mathbb{R} \to A$ defined as $f(x) = x^2 + 1$ is onto, if A is :

(A) $(-\infty, \infty)$ (B) $(1, \infty)$

(C) $[1, \infty)$ (D) $[-1, \infty)$

Ans. (C) $[1, \infty)$ **1**

Q30. **2024** : 65/3/3 **[1 Mark]**

Let Z denote the set of integers, then function $f : Z \to Z$ defined as $f(x) = x^3 - 1$ is :

(A) both one-one and onto (B) one-one but not onto

(C) onto but not one-one (D) neither one-one nor onto

Ans. (B) one-one but not onto **1**

Q31. **2024** : 65/5/1, 65/5/2, 65/5/3 **[1 Mark]**

A function $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = x^2 - 4x + 5$ is :

(A) injective but not surjective. (B) surjective but not injective.

(C) both injective and surjective. (D) neither injective nor surjective.

Ans. (D) neither injective nor surjective **1**

Q32. **2025** : 65/4/1, 65/4/2, 65/4/3 **[1 Mark]**

For real x, let $f(x) = x^3 + 5x + 1$. Then :

(A) f is one-one but not onto on $\mathbb{R}$ (B) f is onto on $\mathbb{R}$ but not one-one

(C) f is one-one and onto on $\mathbb{R}$ (D) f is neither one-one nor onto on $\mathbb{R}$

Ans. (C) f is one-one and onto on $\mathbb{R}$ **1**

Q33. **2025** : 65/4/1, 65/4/2, 65/4/3 **[1 Mark]**

If $f : N \to W$ is defined as $f(n) = \begin{cases} \frac{n}{2}, & \text{if n is even} \\ 0, & \text{if n is odd} \end{cases}$ then f is :

(A) injective only (B) surjective only

(C) a bijection (D) neither surjective nor injective

Ans. (B) surjective only **1**

Assertion – Reason Questions (1 mark)

Q34. **2023** : 65/1/2 **[1 Mark]**

Assertion (A) : The number of onto functions from a set P containing 5 elements to a set Q containing 2 elements is 30.

Reason (R) : Number of onto functions from a set containing m elements to a set containing n elements is n^m.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Ans. (C) Assertion (A) is true, but Reason (R) is false. **1**

Q35. **2025** : 65/1/1, 65/1/2, 65/1/3 **[1 Mark]**

Assertion (A) : Let Z be the set of integers. A function $f : Z \to Z$ defined as $f(x) = 3x - 5$, $\forall x \in Z$ is a bijective.

Reason (R) : A function is a bijective if it is both surjective and injective.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Ans. (D) Assertion (A) is false, but Reason (R) is true. **1**

Q36. **2025** : 65/5/1, 65/5/2, 65/5/3 **[1 Mark]**

Assertion (A) : Let $A = \{x \in \mathbb{R} : -1 \le x \le 1\}$. If $f : A \to A$ be defined as $f(x) = x^2$, then f is not an onto function.

Reason (R) : If $y = -1 \in A$, then $x = \pm\sqrt{-1} \notin A$.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Ans. (A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A). **1**

Q37. **2025 Supp** : 65/S/1, 65/S/2, 65/S/3 **[1 Mark]**

Consider the function $f : \mathbb{R} \to \mathbb{R}$, defined as $f(x) = x^3$.

Assertion (A) : f(x) is a one-one function.

Reason (R) : f(x) is a one-one function, if co-domain = range.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Ans. (C) Assertion (A) is true, but Reason (R) is false. **1**

Very Short Answer Questions (2 marks)

Q38. **2023** : 65/1/1, 65/1/2, 65/1/3 **[2 Marks]**

A function $f : A \to B$ defined as $f(x) = 2x$ is both one-one and onto. If $A = \{1, 2, 3, 4\}$, then find the set B.

Answer (as per CBSE Marking Scheme)	Marks
$f(1) = 2, f(2) = 4, f(3) = 6, f(4) = 8$	**1½**

$\therefore B = \{2, 4, 6, 8\}$	½

Q39. **2023 Supp** : 65/C/1, 65/C/2, 65/C/3 **[2 Marks]**

Prove that the greatest integer function $f : \mathbb{R} \to \mathbb{R}$, given by $f(x) = [x]$, is neither one-one nor onto.

Answer (as per CBSE Marking Scheme)	Marks
For not one-one: $1.1, 1.2 \in \mathbb{R}$ (domain); now, $1.1 \neq 1.2$ but $f(1.1) = f(1.2) = 1 \Rightarrow$ f is not one-one.	**1**
For not onto: Let $\frac{1}{2} \in \mathbb{R}$ (co-domain), but $[x] = \frac{1}{2}$ is not possible for x in domain. So, f is not onto.	**1**

Q40. **2026** : 65/2/1, 65/2/2, 65/2/3 **[2 Marks]**

(a) Check whether $f : \mathbb{R} - \{3\} \to \mathbb{R}$ defined as $f(x) = \frac{x-2}{x-3}$ is onto or not.

OR

(b) Check whether $f : Z \times Z \to Z \times Z$ (where Z is the set of integers) defined as $f(x, y) = (2y, 3x)$ is injective or not.

