CBSE CLASS XII · MATHEMATICS (041)

Chapter 5
Continuity and Differentiability

Chapter-wise Previous Year Questions with Marking Scheme

INFINITY
THINK BEYOND.....
AN EDUCATIONAL INSTITUTE

Board Examinations 2022 – 2026 (including Compartment / Supplementary)

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Continuity & Differentiability

Multiple Choice Questions (1 mark)

Q1. **2023** : 65/1/1 | **2026** : 65/3/1, 65/3/3 **[1 Mark]**

The value of k for which $f(x) = \begin{cases} 3x + 5, & x \geq 2 \\ kx^2, & x < 2 \end{cases}$ is a continuous function, is :

(a) $-\frac{11}{4}$ (b) $\frac{4}{11}$

(c) 11 (d) $\frac{11}{4}$

Ans. (d) $\frac{11}{4}$ **1**

Q2. **2023** : 65/1/2 **[1 Mark]**

For what value of k may the function $f(x) = \begin{cases} k(3x^2 - 5x), & x \leq 0 \\ \cos x, & x > 0 \end{cases}$ become continuous ?

(a) 0 (b) 1

(c) $-\frac{1}{2}$ (d) No value

Ans. (d) No value **1**

Q3. **2023** : 65/1/3 | **2024 Comptt** : 65/S/2 **[1 Mark]**

The function $f(x) = |x| - x$ is :

(a) continuous but not differentiable at x = 0.

(b) continuous and differentiable at x = 0.

(c) neither continuous nor differentiable at x = 0.

(d) differentiable but not continuous at x = 0.

Ans. (a) Continuous but not differentiable at x = 0 **1**

Q4. **2023** : 65/2/1, 65/2/2, 65/2/3 **[1 Mark]**

The function $f(x) = x|x|$ is

(A) continuous and differentiable at x = 0.

(B) continuous but not differentiable at x = 0.

(C) differentiable but not continuous at x = 0.

(D) neither differentiable nor continuous at x = 0.

Ans. (A) continuous and differentiable at x = 0 **1**

Q5. **2023** : 65/3/1, 65/3/2, 65/3/3 **[1 Mark]**

The value of k for which function $f(x) = \begin{cases} x^2, & x \geq 0 \\ kx, & x < 0 \end{cases}$ is differentiable at x = 0 is :

(a) 1 (b) 2

(c) any real number (d) 0

Ans. (d) 0 **1**

Q6. **2023** : 65/4/1, 65/4/2 **[1 Mark]**

The function f(x) = |x| is

(a) continuous and differentiable everywhere.

(b) continuous and differentiable nowhere.

(c) continuous everywhere, but differentiable everywhere except at x = 0.

(d) continuous everywhere, but differentiable nowhere.

Ans. (c) continuous everywhere, but differentiable everywhere except at x = 0 **1**

Q7. **2023** : 65/4/3 **[1 Mark]**

If f(x) = 2|x| + 3|sin x| + 6, then the right hand derivative of f(x) at x = 0 is :

(a) 6 (b) 5

(c) 3 (d) 2

Ans. (b) 5 **1**

Q8. **2023** : 65/5/1, 65/5/2, 65/5/3 **[1 Mark]**

The function f(x) = [x], where [x] denotes the greatest integer less than or equal to x, is continuous at

(a) x = 1 (b) x = 1.5

(c) x = −2 (d) x = 4

Ans. (b) x = 1.5 **1**

Q9. **2023** : 65/5/2 **[1 Mark]**

The function f(x) = x|x|, x ∈ ℝ is differentiable

(a) only at x = 0 (b) only at x = 1

(c) in ℝ (d) in ℝ − {0}

Ans. (c) in ℝ **1**

Q10. **2024** : 65/1/1, 65/1/3 **[1 Mark]**

Which of the following statements is true for the function f(x) = $\begin{cases} x^2 + 3, & x \neq 0 \\ 1, & x = 0 \end{cases}$?

(A) f(x) is continuous and differentiable ∀ x ∈ ℝ

(B) f(x) is continuous ∀ x ∈ ℝ

(C) f(x) is continuous and differentiable ∀ x ∈ ℝ − {0}

(D) f(x) is discontinuous at infinitely many points

Ans. (C) f(x) is continuous and differentiable ∀ x ∈ ℝ − {0} **1**

Q11. **2024** : 65/1/2 **[1 Mark]**

The number of points, where f(x) = [x], 0 < x < 3 ([·] denotes greatest integer function) is not differentiable is :

(A) 1 (B) 2

(C) 3 (D) 4

Ans. (B) 2 **1**

Q12. **2024** : 65/3/1, 65/3/2, 65/3/3 **[1 Mark]**

A function $f(x) = |1 - x + |x||$ is :

(A) discontinuous at x = 1 only
(B) discontinuous at x = 0 only
(C) discontinuous at x = 0, 1
(D) continuous everywhere

Ans. (D) continuous everywhere **1**

Q13. **2024** : 65/4/1 **[1 Mark]**

For what value of k, the function given below is continuous at x = 0 ? f(x) =

$$\begin{cases} \frac{\sqrt{4+x}-2}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$$

(A) 0
(B) $\frac{1}{4}$
(C) 1
(D) 4

Ans. (B) $\frac{1}{4}$ **1**

Q14. **2024** : 65/4/2 **[1 Mark]**

The value of constant c that makes the function f defined by $f(x) = \begin{cases} x^2 - c^2, & \text{if } x < 4 \\ cx + 20, & \text{if } x \geq 4 \end{cases}$ continuous for all real numbers is

(A) −2
(B) −1
(C) 0
(D) 2

Ans. (A) −2 **1**

Q15. **2024** : 65/4/3 **[1 Mark]**

The function f(x) = |x| + |x − 2| is

(A) continuous, but not differentiable at x = 0 and x = 2.
(B) differentiable but not continuous at x = 0 and x = 2.
(C) continuous but not differentiable at x = 0 only.
(D) neither continuous nor differentiable at x = 0 and x = 2.

Ans. (A) continuous, but not differentiable at x = 0 and x = 2 **1**

Q16. **2024 Comptt** : 65/S/1, 65/S/2, 65/S/3 **[1 Mark]**

The number of discontinuities of the function f given by $f(x) = \begin{cases} x + 2, & \text{if } x < 0 \\ e^x, & \text{if } 0 \leq x \leq 1 \\ 2 - x, & \text{if } x > 1 \end{cases}$ is :

(A) 0
(B) 1
(C) 2
(D) 3

Ans. (C) 2 **1**

Q17. **2024 Comptt** : 65/S/1, 65/S/2, 65/S/3 **[1 Mark]**

The greatest integer function defined by $f(x) = [x]$, $1 < x < 3$ is not differentiable at $x =$

(A) 0 (B) 1

(C) 2 (D) $\frac{3}{2}$

Ans. (C) 2 **1**

Q18. **2025** : 65/1/1, 65/1/2, 65/1/3 **[1 Mark]**

If $f(x) = |x| + |x - 1|$, then which of the following is correct ?

(A) f(x) is both continuous and differentiable, at x = 0 and x = 1.

(B) f(x) is differentiable but not continuous, at x = 0 and x = 1.

(C) f(x) is continuous but not differentiable, at x = 0 and x = 1.

(D) f(x) is neither continuous nor differentiable, at x = 0 and x = 1.

Ans. (C) f(x) is continuous but not differentiable, at x = 0 and x = 1. **1**

Q19. **2025** : 65/4/1, 65/4/2, 65/4/3 **[1 Mark]**

If $f(x) = \begin{cases} 3x - 2, & 0 < x \le 1 \\ 2x^2 + ax, & 1 < x < 2 \end{cases}$ is continuous for $x \in (0, 2)$, then a is equal to :

(A) -4 (B) $-\frac{7}{2}$

(C) -2 (D) -1

Ans. (D) -1 **1**

Q20. **2025** : 65/4/2 **[1 Mark]**

If $f(x) = \begin{cases} \frac{1 - \sin^3 x}{3\cos^2 x}, & \text{for } x \ne \frac{\pi}{2} \\ k, & \text{for } x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then the value of k is :

(A) $\frac{3}{2}$ (B) $\frac{1}{6}$

(C) $\frac{1}{2}$ (D) 1

Ans. (C) $\frac{1}{2}$ **1**

Q21. **2025** : 65/5/2, 65/7/2 **[1 Mark]**

If $f(x) = \begin{cases} \frac{\sin^2 ax}{x^2}, & x \ne 0 \\ 1, & x = 0 \end{cases}$ is continuous at x = 0, then the value of a is :

(A) 1 (B) -1

(C) ± 1 (D) 0

Ans. (C) ± 1 **1**

Q22. **2025** : 65/5/1, 65/5/2 **[1 Mark]**

If f(x) = {[x], x ∈ ℝ} is the greatest integer function, then the correct statement is :

(A) f is continuous but not differentiable at x = 2.

(B) f is neither continuous nor differentiable at x = 2.

(C) f is continuous as well as differentiable at x = 2.

(D) f is not continuous but differentiable at x = 2.

Ans. (B) f is neither continuous nor differentiable at x = 2. **1**

Q23. **2025** : 65/6/1, 65/6/2, 65/6/3 **[1 Mark]**

If $f(x) = \begin{cases} \frac{\log(1+ax)+\log(1-bx)}{x}, & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$ is continuous at x = 0, then the value of k is :

(A) a (B) a + b

(C) a − b (D) b

Ans. (C) a − b **1**

Q24. **2026** : 65/3/2 **[1 Mark]**

The value of k for which the function $f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & x \neq 0 \\ k(x+1), & x = 0 \end{cases}$ is a continuous function, is :

(A) $\frac{1}{4}$ (B) 2

(C) $\frac{1}{2}$ (D) 0

Ans. (D) 0 **1**

Q25. **2026** : 65/4/1, 65/4/2, 65/4/3 **[1 Mark]**

If $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at x = 0, then the value of k is :

(A) 0 (B) −2

(C) −1 (D) 2

Ans. (D) 2 **1**

Q26. **2026** : 65/5/1, 65/5/2, 65/5/3 **[1 Mark]**

If $f(x) = \begin{cases} \frac{x^2-4x-5}{x+1}, & x \neq -1 \\ k, & x = -1 \end{cases}$ is continuous at x = −1, then the value of k is :

(A) Any real value (B) 6

(C) −1 (D) −6

Ans. (D) −6 **1**

Q27. **2026** : 65/4/1, 65/4/2, 65/4/3 **[1 Mark]**

The greatest integer function, f(x) = [x], 0 < x < 3 is ***not*** differentiable at how many points ?

(A) At only one point (B) At only two points

(C) At no point (D) At three points

Ans. (B) At only two points **1**

Q28. **2026 Supp** : 65/7/1, 65/7/2, 65/7/3 **[1 Mark]**

If $f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then which of the following statements is ***not*** correct ?

(A) f is continuous for every x < 0.

(B) f is continuous for every x ∈ ℝ.

(C) f is continuous for every x > 0.

(D) f is continuous for each point except x = 0.

Ans. (B) f is continuous for every x ∈ ℝ. **1**

Assertion – Reason Questions (1 mark)

Q29. **2025** : 65/1/1, 65/1/2, 65/1/3 **[1 Mark]**

Assertion (A) : $f(x) = \begin{cases} 3x - 8, & x \leq 5 \\ 2k, & x > 5 \end{cases}$ is continuous at x = 5 for $k = \frac{5}{2}$.

