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Board Booster – Perfect 100

041 Mathematics • CBSE Class X

For CBSE Board 2026-27

By Deepika Bhati – Maths Passionate Since 2009

Master your Mother Book – NCERT before attempting this assignment.

Chapter 1 – Real Numbers

1. Three sets of English, Hindi and Mathematics books have to be stacked in such a way that all the books are stored topic-wise and the height of each stack is the same. The number of English books is 96, Hindi books is 240 and Mathematics books is 336. Assuming the books are of the same thickness, determine the number of stacks of English, Hindi and Mathematics books.

Solution : For equal heights with the fewest stacks, the number of books in each stack must be the HCF of 96, 240 and 336.

Prime factorisation:

$96 = 2^5 \times 3$

$240 = 2^4 \times 3 \times 5$

$336 = 2^4 \times 3 \times 7$

HCF = product of the smallest powers of common prime factors:

$\text{HCF} = 2^4 \times 3 = 48$

Number of stacks of each subject:

English = $96 \div 48 = 2$

Hindi = $240 \div 48 = 5$

Mathematics = $336 \div 48 = 7$

$\therefore$ Stacks: English 2, Hindi 5, Mathematics 7 (48 books in each stack).

2. The traffic lights at three different road crossings change after every 48 s, 72 s and 108 s respectively. If they change simultaneously at 7 a.m., at what time will they change together next?

Solution : The lights change together again after a time that is a multiple of 48, 72 and 108 — the least such time is their LCM.

$48 = 2^4 \times 3$

$72 = 2^3 \times 3^2$

$108 = 2^2 \times 3^3$

LCM = product of the greatest powers of all prime factors:

$LCM = 2^4 \times 3^3 = 16 \times 27 = 432$ s

Convert to minutes:

432 s = 7 min 12 s

∴ They change together next at 7:07:12 a.m.

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3. In a school there are two sections, A and B, of Class X. There are 30 students in Section A and 28 students in Section B. Find the minimum number of books required for their class library so that they can be distributed equally among the students of Section A or Section B.

Solution : The number of books must be divisible by both 30 and 28; the minimum such number is LCM(30, 28).

$30 = 2 \times 3 \times 5$

$28 = 2^2 \times 7$

$LCM = 2^2 \times 3 \times 5 \times 7 = 420$

∴ Minimum number of books = 420.

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4. Find the greatest number that will divide 445, 572 and 699 leaving remainders 4, 5 and 6 respectively.

Solution : Subtract the remainders; the required number divides the results exactly.

$445 - 4 = 441$, $572 - 5 = 567$, $699 - 6 = 693$

Required number = HCF(441, 567, 693).

$441 = 3^2 \times 7^2$

$567 = 3^4 \times 7$

$693 = 3^2 \times 7 \times 11$

$HCF = 3^2 \times 7 = 63$

∴ The greatest number is 63.

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5. There are 156, 208 and 260 students in groups A, B and C respectively. Buses are hired for an educational trip. Find the minimum number of buses to be hired if all buses carry the same number of students.

Solution : For the fewest buses, each bus should carry the largest possible equal number of students = HCF(156, 208, 260).

$156 = 2^2 \times 3 \times 13$

$208 = 2^4 \times 13$

$260 = 2^2 \times 5 \times 13$

$HCF = 2^2 \times 13 = 52$

Number of buses for each group:

A: 156 ÷ 52 = 3, B: 208 ÷ 52 = 4, C: 260 ÷ 52 = 5

Total = 3 + 4 + 5 = 12

∴ Minimum number of buses = 12.

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6. Prove that $\frac{2+\sqrt{3}}{5}$ is an irrational number.

Solution : Assume, on the contrary, that $\frac{2+\sqrt{3}}{5}$ is rational.

Then there exist co-prime integers a, b (b ≠ 0) such that

$$\frac{2+\sqrt{3}}{5} = \frac{a}{b}$$

$$2+\sqrt{3} = \frac{5a}{b}$$

$$\sqrt{3} = \frac{5a}{b} - 2 = \frac{5a-2b}{b}$$

Since a, b are integers, $\frac{5a-2b}{b}$ is rational, so $\sqrt{3}$ would be rational.

This contradicts the fact that $\sqrt{3}$ is irrational.

∴ Our assumption is wrong; $\frac{2+\sqrt{3}}{5}$ is irrational.

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7. A hall has a length of 9.75 m, breadth of 6.75 m and height of 5.25 m. What is the length of the longest unmarked ruler that can exactly measure all three dimensions of the hall?

Solution : Convert to centimetres: 975 cm, 675 cm, 525 cm.

The longest ruler measuring each exactly = HCF(975, 675, 525).

$975 = 3 \times 5^2 \times 13$

$675 = 3^3 \times 5^2$

$525 = 3 \times 5^2 \times 7$

$HCF = 3 \times 5^2 = 75$

∴ Length of the longest ruler = 75 cm.

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8. Find the smallest number which is divisible by both 644 and 462.

Solution : Required number = LCM(644, 462).

$644 = 2^2 \times 7 \times 23$

$462 = 2 \times 3 \times 7 \times 11$

$LCM = 2^2 \times 3 \times 7 \times 11 \times 23 = 21252$

∴ The smallest number is 21252.

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9. Find the least number which, when divided by 12, 16 and 24, leaves remainder 7 in each case.

Solution : First find the least number exactly divisible by 12, 16 and 24.

$12 = 2^2 \times 3$, $16 = 2^4$, $24 = 2^3 \times 3$

$\text{LCM} = 2^4 \times 3 = 48$

Add the common remainder:

$48 + 7 = 55$

$\therefore$ The required number is 55.

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10. If the HCF of 210 and 55 is expressible in the form $210 \times 5 - 55x$, find the value of x.

Solution : Find HCF(210, 55) by Euclid's division algorithm:

$210 = 55 \times 3 + 45$

$55 = 45 \times 1 + 10$

$45 = 10 \times 4 + 5$

$10 = 5 \times 2 + 0 \Rightarrow \text{HCF} = 5$

Now use the given form:

$210 \times 5 - 55x = 5$

$1050 - 55x = 5$

$55x = 1045 \Rightarrow x = 19$

$\therefore x = 19$.

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11. Two tanks contain 504 litres and 735 litres of milk respectively. Find the maximum capacity of a container which can measure the milk of either tank an exact number of times.

Solution : Required capacity = HCF(504, 735).

$504 = 2^3 \times 3^2 \times 7$

$735 = 3 \times 5 \times 7^2$

$\text{HCF} = 3 \times 7 = 21$

$\therefore$ Maximum capacity = 21 litres.

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12. On a morning walk, three persons step off together and their steps measure 40 cm, 42 cm and 45 cm respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps?

Solution : The distance must be a multiple of each step length; the minimum is LCM(40, 42, 45).

$40 = 2^3 \times 5$

$42 = 2 \times 3 \times 7$

$45 = 3^2 \times 5$

$\text{LCM} = 2^3 \times 3^2 \times 5 \times 7 = 2520$

$\therefore$ Minimum distance = 2520 cm = 25.2 m.

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13. Show that 14^n cannot end with the digit 0 or 5 for any natural number n.

Solution : A number ending in 0 or 5 is divisible by 5, so its prime factorisation must contain 5.

Prime factorisation of 14^n:

$14^n = (2 \times 7)^n = 2^n \times 7^n$

The only primes are 2 and 7. By the Fundamental Theorem of Arithmetic this factorisation is unique, so 5 can never be a factor.

$\therefore$ 14^n cannot end with the digit 0 or 5 for any natural number n.

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14. Prove that $\sqrt{p} + \sqrt{q}$ is irrational, where p and q are primes.

Solution : Assume, on the contrary, that $\sqrt{p} + \sqrt{q} = r$, where r is rational ($r \neq 0$).

$\sqrt{q} = r - \sqrt{p}$

Square both sides:

$q = r^2 - 2r\sqrt{p} + p$

$2r\sqrt{p} = r^2 + p - q$

$$\sqrt{p} = \frac{r^2 + p - q}{2r}$$

The right-hand side is rational, so $\sqrt{p}$ would be rational.

But $\sqrt{p}$ is irrational for every prime p — a contradiction.

$\therefore$ $\sqrt{p} + \sqrt{q}$ is irrational.

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15. In a teachers' workshop, the numbers of teachers teaching French, Hindi and English are 48, 80 and 144 respectively. Find the minimum number of rooms required if in each room the same number of teachers are seated and all of them are of the same subject.

Solution : For the fewest rooms, each room should seat the largest equal number = HCF(48, 80, 144).

$48 = 2^4 \times 3$, $80 = 2^4 \times 5$, $144 = 2^4 \times 3^2$

HCF = $2^4 = 16$

Rooms needed:

French $48 \div 16 = 3$, Hindi $80 \div 16 = 5$, English $144 \div 16 = 9$

Total = $3 + 5 + 9 = 17$

$\therefore$ Minimum number of rooms = 17.

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16. Show that $2 \times 5 \times 7 \times 11 + 11 \times 13$ is a composite number.

Solution : Take 11 common:

$2 \times 5 \times 7 \times 11 + 11 \times 13 = 11 \times (2 \times 5 \times 7 + 13)$

$= 11 \times (70 + 13) = 11 \times 83$

The number has factors 11 and 83 besides 1 and itself.

$\therefore$ It is a composite number.

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17. Can two numbers have 15 as their HCF and 175 as their LCM? Give reasons.

Solution : The HCF of two numbers always divides their LCM exactly.

Check:

$175 \div 15 = 11.67$ (not an integer)

$\therefore$ No, two numbers cannot have HCF 15 and LCM 175.

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18. A, B and C start cycling around a circular path in the same direction and at the same time. The circumference of the path is 1980 m. If the speeds of A, B and C are 330 m/min, 198 m/min and 220 m/min respectively and they start from the same point, after how much time will they meet again at the starting point?

Solution : Time taken by each for one round = circumference ÷ speed:

A: $1980 \div 330 = 6$ min

B: $1980 \div 198 = 10$ min

C: $1980 \div 220 = 9$ min

They meet at the start after LCM(6, 10, 9):

$6 = 2 \times 3$, $10 = 2 \times 5$, $9 = 3^2$

LCM $= 2 \times 3^2 \times 5 = 90$

$\therefore$ They meet again at the starting point after 90 minutes.

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19. Prove that $(\sqrt{2} + \sqrt{3})^2$ is an irrational number, given that $\sqrt{6}$ is irrational.

Solution : Expand using $(a + b)^2 = a^2 + 2ab + b^2$:

$(\sqrt{2} + \sqrt{3})^2 = 2 + 2\sqrt{6} + 3 = 5 + 2\sqrt{6}$

Assume $5 + 2\sqrt{6} = r$, a rational number.

$$\sqrt{6} = \frac{r - 5}{2}$$

The right side is rational, so $\sqrt{6}$ would be rational — a contradiction.

$\therefore (\sqrt{2} + \sqrt{3})^2$ is irrational.

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20. The HCF of 2472, 1284 and a third number N is 12. If their LCM is $2^3 \times 3^2 \times 5 \times 103 \times 107$, find the number N.

Solution : Prime factorise the known numbers:

$2472 = 2^3 \times 3 \times 103$

$1284 = 2^2 \times 3 \times 107$

HCF $= 12 = 2^2 \times 3$, so N must contain 2^2 and 3.

The LCM contains 3^2 and 5, which do not come from 2472 or 1284, so they must come from N.

The least such N is

$N = 2^2 \times 3^2 \times 5 = 180$

$\therefore N = 180$.

Chapter 2 – Polynomials

1. If α and β are zeroes of $x^2 - p(x + 1) - c$, show that $(\alpha + 1)(\beta + 1) = 1 - c$. Also show that

$$\frac{\alpha^2 + 2\alpha + 1}{\alpha^2 + 2\alpha + c} + \frac{\beta^2 + 2\beta + 1}{\beta^2 + 2\beta + c} = 1.$$

Solution : Write the polynomial in standard form:

$x^2 - px - (p + c)$

Relationship between zeroes and coefficients:

$\alpha + \beta = p$, $\alpha\beta = -(p + c)$

Part (i):

$(\alpha + 1)(\beta + 1) = \alpha\beta + (\alpha + \beta) + 1$

$= -p - c + p + 1 = 1 - c$

Part (ii): from part (i), $c - 1 = -(\alpha + 1)(\beta + 1)$. Hence

$\alpha^2 + 2\alpha + c = (\alpha + 1)^2 + (c - 1)$

$= (\alpha + 1)^2 - (\alpha + 1)(\beta + 1) = (\alpha + 1)(\alpha - \beta)$

Similarly $\beta^2 + 2\beta + c = (\beta + 1)(\beta - \alpha)$.