Answer (as per CBSE Marking Scheme)	Marks
(a) Let $y = \frac{x-2}{x-3}$ i.e. $x = \frac{3y-2}{y-1}$	**1**
Here $y \neq 1 \therefore R_f = \mathbb{R} - \{1\} \neq$ Codomain $\therefore$ f is not onto.	**1**
OR	
(b) Let $(x_1, y_1), (x_2, y_2) \in Z \times Z$ such that $f(x_1, y_1) = f(x_2, y_2) \Rightarrow (2y_1, 3x_1) = (2y_2, 3x_2) \Rightarrow y_1 = y_2, x_1 = x_2$	**1**
So, $(x_1, y_1) = (x_2, y_2) \therefore$ f is injective.	**1**

Short Answer Questions (3 marks)

Q41. **2025** : 65/2/2 **[3 Marks]**

Prove that $f : N \to N$ defined as $f(x) = ax + b$ $(a, b \in N)$ is one-one but not onto.

Answer (as per CBSE Marking Scheme)	Marks
Let $x_1, x_2 \in N$ (Domain) such that $f(x_1) = f(x_2) \Rightarrow ax_1 + b = ax_2 + b \Rightarrow x_1 = x_2$. Therefore, f is one-one.	**1½**
Let $y \in N$ (codomain). Then $f(x) = y$ if, $ax + b = y$ i.e., if, $x = \frac{y-b}{a}$, which may not belong to N (domain). Therefore, f is not onto.	**1½**

Q42. **2025** : 65/4/1, 65/4/2, 65/4/3 **[3 Marks]**

(a) If $f : \mathbb{R}^+ \to \mathbb{R}$ is defined as $f(x) = \log_a x$ $(a > 0$ and $a \neq 1)$, prove that f is a bijection. ($\mathbb{R}^+$ is a set of all positive real numbers.)

OR

(b) Let $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$. A relation R from A to B is defined as $R = \{(x, y) : x + y = 6, x \in A, y \in B\}$. (i) Write all elements of R. (ii) Is R a function ? Justify. (iii) Determine domain and range of R.

Answer (as per CBSE Marking Scheme)	Marks

(a) $f(x) = \log_a x$ $(a > 0, a \neq 1)$. Let $x_1, x_2 \in \mathbb{R}^+$ such that $f(x_1) = f(x_2) \Rightarrow \log_a x_1 = \log_a x_2 \Rightarrow x_1 = x_2 \Rightarrow$ f is one-one.	**1½**
Let $f(x) = y \Rightarrow \log_a x = y \Rightarrow a^y = x \therefore$ for every $y \in \mathbb{R}$, there exists $x \in \mathbb{R}^+ \therefore$ f is onto. f is a bijection.	**1½**
OR	
(b) (i) $R = \{(1, 5), (2, 4)\}$	**1**
(ii) R is not a function as $3 \in A$ do not have an image in co-domain.	**1**
(iii) Domain of $R = \{1, 2\}$, Range of $R = \{4, 5\}$	**1**

Q43. **2025** : 65/7/1, 65/7/2, 65/7/3 **[3 Marks]**

(a) Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 4x^3 - 5$, $\forall x \in \mathbb{R}$ is one-one and onto.

OR

(b) Let R be a relation defined on a set N of natural numbers such that $R = \{(x, y) : xy$ is a square of a natural number, $x, y \in N\}$. Determine if the relation R is an equivalence relation.

Answer (as per CBSE Marking Scheme)	Marks
(a) **One-One:** Let $x_1, x_2 \in \mathbb{R}$ such that $f(x_1) = f(x_2) \Rightarrow 4{x_1}^3 - 5 = 4{x_2}^3 - 5 \Rightarrow {x_1}^3 = {x_2}^3 \Rightarrow x_1 = x_2$, $\therefore$ 'f' is one-one	**1½**
Onto: $x \in \mathbb{R}$ $(D_f) \Rightarrow x^3 \in \mathbb{R} \Rightarrow 4x^3 - 5 \in \mathbb{R} \Rightarrow f(x) \in \mathbb{R}$, $\therefore R_f =$ Co-domain(f) $\therefore$ 'f' is an onto function $\Rightarrow$ 'f' is one-one & onto both	**1½**
OR	
(b) **Reflexive:** For any $x \in N$, $x \cdot x = x^2$, which is square of the natural number 'x'. $\Rightarrow (x, x) \in R \therefore$ 'R' is a reflexive relation.	**1**
Symmetric: Let $(x, y) \in R \Rightarrow xy$ is a square of a natural number $\Rightarrow yx$ is a square of a natural number, $\because xy = yx \Rightarrow (y, x) \in R \therefore$ 'R' is a symmetric relation.	**1**
Transitive: Let $(x, y), (y, z) \in R \Rightarrow xy = a^2$, $yz = b^2$ for some $a, b \in N$, $\therefore \frac{a^2}{y} = x$, $\frac{b^2}{y} = z \in N \Rightarrow xz = \frac{a^2}{y} \cdot \frac{b^2}{y} = (\frac{ab}{y})^2$, $\frac{ab}{y} \in N \Rightarrow (x, z) \in R \therefore$ 'R' is a transitive relation.	**½**
Hence, R is an equivalence relation	**½**

Q44. **2026** : 65/4/1, 65/4/2, 65/4/3 **[3 Marks]**

(a) Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. A function $f : A \to B$ is defined by $f(x) = (\frac{x - 2}{x - 3})$. Find whether f is one-one and onto.