Reason (R) : For a function f to be continuous at x = a, $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Ans. (D) Assertion (A) is false, but Reason (R) is true. **1**

Q30. **2025** : 65/7/1, 65/7/2, 65/7/3 **[1 Mark]**

Assertion (A) : $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous at x = 0.

Reason (R) : When $x \to 0$, $\sin \frac{1}{x}$ is a finite value between −1 and 1.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Ans. (A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A). **1**

Very Short Answer Questions (2 marks)

Q31. **2023** : 65/2/1, 65/2/2, 65/2/3 **[2 Marks]**

Question & answer as per CBSE Marking Scheme

25.	(a) If $y = \sqrt{ax+b}$, prove that $y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 = 0$. **OR** (b) If $f(x) = \begin{cases} ax+b & ; \ 0 < x \le 1 \\ 2x^2 - x & ; \ 1 < x < 2 \end{cases}$ is a differentiable function in (0, 2), then find the values of a and b.	
Ans	**(a)** $y = \sqrt{ax+b} \Rightarrow y^2 = ax+b$ **Differentiate with respect to 'x',** $2y\frac{dy}{dx} = a$ **Differentiate with respect to 'x',** $2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 = 0 \Rightarrow y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ **Or** **(b)** $f(x)$ **is differentiable in** $(0,2) \Rightarrow f(x)$ **is continuous on** $(0,2)$ $\Rightarrow f(x)$ **is continuous at** $x=1 \therefore \lim_{x\to 1^-}(ax+b) = \lim_{x\to 1^+}(2x^2 - x) \Rightarrow a+b=1$ **Also,** $f(x)$ **is differentiable at** $x=1, \therefore$ **L.H.D.**$(x=1)$ **= R.H.D.**$(x=1)$ $\Rightarrow a = 4(1) - 1 \therefore a = 3 \& b = 1 - a = -2$	1 1 ½ ½ ½ + ½

Q32. **2023** : 65/5/1, 65/5/2 **[2 Marks]**

Question & answer as per CBSE Marking Scheme

22(a).	If $f(x) = \begin{cases} x^2, & \text{if } x \ge 1 \\ x, & \text{if } x < 1 \end{cases}$, then show that f is not differentiable at $x = 1$.	
Sol.	Here RHD $= \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = 2$ LHD $= \lim_{h \to 0} \left[\frac{f(1-h) - f(1)}{-h}\right] = 1$ Since RHD ≠ LHD ∴ f is not differentiable at x = 1.	$1\frac{1}{2}$ $\frac{1}{2}$
22(b).	Find the value(s) of 'λ', if the function $f(x) = \begin{cases} \frac{\sin^2 \lambda x}{x^2}, & \text{if } x \ne 0 \text{ is continuous at } x = 0. \\ 1, & \text{if } x = 0 \end{cases}$	
Sol.	$\lim_{x \to 0} f(x) = \lim_{x \to 0}\left(\frac{\sin^2 \lambda x}{x^2}\right) = \lim_{x \to 0}\left[\frac{\sin^2 \lambda x}{(\lambda x)^2} \cdot \lambda^2\right] = \lambda^2$	1

		Since f(x) is continuous at x = 0 $\lim_{x \to 0} f(x) = f(0)$ $\Rightarrow \lambda^2 = 1 \Rightarrow \lambda = \pm 1$	$\frac{1}{2}$ $\frac{1}{2}$

Q33. **2023** : 65/5/3 **[2 Marks]**

Question & answer as per CBSE Marking Scheme

21(a).	(a) Find the value of k for which the function f given as $f(x) = \begin{cases} \frac{1-\cos x}{2x^2}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$.	
Sol.	Here, $f(x) = \frac{1-\cos x}{2x^2} = \frac{2\sin^2 \frac{x}{2}}{2x^2} = \left(\frac{\sin \frac{x}{2}}{2\frac{x}{2}}\right)^2$ $\Rightarrow \lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{1}{4}\left(\frac{\sin \frac{x}{2}}{\frac{x}{2}}\right) = \frac{1}{4}$ So, if f is continuous at x = 0, then $f(0) = \lim_{x \to 0} f(x)$ $\Rightarrow k = \frac{1}{4}$	**1** **$\frac{1}{2}$** **$\frac{1}{2}$**
21(b).	If $x = a\cos t$ and $y = b\sin t$, then find $\frac{d^2y}{dx^2}$.	
Sol.	Given $x = a\cos t$ and $y = b\sin t$, we have $\frac{dx}{dt} = -a\sin t$ and $\frac{dy}{dt} = b\cos t$ $\therefore \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{b\cos t}{-a\sin t} = -\frac{b}{a}\cot t$ $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(-\frac{b}{a}\cot t\right) \cdot \frac{dt}{dx}$ $= \frac{b}{a}\operatorname{cosec}^2 t \cdot \frac{1}{-a\sin t}$	**1** **$\frac{1}{2}$**

		$= -\frac{b}{a^2} \cdot \frac{1}{\sin^3 t}$ or $-\frac{b}{a^2}\operatorname{cosec}^3 t$	$\frac{1}{2}$

Q34. **2023 Supp** : 65/C/1 **[2 Marks]**

Question & answer as per CBSE Marking Scheme

23.	**Function f is defined as** $f(x) = \begin{cases} 2x+2, & \text{if } x < 2 \\ k, & \text{if } x = 2 \\ 3x, & \text{if } x > 2 \end{cases}$ **Find the value of k for which the function f is continuous at x = 2.**	
Sol.	As f is continuous at $x=2 \Rightarrow \lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x) = f(2)$ $\lim_{x\to 2^+} 3x = \lim_{x\to 2^-} (2x+2) = k$ $\Rightarrow k = 6$	1 1

Q35. **2023 Supp** : 65/C/2 **[2 Marks]**

Question & answer as per CBSE Marking Scheme

22.	**Find the points at which the function f(x) =** $\frac{4+x^2}{4x-x^3}$ **is discontinuous.**	

Sol.	$f(x) = \frac{4+x^2}{x(2-x)(2+x)}$ clearly f is not continuous when $x(2-x)(2+x) = 0$ $\Rightarrow x = 0, 2, -2$	½ ½ 1

Q36. **2023 Supp** : 65/C/3 **[2 Marks]**

Question & answer as per CBSE Marking Scheme

23.	The function $f(x) = \begin{cases} 3ax+b, & \text{if } x > 1 \\ 11, & \text{if } x = 1 \\ 5ax-2b, & \text{if } x < 1 \end{cases}$ **is continuous at x = 1. Find the values of a and b.**	
Sol.	As f is continuous at $x=1 \Rightarrow \lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x) = f(1)$ $\lim_{x\to 1^+} (3ax+b) = \lim_{x\to 1^-} (5ax-2b) = 11$ $\Rightarrow 3a+b = 11$ and $5a-2b = 11$ solving, we get $a=3, b=2$	1 ½ ½

Q37. 2024 : 65/1/1 [2 Marks]

Question & answer as per CBSE Marking Scheme

21(a).	Check whether the function $f(x) = x^2 \|x\|$ is differentiable at $x = 0$ or not.	
Sol.	$f(x) = \begin{cases} x^3, x \geq 0 \\ -x^3, x \leq 0 \end{cases}$ $RHD = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} h^2 = 0$ $LHD = \lim_{h\to 0} \frac{f(0-h)-f(0)}{-h} = \lim_{h\to 0}(-h^2) = 0$ $\because RHD = LHD = 0$, $So\ f(x)\ is\ differentiable\ at\ x = 0$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
	OR	
21(b).	If $y = \sqrt{\tan\sqrt{x}}$, prove that $\sqrt{x}\ \frac{dy}{dx} = \frac{1+y^4}{4y}$.	
Sol.	$y = \sqrt{tan\sqrt{x}}$ $\frac{dy}{dx} = \frac{sec^2\sqrt{x}}{2\sqrt{tan\sqrt{x}}} \times \frac{1}{2\sqrt{x}}$ $\sqrt{x}\frac{dy}{dx} = \frac{sec^2\sqrt{x}}{4\sqrt{tan\sqrt{x}}}$ $= \frac{1+(tan\sqrt{x})^2}{4\sqrt{tan\sqrt{x}}} = \frac{1+y^4}{4y}$	1 1

Q38. 2024 : 65/1/2 [2 Marks]

Question & answer as per CBSE Marking Scheme

24(a).	If $y = \sqrt{\cos x + y}$, prove that $\frac{dy}{dx} = \frac{\sin x}{1-2y}$.	
Ans	$y^2 = \cos x + y$ $(2y-1)\frac{dy}{dx} = -\sin x$ $\Rightarrow \frac{dy}{dx} = \frac{sin\ x}{1-2y}$	$\frac{1}{2}$ 1 $\frac{1}{2}$
	OR	
24(b).	Show that the function $f(x) = \|x\|^3$ is differentiable at all points of its domain.	
Sol.	(b) $f(x) = \begin{cases} x^3, & x \geq 0 \\ -x^3, & x \leq 0 \end{cases}$ At $x = 0$ $LHD = \lim_{h\to 0} \frac{f(0-h)-f(0)}{-h} = \lim_{h\to 0}\left(\frac{h^3}{-h}\right) = \lim_{h\to 0}(-h^2) = 0$ $RHD = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0}\left(\frac{h^3}{h}\right) = \lim_{h\to 0}(h^2) = 0$ $\because$ LHD = RHD at $x = 0$; when $x \neq 0$, $f(x)$ is a polynomial and hence differentiable. $\therefore f(x)$ is differentiable at all points.	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

Q39. **2024** : 65/1/3 **[2 Marks]**

Question & answer as per CBSE Marking Scheme

22(a).	Check the differentiability of function f(x) = [x] at x = – 3, where [·] denotes greatest integer function.	
Ans	f(x) = [x] at x = −3 RHD = $\lim_{h\to 0}\frac{f(-3+h)-f(-3)}{h}$ $= \lim_{h\to 0}\frac{-3-(-3)}{h} = 0$ LHD = $\lim_{h\to 0}\frac{f(-3-h)-f(-3)}{-h}$ $= \lim_{h\to 0}\frac{-4-(-3)}{h} = \lim_{h\to 0}\left(\frac{-1}{h}\right)$ = not defined ∵ LHD ≠ RHD So f is not differentiable at x = −3	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
	OR	
22(b).	If $x^{1/3} + y^{1/3} = 1$, find $\frac{dy}{dx}$ at the point $\left(\frac{1}{8}, \frac{1}{8}\right)$.	
Ans	$\frac{1}{3}x^{\frac{-2}{3}} + \frac{1}{3}y^{\frac{-2}{3}}\frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{-x^{\frac{-2}{3}}}{y^{\frac{-2}{3}}}$ $\left(\frac{dy}{dx}\right)_{\left(\frac{1}{8},\frac{1}{8}\right)} = \frac{-4}{4} = -1$	$1\frac{1}{2}$ $\frac{1}{2}$