The numerators are $(\alpha + 1)^2$ and $(\beta + 1)^2$, so

$$\text{LHS} = \frac{\alpha + 1}{\alpha - \beta} + \frac{\beta + 1}{\beta - \alpha}$$

$$= \frac{(\alpha + 1) - (\beta + 1)}{\alpha - \beta} = \frac{\alpha - \beta}{\alpha - \beta} = 1$$

$\therefore (\alpha + 1)(\beta + 1) = 1 - c$ and the given sum equals 1.

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2. If α and β are the zeroes of the quadratic polynomial $f(x) = 3x^2 - 5x - 2$, find the value of $\alpha^3 + \beta^3$.

Solution : Here $a = 3$, $b = -5$, $c = -2$.

$$\alpha + \beta = -\frac{b}{a} = \frac{5}{3}$$

$$\alpha\beta = \frac{c}{a} = -\frac{2}{3}$$

Use the identity $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$:

$$\alpha^3 + \beta^3 = \left(\frac{5}{3}\right)^3 - 3\left(-\frac{2}{3}\right)\left(\frac{5}{3}\right)$$

$$= \frac{125}{27} + \frac{10}{3}$$

$$= \frac{125}{27} + \frac{90}{27} = \frac{215}{27}$$

$\therefore \alpha^3 + \beta^3 = \frac{215}{27}$.

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3. Given that the sum of the zeroes of the polynomial $(a + 1)x^2 + (2a + 3)x + (3a + 4)$ is -1, find the product of its zeroes.

Solution : Sum of zeroes $= -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$:

$$-\frac{2a+3}{a+1} = -1$$

$2a + 3 = a + 1 \Rightarrow a = -2$

Product of zeroes $= \frac{\text{constant}}{\text{coefficient of } x^2}$:

$$\frac{3a+4}{a+1} = \frac{-6+4}{-2+1} = -\frac{2}{-1} = 2$$

$\therefore$ Product of zeroes = 2.

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4. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and -3, find the values of a and b.

Solution : Sum of zeroes:

$2 + (-3) = -(a + 1)$

$-1 = -a - 1 \Rightarrow a = 0$

Product of zeroes:

$2 \times (-3) = b \Rightarrow b = -6$

$\therefore a = 0, b = -6$.

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5. If α and β are the zeroes of $f(x) = 3x^2 - 4x + 1$, find a quadratic polynomial whose zeroes are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$.

Solution :

$$\alpha + \beta = \frac{4}{3}, \alpha\beta = \frac{1}{3}$$

Sum of new zeroes:

$$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta}$$

$$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = \frac{64}{27} - 3(\frac{1}{3})(\frac{4}{3}) = \frac{64}{27} - \frac{36}{27} = \frac{28}{27}$$

$$\text{Sum} = (\frac{28}{27}) \div (\frac{1}{3}) = \frac{28}{9}$$

Product of new zeroes:

$$(\frac{\alpha^2}{\beta})(\frac{\beta^2}{\alpha}) = \alpha\beta = \frac{1}{3}$$

Required polynomial = $k[x^2 - (\text{sum})x + \text{product}]$:

$$k(x^2 - (\frac{28}{9})x + \frac{1}{3})$$

$\therefore$ Taking $k = 9$: $9x^2 - 28x + 3$.

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6. Find the zeroes of $p(x) = 3x^2 - 4x - 4$. Hence write a polynomial whose each zero is 2 more than the zeroes of p(x).

Solution : Factorise by splitting the middle term (product = −12, sum = −4: −6 and +2):

$3x^2 - 6x + 2x - 4 = 3x(x - 2) + 2(x - 2) = (x - 2)(3x + 2)$

Zeroes: $x = 2$ and $x = -\frac{2}{3}$

New zeroes (each increased by 2):

$2 + 2 = 4$ and $-\frac{2}{3} + 2 = \frac{4}{3}$

Sum $= 4 + \frac{4}{3} = \frac{16}{3}$, Product $= 4 \times \frac{4}{3} = \frac{16}{3}$

Polynomial: $k(x^2 - (\frac{16}{3})x + \frac{16}{3})$; take $k = 3$.

$\therefore$ Required polynomial: $3x^2 - 16x + 16$.

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7. Find a quadratic polynomial whose zeroes are $\frac{1}{2} + 2\sqrt{3}$ and $\frac{1}{2} - 2\sqrt{3}$.

Solution : Sum of zeroes:

$$(\frac{1}{2} + 2\sqrt{3}) + (\frac{1}{2} - 2\sqrt{3}) = 1$$

Product of zeroes (using $(a + b)(a - b) = a^2 - b^2$):

$$(\frac{1}{2})^2 - (2\sqrt{3})^2 = \frac{1}{4} - 12 = -\frac{47}{4}$$

Polynomial: $x^2 - (\text{sum})x + \text{product}$

$$x^2 - x - \frac{47}{4}$$

$\therefore$ Multiplying by 4: $4x^2 - 4x - 47$.

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8. If α and β are the zeroes of $f(x) = 2x^2 + 5x + k$ satisfying $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$, find the value of k.

Solution :

$$\alpha + \beta = -\frac{5}{2},\ \alpha\beta = \frac{k}{2}$$

Rewrite the given relation:

$$\alpha^2 + \beta^2 + \alpha\beta = (\alpha + \beta)^2 - 2\alpha\beta + \alpha\beta = (\alpha + \beta)^2 - \alpha\beta$$

$$\frac{25}{4} - \frac{k}{2} = \frac{21}{4}$$

$$\frac{k}{2} = \frac{4}{4} = 1$$

$\therefore k = 2.$

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9. If α and β are the zeroes of $25p^2 - 15p + 2$, find a quadratic polynomial whose zeroes are $\frac{1}{2\alpha}$ and $\frac{1}{2\beta}$.

Solution :

$$\alpha + \beta = \frac{15}{25} = \frac{3}{5},\ \alpha\beta = \frac{2}{25}$$

Sum of new zeroes:

$$\frac{1}{2\alpha} + \frac{1}{2\beta} = \frac{\alpha + \beta}{2\alpha\beta} = (\frac{3}{5}) \div (\frac{4}{25}) = \frac{15}{4}$$

Product of new zeroes:

$$\frac{1}{4\alpha\beta} = 1 \div (\frac{8}{25}) = \frac{25}{8}$$

Polynomial: $k(x^2 - (\frac{15}{4})x + \frac{25}{8})$; take $k = 8$.

$\therefore$ Required polynomial: $8x^2 - 30x + 25$.

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10. Find the value of p for which one zero of the quadratic polynomial $px^2 - 14x + 8$ is 6 times the other.

Solution : Let the zeroes be α and 6α.

Sum of zeroes:

$$\alpha + 6\alpha = \frac{14}{p} \Rightarrow \alpha = \frac{2}{p}$$

Product of zeroes:

$$\alpha \times 6\alpha = \frac{8}{p} \Rightarrow 6\alpha^2 = \frac{8}{p}$$

Substitute $\alpha = \frac{2}{p}$:

$$6 \times \frac{4}{p^2} = \frac{8}{p}$$

$$\frac{24}{p} = 8 \Rightarrow p = 3$$

$\therefore p = 3.$

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11. If the sum of the squares of the zeroes of $f(x) = x^2 - 8x + k$ is 40, find the value of k.

Solution :

$\alpha + \beta = 8, \alpha\beta = k$

Use $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$:

$64 - 2k = 40$

$2k = 24$

$\therefore k = 12.$

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12. If α and β are the zeroes of $2x^2 - 8x + 5$, find the value of $(\alpha + \frac{1}{\beta})(\beta + \frac{1}{\alpha})$.

Solution :

$$\alpha + \beta = 4, \alpha\beta = \frac{5}{2}$$

Expand the product:

$$(\alpha + \frac{1}{\beta})(\beta + \frac{1}{\alpha}) = \alpha\beta + 1 + 1 + \frac{1}{\alpha\beta}$$

$$= \frac{5}{2} + 2 + \frac{2}{5}$$

$$= \frac{25}{10} + \frac{20}{10} + \frac{4}{10} = \frac{49}{10}$$

$\therefore$ Value $= \frac{49}{10}.$

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13. If the product of the zeroes of $ax^2 - 6x - 6$ is 4, find the value of a.

Solution : Product of zeroes = constant term ÷ coefficient of x^2:

$$-\frac{6}{a} = 4$$

$\therefore a = -\frac{3}{2}$.

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14. Find the value of k such that the polynomial $x^2 - (k + 6)x + 2(2k - 1)$ has the sum of its zeroes equal to half of their product.

Solution :

Sum of zeroes = $k + 6$

Product of zeroes = $2(2k - 1)$

Given: sum = ½ × product

$k + 6 = \frac{1}{2} \times 2(2k - 1) = 2k - 1$

$k = 7$

$\therefore k = 7$.

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15. If one zero of the polynomial $5z^2 + 13z - p$ is the reciprocal of the other, find p.

Solution : Let the zeroes be α and $\frac{1}{\alpha}$.

Product of zeroes = constant term ÷ coefficient of z^2:

$$\alpha \times \frac{1}{\alpha} = -\frac{p}{5}$$

$$1 = -\frac{p}{5}$$

$\therefore p = -5$.

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16. If one zero of the quadratic polynomial $f(x) = 4x^2 - 8kx + 8x - 9$ is the negative of the other, find the zeroes of $kx^2 + 3kx + 2$.

Solution : Write $f(x) = 4x^2 + (8 - 8k)x - 9$. Let the zeroes be α and $-\alpha$.

Sum of zeroes = 0:

$$-\frac{8 - 8k}{4} = 0 \Rightarrow 8 - 8k = 0 \Rightarrow k = 1$$

Now $kx^2 + 3kx + 2 = x^2 + 3x + 2$:

$x^2 + 3x + 2 = (x + 1)(x + 2)$

$\therefore k = 1$ and the zeroes of $kx^2 + 3kx + 2$ are -1 and -2.

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17. If $(x + a)$ is a factor of both the polynomials $x^2 + px + q$ and $x^2 + mx + n$, prove that $a = \frac{n - q}{m - p}$.

Solution : If $(x + a)$ is a factor, then $x = -a$ is a zero of each polynomial (Factor Theorem).

$(-a)^2 + p(-a) + q = 0 \Rightarrow a^2 - pa + q = 0$ …(1)

$(-a)^2 + m(-a) + n = 0 \Rightarrow a^2 - ma + n = 0$ …(2)

Subtract (2) from (1):

$-pa + ma + q - n = 0$

$a(m - p) = n - q$

$\therefore a = \dfrac{n - q}{m - p}$.

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18. If one zero of the quadratic polynomial $(k^2 + k)x^2 + 68x + 6k$ is the reciprocal of the other, find k.

Solution : Let the zeroes be α and $\dfrac{1}{\alpha}$; their product is 1.

$\dfrac{6k}{k^2 + k} = 1$

$6k = k^2 + k \Rightarrow k^2 - 5k = 0 \Rightarrow k(k - 5) = 0$

$k = 0$ makes the coefficient of x^2 zero (not a quadratic), so it is rejected.

$\therefore k = 5$.

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19. If α and β are the zeroes of the polynomial $x^2 - 5x + m$ such that $\alpha - \beta = 1$, find m.

Solution :

$\alpha + \beta = 5, \alpha\beta = m$

Use the identity $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$:

$1^2 = 5^2 - 4m$

$1 = 25 - 4m \Rightarrow 4m = 24$

$\therefore m = 6$.

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20. Find the zeroes of the polynomial $2s^2 + (1 + 2\sqrt{2})s + \sqrt{2}$ by factorisation and verify the relationship between the zeroes and the coefficients.

Solution : Split the middle term (product of extremes $= 2\sqrt{2} = 1 \times 2\sqrt{2}$):

$2s^2 + s + 2\sqrt{2}\ s + \sqrt{2} = s(2s + 1) + \sqrt{2}(2s + 1)$

$= (2s + 1)(s + \sqrt{2})$

Zeroes:

$s = -\dfrac{1}{2}$ and $s = -\sqrt{2}$

Verification:

$\text{Sum} = -\dfrac{1}{2} - \sqrt{2} = -\dfrac{1 + 2\sqrt{2}}{2} = -\dfrac{\text{coefficient of } s}{\text{coefficient of } s^2}$ ✓

Product $= (-\frac{1}{2})(-\sqrt{2}) = \frac{\sqrt{2}}{2} = \frac{\text{constant term}}{\text{coefficient of } s^2}$ ✓

$\therefore$ Zeroes are $-\frac{1}{2}$ and $-\sqrt{2}$; the relationships are verified.