OR

(b) Let n be a fixed positive integer. A relation R is defined in set Z such that $R = \{(x, y) : (x - y)$ is divisible by n, $x, y \in Z\}$. Determine if R is an equivalence relation.

Answer (as per CBSE Marking Scheme)	Marks
(a) Let $f(a) = f(b)$ for some $a, b \in \mathbb{R} - \{3\}$, so, $(\frac{a - 2}{a - 3}) = (\frac{b - 2}{b - 3}) \Rightarrow ab - 2b - 3a + 6 = ab - 3b - 2a + 6$	**1**
$\Rightarrow a = b$, Thus f is one-one function.	**½**
Now, let us assume for some $y \in \mathbb{R} - \{1\}$, $y = f(x) = \frac{x - 2}{x - 3}$. For getting, $x = \frac{3y - 2}{y - 1}$	**1**
Range $= \mathbb{R} - \{1\}$ = co-domain, Thus f is onto function.	**½**
OR	

(b) Let for some a ∈ Z, (a, a) ∈ R ⇒ 0 is divisible by n, which is true ∴ (a, a) ∈ R, so R is reflexive.	½
Let (a, b) ∈ R ⇒ a − b = np, p ∈ Z ⇒ b − a = n(−p) = nq, q ∈ Z ⇒ b − a is also divisible by n. ∴ (b, a) ∈ R, so R is symmetric.	1
Let (a, b) ∈ R, (b, c) ∈ R for some a, b, c ∈ Z ⇒ a − b = np and b − c = nr, where p, r ∈ Z ⇒ a − b + b − c = n(p + r) ⇒ (a − c) is divisible by n. ∴ (a, c) ∈ R, so R is transitive.	1
Thus, R is an equivalence relation.	½

Long Answer Questions (5 marks)

Q45. **2023** : 65/3/1 **[5 Marks]**

Show that a function $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = \frac{5x - 3}{4}$ is both one-one and onto.

Answer (as per CBSE Marking Scheme)	Marks
Let $x_1, x_2 \in \mathbb{R}$ (Domain) such that $f(x_1) = f(x_2) \Rightarrow \frac{5x_1 - 3}{4} = \frac{5x_2 - 3}{4} \Rightarrow 5x_1 = 5x_2 \Rightarrow x_1 = x_2$. ∴ f is one-one.	2½
Let $y \in \mathbb{R}$ (co-domain). Then f(x) = y for some x, if $y = \frac{5x - 3}{4}$, i.e., if $x = \frac{4y + 3}{5}$, which ∈ ℝ (Domain). Thus, for every $y \in \mathbb{R}$ (co-domain), there exists $\frac{4y + 3}{5} \in \mathbb{R}$ (domain) such that $f(\frac{4y + 3}{5}) = y$. ∴ Range of f = ℝ = codomain of f. Hence, f is onto.	2½

Q46. **2023** : 65/3/2 **[5 Marks]**

Prove that a function $f : [0, \infty) \to [-5, \infty)$ defined as $f(x) = 4x^2 + 4x - 5$ is both one-one and onto.

Answer (as per CBSE Marking Scheme)	Marks
Let $x_1, x_2 \in [0, \infty)$ such that $f(x_1) = f(x_2) \Rightarrow 4{x_1}^2 + 4x_1 - 5 = 4{x_2}^2 + 4x_2 - 5 \Rightarrow (x_1 + x_2)(x_1 - x_2) + (x_1 - x_2) = 0 \Rightarrow (x_1 - x_2)[(x_1 + x_2) + 1] = 0 \Rightarrow x_1 - x_2 = 0$ or $x_1 + x_2 = -1$, which is rejected as $x_1, x_2 \geq 0 \Rightarrow x_1 = x_2$. ∴ f is one-one.	2½
Let $f(x) = y \Rightarrow y = 4x^2 + 4x - 5$ for $x \in [0, \infty) \Rightarrow 4x^2 + 4x - 5 - y = 0 \Rightarrow x = \frac{-4 \pm \sqrt{16 - 16(-5 - y)}}{8} \Rightarrow x = \frac{-4 + 4\sqrt{6 + y}}{8} = \frac{-1 + \sqrt{6 + y}}{2}$. Since $x \geq 0$, we have $y + 6 \geq 1 \Rightarrow y \in [-5, \infty)$. ∴ Range = Codomain = $[-5, \infty)$. Hence f is onto.	2½

Q47. **2023** : 65/3/3 **[5 Marks]**

Check whether a function $f : \mathbb{R} \to [-\frac{1}{2}, \frac{1}{2}]$ defined as $f(x) = \frac{x}{1 + x^2}$ is one-one and onto or not.