Q40. **2024** : 65/2/2 **[2 Marks]**

Question & answer as per CBSE Marking Scheme

25.	(a) Check the differentiability of f(x) = $\lvert\cos x\rvert$ at x = $\frac{\pi}{2}$.	
Sol.	f(x) = $\lvert\cos x\rvert = \begin{cases} \cos x & 0 \le x \le \frac{\pi}{2} \\ -\cos x & \frac{\pi}{2} \le x \le \pi \end{cases}$ LHD at $\frac{\pi}{2} = \lim_{h\to 0}\frac{f\left(\frac{\pi}{2}-h\right)-f\left(\frac{\pi}{2}\right)}{-h} = \lim_{h\to 0}\frac{\cos\left(\frac{\pi}{2}-h\right)-0}{-h} = \lim_{h\to 0}\frac{\sin h}{-h} = -1$	1
	RHD at $\frac{\pi}{2} = \lim_{h\to 0}\frac{f\left(\frac{\pi}{2}+h\right)-f\left(\frac{\pi}{2}\right)}{h} = \lim_{h\to 0}\frac{-\cos\left(\frac{\pi}{2}+h\right)-0}{h} = \lim_{h\to 0}\frac{\sin h}{h} = 1$	$\frac{1}{2}$

	LHD $\neq$ RHD $\therefore$ f is not differentiable at $x = \frac{\pi}{2}$	$\frac{1}{2}$
	OR	
	(b) If $y = A \sin 2x + B \cos 2x$ and $\frac{d^2y}{dx^2} - ky = 0$, find the value of k.	
Sol.	$\frac{dy}{dx} = 2A\cos 2x - 2B\sin 2x$ $\Rightarrow \frac{d^2y}{dx^2} = -4A\sin 2x - 4B\cos 2x = -4y$ $\Rightarrow \frac{d^2y}{dx^2} + 4y = 0$ $\Rightarrow k = -4$	1 $\frac{1}{2}$ $\frac{1}{2}$

Q41. 2024 : 65/3/1, 65/3/2, 65/3/3 [2 Marks]

Question & answer as per CBSE Marking Scheme

Q22(a)	If $x = e^{x/y}$, prove that $\frac{dy}{dx} = \frac{\log x - 1}{(\log x)^2}$	
Ans	$x = e^{\frac{x}{y}} \Rightarrow \log x = \frac{x}{y} \Rightarrow y = \frac{x}{\log x}$ $\Rightarrow \frac{dy}{dx} = \frac{(\log x)(1) - x\left(\frac{1}{x}\right)}{(\log x)^2} = \frac{\log x - 1}{(\log x)^2}$	1 1
	OR	
Q22(b)	Check the differentiability of $f(x) = \begin{cases} x^2 + 1, & 0 \le x < 1 \\ 3 - x, & 1 \le x \le 2 \end{cases}$ at x = 1.	
Ans	LHD at $x = 1$ $= \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h \to 0} \frac{\left[(1-h)^2 + 1\right] - 2}{-h} = 2$ RHD at $x = 1$ $= \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{\left[3 - (1+h)\right] - 2}{h} = -1$ as LHD $\neq$ RHD, so $f(x)$ is not differentiable at $x = 1$	1 ½ ½

Q42. 2024 : 65/5/1, 65/5/3 [2 Marks]

Question & answer as per CBSE Marking Scheme

22.	(a) Verify whether the function f defined by $f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous at $x = 0$ or not. **OR**

	(b) Check for differentiability of the function f defined by $f(x) = \|x-5\|$, at the point $x = 5$.	
Ans	**(a)** $\lim_{x\to 0} f(x) = \lim_{x\to 0} x.\sin\frac{1}{x} = 0 \times$ Finite value in $[-1,1] = 0 = f(0)$	$1\frac{1}{2}$
	$\therefore$ f is a continuous function.	$\frac{1}{2}$
	Or	
	(b) $LHD = \lim_{x\to 5^-} \frac{\|x-5\|-0}{x-5} = \lim_{x\to 5^-} \frac{-(x-5)}{x-5} = -1$	1
	$RHD = \lim_{x\to 5^+} \frac{\|x-5\|-0}{x-5} = \lim_{x\to 5^+} \frac{(x-5)}{x-5} = 1$	$\frac{1}{2}$
	$LHD \neq RHD$, $\therefore$ f is not differentiable at $x = 5$	$\frac{1}{2}$

Q43. 2026 : 65/1/1, 65/1/2, 65/1/3 [2 Marks]

Question & answer as per CBSE Marking Scheme

21.(a)	Check whether function f(x) defined as $f(x) = \begin{cases} \frac{\|x-3\|}{2(x-3)}, & x < 3 \\ \frac{x-6}{6}, & x \geq 3 \end{cases}$ is continuous at $x = 3$ or not ?		
Sol.	$f(x) = \begin{cases} \frac{-(x-3)}{2(x-3)}, x < 3 \\ \frac{x-6}{6}, x \geq 3 \end{cases} \Rightarrow f(x) = \begin{cases} \frac{-1}{2}, x < 3 \\ \frac{x-6}{6}, x \geq 3 \end{cases}$		
	$f(3) = \frac{-1}{2}$		
	$LHL = \lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{-1}{2} = \frac{-1}{2}$	I	1
	$RHL = \lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} \frac{x-6}{6} = \frac{-1}{2}$	II	½
	LHL = RHL = $f(3)$, hence the function $f(x)$ is continuous at $x = 3$.	III	½
	OR		
21.(b)	(b) If $\sqrt{3}(x^2 + y^2) = 4xy$, then find $\frac{dy}{dx}$ at $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.		
Sol.	Differentiating both sides w.r.to x, we get $\sqrt{3}\left(2x + 2y\frac{dy}{dx}\right) = 4(x\frac{dy}{dx} + y)$	I	1
	$\Rightarrow \frac{dy}{dx} = \frac{2y - \sqrt{3}x}{\sqrt{3}y - 2x}$	II	½
	$\Rightarrow \left(\frac{dy}{dx}\right)_{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)} = \sqrt{3}$	III	½

Q44. **2026** : 65/4/1, 65/4/2, 65/4/3 **[2 Marks]**

Question & answer as per CBSE Marking Scheme

22. (a)	Show that the function $f(x)=\begin{cases}\dfrac{\cos x}{-x+\dfrac{\pi}{2}}, & x\neq\dfrac{\pi}{2}\\ 1, & x=\dfrac{\pi}{2}\end{cases}$ is continous at $x=\dfrac{\pi}{2}$.		
Sol.	For showing: $\lim\limits_{x\to\frac{\pi}{2}} f(x)=1$	I	1
	For showing: $f\left(\dfrac{\pi}{2}\right)=1$	II	½
	$\therefore f(x)$ is continuous at $x=\dfrac{\pi}{2}$.	III	½
	OR		
22. (b)	**Find whether the function f(x) = $\begin{cases}x-1, & x<2\\ 2x-3, & x\geq 2\end{cases}$ at x = 2 is differentiable or not.**		
Sol.	Getting $LHD=1$	I	1
	Getting $RHD=2$	II	½
	$\therefore LHD\neq RHD$, so $f(x)$ is not differentiable at $x=2$.	III	½

Q45. **2026 Supp** : 65/7/1, 65/7/2, 65/7/3 **[2 Marks]**

Question & answer as per CBSE Marking Scheme

22.	(a) Determine a and b if f(x) is a continuous function, where $f(x)=\begin{cases}2, & x\leq 3\\ ax+b, & 3<x<5\\ 5, & x\geq 5\end{cases}$		
	OR		
	(b) If $2y^3+y=2x$, then show $\dfrac{d^2y}{dx^2}=\dfrac{-48y}{(6y^2+1)^3}$.		
Sol (a)	For getting $3a+b=2$	I	½
	For getting $5a+b=5$	II	½
	Thus, $a=\dfrac{3}{2}$ and $b=\dfrac{-5}{2}$	III	½ + ½
	OR		
Sol (b)	As, $2y^3+y=2x \Rightarrow (6y^2+1)\frac{dy}{dx}=2$	I	½

		Thus, $\frac{dy}{dx} = \frac{2}{6y^2+1}$	II	½
		On again Differentiating w.r.t. x, we get		
		$\frac{d^2y}{dx^2} = \frac{-24y}{(6y^2+1)^2} \times \frac{dy}{dx}$	III	½
		$\frac{d^2y}{dx^2} = \frac{-24y}{(6y^2+1)^2} \times \frac{2}{6y^2+1} = \frac{-48y}{(6y^2+1)^3}$	IV	½

Short Answer Questions (3 marks)

Q46. **2024** : 65/3/1, 65/3/2, 65/3/3 **[3 Marks]**

Question & answer as per CBSE Marking Scheme

Q26(a)	If x cos (p + y) + cos p sin (p + y) = 0, prove that $\cos p \frac{dy}{dx} = -\cos^2(p+y)$, where p is a constant.	
Ans	$x\cos(p+y)+\cos p\sin(p+y)=0$	
	$\Rightarrow x=\frac{-\cos p\sin(p+y)}{\cos(p+y)} \Rightarrow x=-\cos p.\tan(p+y)$	1
	$\Rightarrow \frac{dx}{dy}=-\cos p.\sec^2(p+y)$	1
	$\Rightarrow \frac{dy}{dx}=\frac{-1}{\cos p.\sec^2(p+y)}$	½
	$\Rightarrow \cos p\frac{dy}{dx}=-\cos^2(p+y)$	½
	OR	
Q26(b)	Find the value of a and b so that function f defined as : $f(x) = \begin{cases} \frac{x-2}{\lvert x-2\rvert}+a, & \text{if } x<2 \\ a+b, & \text{if } x=2 \\ \frac{x-2}{\lvert x-2\rvert}+b, & \text{if } x>2 \end{cases}$ is a continuous function.	
Ans	$f(x)=\begin{cases} \frac{x-2}{-(x-2)}+a & ;x<2 \\ a+b & ;x=2 \\ \frac{x-2}{(x-2)}+b & ;x>2 \end{cases} \Rightarrow f(x)=\begin{cases} -1+a & ;x<2 \\ a+b & ;x=2 \\ 1+b & ;x>2 \end{cases}$	
	$\lim_{x\to 2^-} f(x)=-1+a, \lim_{x\to 2^+} f(x)=1+b$ and $f(2)=a+b$	1
	as f is continous at $x=2 \therefore -1+a=1+b=a+b$	1
	$\Rightarrow a=1, b=-1$	½+½