Chapter 3 – Pair of Linear Equations in Two Variables

1. For what value of p will the system $-x + py = 1$ and $px - y = 1$ represent parallel lines?

Solution : Write in the form $a_1x + b_1y + c_1 = 0$:

$-x + py - 1 = 0$ and $px - y - 1 = 0$

For parallel lines: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$-\frac{1}{p} = \frac{p}{-1} \Rightarrow p^2 = 1 \Rightarrow p = \pm 1$

Check the condition with $\frac{c_1}{c_2} = \frac{-1}{-1} = 1$:

$p = 1$: $\frac{a_1}{a_2} = -1 \neq 1$ ✓ (parallel)

$p = -1$: $\frac{a_1}{a_2} = 1 = \frac{c_1}{c_2}$ ✗ (coincident lines)

$\therefore p = 1$.

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2. The area of a rectangle gets reduced by 9 square units if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Solution : Let length = l and breadth = b. Area = lb.

First condition:

$(l - 5)(b + 3) = lb - 9$

$lb + 3l - 5b - 15 = lb - 9 \Rightarrow 3l - 5b = 6$ …(1)

Second condition:

$(l + 3)(b + 2) = lb + 67$

$lb + 2l + 3b + 6 = lb + 67 \Rightarrow 2l + 3b = 61$ …(2)

Solve: (1) × 3 + (2) × 5

$9l - 15b + 10l + 15b = 18 + 305$

$19l = 323 \Rightarrow l = 17$

From (2): $34 + 3b = 61 \Rightarrow b = 9$

$\therefore$ Length = 17 units, breadth = 9 units.

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3. Two people are 16 km apart on a straight road. They start walking at the same time. If they walk towards each other with different speeds, they meet in 2 hours. Had they walked in the same direction with the same speeds as before, they would have met in 8 hours. Find their walking speeds.

Solution : Let the speeds be u km/h (faster) and v km/h.

Towards each other, relative speed = u + v:

$2(u + v) = 16 \Rightarrow u + v = 8$ …(1)

Same direction, relative speed = u − v:

$8(u - v) = 16 \Rightarrow u - v = 2$ …(2)

Adding (1) and (2):

$2u = 10 \Rightarrow u = 5, v = 3$

∴ Speeds are 5 km/h and 3 km/h.

---

4. A two-digit number is four times the sum of its digits. It is also equal to 3 times the product of its digits. Find the number.

Solution : Let the tens digit be a and the units digit b. Number = 10a + b.

First condition:

$10a + b = 4(a + b) \Rightarrow 6a = 3b \Rightarrow b = 2a$ …(1)

Second condition:

$10a + b = 3ab$

Substitute b = 2a:

$12a = 6a^2 \Rightarrow a = 2 \ (a \neq 0)$

$b = 4$

∴ The number is 24.

---

5. The age of the father is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.

Solution : Let father's age = x years and sum of children's ages = y years.

$x = 2y$ …(1)

After 20 years, each child is 20 years older, so the sum increases by 40:

$x + 20 = y + 40$ …(2)

Substitute $y = \frac{x}{2}$ in (2):

$x + 20 = \frac{x}{2} + 40$

$\frac{x}{2} = 20 \Rightarrow x = 40$

∴ Father's age = 40 years.

---

6. There are some students in two examination halls A and B. To make the number of students equal in each hall, 10 students are sent from A to B. But if 20 students are sent from B to A, the number of students in A becomes double the number in B. Find the number of students in both halls.

Solution : Let hall A have x students and hall B have y students.

First condition:

$x - 10 = y + 10 \Rightarrow x - y = 20$ …(1)

Second condition:

$x + 20 = 2(y - 20) \Rightarrow x - 2y = -60$ …(2)

Subtract (2) from (1):

$y = 80$

$x = 100$

∴ Hall A: 100 students, Hall B: 80 students.

---

7. Solve for x and y: $(a - b)x + (a + b)y = a^2 - 2ab - b^2$ and $(a + b)(x + y) = a^2 + b^2$.

Solution : Write the equations clearly:

$(a - b)x + (a + b)y = a^2 - 2ab - b^2$ …(1)

$(a + b)x + (a + b)y = a^2 + b^2$ …(2)

Subtract (1) from (2):

$[(a + b) - (a - b)]x = (a^2 + b^2) - (a^2 - 2ab - b^2)$

$2bx = 2ab + 2b^2 = 2b(a + b)$

$x = a + b$

Put $x = a + b$ in (2):

$(a + b)(a + b) + (a + b)y = a^2 + b^2$

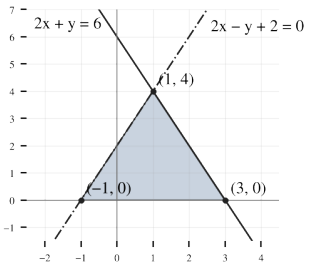
$(a + b)y = a^2 + b^2 - (a^2 + 2ab + b^2) = -2ab$

$$y = -\frac{2ab}{a + b}$$

∴ $x = a + b$, $y = -\frac{2ab}{a + b}$.

---

8. Draw the graphs of $2x + y = 6$ and $2x - y + 2 = 0$. Shade the region bounded by these lines and the x-axis and find the area of the shaded region.



Solution : Table of values:

$2x + y = 6$: (0, 6), (3, 0), (1, 4)

$2x - y + 2 = 0$: (0, 2), (−1, 0), (1, 4)

Plot the points and draw both lines (see figure).

Intersection of the lines: add the equations $2x + y = 6$ and $2x - y = -2$

$4x = 4 \Rightarrow x = 1, y = 4 \Rightarrow$ point $(1, 4)$

The shaded triangle has vertices $(-1, 0)$, $(3, 0)$ and $(1, 4)$.

Base on x-axis $= 3 - (-1) = 4$ units, height $= 4$ units

Area $= \frac{1}{2} \times 4 \times 4 = 8$

$\therefore$ Area of the shaded region $= 8$ square units.

---

9. In a ΔABC, $\angle C = 3\angle B = 2(\angle A + \angle B)$. Find the three angles.

Solution : From the given relations:

$\angle C = 3\angle B$ …(1)

$3\angle B = 2(\angle A + \angle B) \Rightarrow \angle B = 2\angle A$ …(2)

Angle sum property:

$\angle A + \angle B + \angle C = 180°$

$\angle A + 2\angle A + 6\angle A = 180° \Rightarrow 9\angle A = 180° \Rightarrow \angle A = 20°$

$\angle B = 40°$, $\angle C = 120°$

$\therefore \angle A = 20°$, $\angle B = 40°$, $\angle C = 120°$.

---

10. In a cyclic quadrilateral ABCD, $\angle A = (2x + 4)°$, $\angle B = (y + 3)°$, $\angle C = (2y + 10)°$ and $\angle D = (4x - 5)°$. Find the four angles.

Solution : Opposite angles of a cyclic quadrilateral are supplementary.

$\angle A + \angle C = 180°$:

$2x + 4 + 2y + 10 = 180 \Rightarrow x + y = 83$ …(1)

$\angle B + \angle D = 180°$:

$y + 3 + 4x - 5 = 180 \Rightarrow 4x + y = 182$ …(2)

Subtract (1) from (2):

$3x = 99 \Rightarrow x = 33, y = 50$

Angles:

$\angle A = 70°$, $\angle B = 53°$, $\angle C = 110°$, $\angle D = 127°$

$\therefore \angle A = 70°$, $\angle B = 53°$, $\angle C = 110°$, $\angle D = 127°$.

---

11. The ratio of the incomes of two persons A and B is 3 : 4 and the ratio of their expenditures is 5 : 7. If each saves ₹ 15,000 annually, find their annual incomes.

Solution : Let incomes be ₹ 3x and ₹ 4x, and expenditures ₹ 5y and ₹ 7y.

Saving = income − expenditure:

$3x - 5y = 15000$ …(1)

$4x - 7y = 15000$ …(2)

$(1) \times 7 - (2) \times 5$:

$21x - 20x = 105000 - 75000 \Rightarrow x = 30000$

Incomes:

$A = 3 \times 30000 = 90000$, $B = 4 \times 30000 = 120000$

$\therefore$ Annual incomes: A = ₹ 90,000 and B = ₹ 1,20,000.

---

12. For what values of a and b does the pair $2x + 3y = 7$ and $a(x + y) - b(x - y) = 3a + b - 2$ have infinitely many solutions?

Solution : Rewrite the second equation:

$(a - b)x + (a + b)y = 3a + b - 2$

For infinitely many solutions: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\frac{2}{a-b} = \frac{3}{a+b} = \frac{7}{3a+b-2}$$

From the first two ratios:

$2(a + b) = 3(a - b) \Rightarrow a = 5b$ …(1)

From the last two ratios:

$3(3a + b - 2) = 7(a + b) \Rightarrow 2a - 4b = 6 \Rightarrow a - 2b = 3$ …(2)

Put $a = 5b$ in (2):

$3b = 3 \Rightarrow b = 1, a = 5$

$\therefore a = 5, b = 1.$

---

13. Find the value of k for which the pair $(3k + 1)x + 3y - 2 = 0$ and $(k^2 + 1)x + (k - 2)y - 5 = 0$ has no solution.

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Solution : For no solution: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\frac{3k+1}{k^2+1} = \frac{3}{k-2}$$

$(3k + 1)(k - 2) = 3(k^2 + 1)$

$3k^2 - 5k - 2 = 3k^2 + 3$

$-5k = 5 \Rightarrow k = -1$

Check $\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ for $k = -1$:

$\frac{b_1}{b_2} = \frac{3}{-3} = -1$, $\frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$ ✓ (not equal)

$\therefore k = -1.$

---

14. Solve: $152x - 378y = -74$ and $-378x + 152y = -604$.

Solution : The coefficients are interchanged, so add and subtract the equations.

Adding:

$-226x - 226y = -678 \Rightarrow x + y = 3$ …(1)

Subtracting (second from first):

$530x - 530y = 530 \Rightarrow x - y = 1$ …(2)

Adding (1) and (2):

$2x = 4 \Rightarrow x = 2, y = 1$

$\therefore x = 2, y = 1.$

---

15. Pinky scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks deducted for each wrong answer, she would again have scored 40 marks. How many questions were there in the test?

Solution : Let the number of right answers = x and wrong answers = y.

$3x - y = 40$ …(1)

$4x - 2y = 40 \Rightarrow 2x - y = 20$ …(2)

Subtract (2) from (1):

$x = 20$

$y = 3(20) - 40 = 20$

Total questions = x + y:

$20 + 20 = 40$

$\therefore$ There were 40 questions in the test.

---

16. A fraction becomes $\frac{9}{11}$ if 2 is added to both its numerator and denominator. If 3 is added to both the numerator and denominator, it becomes $\frac{5}{6}$. Find the fraction.

Solution : Let the fraction be $\frac{x}{y}$.

$$\frac{x+2}{y+2} = \frac{9}{11} \Rightarrow 11x + 22 = 9y + 18 \Rightarrow 11x - 9y = -4 \text{ …(1)}$$

$$\frac{x+3}{y+3} = \frac{5}{6} \Rightarrow 6x + 18 = 5y + 15 \Rightarrow 6x - 5y = -3 \text{ …(2)}$$

(1) × 5 − (2) × 9:

$55x - 54x = -20 + 27 \Rightarrow x = 7$

From (1): $77 - 9y = -4 \Rightarrow y = 9$

$\therefore$ The fraction is $\frac{7}{9}$.

---

17. The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is ₹ 105 and for a journey of 15 km, the charge paid is ₹ 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?

Solution : Let the fixed charge be ₹ x and the charge per km be ₹ y.

$x + 10y = 105$ …(1)

$x + 15y = 155$ …(2)

Subtract (1) from (2):

$5y = 50 \Rightarrow y = 10$

$x = 105 - 100 = 5$

Charge for 25 km:

$5 + 25 \times 10 = 255$

∴ Fixed charge = ₹ 5, charge per km = ₹ 10; fare for 25 km = ₹ 255.

---

18. Meena went to a bank to withdraw ₹ 2000. She asked the cashier to give her ₹ 50 and ₹ 100 notes only. Meena got 25 notes in all. Find how many notes of ₹ 50 and ₹ 100 she received.

Solution : Let the number of ₹ 50 notes be x and of ₹ 100 notes be y.

$x + y = 25$ …(1)

$50x + 100y = 2000 \Rightarrow x + 2y = 40$ …(2)

Subtract (1) from (2):

$y = 15$

$x = 25 - 15 = 10$

∴ 10 notes of ₹ 50 and 15 notes of ₹ 100.

---

19. A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹ 27 for a book kept for seven days, while Susy paid ₹ 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Solution : Let the fixed charge (first three days) be ₹ x and the charge for each extra day be ₹ y.