Answer (as per CBSE Marking Scheme)	Marks
Let $f(x_1) = f(x_2) \Rightarrow \frac{x_1}{1 + {x_1}^2} = \frac{x_2}{1 + {x_2}^2} \Rightarrow x_1 + x_1{x_2}^2 = x_2 + {x_1}^2x_2 \Rightarrow (x_1 - x_2)(1 - x_1x_2) = 0$. For $x_1 = 2, x_2 = \frac{1}{2}$, we have $(x_1 - x_2)(1 - x_1x_2) = 0$ but $x_1 \neq x_2 \Rightarrow$ f is not one-one.	2½
Let $x \in \mathbb{R}$ such that $f(x) = y \Rightarrow y = \frac{x}{1 + x^2} \Rightarrow x^2y - x + y = 0 \Rightarrow x = \frac{1 \pm \sqrt{1 - 4y^2}}{2y}$ $(y \neq 0)$. [For $y = 0 \in [-\frac{1}{2}, \frac{1}{2}]$, we have $0 \in \mathbb{R}$ such that f(0) = 0.] $x \neq 0, x \in \mathbb{R} \Rightarrow 1 - 4y^2 \geq 0, y \neq 0 \Rightarrow y \in [-\frac{1}{2}, \frac{1}{2}] - \{0\}$. Also, y = 0 when x = 0. ∴ Range = $[-\frac{1}{2}, \frac{1}{2}]$ = Codomain. ∴ f is onto.	2½

Q48. **2023** : 65/2/1, 65/2/2, 65/2/3 **[5 Marks]**

A function f : [−4, 4] → [0, 4] is given by f(x) = $\sqrt{16 - x^2}$. Show that f is an onto function but not a one-one function. Further, find all possible values of 'a' for which f(a) = $\sqrt{7}$.

Answer (as per CBSE Marking Scheme)	Marks
Onto: Let y = $\sqrt{16 - x^2}$ ⇒ y ≥ 0	½
Squaring we get, $x^2 = 16 - y^2$ ⇒ x = ±$\sqrt{16 - y^2}$	½
For each y ∈ [−4, 4], 'x' is a real number, ∴ 0 ≤ y ≤ 4 ⇒ R_f = [0, 4] = Co-domain ∴ 'f' is an onto function.	1½
One-one: f(−1) = f(1) = $\sqrt{15}$ but −1 ≠ 1, ∴ 'f' is not a one-one function.	1½
f(a) = $\sqrt{7}$ ⇒ $\sqrt{16 - a^2}$ = $\sqrt{7}$ ⇒ a = ±3	1

Q49. **2024** : 65/2/1, 65/2/2 **[5 Marks]**

(a) Show that a function f : ℝ → ℝ defined by f(x) = $\frac{2x}{1 + x^2}$ is neither one-one nor onto. Further, find set A so that the given function f : ℝ → A becomes an onto function.

OR

(b) A relation R is defined on N × N (where N is the set of natural numbers) as : (a, b) R (c, d) ⇔ a − c = b − d. Show that R is an equivalence relation.

Answer (as per CBSE Marking Scheme)	Marks
(a) Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in \mathbb{R}$. Then $\frac{2x_1}{1 + x_1^2} = \frac{2x_2}{1 + x_2^2}$ ⇒ $x_1 + x_1x_2^2 = x_2 + x_1^2x_2$ ⇒ $(x_1 - x_2) - x_1x_2(x_1 - x_2) = 0$ ⇒ $(x_1 - x_2)(1 - x_1x_2) = 0$ ⇒ $x_1 = x_2$ or $x_1x_2 = 1$, so if $x_1x_2 = 1$, $x_1 \neq x_2$. Hence f is not one-one.	2
Let y = f(x) where x ∈ ℝ. Then y = $\frac{2x}{1 + x^2}$. Here, for x = 0, y = 0. If y ≠ 0, then $yx^2 - 2x + y = 0$ ⇒ x = $\frac{2 \pm \sqrt{4 - 4y^2}}{2y}$. For x to be real, $4 - 4y^2 \geq 0$ ⇒ $y^2 \leq 1$ ⇒ −1 ≤ y ≤ 1. Hence, range = [−1, 1] ≠ codomain. Hence, f is not onto.	2
For the given function to become onto, A = [−1, 1]	1
OR	
(b) Let (a, b) ∈ N × N. We have a − a = b − b. This implies that (a, b) R (a, b) ∀ (a, b) ∈ N × N. Hence R is reflexive.	1½
Let (a, b) R (c, d) for some (a, b), (c, d) ∈ N × N. Then a − c = b − d ⇒ c − a = d − b ⇒ (c, d) R (a, b). Hence, R is symmetric.	1½
Let (a, b) R (c, d), (c, d) R (e, f) for some (a, b), (c, d), (e, f) ∈ N × N. Then a − c = b − d, c − e = d − f ⇒ a − c + c − e = b − d + d − f ⇒ a − e = b − f ⇒ (a, b) R (e, f). Hence, R is transitive. Thus, R is an equivalence relation.	2

Q50. **2024** : 65/2/3 **[5 Marks]**

(a) Show that a function f : ℝ → ℝ defined as $f(x) = x^2 + x + 1$ is neither one-one nor onto. Also, find all the values of x for which f(x) = 3.

OR

(b) A relation R is defined on N × N (where N is the set of natural numbers) as (a, b) R (c, d) ⇔ $\frac{a}{c} = \frac{b}{d}$. Show that R is an equivalence relation.