Q47. **2024** : 65/5/2 **[3 Marks]**

Question & answer as per CBSE Marking Scheme

27.	Find the values of a and b so that the following function is differentiable for all values of x :	

	$f(x)=\begin{cases} ax+b & , \quad x>-1 \\ bx^2-3 & , \quad x\le -1 \end{cases}$	
Ans	**Since Differentiability $\Rightarrow$ Continuity** $f(x)$ is continuous at $x=-1 \Rightarrow \lim\limits_{x\to -1^-}(bx^2-3)=\lim\limits_{x\to -1^+}(ax+b)$ $\Rightarrow b-3=-a+b,\ \therefore\ a=3$ $f(x)$ is differentiable at $x=-1$ $\Rightarrow \lim\limits_{x\to -1^-}\dfrac{(bx^2-3)-(b-3)}{x+1}=\lim\limits_{x\to -1^+}\dfrac{(ax+b)-(b-3)}{x+1}$ $\Rightarrow \lim\limits_{x\to -1^-} b(x-1)=\lim\limits_{x\to -1^+}\dfrac{3(x+1)}{x+1},\ \therefore\ b=-\dfrac{3}{2}$ **Alternately,** $\text{LHD}=\lim\limits_{h\to 0}\dfrac{f(-1)-f(-1-h)}{h}=\lim\limits_{h\to 0}\dfrac{(b-3)-\left(b(-1-h)^2-3\right)}{h}=\lim\limits_{h\to 0}(-bh-2b)=-2b$ $\text{RHD}=\lim\limits_{h\to 0}\dfrac{f(-1)-f(-1-h)}{h}=\lim\limits_{h\to 0}\dfrac{[a(-1+h)+b]-(b-3)}{h}=\lim\limits_{h\to 0}\dfrac{(-a+3)+ah}{h}=a$ and $-a+3=0\Rightarrow a=3,-2b=a\Rightarrow b=-\frac{3}{2}$	1 ½ 1 ½ 1 1 ½+½

Q48. **2025** : 65/4/1, 65/4/3 **[3 Marks]**

Question & answer as per CBSE Marking Scheme

Q27.	(a) Find k so that $f(x)=\begin{cases}\dfrac{x^2-2x-3}{x+1}, & x\neq -1\\ k, & x=-1\end{cases}$ is continuous at x = – 1. **OR** (b) Check the differentiability of function $f(x)=x\lvert x\rvert$ at x = 0.	
A27(a).	$\lim\limits_{x\to -1}\dfrac{x^2-2x-3}{x+1}=\lim\limits_{x\to -1}\dfrac{(x-3)(x+1)}{x+1}=\lim\limits_{x\to -1}(x-3)=-4$ Also, $f(-1)=k$ as f is continuous, $k=-4$	2 ½ ½
	OR	
A27(b).	$f(x)=x\lvert x\rvert=\begin{cases}-x^2 & ,x\le 0\\ x^2 & ,x>0\end{cases}$ $\text{LHD}=\lim\limits_{h\to 0}\dfrac{f(0-h)-f(0)}{-h}=\lim\limits_{h\to 0}\dfrac{-h^2-0}{-h}=0$ $\text{RHD}=\lim\limits_{h\to 0}\dfrac{f(0+h)-f(0)}{h}=\lim\limits_{h\to 0}\dfrac{h^2-0}{h}=0$ Since LHD = RHD, f is differentiable at $x=0$	1 1 ½ ½

Derivatives – Chain Rule, Implicit, Inverse Trig, Log & Parametric

Multiple Choice Questions (1 mark)

Q49. **2023** : 65/2/3 **[1 Mark]**

If $\tan(\frac{x+y}{x-y}) = k$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{-y}{x}$ (B) $\frac{y}{x}$

(C) $\sec^2(\frac{y}{x})$ (D) $-\sec^2(\frac{y}{x})$

Ans. (B) $\frac{y}{x}$ **1**

Q50. **2023** : 65/3/1, 65/3/2, 65/3/3 **[1 Mark]**

If $y = \frac{\cos x - \sin x}{\cos x + \sin x}$, then $\frac{dy}{dx}$ is :

(a) $-\sec^2(\frac{\pi}{4} - x)$ (b) $\sec^2(\frac{\pi}{4} - x)$

(c) $\log|\sec(\frac{\pi}{4} - x)|$ (d) $-\log|\sec(\frac{\pi}{4} - x)|$

Ans. (a) $-\sec^2(\frac{\pi}{4} - x)$ **1**

Q51. **2023** : 65/4/1, 65/4/3 **[1 Mark]**

If $y = \sin^2(x^3)$, then $\frac{dy}{dx}$ is equal to :

(a) $2 \sin x^3 \cos x^3$ (b) $3x^3 \sin x^3 \cos x^3$

(c) $6x^2 \sin x^3 \cos x^3$ (d) $2x^2 \sin^2(x^3)$

Ans. (c) $6x^2 \sin(x^3) \cos(x^3)$ **1**

Q52. **2023** : 65/4/2 **[1 Mark]**

If $y = \log(\sin e^x)$, then $\frac{dy}{dx}$ is :

(a) $\cot e^x$ (b) $\operatorname{cosec} e^x$

(c) $e^x \cot e^x$ (d) $e^x \operatorname{cosec} e^x$

Ans. (c) $e^x \cot e^x$ **1**

Q53. **2023** : 65/5/1, 65/5/2 | **2024** : 65/3/2 **[1 Mark]**

The derivative of x^{2x} w.r.t. x is

(a) x^{2x-1} (b) $2x^{2x} \log x$

(c) $2x^{2x}(1 + \log x)$ (d) $2x^{2x}(1 - \log x)$

Ans. (c) $2x^{2x}(1 + \log x)$ **1**

Q54. **2023 Supp** : 65/C/1 **[1 Mark]**

If $(\cos x)^y = (\cos y)^x$, then $\frac{dy}{dx}$ is equal to :

(a) $\frac{y \tan x + \log(\cos y)}{x \tan y - \log(\cos x)}$

(b) $\frac{x \tan y + \log(\cos x)}{y \tan x + \log(\cos y)}$

(c) $\frac{y \tan x - \log(\cos y)}{x \tan y - \log(\cos x)}$

(d) $\frac{y \tan x + \log(\cos y)}{x \tan y + \log(\cos x)}$

Ans. (d) $\frac{y \tan x + \log(\cos y)}{x \tan y + \log(\cos x)}$ **1**

Q55. **2023 Supp** : 65/C/2 | **2025 Supp** : 65/S/2 **[1 Mark]**

Derivative of $\tan^{-1}(\frac{x}{\sqrt{1-x^2}})$ with respect to $\sin^{-1}(2x\sqrt{1-x^2})$ is :

(a) $-\frac{1}{4}$

(b) $\frac{1}{2}$

(c) 2

(d) $-\frac{1}{2}$

Ans. (b) $\frac{1}{2}$ **1**

Q56. **2023 Supp** : 65/C/3 **[1 Mark]**

If $y = \log[\tan(\frac{\pi}{4} + \frac{x}{2})]$, then $\frac{dy}{dx}$ is :

(a) sec x

(b) cosec x

(c) tan x

(d) sec x tan x

Ans. (a) sec x **1**

Q57. **2024** : 65/1/1, 65/1/2 **[1 Mark]**

The derivative of $\sin(x^2)$ w.r.t. x, at $x = \sqrt{\pi}$ is :

(A) 1

(B) -1

(C) $-2\sqrt{\pi}$

(D) $2\sqrt{\pi}$

Ans. (C) $-2\sqrt{\pi}$ **1**

Q58. **2024** : 65/2/1 **[1 Mark]**

The derivative of $\tan^{-1}(x^2)$ w.r.t. x is :

(A) $\frac{x}{1+x^4}$

(B) $\frac{2x}{1+x^4}$

(C) $-\frac{2x}{1+x^4}$

(D) $\frac{1}{1+x^4}$

Ans. (B) $\frac{2x}{1+x^4}$ **1**

Q59. **2024** : 65/2/3 **[1 Mark]**

If $y = \cos^{-1}(e^x)$, then $\frac{dy}{dx}$ is :

(A) $\frac{1}{\sqrt{e^{-2x}-1}}$ (B) $-\frac{1}{\sqrt{e^{-2x}-1}}$

(C) $\frac{1}{\sqrt{e^{2x}-1}}$ (D) $-\frac{1}{\sqrt{e^{2x}-1}}$

Ans. (B) $-\frac{1}{\sqrt{e^{-2x}-1}}$ **1**

Q60. **2024** : 65/3/1 **[1 Mark]**

The derivative of 2^x w.r.t. 3^x is :

(A) $(\frac{3}{2})^x \frac{\log 2}{\log 3}$ (B) $(\frac{2}{3})^x \frac{\log 3}{\log 2}$

(C) $(\frac{2}{3})^x \frac{\log 2}{\log 3}$ (D) $(\frac{3}{2})^x \frac{\log 3}{\log 2}$

Ans. (C) $(\frac{2}{3})^x \frac{\log 2}{\log 3}$ **1**

Q61. **2024** : 65/4/1 **[1 Mark]**

Derivative of e^{2x} with respect to e^x, is :

(A) e^x (B) $2e^x$

(C) $2e^{2x}$ (D) $2e^{3x}$

Ans. (B) $2e^x$ **1**

Q62. **2024** : 65/4/3 | **2026** : 65/5/1, 65/5/3 **[1 Mark]**

Derivative of x^2 with respect to x^3, is :

(A) $\frac{2}{3x}$ (B) $\frac{3x}{2}$

(C) $\frac{2x}{3}$ (D) $6x^5$

Ans. (A) $\frac{2}{3x}$ **1**

Q63. **2024** : 65/5/1, 65/5/2, 65/5/3 **[1 Mark]**

Derivative of $e^{\sin^2 x}$ with respect to cos x is :

(A) sin x $e^{\sin^2 x}$ (B) cos x $e^{\sin^2 x}$

(C) -2 cos x $e^{\sin^2 x}$ (D) $-2 \sin^2 x$ cos x $e^{\sin^2 x}$

Ans. (C) -2 cos x $e^{\sin^2 x}$ **1**

Q64. **2024 Comptt** : 65/S/1 **[1 Mark]**

Let $y = f(\frac{1}{x})$ and $f'(x) = x^3$. What is the value of $\frac{dy}{dx}$ at $x = \frac{1}{2}$?

(A) $-\frac{1}{64}$ (B) $-\frac{1}{32}$

(C) -32 (D) -64

Ans. (C) -32 **1**

Q65. **2024 Comptt** : 65/S/1, 65/S/2 **[1 Mark]**

If $y = \log \sqrt{\sec\sqrt{x}}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi^2}{16}$ is :

(A) $\frac{1}{\pi}$ (B) π

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Ans. (A) $\frac{1}{\pi}$ **1**

Q66. **2026** : 65/5/2 **[1 Mark]**

Derivative of $\cos^{-1}(\frac{\sin x + \cos x}{\sqrt{2}})$, $-\frac{\pi}{4} < x < \frac{\pi}{4}$ with respect to x is :

(A) -1 (B) 1

(C) $\frac{\pi}{4}$ (D) $-\frac{\pi}{4}$

Ans. (A) -1 **1**

Second Order Derivatives

Multiple Choice Questions (1 mark)

Q67. **2023 Supp** : 65/C/1, 65/C/2, 65/C/3 **[1 Mark]**

If $x = a\cos\theta + b\sin\theta$, $y = a\sin\theta - b\cos\theta$, then which one of the following is true ?