Saritha: 7 days = 3 days + 4 extra days:

$x + 4y = 27$ …(1)

Susy: 5 days = 3 days + 2 extra days:

$x + 2y = 21$ …(2)

Subtract (2) from (1):

$2y = 6 \Rightarrow y = 3$

$x = 21 - 6 = 15$

∴ Fixed charge = ₹ 15; charge for each extra day = ₹ 3.

---

20. A cricket team coach buys 7 bats and 6 balls for ₹ 3800. Later, she buys 3 bats and 5 balls for ₹ 1750. Find the cost of each bat and each ball.

Solution : Let the cost of a bat be ₹ x and of a ball be ₹ y.

$7x + 6y = 3800$ …(1)

$3x + 5y = 1750$ …(2)

(1) × 5 − (2) × 6:

$35x - 18x = 19000 - 10500$

$17x = 8500 \Rightarrow x = 500$

From (1):

$3500 + 6y = 3800 \Rightarrow y = 50$

∴ Cost of a bat = ₹ 500; cost of a ball = ₹ 50.

---

21. A train covered a certain distance at a uniform speed. If the train had been 10 km/h faster, it would have taken 2 hours less than the scheduled time. If the train were slower by 10 km/h, it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

Solution : Let the speed be x km/h and the scheduled time t hours. Distance = xt.

$(x + 10)(t - 2) = xt \Rightarrow -2x + 10t = 20 \Rightarrow -x + 5t = 10$ …(1)

$(x - 10)(t + 3) = xt \Rightarrow 3x - 10t = 30$ …(2)

(1) × 2 + (2):

$-2x + 3x = 20 + 30 \Rightarrow x = 50$

From (1): $5t = 60 \Rightarrow t = 12$

Distance:

$50 \times 12 = 600$ km

∴ Distance covered = 600 km.

Note: Case-study questions are recommended from this chapter for **CBSE Board 2026-27**.
Practise them here: **Class X Case Studies (click here)**

Chapter 4 – Quadratic Equations

1. Solve by factorisation: $9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$.

Solution : Factorise the constant term:

$2a^2 + 5ab + 2b^2 = (2a + b)(a + 2b)$

Split the middle term, since $3(2a + b) + 3(a + 2b) = 9(a + b)$:

$9x^2 - 3(2a + b)x - 3(a + 2b)x + (2a + b)(a + 2b) = 0$

$3x[3x - (2a + b)] - (a + 2b)[3x - (2a + b)] = 0$

$[3x - (2a + b)][3x - (a + 2b)] = 0$

$\therefore x = \frac{2a + b}{3}$ or $x = \frac{a + 2b}{3}$.

---

2. Solve for x: $9(x^2 + \frac{1}{x^2}) - 9(x + \frac{1}{x}) - 52 = 0$.

Solution : Let $x + \frac{1}{x} = y$. Then $x^2 + \frac{1}{x^2} = y^2 - 2$.

$9(y^2 - 2) - 9y - 52 = 0$

$9y^2 - 9y - 70 = 0$

$(3y - 10)(3y + 7) = 0 \Rightarrow y = \frac{10}{3}$ or $y = -\frac{7}{3}$

Case 1: $x + \frac{1}{x} = \frac{10}{3}$

$3x^2 - 10x + 3 = 0 \Rightarrow (3x - 1)(x - 3) = 0 \Rightarrow x = 3, \frac{1}{3}$

Case 2: $x + \frac{1}{x} = -\frac{7}{3}$

$3x^2 + 7x + 3 = 0 \Rightarrow x = \frac{-7 \pm \sqrt{13}}{6}$

$\therefore x = 3, \frac{1}{3}, \frac{-7 + \sqrt{13}}{6}, \frac{-7 - \sqrt{13}}{6}$.

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3. A train travels at a certain average speed for a distance of 54 km and then travels 63 km at an average speed 6 km/h more than the first speed. If it takes 3 hours to complete the journey, what was its first average speed?

Solution : Let the first speed be x km/h; the second speed is (x + 6) km/h.

Time = distance ÷ speed:

$\frac{54}{x} + \frac{63}{x + 6} = 3$

$54(x + 6) + 63x = 3x(x + 6)$

$117x + 324 = 3x^2 + 18x$

$3x^2 - 99x - 324 = 0 \Rightarrow x^2 - 33x - 108 = 0$

$(x - 36)(x + 3) = 0 \Rightarrow x = 36$ (speed cannot be negative)

$\therefore$ First average speed = 36 km/h.

---

4. Two pipes together can fill a tank in $\frac{15}{8}$ hours. The pipe with the larger diameter takes 2 hours less than the pipe with the smaller diameter to fill the tank separately. Find the time in which each pipe can fill the tank separately.

Solution : Let the smaller pipe take x hours; the larger takes (x − 2) hours.

Work done in 1 hour:

$$\frac{1}{x} + \frac{1}{x-2} = \frac{8}{15}$$

$$15[(x-2) + x] = 8x(x-2)$$

$$30x - 30 = 8x^2 - 16x$$

$$8x^2 - 46x + 30 = 0 \Rightarrow 4x^2 - 23x + 15 = 0$$

$$(x-5)(4x-3) = 0 \Rightarrow x = 5 \text{ or } x = \frac{3}{4}$$

$x = \frac{3}{4}$ gives $x - 2 < 0$, which is not possible.

∴ Smaller pipe: 5 hours, larger pipe: 3 hours.

---

5. A train travelling at a uniform speed for 360 km would have taken 48 minutes less to travel the same distance if its speed were 5 km/h more. Find the original speed of the train.

Solution : Let the original speed be x km/h. 48 min = $\frac{4}{5}$ h.

$$\frac{360}{x} - \frac{360}{x+5} = \frac{4}{5}$$

$$360 \times \frac{5}{x(x+5)} = \frac{4}{5}$$

$$x(x+5) = 2250$$

$$x^2 + 5x - 2250 = 0$$

$$(x+50)(x-45) = 0 \Rightarrow x = 45$$

∴ Original speed = 45 km/h.

---

6. The sum of the areas of two squares is 52 cm² and the difference of their perimeters is 8 cm. Find the lengths of the sides of the two squares.

Solution : Let the sides be x cm and y cm (x > y).

$$4x - 4y = 8 \Rightarrow x = y + 2$$

$$x^2 + y^2 = 52$$

Substitute:

$$(y+2)^2 + y^2 = 52$$

$$2y^2 + 4y - 48 = 0 \Rightarrow y^2 + 2y - 24 = 0$$

$(y + 6)(y - 4) = 0 \Rightarrow y = 4, x = 6$

$\therefore$ Sides are 6 cm and 4 cm.

---

7. In a rectangular park measuring 50 m × 40 m, the gram panchayat plans to construct a rectangular swimming pool in the middle, surrounded by a grass strip of uniform width. Find the dimensions of the swimming pool if the area of the grass strip is 1184 m².

Solution : Let the width of the grass strip be x m.

Area of park = $50 \times 40 = 2000$ m²

Area of pool = $2000 - 1184 = 816$ m²

Pool dimensions are $(50 - 2x)$ and $(40 - 2x)$:

$(50 - 2x)(40 - 2x) = 816$

$4x^2 - 180x + 2000 = 816$

$x^2 - 45x + 296 = 0$

$(x - 8)(x - 37) = 0 \Rightarrow x = 8$ ($x = 37$ is too large)

Pool: $50 - 16 = 34$ m and $40 - 16 = 24$ m

$\therefore$ Swimming pool: 34 m × 24 m.

---

8. In a flight of 2800 km, an aircraft was slowed down due to bad weather. Its average speed was reduced by 100 km/h and the time of flight increased by 30 minutes. Find the original duration of the flight.

Solution : Let the original speed be x km/h.

$$\frac{2800}{x - 100} - \frac{2800}{x} = \frac{1}{2}$$

$$2800 \times \frac{100}{x(x - 100)} = \frac{1}{2}$$

$x^2 - 100x - 560000 = 0$

$(x - 800)(x + 700) = 0 \Rightarrow x = 800$

Original duration:

$2800 \div 800 = 3.5$ h

$\therefore$ Original duration of flight = 3½ hours.

---

9. Find the value of k for which the quadratic equation $(k + 1)x^2 - 6(k + 1)x + 3(k + 9) = 0$, $k \neq -1$, has real and equal roots.

Solution : For equal roots, D = 0:

$[6(k + 1)]^2 - 4(k + 1) \times 3(k + 9) = 0$

$12(k + 1)[3(k + 1) - (k + 9)] = 0$

$12(k + 1)(2k - 6) = 0$

Since $k \neq -1$:

$2k - 6 = 0 \Rightarrow k = 3$

$\therefore k = 3.$

---

10. At present Asha's age (in years) is 2 more than the square of her daughter Nisha's age. When Nisha grows to her mother's present age, Asha's age would be one year less than 10 times the present age of Nisha. Find the present ages of both.

Solution : Let Nisha's present age = x years. Asha's age = $x^2 + 2$.

Nisha reaches $x^2 + 2$ after $(x^2 + 2 - x)$ years. Asha's age then:

$(x^2 + 2) + (x^2 + 2 - x) = 2x^2 - x + 4$

Given condition:

$2x^2 - x + 4 = 10x - 1$

$2x^2 - 11x + 5 = 0$

$(2x - 1)(x - 5) = 0 \Rightarrow x = 5$ ($x = \frac{1}{2}$ is not reasonable here)

Asha = 25 + 2 = 27

$\therefore$ Nisha is 5 years old and Asha is 27 years old.

---

11. Solve by factorisation: $x^2 + (\frac{a}{a+b} + \frac{a+b}{a})x + 1 = 0$.

Solution : Note that $\frac{a}{a+b} \times \frac{a+b}{a} = 1$, which is the constant term.

So the middle term splits as:

$x^2 + [\frac{a}{a+b}]x + [\frac{a+b}{a}]x + 1 = 0$

$x[x + \frac{a}{a+b}] + [\frac{a+b}{a}][x + \frac{a}{a+b}] = 0$

$[x + \frac{a}{a+b}][x + \frac{a+b}{a}] = 0$

$\therefore x = -\frac{a}{a+b}$ or $x = -\frac{a+b}{a}$.

---

12. If the roots of $(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$ are equal, prove that either $a = 0$ or $a^3 + b^3 + c^3 = 3abc$.

Solution : For equal roots, D = 0:

$4(a^2 - bc)^2 - 4(c^2 - ab)(b^2 - ac) = 0$

$(a^2 - bc)^2 = (c^2 - ab)(b^2 - ac)$

Expand both sides:

$a^4 - 2a^2bc + b^2c^2 = b^2c^2 - ac^3 - ab^3 + a^2bc$

$a^4 + ab^3 + ac^3 - 3a^2bc = 0$

$a(a^3 + b^3 + c^3 - 3abc) = 0$

$\therefore$ Either $a = 0$ or $a^3 + b^3 + c^3 = 3abc$.

---

13. Divide 16 into two parts such that twice the square of the larger part exceeds the square of the smaller part by 164.

Solution : Let the larger part be x; the smaller part is $16 - x$.

$2x^2 - (16 - x)^2 = 164$

$2x^2 - 256 + 32x - x^2 = 164$

$x^2 + 32x - 420 = 0$

$(x + 42)(x - 10) = 0 \Rightarrow x = 10$

$\therefore$ The parts are 10 and 6.

---

14. Solve for x: $\frac{1}{2a + b + 2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$.

Solution : Bring the x-terms to one side:

$$\frac{1}{2a + b + 2x} - \frac{1}{2x} = \frac{1}{2a} + \frac{1}{b}$$

$$\frac{2x - (2a + b + 2x)}{2x(2a + b + 2x)} = \frac{b + 2a}{2ab}$$

$$-\frac{2a + b}{2x(2a + b + 2x)} = \frac{2a + b}{2ab}$$

Cancel $(2a + b)$ (non-zero):

$2x(2a + b + 2x) = -2ab$

$4x^2 + 2(2a + b)x + 2ab = 0 \Rightarrow 2x^2 + (2a + b)x + ab = 0$

Factorise:

$2x^2 + 2ax + bx + ab = 0 \Rightarrow 2x(x + a) + b(x + a) = 0$

$(x + a)(2x + b) = 0$

$\therefore x = -a$ or $x = -\frac{b}{2}$.