Answer (as per CBSE Marking Scheme)	Marks

(a) Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in \mathbb{R}$. Then ${x_1}^2 + x_1 + 1 = {x_2}^2 + x_2 + 1 \Rightarrow (x_1 - x_2)(1 + x_1 + x_2) = 0 \Rightarrow x_1 - x_2 = 0$ or $x_1 + x_2 = -1 \Rightarrow x_1 = x_2$ or $x_1 + x_2 = -1$, so if $x_1 + x_2 = -1$, $x_1 \neq x_2$. Hence f is not one-one.	**2**
Let $y = f(x)$ where $x \in \mathbb{R}$. Then $y = x^2 + x + 1 \Rightarrow x^2 + x + 1 - y = 0 \Rightarrow x = \frac{-1 \pm \sqrt{4y - 3}}{2}$. For x to be real, $4y - 3 \geq 0 \Rightarrow y \geq \frac{3}{4}$. Hence, range $= [\frac{3}{4}, \infty) \neq$ codomain. Hence, f is not onto.	**2**
$f(x) = 3 \Rightarrow x^2 + x + 1 = 3 \Rightarrow x^2 + x - 2 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{9}}{2} = -2, 1$	**1**
OR	
(b) Let $(a, b) \in N \times N$. We have $\frac{a}{a} = \frac{b}{b}$. This implies that (a, b) R (a, b) $\forall$ $(a, b) \in N \times N$. Hence R is reflexive.	**1½**
Let (a, b) R (c, d) for some (a, b), (c, d) $\in N \times N$. Then $\frac{a}{c} = \frac{b}{d} \Rightarrow \frac{c}{a} = \frac{d}{b} \Rightarrow$ (c, d) R (a, b). Hence, R is symmetric.	**1½**
Let (a, b) R (c, d), (c, d) R (e, f) for some (a, b), (c, d), (e, f) $\in N \times N$. Then $\frac{a}{c} = \frac{b}{d}, \frac{c}{e} = \frac{d}{f} \Rightarrow \frac{a}{e} = \frac{b}{f} \Rightarrow$ (a, b) R (e, f). Hence, R is transitive. Thus, R is an equivalence relation.	**2**

Q51. **2024** : 65/3/3 **[5 Marks]**

Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{a\}$. Find the value of 'a' such that the function $f : A \to B$ defined by $f(x) = \frac{x - 2}{x - 3}$ is onto. Also, check whether the given function is one-one or not.

Answer (as per CBSE Marking Scheme)	Marks
For onto, let $f(x) = y$: $\frac{x - 2}{x - 3} = y \Rightarrow x - 2 = xy - 3y$	**½**
$\Rightarrow x(1 - y) = 2 - 3y \Rightarrow x = \frac{2 - 3y}{1 - y}$	**1**
For $y = 1$, $x \notin A \therefore$ Range $= \mathbb{R} - \{1\} \therefore a = 1$	**1**
For one-one: Let $f(x_1) = f(x_2)$ where $x_1, x_2 \in A \Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$	**1**
$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_2x_1 - 2x_1 - 3x_2 + 6$	**1**
$\Rightarrow x_1 = x_2 \therefore$ f is one-one.	**½**

Q52. **2024** : 65/4/1, 65/4/2, 65/4/3 **[5 Marks]**

(a) Let $A = \mathbb{R} - \{5\}$ and $B = \mathbb{R} - \{1\}$. Consider the function $f : A \to B$, defined by $f(x) = \frac{x - 3}{x - 5}$. Show that f is one-one and onto.

OR

(b) Check whether the relation S in the set of real numbers $\mathbb{R}$ defined by $S = \{(a, b) :$ where $a - b + \sqrt{2}$ is an irrational number$\}$ is reflexive, symmetric or transitive.

Answer (as per CBSE Marking Scheme)	Marks
(a) Let $f(x_1) = f(x_2)$, for some $x_1, x_2 \in A \Rightarrow \frac{x_1 - 3}{x_1 - 5} = \frac{x_2 - 3}{x_2 - 5} \Rightarrow (x_1 - 3)(x_2 - 5) = (x_2 - 3)(x_1 - 5) \Rightarrow x_1 = x_2$, so f is one-one function.	**2½**
Let $y = f(x) = \frac{x - 3}{x - 5} \Rightarrow y(x - 5) = x - 3 \Rightarrow yx - 5y = x - 3 \Rightarrow x = \frac{5y - 3}{y - 1}$. We observe that x is defined for all values of y except $y = 1$. So, Range $= \mathbb{R} - \{1\}$ and Co-domain is given $\mathbb{R} - \{1\}$ [As, $f : A \to B$]. Since, Range = Co-domain, f is onto function. Thus, f is one-one & onto function.	**2½**