(a) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$ (b) $y^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

(c) $y^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$ (d) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$

Ans. (a) $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$ **1**

Q68. **2024** : 65/1/3 **[1 Mark]**

If $x = at$, $y = \frac{a}{t}$, then $\frac{dy}{dx}$ is :

(A) t^2 (B) $-t^2$

(C) $\frac{1}{t^2}$ (D) $-\frac{1}{t^2}$

Ans. (D) $-\frac{1}{t^2}$ **1**

Q69. **2025** : 65/4/1 **[1 Mark]**

If $y = \sin^{-1}x$, then $(1 - x^2)\frac{d^2y}{dx^2}$ is equal to :

(A) $x\frac{dy}{dx}$ (B) $-x\frac{dy}{dx}$

(C) $x^2\frac{dy}{dx}$ (D) $-x^2\frac{dy}{dx}$

Ans. (A) $x\frac{dy}{dx}$ **1**

Differentiation – Short & Long Answer Questions

Very Short Answer Questions (2 marks)

Q70. **2023** : 65/3/1, 65/3/2, 65/3/3 **[2 Marks]**

Question & answer as per CBSE Marking Scheme

Q25(a)	If $y = x^{\frac{1}{x}}$, then find $\frac{dy}{dx}$ at $x = 1$.	
Ans	(a) $y = x^{1/x}$	
	$\Rightarrow \log y = \frac{1}{x}\log x$	½
	$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -\frac{\log x}{x^2} + \frac{1}{x^2} \Rightarrow \frac{dy}{dx} = x^{\frac{1}{x}}\frac{(1-\log x)}{x^2}$	1
	$\Rightarrow \left(\frac{dy}{dx}\right)_{x=1} = 1$	½
	OR	
Q25(b)	If $x = a\sin 2t, y = a(\cos 2t + \log\tan t)$, then find $\frac{dy}{dx}$.	
Ans	(b) $\frac{dx}{dt} = 2a\cos 2t$	½
	$\frac{dy}{dt} = 2a\left(-\sin 2t + \frac{\sec^2 t}{2\tan t}\right) = 2a\frac{\cos^2 2t}{\sin 2t}$	1
	$\frac{dy}{dx} = \cot 2t$	½

Q71. **2023** : 65/4/1, 65/4/2 **[2 Marks]**

Question & answer as per CBSE Marking Scheme

22.	If $(x^2 + y^2)^2 = xy$, then find $\frac{dy}{dx}$.	
Sol.	$(x^2 + y^2)^2 = xy$ gives	
	$2(x^2 + y^2)\left[2x + 2y\frac{dy}{dx}\right] = x\frac{dy}{dx} + y$	1
	$\Rightarrow [4y(x^2 + y^2) - x]\frac{dy}{dx} = y - 4x(x^2 + y^2)$	½
	$\Rightarrow \frac{dy}{dx} = \frac{y - 4x(x^2 + y^2)}{4y(x^2 + y^2) - x}$	½

Q72. **2023** : 65/4/3 **[2 Marks]**

Question & answer as per CBSE Marking Scheme

21.	If $xy = e^{x-y}$, then show that $\frac{dy}{dx} = \frac{y(x-1)}{x(y+1)}$.	
Sol.	Given $xy = e^{x-y}$, gives $x - y = \log x + \log y$	½
	$\Rightarrow 1 - \frac{dy}{dx} = \frac{1}{x} + \frac{1}{y}\frac{dy}{dx}$	1

	$\Rightarrow \left(\frac{1}{y}+1\right)\frac{dy}{dx}=1-\frac{1}{x}$ $\Rightarrow \frac{dy}{dx} = \frac{x-1}{x} \times \frac{y}{1+y}=\frac{y(x-1)}{x(1+y)}$	$\frac{1}{2}$

Q73. **2023** : 65/1/3 **[2 Marks]**

Question & answer as per CBSE Marking Scheme

25.	If $x=\sqrt{a^{\tan^{-1}t}}$, $y=\sqrt{a^{\cot^{-1}t}}$, then show that $x\frac{dy}{dx}+y=0$.	
Ans	$\frac{dx}{dt}=\frac{\sqrt{a^{\tan^{-1}t}}}{2}\frac{\log a}{1+t^2}$ $\frac{dy}{dt}=-\frac{\sqrt{a^{\cot^{-1}t}}}{2}\frac{\log a}{1+t^2}$ $\frac{dy}{dx}=-\frac{\sqrt{a^{\cot^{-1}t}}}{\sqrt{a^{\tan^{-1}t}}}=-\frac{y}{x}$ $\Rightarrow x\frac{dy}{dx}+y=0$	1 $\frac{1}{2}$ $\frac{1}{2}$

Q74. **2024** : 65/2/1, 65/2/3 **[2 Marks]**

Question & answer as per CBSE Marking Scheme

22.	(a) If $f(x) = \lvert \tan 2x \rvert$, then find the value of $f'(x)$ at $x=\frac{\pi}{3}$.	
Sol.	$f(x)=-\tan 2x,\ \frac{\pi}{4}<x<\frac{\pi}{2}$ $f'(x)=-2\sec^2 2x,\ \frac{\pi}{4}<x<\frac{\pi}{2}$ $f'(\frac{\pi}{3})=-2(-2)^2=-8$	1 $\frac{1}{2}$ $\frac{1}{2}$
	OR	
	(b) If $y=\operatorname{cosec}(\cot^{-1}x)$, then prove that $\sqrt{1+x^2}\frac{dy}{dx}-x=0$.	
Sol.	$y=\sqrt{1+\cot^2(\cot^{-1}x)}=\sqrt{1+x^2}$ $\Rightarrow \frac{dy}{dx}=\frac{x}{\sqrt{1+x^2}}$ $\Rightarrow \sqrt{1+x^2}\frac{dy}{dx}-x=0$	$\frac{1}{2}$ 1 $\frac{1}{2}$

Q75. **2024** : 65/4/1, 65/4/2, 65/4/3 **[2 Marks]**

Question & answer as per CBSE Marking Scheme

22(a).	If $y = \cos^3(\sec^2 2t)$, find $\frac{dy}{dt}$.	
Sol.	$y = \cos^3(\sec^2 2t)$ $\Rightarrow \frac{dy}{dt} = 3\cos^2(\sec^2 2t)[-\sin(\sec^2 2t)] \times \frac{d(\sec^2 2t)}{dt}$ $\Rightarrow \frac{dy}{dt} = -3\cos^2(\sec^2 2t).\sin(\sec^2 2t) \times 2\sec 2t.\sec 2t \tan 2t.2$ $\therefore \frac{dy}{dt} = -12\cos^2(\sec^2 2t) \times \sin(\sec^2 2t) \times \sec^2 2t \times \tan 2t.$	½ 1 ½
	OR	
22(b).	If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.	
Sol.	$As, x^y = e^{x-y} \Rightarrow \log(x^y) = \log(e^{x-y})$ $\Rightarrow y\log x = (x-y) \Rightarrow y = \frac{x}{1+\log x}$ *Now*, Differentiating both the sides wrt x $\frac{dy}{dx} = \frac{(\log x+1).1 - x(\frac{1}{x})}{(\log x+1)^2} = \frac{\log x}{(1+\log x)^2}$	1 1

Q76. **2024 Comptt** : 65/S/1, 65/S/2, 65/S/3 **[2 Marks]**

Question & answer as per CBSE Marking Scheme

22.	(a) If $y = (\sin^{-1} x)^2$, then find $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx}$. **OR** (b) If $y^x = x^y$, then find $\frac{dy}{dx}$.	
Ans	**(a)** $\frac{dy}{dx} = \frac{2\sin^{-1} x}{\sqrt{1-x^2}} \Rightarrow (1-x^2)\left(\frac{dy}{dx}\right)^2 = 4y$ **Differentiating with respect to 'x', we get** $(1-x^2)2\frac{dy}{dx}\frac{d^2y}{dx^2} - 2x\left(\frac{dy}{dx}\right)^2 = 4\frac{dy}{dx}$ $\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$ **Or**	1 ½ ½
	(b) $y^x = x^y \Rightarrow x\log y = y\log x$ **Differentiating with respect to 'x',** $\frac{x}{y}\frac{dy}{dx} + \log y = \frac{y}{x} + \frac{dy}{dx}\log x$	½ 1

	$\Rightarrow \frac{dy}{dx} = \frac{y}{x}\left(\frac{y - x\log y}{x - y\log x}\right)$	½

Q77. **2025** : 65/1/1, 65/1/2, 65/1/3, 65/7/2 **[2 Marks]**

Question & answer as per CBSE Marking Scheme

Q25.	(a) Differentiate $2^{\cos^2 x}$ w.r.t $\cos^2 x$. **OR** (b) If $\tan^{-1}(x^2 + y^2) = a^2$, then find $\frac{dy}{dx}$.	
Ans(a)	$\text{Let } u = 2^{\cos^2 x} \Rightarrow \frac{du}{dx} = 2^{\cos^2 x}(-2\cos x \sin x)\log 2$ $\text{Let } v = \cos^2 x \Rightarrow \frac{dv}{dx} = -2\cos x \sin x$ $\text{Now } \frac{du}{dv} = \frac{\left(\frac{du}{dx}\right)}{\left(\frac{dv}{dx}\right)} = 2^{\cos^2 x}\log 2$	1 ½ ½
	OR	
Ans(b)	$\tan^{-1}(x^2 + y^2) = a^2 \Rightarrow x^2 + y^2 = \tan a^2$ Differentiate both sides wrt x, $2x + 2y\frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = -\frac{x}{y}$	½ 1 ½

Q78. **2025** : 65/5/1, 65/5/2, 65/5/3 **[2 Marks]**

Question & answer as per CBSE Marking Scheme

23.	(a) Differentiate $\frac{\sin x}{\sqrt{\cos x}}$ with respect to x. **OR** (b) If $y = 5\cos x - 3\sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$.	
Ans	(a) Let $y = \frac{\sin x}{\sqrt{\cos x}}$ $\frac{dy}{dx} = \frac{\sqrt{\cos x}.\cos x - \sin x.\left(\frac{-\sin x}{2\sqrt{\cos x}}\right)}{\cos x}$ $\Rightarrow \frac{dy}{dx} = \frac{2\cos^2 x + \sin^2 x}{2(\cos x)^{3/2}}$ or $\frac{1 + \cos^2 x}{2(\cos x)^{3/2}}$ **OR** (b) $y = 5\cos x - 3\sin x$, then $\frac{dy}{dx} = -5.\sin x - 3.\cos x$ $\Rightarrow \frac{d^2y}{dx^2} = -5.\cos x + 3.\sin x = -y$ $\Rightarrow \frac{d^2y}{dx^2} + y = 0$	1½ ½ 1 ½ ½

Q79. **2025** : 65/6/1, 65/6/2, 65/6/3 **[2 Marks]**

Question & answer as per CBSE Marking Scheme

22.	(a) Differentiate $\sqrt{e^{\sqrt{2x}}}$ with respect to $e^{\sqrt{2x}}$ for $x > 0$. **OR** (b) If $(x)^y = (y)^x$, then find $\frac{dy}{dx}$.	
Ans	(a) Let $u = \sqrt{e^{\sqrt{2x}}}$ and $v = e^{\sqrt{2x}}$ Derivative of $\sqrt{v}$ w.r.t. $v = \frac{1}{2\sqrt{v}}$.	½ 1
	Required derivative $= \frac{1}{2\sqrt{e^{\sqrt{2x}}}}$. OR Taking log on both sides, we get $y \log x = x \log y$ Differentiating both sides w.r.t. x, we get $\frac{y}{x} + \log x \frac{dy}{dx} = \frac{x}{y}\frac{dy}{dx} + \log y$ $\Rightarrow \frac{dy}{dx} = \frac{y(x\log y - y)}{x(y\log x - x)}$	½ ½ 1 ½
	Determine the values of x for which $f(x) = \frac{x-4}{\ldots}$ is an increasing	