---

15. Find the values of p for which $(2p + 1)x^2 - (7p + 2)x + (7p - 3) = 0$ has equal roots. Also find these roots.

Solution : For equal roots, $D = 0$:

$(7p + 2)^2 - 4(2p + 1)(7p - 3) = 0$

$49p^2 + 28p + 4 - 4(14p^2 + p - 3) = 0$

$-7p^2 + 24p + 16 = 0 \Rightarrow 7p^2 - 24p - 16 = 0$

$(p - 4)(7p + 4) = 0 \Rightarrow p = 4$ or $p = -\frac{4}{7}$

Equal roots $= -\frac{b}{2a} = \frac{7p+2}{2(2p+1)}$:

$p = 4$: root $= \frac{30}{18} = \frac{5}{3}$

$p = -\frac{4}{7}$: root $= \frac{-2}{2(-\frac{1}{7})} = 7$

$\therefore$ $p = 4$ (roots $\frac{5}{3}, \frac{5}{3}$) or $p = -\frac{4}{7}$ (roots 7, 7).

---

16. Solve: $\sqrt{2x-3} + 1 = x$.

Solution : Isolate the square root:

$\sqrt{2x-3} = x - 1$

Square both sides:

$2x - 3 = x^2 - 2x + 1$

$x^2 - 4x + 4 = 0 \Rightarrow (x-2)^2 = 0 \Rightarrow x = 2$

Verify in the original equation:

$\sqrt{4-3} + 1 = 2 = x$ ✓

$\therefore$ $x = 2$.

---

17. If the roots of $(b-c)x^2 + (c-a)x + (a-b) = 0$ are equal, prove that $2b = a + c$.

Solution : For equal roots, $D = 0$:

$(c-a)^2 - 4(b-c)(a-b) = 0$

Expand:

$c^2 - 2ac + a^2 - 4(ab - b^2 - ac + bc) = 0$

$a^2 + c^2 + 4b^2 + 2ac - 4ab - 4bc = 0$

This is a perfect square:

$(a + c - 2b)^2 = 0$

$a + c - 2b = 0$

$\therefore$ $2b = a + c$.

---

18. Find the value of $\sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots}}}$.

Solution : Let $y = \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots}}}$. The expression under the first root again contains y:

$y = \sqrt{6 + y}$

Square both sides:

$y^2 = 6 + y \Rightarrow y^2 - y - 6 = 0$

$(y-3)(y+2) = 0 \Rightarrow y = 3$ or -2

A square root cannot be negative.

$\therefore$ Value = 3.

---

19. A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Solution : Let the speed of the stream be x km/h.

Upstream speed = (18 − x) km/h, downstream speed = (18 + x) km/h

Time difference is 1 hour:

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$

$24[(18 + x) - (18 - x)] = (18 - x)(18 + x)$

$48x = 324 - x^2$

$x^2 + 48x - 324 = 0$

$(x + 54)(x - 6) = 0 \Rightarrow x = 6$

$\therefore$ Speed of the stream = 6 km/h.

---

20. If the quadratic equation $(1 + m^2)x^2 + 2mcx + (c^2 - a^2) = 0$ has equal roots, prove that $c^2 = a^2(1 + m^2)$.

Solution : For equal roots, D = 0:

$(2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$

$m^2c^2 - (c^2 - a^2 + m^2c^2 - m^2a^2) = 0$

$m^2c^2 - c^2 + a^2 - m^2c^2 + m^2a^2 = 0$

$-c^2 + a^2 + a^2m^2 = 0$

$\therefore c^2 = a^2(1 + m^2)$.

---

21. In the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, one root is four times the other. Prove that $4b^2 = 25ac$.

Solution : Let the roots be α and 4α.

Sum of roots:

$$\alpha + 4\alpha = -\frac{b}{a} \Rightarrow \alpha = -\frac{b}{5a}$$

Product of roots:

$$\alpha \times 4\alpha = \frac{c}{a} \Rightarrow 4\alpha^2 = \frac{c}{a}$$

Substitute α:

$$4 \times \frac{b^2}{25a^2} = \frac{c}{a}$$

$4b^2 = 25ac$

$\therefore 4b^2 = 25ac$.

---

22. One-fourth of a herd of camels was seen in the forest. Twice the square root of the herd had gone to the mountains and the remaining 15 camels were seen on the bank of a river. Find the total number of camels.

Solution : Let the total number of camels be x.

$$\frac{x}{4} + 2\sqrt{x} + 15 = x$$

$$\frac{3x}{4} - 2\sqrt{x} - 15 = 0 \Rightarrow 3x - 8\sqrt{x} - 60 = 0$$

Put $\sqrt{x} = y$:

$3y^2 - 8y - 60 = 0$

$(y - 6)(3y + 10) = 0 \Rightarrow y = 6 \ (y > 0)$

$x = y^2 = 36$

$\therefore$ Total number of camels = 36.

---

23. Determine the nature of the roots of the following quadratic equations: (i) $3a^2b^2x^2 - 16abcx + 4c^2 = 0$ (ii) $4p^2q^2x^2 - 12pqx + 36 = 0$.

Solution : Nature of roots depends on $D = B^2 - 4AC$.

(i) $A = 3a^2b^2$, $B = -16abc$, $C = 4c^2$

$D = 256a^2b^2c^2 - 4 \times 3a^2b^2 \times 4c^2 = 256a^2b^2c^2 - 48a^2b^2c^2 = 208a^2b^2c^2$

$D > 0$ (for a, b, $c \neq 0$) $\Rightarrow$ roots are real and distinct

(ii) $A = 4p^2q^2$, $B = -12pq$, $C = 36$

$D = 144p^2q^2 - 4 \times 4p^2q^2 \times 36 = 144p^2q^2 - 576p^2q^2 = -432p^2q^2$

$D < 0 \Rightarrow$ no real roots

$\therefore$ (i) Real and distinct roots (ii) No real roots.

Note: Case-study questions are recommended from this chapter for **CBSE Board 2026-27**.
Practise them here: **Class X Case Studies (click here)**

Chapter 5 – Arithmetic Progressions

1. If five times the fifth term of an AP is equal to 8 times its eighth term, show that its 13th term is zero.

Solution : Using $a_n = a + (n - 1)d$:

$5(a + 4d) = 8(a + 7d)$

$5a + 20d = 8a + 56d$

$3a + 36d = 0 \Rightarrow a + 12d = 0$

But $a_{13} = a + 12d$.

$\therefore a_{13} = 0$.

---

2. If the pth term of an AP is q and the qth term is p, prove that its nth term is (p + q − n).

Solution :

$a + (p - 1)d = q$ …(1)

$a + (q - 1)d = p$ …(2)

Subtract (2) from (1):

$(p - q)d = q - p \Rightarrow d = -1$

Put $d = -1$ in (1):

$a = q + (p - 1) = p + q - 1$

nth term:

$a_n = a + (n - 1)d = (p + q - 1) - (n - 1) = p + q - n$

$\therefore a_n = p + q - n.$

---

3. Ridhi drew a polygon with n sides. The smallest exterior angle is 8° and each subsequent exterior angle is 4° more than the previous one. Find the number of sides of the polygon.

Solution : The exterior angles form an AP with $a = 8$, $d = 4$.

The sum of exterior angles of any polygon is 360°:

$S_n = (\frac{n}{2})[2a + (n - 1)d] = 360$

$(\frac{n}{2})[16 + 4n - 4] = 360$

$n(6 + 2n) = 360 \Rightarrow n^2 + 3n - 180 = 0$

$(n + 15)(n - 12) = 0 \Rightarrow n = 12$

$\therefore$ The polygon has 12 sides.

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4. The sum of the 2nd and the 7th terms of an AP is 30. If its 15th term is 1 less than twice its 8th term, find the AP.

Solution :

$a_2 + a_7 = 30 \Rightarrow (a + d) + (a + 6d) = 30 \Rightarrow 2a + 7d = 30$ …(1)

$a_{15} = 2a_8 - 1 \Rightarrow a + 14d = 2(a + 7d) - 1 \Rightarrow a = 1$

From (1):

$2 + 7d = 30 \Rightarrow d = 4$

$\therefore$ AP: 1, 5, 9, 13, …

---

5. If S_n denotes the sum of the first n terms of an AP, prove that $S_{12} = 3(S_8 - S_4)$.

Solution : Use $S_n = (\frac{n}{2})[2a + (n - 1)d]$:

$S_8 = 4(2a + 7d) = 8a + 28d$

$S_4 = 2(2a + 3d) = 4a + 6d$

$S_8 - S_4 = 4a + 22d$

$3(S_8 - S_4) = 12a + 66d$

$S_{12} = 6(2a + 11d) = 12a + 66d$

$\therefore S_{12} = 3(S_8 - S_4)$.

---

6. Find the sum of all 11 terms of an AP whose middle-most term is 30.

Solution : With 11 terms, the middle-most term is the 6th term:

$a_6 = a + 5d = 30$

Sum of 11 terms:

$$S_{11} = (\frac{11}{2})(2a + 10d) = 11(a + 5d) = 11 \times 30$$

$\therefore S_{11} = 330$.

---

7. Find the sum of the last ten terms of the AP 8, 10, 12, …, 126.

Solution : Write the AP in reverse: 126, 124, 122, … ($a = 126$, $d = -2$).

The last ten terms of the original AP are the first ten of this one:

$$S_{10} = (\frac{10}{2})[2(126) + 9(-2)] = 5(252 - 18) = 5 \times 234$$

$\therefore$ Sum of the last ten terms = 1170.

---

8. An AP consists of 37 terms. The sum of the three middle-most terms is 225 and the sum of the last three terms is 429. Find the AP.

Solution : Middle-most terms of 37 terms: 18th, 19th, 20th.

$a_{18} + a_{19} + a_{20} = 3a_{19} = 225 \Rightarrow a + 18d = 75$ …(1)

Last three terms: 35th, 36th, 37th.

$3a_{36} = 429 \Rightarrow a + 35d = 143$ …(2)

Subtract (1) from (2):

$17d = 68 \Rightarrow d = 4$

$a = 75 - 72 = 3$

$\therefore$ AP: 3, 7, 11, 15, …

---

9. Divide 32 into four parts which are in AP such that the product of the extremes is to the product of the means as 7 : 15.

Solution : Let the parts be $a - 3d$, $a - d$, $a + d$, $a + 3d$ (common difference $2d$).

Sum = $4a = 32 \Rightarrow a = 8$

Given ratio:

$(8 - 3d)(8 + 3d) : (8 - d)(8 + d) = 7 : 15$

$15(64 - 9d^2) = 7(64 - d^2)$

$960 - 135d^2 = 448 - 7d^2$

$128d^2 = 512 \Rightarrow d^2 = 4 \Rightarrow d = 2$

$\therefore$ The parts are 2, 6, 10, 14.

---

10. The ratio of the sums of n terms of two APs is (7n + 1) : (4n + 27). Find the ratio of their mth terms.

Solution : Let the APs have first terms a, A and common differences d, D.

$$\frac{S_n}{S_n'} = \frac{2a + (n-1)d}{2A + (n-1)D} = \frac{7n+1}{4n+27}$$

Divide numerator and denominator by 2:

$$\frac{a + (\frac{n-1}{2})d}{A + (\frac{n-1}{2})D} = \frac{7n+1}{4n+27}$$

The mth terms are $a + (m-1)d$ and $A + (m-1)D$, so put $\frac{n-1}{2} = m - 1$, i.e. $n = 2m - 1$:

$$\frac{a_m}{A_m} = \frac{7(2m-1)+1}{4(2m-1)+27}$$

$\therefore$ Ratio of mth terms = (14m − 6) : (8m + 23).

---

11. Which term of the AP 3, 15, 27, 39, … will be 132 more than its 54th term?

Solution :

$a = 3, d = 15 - 3 = 12$

54th term:

$a_{54} = 3 + 53 \times 12 = 639$

Required term = 639 + 132 = 771. Let it be the nth term:

$3 + (n-1)12 = 771$

$(n-1)12 = 768 \Rightarrow n - 1 = 64 \Rightarrow n = 65$

$\therefore$ The 65th term is 132 more than the 54th term.

---

12. How many terms of the AP 24, 21, 18, … must be taken so that their sum is 78? Explain the double answer.

Solution :

$a = 24, d = -3, S_n = 78$

$(\frac{n}{2})[48 + (n-1)(-3)] = 78$

$n(51 - 3n) = 156$

$3n^2 - 51n + 156 = 0 \Rightarrow n^2 - 17n + 52 = 0$

$(n-4)(n-13) = 0 \Rightarrow n = 4$ or $n = 13$

Both answers are valid: the 5th to 13th terms are 12, 9, 6, 3, 0, −3, −6, −9, −12, whose sum is 0.

So adding them to the first 4 terms does not change the sum.

$\therefore$ 4 terms or 13 terms.

13. The sum of the first n terms of an AP is given by $S_n = 4n - n^2$. Find the first term, the sum of the first two terms, the second term, the 10th term and the nth term.