OR	
(b) **Reflexive:** For $a \in \mathbb{R} \Rightarrow a - a + \sqrt{2}$ is irrational number $\Rightarrow \sqrt{2}$ is irrational number $\Rightarrow (a, a) \in S$. Thus, S is reflexive relation.	**1½**
Symmetric: Let $(a, b) \in S \Rightarrow a - b + \sqrt{2}$ is irrational number, but $b - a + \sqrt{2}$ may not be irrational number. For example, $(\sqrt{2}, 1) \in S \Rightarrow \sqrt{2} - 1 + \sqrt{2} = 2\sqrt{2} - 1$ is irrational number; $(1, \sqrt{2}) \notin S$ as $1 - \sqrt{2} + \sqrt{2} = 1$ is not irrational number $\therefore (b, a) \notin S$, so S is NOT symmetric relation.	**1½**
Transitive: Let $(a, b) \in S \Rightarrow a - b + \sqrt{2}$ is irrational number & $(b, c) \in S \Rightarrow b - c + \sqrt{2}$ is irrational number, but $a - c + \sqrt{2}$ may not be irrational number. For example, $(1, \sqrt{3}) \in S \Rightarrow 1 - \sqrt{3} + \sqrt{2}$ is irrational number; $(\sqrt{3}, \sqrt{2}) \in S \Rightarrow \sqrt{3} - \sqrt{2} + \sqrt{2} = \sqrt{3}$ is irrational number. But $(1, \sqrt{2}) \notin S$ as $1 - \sqrt{2} + \sqrt{2} = 1$ is not irrational number $\therefore (a, c) \notin S$, so S is NOT transitive relation. Thus, S is reflexive but neither symmetric nor transitive relation.	**2**

Q53. **2026** : 65/1/1, 65/1/2, 65/1/3 **[5 Marks]**

A function $f : \mathbb{R} - \{\frac{3}{5}\} \to \mathbb{R} - \{\frac{3}{5}\}$ is defined as $f(x) = \frac{3x + 2}{5x - 3}$. Show that f is one-one and onto.

Answer (as per CBSE Marking Scheme)	Marks
Let $x_1, x_2 \in \mathbb{R} - \{\frac{3}{5}\}$ such that $f(x_1) = f(x_2) \Rightarrow \frac{3x_1 + 2}{5x_1 - 3} = \frac{3x_2 + 2}{5x_2 - 3} \Rightarrow 15x_1x_2 - 9x_1 + 10x_2 - 6 = 15x_1x_2 + 10x_1 - 9x_2 - 6 \Rightarrow x_1 = x_2$. Hence, f is one-one.	**2½**
Let $y \in \mathbb{R} - \{\frac{3}{5}\}$ (Codomain). Then $f(x) = y$ or, $\frac{3x + 2}{5x - 3} = y \Rightarrow x = \frac{3y + 2}{5y - 3} \in$ Domain. Hence, for every $y \in \mathbb{R} - \{\frac{3}{5}\}$ (codomain), there exists $x = \frac{3y + 2}{5y - 3} \in \mathbb{R} - \{\frac{3}{5}\}$ (domain) such that $f(\frac{3y + 2}{5y - 3}) = y$. Thus, f is onto.	**2½**

Q54. **2026** : 65/3/1 **[5 Marks]**

Show that $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = \frac{x}{\sqrt{1 + x^2}}$ is one-one but not onto.

Answer (as per CBSE Marking Scheme)	Marks
One-one: For $x_1, x_2 \in \mathbb{R}$; let $f(x_1) = f(x_2) \Rightarrow \frac{x_1}{\sqrt{1 + {x_1}^2}} = \frac{x_2}{\sqrt{1 + {x_2}^2}}$ [$\Rightarrow x_1$ and x_2 must be of same sign.]	**1**
$\Rightarrow {x_1}^2(1 + {x_2}^2) = {x_2}^2(1 + {x_1}^2) \Rightarrow {x_1}^2 = {x_2}^2 \Rightarrow x_1 = x_2$ (Rejecting $x_1 = -x_2$). Hence, f is one-one.	**1½**
Onto: $y = \frac{x}{\sqrt{1 + x^2}} \Rightarrow x = \pm\sqrt{\frac{y^2}{1 - y^2}}$	**1**
x is defined $\forall\ y \in (-1, 1) \Rightarrow$ Range $= (-1, 1)$	**1**
$\because$ Range $\neq$ codomain $\Rightarrow$ f is not onto.	**½**

Q55. **2026** : 65/3/2 **[5 Marks]**

Show that $f : \mathbb{R}_+ \to [-5, \infty)$ given by $f(x) = 4x^2 + 4x - 5$ is both one-one and onto where $\mathbb{R}_+ = [0, \infty)$. Also, find $p \in \mathbb{R}_+$ such that $f(p) = 3$.

Answer (as per CBSE Marking Scheme)	Marks
One-one: For $x_1, x_2 \in \mathbb{R}_+$, let $f(x_1) = f(x_2) \Rightarrow 4{x_1}^2 + 4x_1 - 5 = 4{x_2}^2 + 4x_2 - 5$	**1**
$\Rightarrow (x_1 - x_2)(x_1 + x_2 + 1) = 0 \Rightarrow x_1 = x_2$ as $x_1 + x_2 + 1 \neq 0$ as $x_1, x_2 \in \mathbb{R}_+$. Hence f is one-one.	**1**
Onto: $y = 4x^2 + 4x - 5 \Rightarrow x = \frac{\sqrt{y + 6} - 1}{2}$	**1**
As $x \geq 0 \Rightarrow \sqrt{y + 6} \geq 1 \Rightarrow y \geq -5 \Rightarrow$ Range $= [-5, \infty)$	**½**

As Range = codomain = $[-5, \infty) \Rightarrow$ f is onto.	½
Given that $f(p) = 3 \Rightarrow 4p^2 + 4p - 5 = 3 \Rightarrow p = 1$ as $p \neq -2$	1

Case Study Based Questions

Q56. **2023** : 65/5/1, 65/5/2, 65/5/3 **[4 Marks]**

Case Study: An organization conducted bike race under two different categories – Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project. Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.
Based on the above information, answer the following questions :

(i) How many relations are possible from B to G ? *1*

(ii) Among all the possible relations from B to G, how many functions can be formed from B to G ? *1*

(iii) Let $R : B \to B$ be defined by R = {(x, y) : x and y are students of the same sex}. Check if R is an equivalence relation. *2*

OR

A function $f : B \to G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$. Check if f is bijective. Justify your answer.