Q80. **2025** : 65/7/1, 65/7/3 **[2 Marks]**

Question & answer as per CBSE Marking Scheme

23.	(a) Differentiate $\left(\frac{5^x}{x^5}\right)$ with respect to x. **OR** (b) If $-2x^2 - 5xy + y^3 = 76$, then find $\frac{dy}{dx}$.	
Ans	(a) Let, $y = \frac{5^x}{x^5} = 5^x \cdot x^{-5} \Rightarrow \frac{dy}{dx} = \left(5^x\right)' \cdot x^{-5} + 5^x \cdot \left(x^{-5}\right)'$ $= \frac{5^x}{x^5}\log 5 - \frac{5^{x+1}}{x^6}$ **OR** (b) Differentiating $-2x^2 - 5xy + y^3 = 76$, with respect to 'x' $-4x - 5y - 5x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = \frac{4x+5y}{3y^2-5x}$	1 1 1½ ½
	Linear Programming Problem: objective function $Z = 5x + 4y$ and	

Q81. **2026** : 65/5/1, 65/5/2, 65/5/3 **[2 Marks]**

Question & answer as per CBSE Marking Scheme

22.	(a) Differentiate x^x with respect to $x \log x$. **OR**	

	(b) If $y = P\cos ux + Q\sin ux$, show that $\frac{d^2y}{dx^2} + u^2y = 0$.		
Sol.	(a) Let $u = x^x$ and $v = x\log x$	I	½
	$\frac{du}{dx} = x^x(1+\log x)$	II	½
	$\frac{dv}{dx} = 1 + \log x$	III	½
	$\therefore \frac{du}{dv} = x^x$	IV	½
	OR		
	(b) $\frac{dy}{dx} = -P\sin ux + Q u\cos ux$	I	½
	$\frac{d^2y}{dx^2} = -P u^2\cos ux - Q u^2\sin ux$	II	½
	$\frac{d^2y}{dx^2} = -u^2(P\cos ux + Q\sin ux)$	III	½
	$\therefore \frac{d^2y}{dx^2} + u^2y = 0$	IV	½

Q82. **2026** : 65/2/3 **[2 Marks]**

Question & answer as per CBSE Marking Scheme

23.	If $x = e^{\sin^{-1}t}$, $y = e^{\cos^{-1}t}$, find $\frac{dy}{dx}$ at $t = \frac{1}{\sqrt{2}}$		
Sol.	$x = e^{\sin^{-1}t} \Rightarrow \frac{dx}{dt} = e^{\sin^{-1}t} \times \frac{1}{\sqrt{1-t^2}}$ $y = e^{\cos^{-1}t} \Rightarrow \frac{dy}{dt} = e^{\cos^{-1}t} \times \frac{-1}{\sqrt{1-t^2}}$	I	1
	Now, $\frac{dy}{dx} = -\frac{e^{\cos^{-1}t}}{e^{\sin^{-1}t}}$ or $-e^{\cos^{-1}t - \sin^{-1}t}$	II	½
	$\left[\frac{dy}{dx}\right]_{t=\frac{1}{\sqrt{2}}} = -1$	III	½
	Note: Proportionate marks may be allotted if a student uses property $\sin^{-1}t + \cos^{-1}t = \frac{\pi}{2}$ to solve this question.		

Short Answer Questions (3 marks)

Q83. 2023 : 65/5/1, 65/5/3 [3 Marks]

Question & answer as per CBSE Marking Scheme

27(a).	Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}(2x\sqrt{1-x^2})$.	
Sol.	Let x = sin θ. Then $U = \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right) = \sec^{-1}\left(\frac{1}{\cos\theta}\right)$	
	$= \sec^{-1}(\sec\theta) = \theta = sin^{-1}x$ $\Rightarrow \frac{dU}{dx} = \frac{1}{\sqrt{1-x^2}}$ and $V = \sin^{-1}\{2\sin\theta\sqrt{1-\sin^2\theta}\}$ $= \sin^{-1}[2\sin\theta\cos\theta] = 2\theta = 2\,sin^{-1}x$ $\Rightarrow \frac{dV}{dx} = \frac{2}{\sqrt{1-x^2}}$ $\Rightarrow \frac{dU}{dV} = \frac{dU/dx}{dV/dx} = \frac{1}{2}$ **Note: If the substitution is made as $x = cos\,\theta$, answer will be $-\frac{1}{2}$**	**1** **1** **1**
27(b).	If $y = \tan x + \sec x$, then prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$.	
Sol.	$y = \tan x + \sec x = \frac{\sin x + 1}{\cos x}$ $\Rightarrow \frac{dy}{dx} = \frac{cos\ x(\cos x) + (\sin x + 1)\sin x}{cos^2\ x}$ $= \frac{\cos^2 x + \sin^2 x + \sin x}{\cos^2 x} = \frac{1+\sin x}{1-\sin^2 x} = \frac{1}{1-\sin x}$ $\Rightarrow \frac{d^2y}{dx^2} = \frac{(1-\sin x).0 - 1(0-\cos x)}{(1-\sin x)^2} = \frac{\cos x}{(1-\sin x)^2}$	 **$1\frac{1}{2}$** **$1\frac{1}{2}$**

Q84. 2024 : 65/1/1, 65/1/2, 65/1/3 [3 Marks]

Question & answer as per CBSE Marking Scheme

27(a).	If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.	
Sol.	$\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ **Put $x = \sin\theta, y = \sin\phi$** $\Rightarrow \cos\theta + \cos\phi = a(\sin\theta - \sin\phi)$ $\Rightarrow 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right) = 2a\sin\left(\frac{\theta-\phi}{2}\right)\cos\left(\frac{\theta+\phi}{2}\right)$	 **$\frac{1}{2}$** **1**

	$\Rightarrow \cot\left(\frac{\theta-\phi}{2}\right) = a$ $\Rightarrow \theta - \phi = 2\cot^{-1} a$ $\Rightarrow \sin^{-1} x - \sin^{-1} y = 2\cot^{-1} a$ $\Rightarrow \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$	$\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$
	OR	
27(b).	If $y = (\tan x)^x$, then find $\frac{dy}{dx}$.	
Sol.	$y = (\tan x)^x$ $\log y = x \log(\tan x)$ $\frac{1}{y}\frac{dy}{dx} = x\left(\frac{sec^2 x}{\tan x}\right) + \log(\tan x)$ $\frac{dy}{dx} = (\tan x)^x\left[\left(\frac{x\sec^2 x}{\tan x}\right) + \log(\tan x)\right]$	$\frac{1}{2}$ 2 $\frac{1}{2}$

Q85. 2024 : 65/2/3 [3 Marks]

Question & answer as per CBSE Marking Scheme

28.	(a) If $y = (\log x)^2$, prove that $x^2y'' + xy' = 2$.	
Sol.	Differentiating both sides w.r.t. x, we get $y' = \frac{2\log x}{x}$ $\Rightarrow xy' = 2\log x$ $\Rightarrow xy'' + y' = \frac{2}{x}$ $\Rightarrow x^2y'' + xy' = 2$	1 $1\frac{1}{2}$ $\frac{1}{2}$

	OR	
	(b) If $y = \sin(\tan^{-1} e^x)$, then find $\frac{dy}{dx}$ at $x = 0$.	
Sol.	$\frac{dy}{dx} = \cos(tan^{-1}(e^x)) \times \frac{e^x}{1+e^{2x}}$ $\left(\frac{dy}{dx}\right)_{x=0} = cos\frac{\pi}{4} \times \frac{1}{2} = \frac{1}{2\sqrt{2}}$	2 1

Q86. 2024 : 65/2/1 [3 Marks]

Question & answer as per CBSE Marking Scheme

26.	(a) If $x = e^{\cos 3t}$ and $y = e^{\sin 3t}$, prove that $\frac{dy}{dx} = -\frac{y\log x}{x\log y}$.	
Sol	$\frac{dx}{dt} = e^{cos3t} \times (-sin3t) \times 3$ $\frac{dy}{dt} = e^{sin3t} \times (cos3t) \times 3$	1 1

	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{e^{sin3t} \times (cos3t)}{-e^{cos3t} \times (sin3t)}$ $x = e^{cos3t} \Rightarrow cos3t = logx$ $y = e^{sin3t} \Rightarrow sin3t = logy$ $\therefore \frac{dy}{dx} = \frac{-ylogx}{xlogy}$	1

	OR	
	(b) Show that : $\frac{d}{dx}(\lvert x \rvert) = \frac{x}{\lvert x \rvert}, x \neq 0$	
Sol.	$\frac{d(\lvert x \rvert)}{dx} = \frac{d(\sqrt{x^2})}{dx}, x \neq 0$ $= \frac{1}{2}(x^2)^{-\frac{1}{2}} \times \frac{d(x^2)}{dx}$ $= \frac{1}{2\sqrt{x^2}} 2x = \frac{x}{\lvert x \rvert}$	1 $1\frac{1}{2}$ $\frac{1}{2}$

Q87. **2024** : 65/2/2 **[3 Marks]**

Question & answer as per CBSE Marking Scheme

30.	(a) If $x^{30} y^{20} = (x + y)^{50}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.	
Sol.	(a)Taking log of both sides, we get $30 \log x + 20 \log y = 50 \log (x + y)$ Differentiating both sides w.r.t. x, we get $\frac{30}{x} + \frac{20}{y}\frac{dy}{dx} = \frac{50}{x+y}(1 + \frac{dy}{dx})$ $\Rightarrow \frac{dy}{dx}\left(\frac{20x - 30y}{y(x+y)}\right) = \frac{20x - 30y}{x(x+y)}$ $\Rightarrow \frac{dy}{dx} = \frac{y}{x}$	1 1 1
	OR	
	(b) Find $\frac{dy}{dx}$, if $5^x + 5^y = 5^{x+y}$.	
Sol.	Differentiating both sides w.r.t. x, we get $5^x log5 + 5^y log5 \frac{dy}{dx} = 5^{x+y} log5 (1 + \frac{dy}{dx})$ $\Rightarrow 5^x + 5^y \frac{dy}{dx} = (5^x + 5^y)(1 + \frac{dy}{dx})$ $\Rightarrow 5^x + 5^y \frac{dy}{dx} = 5^x + 5^x \frac{dy}{dx} + 5^y + 5^y \frac{dy}{dx}$	2

	$\Rightarrow \frac{dy}{dx} = -\frac{5^y}{5^x} = -5^{y-x}$	1

Q88. 2024 : 65/4/2 [3 Marks]

Question & answer as per CBSE Marking Scheme

26.	Given that $x^y + y^x = a^b$, where a and b are positive constants, find $\frac{dy}{dx}$.	
Sol.	Let $u = x^y \Rightarrow \frac{du}{dx} = x^y(\frac{y}{x} + \log x.\frac{dy}{dx})$, $v = y^x \Rightarrow \frac{dv}{dx} = y^x(\frac{x}{y}\frac{dy}{dx} + \log y)$	1 + 1
	Since, $u + v = a^b \Rightarrow \frac{du}{dx} + \frac{dv}{dx} = 0$	
	$\therefore \frac{dy}{dx} = \frac{-\left(x^{y-1}y + y^x \log y\right)}{\left(x^y \log x + y^{x-1}x\right)}$	1