Solution :

$S_1 = 4 - 1 = 3 \Rightarrow$ first term $a_1 = 3$

$S_2 = 8 - 4 = 4 \Rightarrow$ sum of first two terms = 4

$a_2 = S_2 - S_1 = 4 - 3 = 1$

Common difference:

$d = a_2 - a_1 = -2$

10th term:

$a_{10} = 3 + 9(-2) = -15$

nth term:

$a_n = 3 + (n - 1)(-2) = 5 - 2n$

$\therefore a_1 = 3, S_2 = 4, a_2 = 1, a_{10} = -15, a_n = 5 - 2n.$

14. 200 logs are stacked such that there are 20 logs in the bottom row, 19 in the next row, 18 in the row next to it, and so on. In how many rows are the 200 logs placed and how many logs are in the top row?

Solution : The rows form an AP: $a = 20, d = -1, S_n = 200.$

$(\frac{n}{2})[40 + (n - 1)(-1)] = 200$

$n(41 - n) = 400$

$n^2 - 41n + 400 = 0 \Rightarrow (n - 16)(n - 25) = 0$

$n = 16$ or $n = 25$

For n = 25 the top row would have $20 - 24 = -4$ logs, which is impossible.

$n = 16$, top row $= a_{16} = 20 - 15 = 5$

$\therefore$ 16 rows; 5 logs in the top row.

15. Find the sum of all three-digit natural numbers which are divisible by 7.

Solution : Three-digit multiples of 7 form an AP: 105, 112, …, 994 ($a = 105, d = 7$).

Number of terms:

$994 = 105 + (n - 1)7 \Rightarrow n - 1 = 127 \Rightarrow n = 128$

Sum:

$S_{128} = (\frac{128}{2})(105 + 994) = 64 \times 1099 = 70336$

$\therefore$ Sum = 70336.

16. If the sum of the first 7 terms of an AP is 49 and that of the first 17 terms is 289, find the sum of the first n terms.

Solution :

$S_7 = (\frac{7}{2})(2a + 6d) = 49 \Rightarrow a + 3d = 7$ …(1)

$S_{17} = (\frac{17}{2})(2a + 16d) = 289 \Rightarrow a + 8d = 17$ …(2)

Subtract (1) from (2):

$5d = 10 \Rightarrow d = 2, a = 1$

Sum of n terms:

$S_n = (\frac{n}{2})[2 + (n - 1)2] = (\frac{n}{2})(2n) = n^2$

$\therefore S_n = n^2$.

---

17. A sum of ₹ 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹ 20 less than its preceding prize, find the value of each of the prizes.

Solution : Let the first prize be ₹ a; prizes form an AP with d = –20, n = 7.

$S_7 = (\frac{7}{2})[2a + 6(-20)] = 700$

$2a - 120 = 200 \Rightarrow a = 160$

Prizes:

160, 140, 120, 100, 80, 60, 40

$\therefore$ ₹ 160, ₹ 140, ₹ 120, ₹ 100, ₹ 80, ₹ 60 and ₹ 40.

---

18. If the sum of the first p terms of an AP is q and the sum of the first q terms is p, prove that the sum of the first (p + q) terms is −(p + q).

Solution :

$S(p) = (\frac{p}{2})[2a + (p - 1)d] = q \Rightarrow 2a + (p - 1)d = \frac{2q}{p}$ …(1)

$S(q) = (\frac{q}{2})[2a + (q - 1)d] = p \Rightarrow 2a + (q - 1)d = \frac{2p}{q}$ …(2)

Subtract (2) from (1):

$(p - q)d = \frac{2q}{p} - \frac{2p}{q} = \frac{2(q^2 - p^2)}{pq}$

$d = -\frac{2(p + q)}{pq}$

Now

$$S(p+q) = \left(\frac{p+q}{2}\right)[2a + (p-1)d + qd]$$

$$= \left(\frac{p+q}{2}\right)\left[\frac{2q}{p} - \frac{2(p+q)}{p}\right] \text{ (using (1))}$$

$$= \left(\frac{p+q}{2}\right) \times \left(-\frac{2p}{p}\right) = -(p+q)$$

$\therefore S(p+q) = -(p+q)$.

---

19. Find the 20th term from the end of the AP 3, 8, 13, …, 253.

Solution : Reverse the AP: 253, 248, 243, … (a = 253, d = −5).

The 20th term from the end of the original AP is the 20th term of the reversed AP:

$253 + 19(-5) = 253 - 95 = 158$

$\therefore$ The 20th term from the end is 158.

---

20. The ratio of the 11th term to the 18th term of an AP is 2 : 3. Find the ratio of the 5th term to the 21st term, and the ratio of the sum of the first 5 terms to the sum of the first 21 terms.

Solution :

$$\frac{a+10d}{a+17d} = \frac{2}{3}$$

$3a + 30d = 2a + 34d \Rightarrow a = 4d$

Ratio of terms:

$a_5 = a + 4d = 8d$, $a_{21} = a + 20d = 24d \Rightarrow a_5 : a_{21} = 1 : 3$

Ratio of sums:

$$S_5 = \left(\frac{5}{2}\right)(2a + 4d) = \left(\frac{5}{2}\right)(12d) = 30d$$

$$S_{21} = \left(\frac{21}{2}\right)(2a + 20d) = \left(\frac{21}{2}\right)(28d) = 294d$$

$S_5 : S_{21} = 30 : 294 = 5 : 49$

$\therefore a_5 : a_{21} = 1 : 3$ and $S_5 : S_{21} = 5 : 49$.

---

21. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: ₹ 200 for the first day, ₹ 250 for the second day, ₹ 300 for the third day, etc., the penalty for each succeeding day being ₹ 50 more than for the preceding day. How much money does the contractor have to pay as penalty if he has delayed the work by 30 days?

Solution : Penalties form an AP: a = 200, d = 50, n = 30.

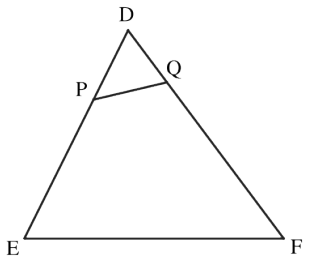
$$S_{30} = \left(\frac{30}{2}\right)[2 \times 200 + 29 \times 50]$$

= 15(400 + 1450) = 15 × 1850 = 27750

∴ Penalty = ₹ 27,750.

Chapter 6 – Triangles

1. P and Q are points on the sides DE and DF of a triangle DEF such that DP = 5 cm, DE = 15 cm, DQ = 6 cm and QF = 18 cm. Is PQ || EF? Give reasons.



Solution : Find the remaining part of DE:

PE = DE – DP = 15 – 5 = 10 cm

Compare the ratios in which P and Q divide the sides:

$$\frac{DP}{PE} = \frac{5}{10} = \frac{1}{2}$$

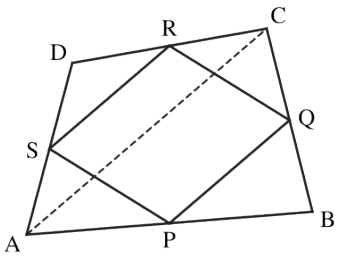
$$\frac{DQ}{QF} = \frac{6}{18} = \frac{1}{3}$$

Since $\frac{1}{2} \neq \frac{1}{3}$, the sides are not divided in the same ratio.

By the converse of the Basic Proportionality Theorem, PQ is parallel to EF only if $\frac{DP}{PE} = \frac{DQ}{QF}$.

∴ No, PQ is not parallel to EF.

2. Prove that the line segments joining the mid-points of the adjacent sides of a quadrilateral form a parallelogram.



Solution : Given: ABCD is a quadrilateral; P, Q, R, S are the mid-points of AB, BC, CD, DA.

To prove: PQRS is a parallelogram.

Construction: Join the diagonal AC.

In ΔABC, P and Q are mid-points of AB and BC. By the mid-point theorem,

PQ ∥ AC and PQ = ½AC …(1)

In ΔADC, S and R are mid-points of AD and DC. Similarly,

SR ∥ AC and SR = ½AC …(2)

From (1) and (2):

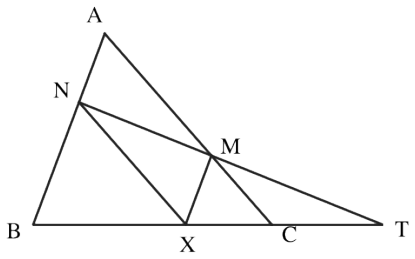
PQ ∥ SR and PQ = SR

A quadrilateral with one pair of opposite sides equal and parallel is a parallelogram.

$\therefore$ PQRS is a parallelogram.

---

3. Let X be any point on the side BC of a triangle ABC. If XM and XN are drawn parallel to BA and CA meeting CA and BA in M and N respectively, and MN meets BC produced in T, prove that $TX^2 = TB \times TC$.



Solution : Given: XM ∥ BA, XN ∥ CA; MN produced meets BC produced at T.

In ΔTXM, BN ∥ XM (since BA ∥ XM). By BPT:

$$\frac{TB}{TX} = \frac{TN}{TM} \quad \ldots(1)$$

In ΔTCM, XN ∥ CM (since CA ∥ XN). By BPT:

$$\frac{TX}{TC} = \frac{TN}{TM} \quad \ldots(2)$$

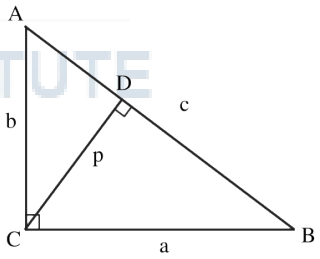
From (1) and (2):

$$\frac{TB}{TX} = \frac{TX}{TC}$$

$\therefore TX^2 = TB \times TC$.

---

4. ABC is a triangle right-angled at C, and p is the length of the perpendicular from C to AB. If AB = c, CA = b and BC = a, prove that (i) pc = ab (ii) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.



Solution : Let CD ⊥ AB with CD = p.

(i) Area of ΔABC computed in two ways:

Taking base AB: Area = ½ × AB × CD = ½ × c × p

Taking base BC: Area = ½ × BC × AC = ½ × a × b

½cp = ½ab ⇒ pc = ab

(ii) From (i), $p = \frac{ab}{c}$, so

$$\frac{1}{p^2} = \frac{c^2}{a^2b^2}$$

By Pythagoras theorem in ΔACB, $c^2 = a^2 + b^2$:

$$\frac{1}{p^2} = \frac{a^2 + b^2}{a^2b^2} = \frac{a^2}{a^2b^2} + \frac{b^2}{a^2b^2} = \frac{1}{b^2} + \frac{1}{a^2}$$

$\therefore$ pc = ab and $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

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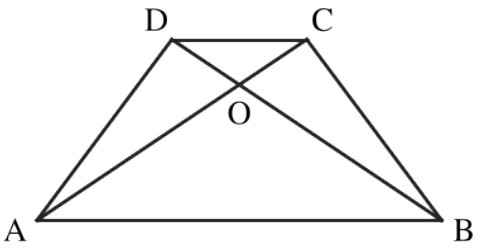
5. If one diagonal of a trapezium divides the other diagonal in the ratio 1 : 3, prove that one of the parallel sides is three times the other.



Solution : Given: ABCD is a trapezium with AB ∥ DC; diagonals AC and BD meet at O such that DO : OB = 1 : 3.

To prove: AB = 3 DC.

In ΔAOB and ΔCOD:

$\angle OAB = \angle OCD$ (alternate angles, AB ∥ DC)

$\angle AOB = \angle COD$ (vertically opposite angles)

$\Delta AOB \sim \Delta COD$ (AA similarity)

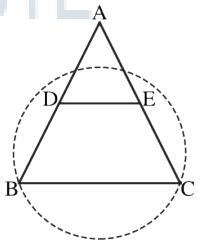
Corresponding sides are proportional:

$$\frac{AB}{CD} = \frac{OB}{OD} = \frac{3}{1}$$

$\therefore$ AB = 3 CD, i.e. one parallel side is three times the other.

---

6. ABC is a triangle in which AB = AC. D and E are points on AB and AC such that AD = AE. Show that B, C, E and D are concyclic.



Solution : Since AB = AC and AD = AE,

DB = AB − AD = AC − AE = EC

$$\frac{AD}{DB} = \frac{AE}{EC}$$

By the converse of BPT:

DE ∥ BC ⇒ $\angle ADE = \angle ABC$ (corresponding angles)

AB = AC ⇒ $\angle ABC = \angle ACB$.

D lies on AB, so $\angle ADE + \angle BDE = 180°$:

$\angle BDE = 180° - \angle ADE = 180° - \angle ACB$

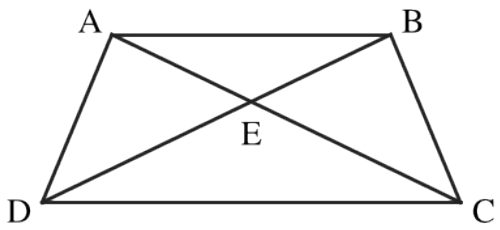
$\angle BDE + \angle ECB = 180°$

In quadrilateral BCED, a pair of opposite angles is supplementary.