Answer (as per CBSE Marking Scheme)	Marks
(i) Number of relations $= 2^6 = 64$	1
(ii) Number of possible functions $= 2^3 = 8$	1
(iii) R is an equivalence relation as it is reflexive, symmetric and transitive	2
OR	
(iii) Since f is not one-one function	1
$\therefore$ f is not bijective	1

Q57. **2025** : 65/1/1, 65/1/2, 65/1/3 **[4 Marks]**

Case Study: A class-room teacher is keen to assess the learning of her students the concept of "relations" taught to them. She writes the following five relations each defined on the set A = {1, 2, 3} :
$R_1 = \{(2, 3), (3, 2)\}$; $R_2 = \{(1, 2), (1, 3), (3, 2)\}$; $R_3 = \{(1, 2), (2, 1), (1, 1)\}$; $R_4 = \{(1, 1), (1, 2), (3, 3), (2, 2)\}$; $R_5 = \{(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)\}$.
The students are asked to answer the following questions about the above relations :

(i) Identify the relation which is reflexive, transitive but not symmetric. *1*

(ii) Identify the relation which is reflexive and symmetric but not transitive. *1*

(iii) (a) Identify the relations which are symmetric but neither reflexive nor transitive. *2*

OR

(b) What pairs should be added to the relation R_2 to make it an equivalence relation ?

Answer (as per CBSE Marking Scheme)	Marks
(i) R_4	1
(ii) R_5	1
(iii)(a) R_1 and R_3	1+1
OR	

(iii)(b) Required pairs to be added to make the relation R_2 as an equivalence relation are: (1, 1), (2, 2), (3, 3), (2, 1), (3, 1) and (2, 3)	**2**

Q58. **2025** : 65/2/1, 65/2/2, 65/2/3 **[4 Marks]**

Case Study: A school is organizing a debate competition with participants as speakers S = {S_1, S_2, S_3, S_4} and these are judged by judges J = {J_1, J_2, J_3}. Each speaker can be assigned one judge. Let R be a relation from set S to J defined as R = {(x, y) : speaker x is judged by judge y, x ∈ S, y ∈ J}.

Based on the above, answer the following :

(i) How many relations can be there from S to J ? *1*

(ii) A student identifies a function from S to J as f = {(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)}. Check if it is bijective. *1*

(iii) (a) How many one-one functions can be there from set S to set J ? *2*

OR

(b) Another student considers a relation R_1 = {(S_1, S_2), (S_2, S_4)} in set S. Write minimum ordered pairs to be included in R_1 so that R_1 is reflexive but not symmetric.

Answer (as per CBSE Marking Scheme)	Marks
(i) The number of relations = $2^{4\times3} = 2^{12}$	**1**
(ii) Since, S_2 and S_3 have been assigned the same judge J_2, the function is not one-one. Hence, it is not bijective.	**1**
(iii)(a) There cannot exist any one-one function from S to J as n(S) > n(J). Hence, the number of one-one functions from S to J is 0.	**2**
OR	
(iii)(b) To make R_1 reflexive and not symmetric we need to add the following ordered pairs: (S_1, S_1), (S_2, S_2), (S_3, S_3), (S_4, S_4)	**2**

Q59. **2025** : 65/5/1, 65/5/2, 65/5/3 **[4 Marks]**

Case Study: Let A be the set of 30 students of class XII in a school. Let f : A → N, N is a set of natural numbers such that function f(x) = Roll Number of student x.

On the basis of the given information, answer the following :

(i) Is f a bijective function ? *1*

(ii) Give reasons to support your answer to (i). *1*

(iii) (a) Let R be a relation defined by the teacher to plan the seating arrangement of students in pairs, where R = {(x, y) : x, y are Roll Numbers of students such that y = 3x}. List the elements of R. Is the relation R reflexive, symmetric and transitive ? Justify your answer. *2*

OR

(b) Let R be a relation defined by R = {(x, y) : x, y are Roll Numbers of students such that y = x^3}. List the elements of R. Is R a function ? Justify your answer.

Answer (as per CBSE Marking Scheme)	Marks
(i) No, f is not bijective function	**1**
(ii) Range = {1, 2, 3, 4, ……., 30} and codomain = N	**½**
Since, Range ≠ codomain ⇒ f is not onto and hence f is not bijective.	**½**
(iii)(a) R = {(1, 3), (2, 6), (3, 9), (4, 12), (5, 15), (6, 18), (7, 21), (8, 24), (9, 27), (10, 30)}	**1**
Since (1, 1) ∉ R ⇒ R is not reflexive. (1, 3) ∈ R but (3, 1) ∉ R ⇒ R is not symmetric. (1, 3) ∈ R, (3, 9) ∈ R but (1, 9) ∉ R ⇒ R is not transitive.	**1**

OR	
(iii)(b) R = {(1, 1), (2, 8), (3, 27)}	**1**
∵ elements 4, 5, 6 ... 30 do not have an image. Hence the above relation is not a function.	**1**

Q60. **2025 Supp** : 65/S/1, 65/S/2, 65/S/3 **[4 Marks]**

Case Study: During the festival season, there was a mela organized by the Resident Welfare Association at a park, near the society. The main attraction of the mela was a huge swing installed at one corner of the park. The swing is traced to follow the path of a parabola given by $x^2 = y$.