Q89. 2024 : 65/4/3 [3 Marks]

Question & answer as per CBSE Marking Scheme

26.	Find $\frac{dy}{dx}$, if $y = (\cos x)^x + \cos^{-1}\sqrt{x}$ is given.	
Sol.	Let $u = (\cos x)^x \Rightarrow \frac{du}{dx} = (\cos x)^x(-x\tan x + \log(\cos x))$,	1
	$v = \cos^{-1}\sqrt{x} \Rightarrow \frac{dv}{dx} = \frac{-1}{2\sqrt{x - x^2}}$	1
	Since, $y = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} = (\cos x)^x(-x\tan x + \log(\cos x)) + \frac{-1}{2\sqrt{x - x^2}}$	1

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Q90. 2024 Comptt : 65/S/2 [3 Marks]

Question & answer as per CBSE Marking Scheme

27.	(a) If $y = \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right)$, $0 < x < 1$, then find $\frac{dy}{dx}$.	
	OR	
	(b) If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{\{\log(xe)\}^2}$.	
Ans	(a) $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, Put $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$	1
	$\Rightarrow y = \cos^{-1}(\cos 2\theta) = 2\theta = 2\tan^{-1}x$, $\therefore \frac{dy}{dx} = \frac{2}{1+x^2}$	2
	Or	
	(b) $x^y = e^{x-y} \Rightarrow y\log x = x - y \Rightarrow y = \frac{x}{\log x + 1}$.	1
	$\Rightarrow \frac{dy}{dx} = \frac{\log x}{(\log x + 1)^2} = \frac{\log x}{\{\log(xe)\}^2}$	2

Q91. **2024 Comptt** : 65/S/1 **[3 Marks]**

Question & answer as per CBSE Marking Scheme

27.	(a) If $x \sin(a+y) - \sin y = 0$, prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ **OR** (b) Find $\frac{dy}{dx}$, if $y = (\cos x)^x + \cos^{-1}\sqrt{x}$.	
Ans	(a) $x = \frac{\sin y}{\sin(a+y)} \Rightarrow \frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y\cos(a+y)}{\sin^2(a+y)}$ $\Rightarrow \frac{dx}{dy} = \frac{\sin a}{\sin^2(a+y)}, \therefore \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ **Or** (b) Let $y = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ $u = (\cos x)^x \Rightarrow \log u = x\log\cos x$ $\Rightarrow \frac{du}{dx} = (\cos x)^x[\log\cos x - x\tan x]$ $v = \cos^{-1}\sqrt{x} \Rightarrow \frac{dv}{dx} = -\frac{1}{2\sqrt{x}\sqrt{1-x}}$ $\therefore \frac{dy}{dx} = (\cos x)^x[\log\cos x - x\tan x] - \frac{1}{2\sqrt{x}\sqrt{1-x}}$	2 1 ½ 1 1 ½

Q92. **2024 Comptt** : 65/S/3 **[3 Marks]**

Question & answer as per CBSE Marking Scheme

27.	(a) If $y = e^{a\cos^{-1}x}$, then show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$. **OR** (b) Find $\frac{dy}{dx}$, if $y = x^{\cos x} - 2^{\sin x}$.	
Ans	(a) $\frac{dy}{dx} = e^{a\cos^{-1}x}\frac{-a}{\sqrt{1-x^2}} = -\frac{ay}{\sqrt{1-x^2}}$, squaring we get $\left(\frac{dy}{dx}\right)^2(1-x^2) = a^2y^2$, differentiating again with respect to 'x'. $(1-x^2)2\frac{dy}{dx}\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2(-2x) = 2a^2y\frac{dy}{dx} \Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$ **Or** (b) Let $y = u - v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$, where $u = x^{\cos x}, v = 2^{\sin x}$	1½ 1½

	$\log u = \cos x \log x \Rightarrow \frac{du}{dx} = x^{\cos x}\left[\frac{\cos x}{x} - \sin x \log x\right], \frac{dv}{dx} = 2^{\sin x}\cos x \log 2$ $\frac{dy}{dx} = x^{\cos x}\left[\frac{\cos x}{x} - \sin x \log x\right] - 2^{\sin x}\cos x \log 2$	$1\frac{1}{2}+1$ $\frac{1}{2}$

Q93. **2025** : 65/6/1, 65/6/2, 65/6/3 **[3 Marks]**

Question & answer as per CBSE Marking Scheme

27.	**27.** (a) Differentiate $y = \sin^{-1}(3x - 4x^3)$ w.r.t. x, if $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$. **OR** (b) Differentiate $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to x, when $x \in (0, 1)$.	
Ans	(a) $x = \sin t$ gives $y = \sin^{-1}(\sin 3t) = 3t = 3\sin^{-1}x$ $\frac{dy}{dx} = \frac{3}{\sqrt{1-x^2}}$ Aliter: $\frac{dy}{dx} = \frac{3-12x^2}{\sqrt{1-(3x-4x^3)^2}}$ OR (b) $x = \tan t$ gives $y = \cos^{-1}(\cos 2t) = 2t = 2\tan^{-1}x$ $\frac{dy}{dx} = \frac{2}{1+x^2}$ Aliter: $\frac{dy}{dx} = \frac{-1}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}} \cdot \frac{-4x}{(1+x^2)^2}$.	$\frac{1}{2}+1+\frac{1}{2}$ 1 3 $\frac{1}{2}+1+\frac{1}{2}$ 1 3

Q94. **2025** : 65/7/1 **[3 Marks]**

Question & answer as per CBSE Marking Scheme

28.	Differentiate $y = \sqrt{\log\left\{\sin\left(\frac{x^3}{3} - 1\right)\right\}}$ with respect to x.	
Ans	$\frac{dy}{dx} = \frac{1}{2\sqrt{\log\left\{\sin\left(\frac{x^3}{3}-1\right)\right\}}} \cdot \frac{1}{\sin\left(\frac{x^3}{3}-1\right)} \cdot \cos\left(\frac{x^3}{3}-1\right) \cdot \frac{3x^2}{3}$ $= \frac{x^2 \cot\left(\frac{x^3}{3}-1\right)}{2\sqrt{\log\left\{\sin\left(\frac{x^3}{3}-1\right)\right\}}}$	$1+1+\frac{1}{2}$ $\frac{1}{2}$

Q95. **2025** : 65/7/2 **[3 Marks]**

Question & answer as per CBSE Marking Scheme

28.	Show that the derivative of $\tan^{-1}(\sec x + \tan x), \left[-\frac{\pi}{2} < x < \frac{\pi}{2}\right]$ with respect to x is equal to $\frac{1}{2}$.	
Ans	$\tan^{-1}(\sec x+\tan x)=\tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)=\tan^{-1}\left(\frac{1+\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)}\right)$	½
	$=\tan^{-1}\left(\frac{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right)$	1
	$=\tan^{-1}\left(\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)$	
	$=\tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)=\frac{\pi}{4}+\frac{x}{2}$	1
	$\therefore \left(\tan^{-1}(\sec x+\tan x)\right)'=\frac{1}{2}$	½

Q96. **2026** : 65/3/2 **[3 Marks]**

Question & answer as per CBSE Marking Scheme

28.	If $(\sin x)^y = y^{\cos x}$, then find $\frac{dy}{dx}$.		
Sol. 28	$(\sin x)^y = y^{\cos x} \Rightarrow y\log\sin x = \cos x \log y$	**I**	1
	On diff. wrt x both sides, we get		
	$\frac{y\cos x}{\sin x} + \log\sin x\frac{dy}{dx} = \frac{\cos x}{y}\frac{dy}{dx} - \log y.\sin x$	**II**	$1\frac{1}{2}$
	$\Rightarrow \frac{dy}{dx} = \frac{y(y\cos x + \log y.\sin^2 x)}{\sin x(\cos x - y\log\sin x)}$ or $\frac{(y\cot x + \sin x.\log y)y}{\cos x - y\log\sin x}$	**III**	½

Q97. **2026** : 65/4/1, 65/4/2, 65/4/3 **[3 Marks]**

Question & answer as per CBSE Marking Scheme

28. (a)	If $xy = e^{x-y}$, then find $\frac{dy}{dx}$.		
Sol.	$xy = e^{x-y} \Rightarrow x\frac{dy}{dx} + y = e^{x-y}\left(1 - \frac{dy}{dx}\right)$	I	1½

	$\Rightarrow \left(x+e^{x-y}\right)\frac{dy}{dx}=e^{x-y}-y \Rightarrow \frac{dy}{dx}=\frac{e^{x-y}-y}{x+e^{x-y}}$ **or** $\frac{y(x-1)}{x(y+1)}$	II	1½
	Alternative method:		
	$xy=e^{x-y} \Rightarrow \log x+\log y=(x-y)$	I	1
	$\Rightarrow \frac{1}{x}+\frac{1}{y}\frac{dy}{dx}=1-\frac{dy}{dx}$	II	1
	$\Rightarrow \frac{dy}{dx}=\frac{1-\frac{1}{x}}{\frac{1}{y}+1}=\frac{y(x-1)}{x(y+1)}$	III	1
	OR		
28. (b)	Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ with respect to $\cos^{-1}x^2$.		
Sol.	Let $y=\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ & put $x^2=\cos\theta$		
	$y=\tan^{-1}\left(\frac{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}\right)$	I	½
	$y=\tan^{-1}\left(\frac{\cos\left(\frac{\theta}{2}\right)-\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)+\sin\left(\frac{\theta}{2}\right)}\right)=\tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\right)$	II	1½
	Thus, $y=\left(\frac{\pi}{4}-\frac{\theta}{2}\right) \Rightarrow \frac{dy}{d\theta}=\frac{-1}{2}$	III	1

Q98. **2026 Supp** : 65/7/1 **[3 Marks]**

Differentiate $y = e^{\cos^{-1}\sqrt{(1-x^2)}}$ with respect to x and hence show that $dy/dx = y/\sqrt{(1-x^2)}$.

Question & answer as per CBSE Marking Scheme

27.	Differentiate $y-e^{\cos^{-1}\sqrt{1-x^2}}$ with respect to x and hence show that $\frac{dy}{dx}-\frac{y}{\sqrt{1-x^2}}$.		
Sol.	$y=e^{\cos^{-1}\sqrt{1-x^2}}$		
	$\frac{dy}{dx}=e^{\cos^{-1}\sqrt{1-x^2}}\times\frac{-1}{\sqrt{1-\left(\sqrt{1-x^2}\right)^2}}\times\frac{1}{2\sqrt{1-x^2}}\times(-2x)$	I	1½
	$\Rightarrow \frac{dy}{dx}=e^{\cos^{-1}\sqrt{1-x^2}}\times\frac{1}{\sqrt{1-x^2}}$	II	1
	$\Rightarrow \frac{dy}{dx}=\frac{y}{\sqrt{1-x^2}}$	III	½

Long Answer Questions (5 marks)

Q99. **2025** : 65/1/2 **[5 Marks]**

Question & answer as per CBSE Marking Scheme

Q32. (a) Differentiate $\tan^{-1}\dfrac{\sqrt{1-x^2}}{x}$ w.r.t. $\cos^{-1}(2x\sqrt{1-x^2})$, $x \in \left(\dfrac{1}{\sqrt{2}}, 1\right)$

OR

(b) Find $\dfrac{dy}{dx}$, if $y = x^{\tan x} + \dfrac{\sqrt{x^2+1}}{2}$.