$\therefore$ B, C, E and D are concyclic.

---

7. ABCD is a trapezium with AB || DC in which diagonals AC and BD intersect at E and $\Delta AED \sim \Delta BEC$. Prove that AD = BC.



Solution : In ΔAEB and ΔCED:

$\angle EAB = \angle ECD$ (alternate angles), $\angle AEB = \angle CED$ (vertically opposite)

$$\Delta AEB \sim \Delta CED \Rightarrow \frac{AE}{CE} = \frac{BE}{DE} \Rightarrow \frac{AE}{BE} = \frac{CE}{DE} \ldots(1)$$

Given $\Delta AED \sim \Delta BEC$:

$$\frac{AE}{BE} = \frac{ED}{EC} = \frac{AD}{BC} \ldots(2)$$

From (1) and (2):

$$\frac{CE}{DE} = \frac{DE}{CE} \Rightarrow CE^2 = DE^2 \Rightarrow CE = DE$$

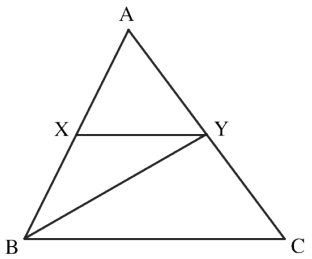
Then from (2):

$$\frac{AD}{BC} = \frac{ED}{EC} = 1$$

$\therefore$ AD = BC.

---

8. In ΔABC, a line XY parallel to BC cuts AB at X and AC at Y. If BY bisects $\angle XYC$, show that BC = CY.



Solution : XY || BC and BY is a transversal:

$\angle XYB = \angle YBC$ (alternate angles) …(1)

BY bisects $\angle XYC$:

$\angle XYB = \angle BYC$ …(2)

From (1) and (2):

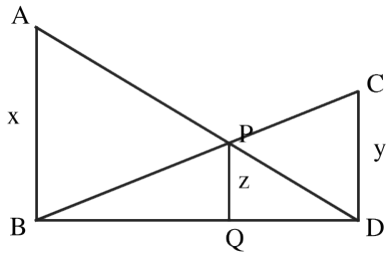
$\angle YBC = \angle BYC$

In ΔBCY, sides opposite to equal angles are equal:

$\therefore$ CY = BC.

---

9. In the figure, AB || PQ || CD, AB = x units, CD = y units and PQ = z units. Prove that $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$.



Solution : In ΔBDC, PQ ∥ CD:

$$\Delta BQP \sim \Delta BDC \text{ (AA)} \Rightarrow \frac{PQ}{CD}=\frac{BQ}{BD} \Rightarrow \frac{z}{y}=\frac{BQ}{BD} \ldots(1)$$

In ΔDBA, PQ ∥ AB:

$$\Delta DQP \sim \Delta DBA \text{ (AA)} \Rightarrow \frac{PQ}{AB}=\frac{DQ}{DB} \Rightarrow \frac{z}{x}=\frac{DQ}{BD} \ldots(2)$$

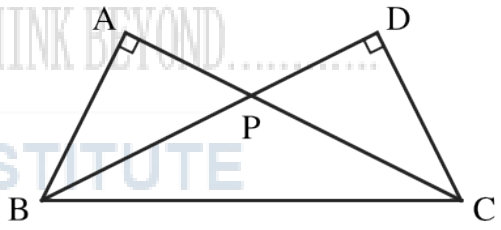
Add (1) and (2):

$$\frac{z}{x}+\frac{z}{y}=\frac{BQ+DQ}{BD}=\frac{BD}{BD}=1$$

Divide by z:

$$\therefore \frac{1}{x}+\frac{1}{y}=\frac{1}{z}.$$

10. Two right triangles ABC and DBC are drawn on the same hypotenuse BC and on the same side of BC. If AC and BD intersect at P, prove that AP × PC = BP × PD.



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Solution : In ΔAPB and ΔDPC:

∠BAP = ∠CDP = 90° (given right angles at A and D)

∠APB = ∠DPC (vertically opposite angles)

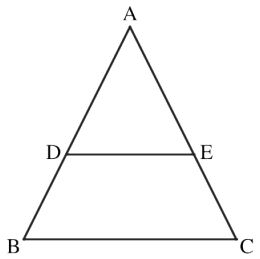
ΔAPB ~ ΔDPC (AA similarity)

Corresponding sides are proportional:

$$\frac{AP}{DP}=\frac{BP}{CP}$$

∴ AP × PC = BP × PD.

11. D and E are points on sides AB and AC of ΔABC such that ∠ADE = ∠AED and $\frac{AD}{DB} = \frac{AE}{EC}$. Prove that ΔBAC is isosceles.



Solution : In ΔADE, ∠ADE = ∠AED, so sides opposite equal angles are equal:

AD = AE …(1)

Given $\frac{AD}{DB} = \frac{AE}{EC}$; using (1):

DB = EC …(2)

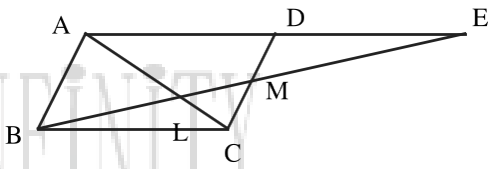
Add (1) and (2):

AD + DB = AE + EC ⇒ AB = AC

∴ ΔBAC is isosceles.

---

12. M is the mid-point of side CD of a parallelogram ABCD. The line BM intersects AC at L and AD produced at E. Prove that EL = 2BL.



Solution : In ΔBMC and ΔEMD:

CM = DM (M is the mid-point of CD)

∠BMC = ∠EMD (vertically opposite)

∠MCB = ∠MDE (alternate angles, BC ∥ AE)

ΔBMC ≅ ΔEMD (ASA) ⇒ BC = DE

Since AD = BC (opposite sides of a parallelogram):

AE = AD + DE = 2BC

In ΔAEL and ΔCBL:

∠ALE = ∠CLB (vertically opposite), ∠EAL = ∠BCL (alternate angles)

$\Delta AEL \sim \Delta CBL \Rightarrow \frac{EL}{BL} = \frac{AE}{CB} = \frac{2BC}{BC} = 2$

∴ EL = 2BL.

---

13. The perimeters of two similar triangles ABC and LMN are 60 cm and 48 cm respectively. If LM = 8 cm, find the length of AB.

Solution : For similar triangles, the ratio of perimeters equals the ratio of corresponding sides:

$\frac{AB}{LM} = \frac{\text{perimeter of } \Delta ABC}{\text{perimeter of } \Delta LMN} = \frac{60}{48} = \frac{5}{4}$

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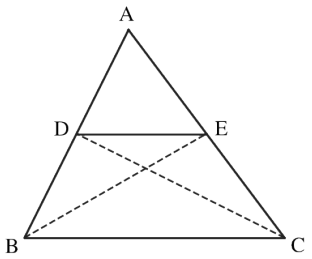
$$AB = (\frac{5}{4}) \times 8$$

$\therefore$ AB = 10 cm.

---

14. Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. (Basic Proportionality Theorem)



Solution : Given: In ΔABC, DE $\parallel$ BC, with D on AB and E on AC.

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$.

Construction: Join BE and CD; draw EN $\perp$ AB and DM $\perp$ AC.

Area of a triangle = ½ × base × height:

$$ar\frac{ADE}{ar}(BDE) = \frac{½ \times AD \times EN}{½ \times DB \times EN} = \frac{AD}{DB} \ldots(1)$$

$$ar\frac{ADE}{ar}(DEC) = \frac{½ \times AE \times DM}{½ \times EC \times DM} = \frac{AE}{EC} \ldots(2)$$

ΔBDE and ΔDEC are on the same base DE and between the same parallels DE and BC:

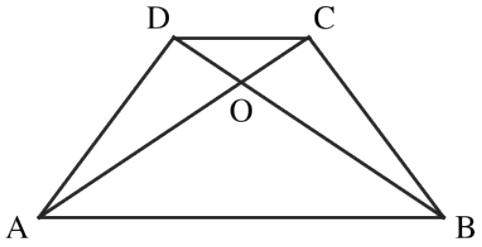
$$ar(BDE) = ar(DEC) \ldots(3)$$

From (1), (2) and (3):

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}.$$

---

15. ABCD is a trapezium in which AB $\parallel$ DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.



Solution : In ΔAOB and ΔCOD:

$\angle OAB = \angle OCD$ (alternate angles, AB $\parallel$ DC)

$\angle OBA = \angle ODC$ (alternate angles)

$\Delta AOB \sim \Delta COD$ (AA similarity)

Corresponding sides are proportional:

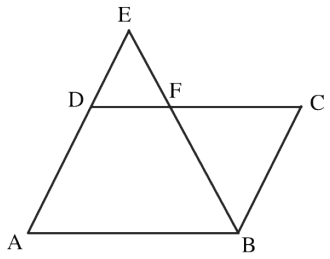
$$\frac{AO}{CO} = \frac{BO}{DO}$$

Rearranging:

$\therefore \frac{AO}{BO} = \frac{CO}{DO}$.

---

16. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that ΔABE ~ ΔCFB.



Solution : In ΔABE and ΔCFB:

$\angle A = \angle C$ (opposite angles of a parallelogram)

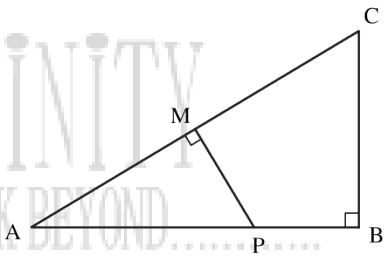
$\angle AEB = \angle CBF$ (alternate angles, AE ∥ BC with transversal BE)

ΔABE ~ ΔCFB (AA similarity)

$\therefore$ ΔABE ~ ΔCFB.

---

17. In the figure, ABC and AMP are two right triangles, right-angled at B and M respectively. Prove that (i) ΔABC ~ ΔAMP (ii) $\frac{CA}{PA} = \frac{BC}{MP}$.



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Solution : (i) In ΔABC and ΔAMP:

$\angle ABC = \angle AMP = 90°$

$\angle BAC = \angle MAP$ (common angle)

ΔABC ~ ΔAMP (AA similarity)

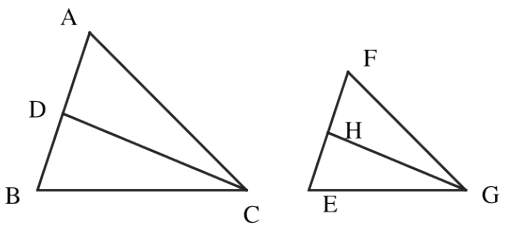
(ii) Corresponding sides of similar triangles are proportional:

$\frac{CA}{PA} = \frac{BC}{MP}$

$\therefore$ ΔABC ~ ΔAMP and $\frac{CA}{PA} = \frac{BC}{MP}$.

---

18. CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If ΔABC ~ ΔFEG, show that (i) $\frac{CD}{GH} = \frac{AC}{FG}$ (ii) ΔDCB ~ ΔHGE.



Solution : ΔABC ~ ΔFEG gives

$\angle A = \angle F, \angle B = \angle E, \angle ACB = \angle FGE$

Halving the equal angles (CD, GH are bisectors):

$\angle ACD = \angle FGH$ and $\angle DCB = \angle HGE$

(i) In ΔACD and ΔFGH:

$\angle A = \angle F, \angle ACD = \angle FGH \Rightarrow \Delta ACD \sim \Delta FGH$ (AA)

$$\frac{CD}{GH} = \frac{AC}{FG}$$

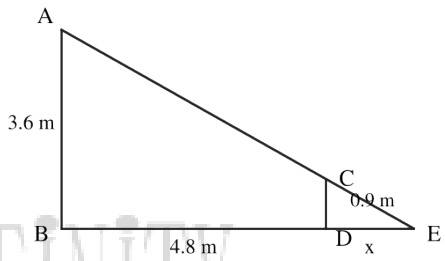
(ii) In ΔDCB and ΔHGE:

$\angle B = \angle E, \angle DCB = \angle HGE \Rightarrow \Delta DCB \sim \Delta HGE$ (AA)

$\therefore \frac{CD}{GH} = \frac{AC}{FG}$ and $\Delta DCB \sim \Delta HGE$.

---

19. A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/s. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.



Solution : Let AB be the lamp-post (3.6 m), CD the girl (0.9 m) and DE her shadow (x m).