Based on the above information, answer the following questions :

(i) Let $f : N \to \mathbb{R}$ is defined by $f(x) = x^2$. What will be the range ? *1*

(ii) Let $f : N \to N$ is defined by $f(x) = x^2$. Check if the function is injective or not. *1*

(iii) (a) Let $f : \{1, 2, 3, 4, ...\} \to \{1, 4, 9, 16, ...\}$ be defined by $f(x) = x^2$. Prove that the function is bijective. *2*

OR

(b) Let $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x^2$. Show that f is neither injective nor surjective.

Answer (as per CBSE Marking Scheme)	Marks
(i) $R_f = \{1, 4, 9, 16, ...\}$ i.e. set of perfect squares of natural numbers.	**1**
(ii) Let $x_1, x_2 \in N$ (domain). Let $f(x_1) = f(x_2) \Rightarrow {x_1}^2 = {x_2}^2 \Rightarrow x_1 = \pm x_2 \Rightarrow x_1 = x_2$ as $x_1, x_2 \in N$ $\therefore$ f is injective.	**1**
(iii)(a) $f(x) = x^2$. Let $x_1, x_2 \in \{1, 2, 3, 4, ...\}$. Let $f(x_1) = f(x_2) \Rightarrow {x_1}^2 = {x_2}^2 \Rightarrow x_1 = x_2$ $\therefore$ f is one-one.	**1**
As Co-domain = Range = {1, 4, 9, 16, ...} $\therefore$ f is onto. Since, f is one-one and onto, so f is bijective.	**1**
OR	
(iii)(b) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$. $-1, 1 \in \mathbb{R}$ (domain). As $f(-1) = f(1) = 1$ but $-1 \neq 1$ $\therefore$ f is not injective.	**1**
Co-domain = $\mathbb{R}$, but Range = $[0, \infty)$. Since Co-domain ≠ Range, f is not surjective.	**1**

Q61. **2026** : 65/5/1, 65/5/2, 65/5/3 **[4 Marks]**

Case Study: A school wants the students of class XII to do a project on 'Sustainability' keeping the world environment in mind. They select the student participants on the basis of an essay writing competition. 7 students out of 80 are selected for the project and are categorized into two sets such that : Girl students belong to Set A = $\{G_1, G_2, G_3, G_4\}$, Boy students belong to Set B = $\{B_1, B_2, B_3\}$.

Based on the above information, answer the following questions :

(i) How many relations are possible from Set A → Set B ? *1*

(ii) Let R be a relation from A → B such that R = $\{(G_1, B_1), (G_2, B_2), (G_3, B_2), (G_4, B_3), (G_1, B_2)\}$. Is R an injective function ? Justify your answer. *1*

(iii) (a) Let the relation R from A → A be such that R = {(x, y), x, y ∈ A, x and y are students from the same colony in the city}. Verify if R is an equivalence relation. *2*

OR

(b) Verify if any function f : B → A is bijective. Give reason to support your answer.

Answer (as per CBSE Marking Scheme)	Marks
(i) Number of relations from A to B is 2^{12} or 4096	**1**
(ii) No, R is not injective.	**½**

It is not a function as G_1 has two images B_1 and B_2	½
(iii)(a) R is reflexive as (x, x) ∈ R ∀ x ∈ A because x and x are students from same colony.	½
Let (x, y) ∈ R so x and y are students from same colony. Hence y and x are students from same colony. Thus, (y, x) ∈ R ∴ R is symmetric.	½
Let (x, y) ∈ R and (y, z) ∈ R so x and y are students from same colony, y and z are students from same colony. Thus, x and z are students from same colony ∴ (x, z) ∈ R. Thus, R is transitive.	½
As R is reflexive, symmetric and transitive, hence R is an equivalence relation.	½
OR	
(iii)(b) Any function f from B to A will not be bijective	**1**
As f from B to A cannot be surjective because n(B) < n(A) or A and B are finite sets and n(A) ≠ n(B)	**1**

INFINITY THINK BEYOND..... AN EDUCATIONAL INSTITUTE

Pre-Booster Test Series 2026-27

Chapter-wise & full-syllabus tests with detailed answer keys — practise the board pattern before the boards.

Test Series	https://infinitythinkbeyond.co.in/mind-curve-class-x-test-series-2026-27/
Mind Curve	The Mathematics Forum for students and teachers
Website	For students & teachers: www.infinitythinkbeyond.co.in
WhatsApp	Teacher–Student Forum: https://chat.whatsapp.com/E72LRTSsNjb6LPEGW2fVLt
References	CBSE Question Papers: https://www.cbse.gov.in/cbsenew/question-paper.html CBSE Marking Schemes: https://www.cbse.gov.in/cbsenew/marking-scheme.html

AN EDUCATIONAL INSTITUTE