Ans (a)

Put $x = \cos\theta \Rightarrow \theta = \cos^{-1} x$ — ½

Let $u = \tan^{-1}\dfrac{\sqrt{1-x^2}}{x} = \tan^{-1}\left(\dfrac{\sin\theta}{\cos\theta}\right) = \tan^{-1}(\tan\theta) = \theta = \cos^{-1} x$ — 1½

$\Rightarrow \dfrac{du}{dx} = -\dfrac{1}{\sqrt{1-x^2}}$ — ½

Let $v = \cos^{-1}\left(2x\sqrt{1-x^2}\right) = \cos^{-1}(\sin 2\theta) = \cos^{-1}\left(\cos\left(\dfrac{\pi}{2} - 2\theta\right)\right) = \dfrac{\pi}{2} - 2\cos^{-1} x$ — 1½

$\Rightarrow \dfrac{dv}{dx} = \dfrac{2}{\sqrt{1-x^2}}$ — ½

$\therefore \dfrac{du}{dv} = \dfrac{du/dx}{dv/dx} = -\dfrac{1}{2}$ — ½

OR

Ans (b)

Let $y = u + v \Rightarrow \dfrac{dy}{dx} = \dfrac{du}{dx} + \dfrac{dv}{dx}$, where $u = x^{\tan x}, v = \dfrac{\sqrt{x^2+1}}{2}$ — 1

$u = x^{\tan x} \Rightarrow \log u = \tan x \log x$, differentiating with respect to 'x', we get

$\Rightarrow \dfrac{1}{u}\dfrac{du}{dx} = \dfrac{\tan x}{x} + \sec^2 x \log x$ — 1½

$\Rightarrow \dfrac{du}{dx} = u\left(\dfrac{\tan x}{x} + \sec^2 x \log x\right) = x^{\tan x}\left(\dfrac{\tan x}{x} + \sec^2 x \log x\right)$ — ½

$v = \dfrac{\sqrt{x^2+1}}{2} \Rightarrow \dfrac{dv}{dx} = \dfrac{2x}{4\sqrt{x^2+1}} = \dfrac{x}{2\sqrt{x^2+1}}$ — 1½

$\Rightarrow \dfrac{dy}{dx} = x^{\tan x}\left(\dfrac{\tan x}{x} + \sec^2 x \log x\right) + \dfrac{x}{2\sqrt{x^2+1}}$ — ½

Q100. **2025** : 65/5/1, 65/5/2, 65/5/3 **[5 Marks]**

Question & answer as per CBSE Marking Scheme

34.

(a) For a positive constant 'a', differentiate $a^{t+\frac{1}{t}}$ with respect to $\left(t+\frac{1}{t}\right)^a$, where t is a non-zero real number.

OR

(b) Find $\frac{dy}{dx}$ if $y^x + x^y + x^x = a^b$, where a and b are constants.

Ans

(a) Let $u = a^{t+\frac{1}{t}} \Rightarrow \frac{du}{dt} = a^{t+\frac{1}{t}}.\log a.\left(1-\frac{1}{t^2}\right)$ — 2

$v = \left(t+\frac{1}{t}\right)^a \Rightarrow \frac{dv}{dt} = a\left(t+\frac{1}{t}\right)^{a-1}.\left(1-\frac{1}{t^2}\right)$ — 2

$\frac{du}{dv} = \frac{du/dt}{dv/dt} = \frac{a^{t+\frac{1}{t}}.\log a}{a\left(t+\frac{1}{t}\right)^{a-1}}$ — 1

OR

(b) Let $u = y^x$, $v = x^y$ and $w = x^x$

$\Rightarrow \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} = 0$ .....(i) — 1

$u = y^x \Rightarrow \log u = x.\log y \Rightarrow \frac{1}{u}.\frac{du}{dx} = \frac{x}{y}.\frac{dy}{dx} + \log y$

$\Rightarrow \frac{du}{dx} = y^x\left(\frac{x}{y}.\frac{dy}{dx} + \log y\right) = xy^{x-1}\frac{dy}{dx} + y^x \log y$ — 1

$v = x^y \Rightarrow \log v = y.\log x \Rightarrow \frac{1}{v}.\frac{dv}{dx} = \frac{y}{x} + \log x.\frac{dy}{dx}$

$\Rightarrow \frac{dv}{dx} = x^y\left(\frac{y}{x} + \log x.\frac{dy}{dx}\right) = yx^{y-1} + x^y \log x \frac{dy}{dx}$ — 1

$w = x^x \Rightarrow \log w = x.\log x \Rightarrow \frac{1}{w}.\frac{dw}{dx} = 1 + \log x$

$\Rightarrow \frac{dw}{dx} = x^x.(1+\log x)$ — 1

∴ From (i), we get

$xy^{x-1}.\frac{dy}{dx} + y^x.\log y + yx^{y-1} + x^y.\log x.\frac{dy}{dx} + x^x.(1+\log x) = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{x^x.(1+\log x) + y^x.\log y + yx^{y-1}}{x.y^{x-1} + x^y.\log x}$ — 1

Q101. **2026** : 65/1/2 **[5 Marks]**

Question & answer as per CBSE Marking Scheme

32. If $x = 3\sin t - \sin 3t$, $y = 3\cos t - \cos 3t$

find $\frac{dy}{dx}$ and prove that $\frac{d^2y}{dx^2} = \frac{-\text{cosec}^3 2t \cdot \text{cosec } t}{3}$

Sol.

$\frac{dx}{dt} = 3cost - 3cos3t,$ — I — 1

$\frac{dy}{dt} = -3sint + 3sin3t$ — II — 1

$dy \quad 3(sin3t - sint)$ — III — ½

	$\frac{dy}{dx} = \frac{}{3(cost - cos3t)}$ $= \frac{2cos2t\ sint}{2sin2t\ sint}$ $= cot2t$ $\frac{d^2y}{dx^2} = -2cosec^2 2t \times \frac{dt}{dx}$ $= -\frac{2cosec^2 2t}{6sin2t\ sint}$	IV V	½ 1
	$= -\frac{cosec^3 2t\ cosect}{3}$	VI	1

Q102. **2026** : 65/5/1, 65/5/2, 65/5/3 **[5 Marks]**

Question & answer as per CBSE Marking Scheme

33.	(a) If $y\sqrt{x^2+1} = \log\ \sqrt{x^2+1} - x$, show that $(x^2 + 1)\frac{dy}{dx} + xy + 1 = 0.$ **OR** (b) Find the differential of $x^{\cot x} + \frac{2x^2-3}{2x^2-x+2}$ with respect to x.		
Sol.	(a) Differentiating both sides with respect to x, we get $\sqrt{x^2+1}\frac{dy}{dx} + y.\frac{x}{\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}}.\frac{x}{\sqrt{x^2+1}} - 1$ $(x^2+1)\frac{dy}{dx} + y.x = \frac{x}{\sqrt{x^2+1}} - \sqrt{x^2+1}$ Note: In case student, considers $\sqrt{x^2+1} - x$ as part of log and writes following solution Differentiating both sides with respect to x, we get $\sqrt{x^2+1}\ \frac{dy}{dx} + y.\frac{x}{\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}-x}.\left(\frac{x}{\sqrt{x^2+1}} - 1\right)$ $\sqrt{x^2+1}\ \frac{dy}{dx} + y.\frac{x}{\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}-x}.\left(\frac{x-\sqrt{x^2+1}}{\sqrt{x^2+1}}\right)$ $\sqrt{x^2+1}\frac{dy}{dx} + y.\frac{x}{\sqrt{x^2+1}} = \frac{-1}{\sqrt{x^2+1}}$ gives $(x^2+1)\frac{dy}{dx} + y.x = -1$ or $(x^2+1)\frac{dy}{dx} + x.y + 1 = 0$ **OR** (b) Let $u = x^{\cot x}$ and $v = \frac{2x^2-3}{2x^2-x+2}$ $\log u = \cot x.\log x$ $\frac{1}{u}\frac{du}{dx} = \frac{\cot x}{x} + \log x(-\text{cosec}^2 x)$	I II I II III IV I II III	2 + 2 1 2 1 1 1 ½ ½ 1

	$\frac{du}{dx} = x^{\cot x}\left(\frac{\cot x}{x} - \operatorname{cosec}^2 x.\log x\right)$	IV	½
	$\frac{dv}{dx} = \frac{(2x^2-x+2)(4x)-(2x^2-3)(4x-1)}{(2x^2-x+2)^2}$	V	1
	$= \frac{-2x^2+20x-3}{(2x^2-x+2)^2}$	VI	1
	Required derivative $= x^{\cot x}\left(\frac{\cot x}{x} - \operatorname{cosec}^2 x.\log x\right) + \frac{-2x^2+20x-3}{(2x^2-x+2)^2}$	VII	½

Case Study Based Questions

Q103. **2023** : 65/4/3 **[4 Marks]**

Question & answer as per CBSE Marking Scheme

37. Let f(x) be a real valued function. Then its

- Left Hand Derivative (L.H.D.) : $Lf'(a) = \lim_{h\to 0} \frac{f(a-h)-f(a)}{-h}$
- Right Hand Derivative (R.H.D.) : $Rf'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

Also, a function f(x) is said to be differentiable at x = a if its L.H.D. and R.H.D. at x = a exist and both are equal.

For the function $f(x) = \begin{cases} |x-3|, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$

answer the following questions :

(i) What is R.H.D. of f(x) at x = 1 ? *1*

(ii) What is L.H.D. of f(x) at x = 1 ? *1*

(iii) (a) Check if the function f(x) is differentiable at x = 1. *2*

OR

(iii) (b) Find the f'(2) and f'(– 1). *2*

Sol. (i) R.H.D. of f(x) at $x = 1 = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$

$= \lim_{h\to 0} \frac{|1+h-3|-|1-3|}{h} = \lim_{h\to 0} \frac{|h-2|-2}{h} = -1$ 1

(ii) L.H.D. of f(x) at $x = 1 = \lim_{h\to 0} \frac{f(1-h)-f(1)}{-h}$

$= \lim_{h\to 0} \frac{-1}{h}\left[\frac{(1-h)^2}{4} - \frac{3(1-h)}{2} + \frac{13}{4} - 2\right]$

$= \lim_{h\to 0} \left[\frac{h^2-2h+1-6+6h+13-8}{-4h}\right]$

$= \lim_{h\to 0} \left[\frac{h^2+4h}{-4h}\right] = -1.$ 1

(iii) (a) Since L.H.D. of f(x) at x = 1

= R.H.D. of f(x) at x = 1,

f(x) is differentiable at x = 1. 2

OR

(iii) (b) $f(x) = \begin{cases} x-3, & x \geq 3 \\ 3-x, & 1 \leq x < 3 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$

	$[f'(x)]_{x=2} = 0 - 1 = -1$ $[f'(x)]_{x=-1} = \frac{2(-1)}{4} - \frac{3}{2} = -2$	1 1

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