Distance walked in 4 s:

$BD = 1.2 \times 4 = 4.8$ m

$\Delta ABE \sim \Delta CDE$ (both right-angled, $\angle E$ common):

$$\frac{AB}{CD} = \frac{BE}{DE}$$

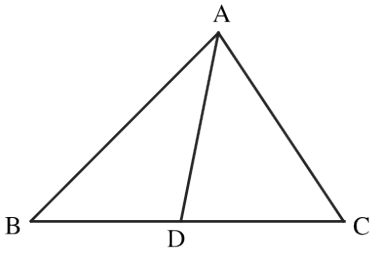
$$\frac{3.6}{0.9} = \frac{4.8 + x}{x}$$

$4x = 4.8 + x \Rightarrow 3x = 4.8 \Rightarrow x = 1.6$

$\therefore$ Length of her shadow = 1.6 m.

---

20. D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB \times CD$.



Solution : In ΔADC and ΔBAC:

$\angle ADC = \angle BAC$ (given)

$\angle ACD = \angle BCA$ (common angle)

$\Delta ADC \sim \Delta BAC$ (AA similarity)

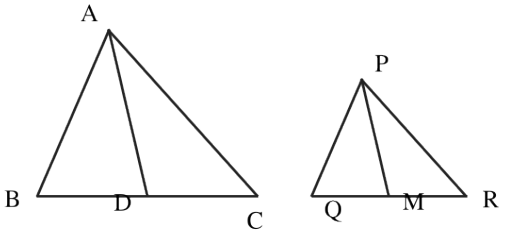
Corresponding sides are proportional:

$$\frac{CA}{CB} = \frac{CD}{CA}$$

$\therefore CA^2 = CB \times CD.$

---

21. If AD and PM are medians of triangles ABC and PQR respectively, where ΔABC ~ ΔPQR, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.



Solution : ΔABC ~ ΔPQR gives

$$\frac{AB}{PQ} = \frac{BC}{QR} \text{ and } \angle B = \angle Q \ldots(1)$$

D and M are mid-points of BC and QR:

$$\frac{BC}{QR} = \frac{2BD}{2QM} = \frac{BD}{QM}$$

$$\frac{AB}{PQ} = \frac{BD}{QM} \ldots(2)$$

In ΔABD and ΔPQM:

$$\frac{AB}{PQ} = \frac{BD}{QM} \text{ and } \angle B = \angle Q \Rightarrow \Delta ABD \sim \Delta PQM \text{ (SAS)}$$

$$\therefore \frac{AB}{PQ} = \frac{AD}{PM}.$$

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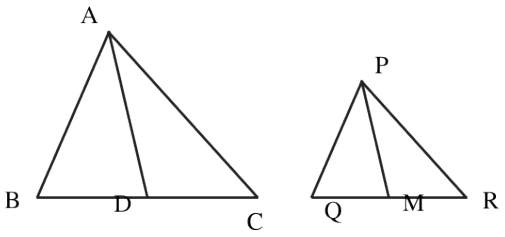
22. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR. Show that ΔABC ~ ΔPQR.



Solution : Given:

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

D and M are mid-points, so $\frac{BC}{QR} = \frac{2BD}{2QM} = \frac{BD}{QM}$:

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

ΔABD ~ ΔPQM (SSS similarity) ⇒ ∠B = ∠Q

Now in ΔABC and ΔPQR:

$\frac{AB}{PQ} = \frac{BC}{QR}$ and $\angle B = \angle Q$

$\Delta ABC \sim \Delta PQR$ (SAS similarity)

$\therefore \Delta ABC \sim \Delta PQR$.

Chapter 7 – Coordinate Geometry

1. Find the ratio in which the line $2x + y = 4$ divides the join of A(2, −2) and B(3, 7). Also find the coordinates of the point of intersection.

Solution : Let the line divide AB in the ratio k : 1 at P. By the section formula:

$$P = \left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$$

P lies on $2x + y = 4$:

$$\frac{2(3k+2)}{k+1} + \frac{7k-2}{k+1} = 4$$

$$6k + 4 + 7k - 2 = 4k + 4$$

$$9k = 2 \Rightarrow k = \frac{2}{9}$$

Ratio 2 : 9. Coordinates of P:

$$x = \frac{2\times3 + 9\times2}{11} = \frac{24}{11}$$

$$y = \frac{2\times7 + 9\times(-2)}{11} = -\frac{4}{11}$$

$\therefore$ Ratio 2 : 9; point of intersection $\left(\frac{24}{11}, -\frac{4}{11}\right)$.

---

2. The line segment joining A(4, −5) and B(4, 5) is divided by the point P such that AP : AB = 2 : 5. Find the coordinates of P.

Solution : AP : AB = 2 : 5 means AP = 2 parts and PB = 5 − 2 = 3 parts.

AP : PB = 2 : 3

Section formula with m : n = 2 : 3:

$$x = \frac{2\times4 + 3\times4}{5} = 4$$

$$y = \frac{2\times5 + 3\times(-5)}{5} = -\frac{5}{5} = -1$$

$\therefore P = (4, -1)$.

---

3. If (−3, 2), (1, −2) and (5, 6) are the mid-points of the sides of a triangle, find the coordinates of the vertices of the triangle.

Solution : Let the vertices be $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$, with D(−3, 2), E(1, −2), F(5, 6) the mid-points of AB, BC, CA.

Mid-point formula gives:

$x_1 + x_2 = -6$, $x_2 + x_3 = 2$, $x_3 + x_1 = 10$

Adding: $2(x_1 + x_2 + x_3) = 6 \Rightarrow x_1 + x_2 + x_3 = 3$

$x_3 = 3 - (-6) = 9$, $x_1 = 3 - 2 = 1$, $x_2 = 3 - 10 = -7$

Similarly for y:

$y_1 + y_2 = 4$, $y_2 + y_3 = -4$, $y_3 + y_1 = 12 \Rightarrow y_1 + y_2 + y_3 = 6$

$y_3 = 2$, $y_1 = 10$, $y_2 = -6$

∴ Vertices: (1, 10), (−7, −6) and (9, 2).

---

4. If (−5, 3) and (5, 3) are two vertices of an equilateral triangle, find the coordinates of the third vertex, given that the origin lies inside the triangle. (Take $\sqrt{3} = 1.7$)

Solution : Let A(−5, 3), B(5, 3) and the third vertex C(x, y).

AB = 10 units

C lies on the perpendicular bisector of AB, which is the line x = 0.

Height of an equilateral triangle of side 10:

$$h = \left(\frac{\sqrt{3}}{2}\right) \times 10 = 5\sqrt{3}$$

So C is $5\sqrt{3}$ units above or below the line y = 3:

$y = 3 + 5\sqrt{3}$ or $y = 3 - 5\sqrt{3}$

The origin must lie inside, so C is below AB:

$y = 3 - 5\sqrt{3} = 3 - 8.5 = -5.5$

∴ Third vertex = (0, −5.5).

---

5. ∠AOB is a right angle, where O is the origin and OA = OB = $\sqrt{8}$ units. If OA makes an angle of 45° with the x-axis, find the coordinates of B.

Solution : OA makes 45° with the x-axis and ∠AOB = 90°, so OB makes 45° + 90° = 135° with the positive x-axis.

Coordinates of B = (OB cos 135°, OB sin 135°):

$$x = \sqrt{8} \times \left(-\frac{1}{\sqrt{2}}\right) = -\sqrt{4} = -2$$

$$y = \sqrt{8} \times \left(\frac{1}{\sqrt{2}}\right) = 2$$

∴ B = (−2, 2).

---

6. If P(9a − 2, −b) divides the line segment joining A(3a + 1, −3) and B(8a, 5) in the ratio 3 : 1, find the values of a and b.

Solution : Section formula with m : n = 3 : 1:

$$x = \frac{3(8a) + 1(3a + 1)}{4} = \frac{27a + 1}{4}$$

$$y = \frac{3(5) + 1(-3)}{4} = \frac{12}{4} = 3$$

Compare with P(9a − 2, −b):

$-b = 3 \Rightarrow b = -3$

$$9a - 2 = \frac{27a + 1}{4} \Rightarrow 36a - 8 = 27a + 1 \Rightarrow a = 1$$

$\therefore a = 1, b = -3.$

---

7. Find the value of a if the points (3, 5) and (7, 1) are equidistant from the point (a, 0).

Solution : Equal distances ⇒ equal squares of distances:

$(a - 3)^2 + (0 - 5)^2 = (a - 7)^2 + (0 - 1)^2$

$a^2 - 6a + 9 + 25 = a^2 - 14a + 49 + 1$

$8a = 16$

$\therefore a = 2.$

---

8. Name the type of triangle formed by the points A(−5, 6), B(−4, −2) and C(7, 5).

Solution : Find the squares of the sides:

$AB^2 = (-4 + 5)^2 + (-2 - 6)^2 = 1 + 64 = 65$

$BC^2 = (7 + 4)^2 + (5 + 2)^2 = 121 + 49 = 170$

$AC^2 = (7 + 5)^2 + (5 - 6)^2 = 144 + 1 = 145$

All three sides are unequal, so the triangle is scalene.

Check the largest side: $AB^2 + AC^2 = 65 + 145 = 210 > 170 = BC^2$, so the angle opposite BC is acute.

$\therefore \Delta ABC$ is a scalene (acute-angled) triangle.

---

9. Points A(−1, y) and B(5, 7) lie on a circle with centre O(2, −3y). Find the values of y. Hence find the radius of the circle.

Solution : OA = OB (radii):

$(2 + 1)^2 + (-3y - y)^2 = (2 - 5)^2 + (-3y - 7)^2$

$9 + 16y^2 = 9 + 9y^2 + 42y + 49$

$7y^2 - 42y - 49 = 0 \Rightarrow y^2 - 6y - 7 = 0$

$(y - 7)(y + 1) = 0 \Rightarrow y = 7$ or $y = -1$

Radius:

$y = 7$: $O(2, -21)$, $r^2 = 9 + (-21 - 7)^2 = 793 \Rightarrow r = \sqrt{793}$ units

$y = -1$: O(2, 3), $r^2 = 9 + (3 - 7)^2 = 25 \Rightarrow r = 5$ units

$\therefore$ $y = 7$ (radius $\sqrt{793}$) or $y = -1$ (radius 5 units).

---

10. The mid-point P of the line segment joining A(−10, 4) and B(−2, 0) lies on the line segment joining C(−9, −4) and D(−4, y). Find the ratio in which P divides CD. Also find the value of y.

Solution : Mid-point of AB:

$$P = \left(\frac{-10-2}{2}, \frac{4+0}{2}\right) = (-6, 2)$$

Let P divide CD in the ratio k : 1. Using x-coordinates:

$$\frac{-4k-9}{k+1} = -6$$

$$-4k - 9 = -6k - 6 \Rightarrow 2k = 3 \Rightarrow k = \frac{3}{2} \Rightarrow \text{ratio } 3:2$$

Using y-coordinates with ratio 3 : 2:

$$\frac{3y + 2(-4)}{5} = 2 \Rightarrow 3y - 8 = 10 \Rightarrow y = 6$$

$\therefore$ Ratio 3 : 2 and $y = 6$.

---

11. A point P divides the line segment joining A(3, −5) and B(−4, 8) such that $\frac{AB}{PB} = \frac{k}{1}$. If P lies on the line $x + y = 0$, find the value of k.

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Solution : $\frac{AB}{PB} = k \Rightarrow \frac{AP}{PB} = \frac{AB - PB}{PB} = k - 1$. So P divides AB in the ratio $(k - 1) : 1$.

$$P = \left(\frac{(k-1)(-4)+3}{k}, \frac{(k-1)(8)-5}{k}\right) = \left(\frac{7-4k}{k}, \frac{8k-13}{k}\right)$$

P lies on $x + y = 0$:

$$(7 - 4k) + (8k - 13) = 0 \Rightarrow 4k = 6$$

$\therefore$ $k = \frac{3}{2}$.

---

12. Find a point on the y-axis which is equidistant from the points A(5, −2) and B(−3, 2).

Solution : Let the point be P(0, y). PA = PB:

$$(0 - 5)^2 + (y + 2)^2 = (0 + 3)^2 + (y - 2)^2$$

$$25 + y^2 + 4y + 4 = 9 + y^2 - 4y + 4$$

$$8y = -16 \Rightarrow y = -2$$

$\therefore$ The required point is (0, −2).

---

13. Show that the points (1, 7), (4, 2), (−1, −1) and (−4, 4) are the vertices of a square.

Solution : Let A(1, 7), B(4, 2), C(−1, −1), D(−4, 4).

Sides:

$AB^2 = 9 + 25 = 34$, $BC^2 = 25 + 9 = 34$, $CD^2 = 9 + 25 = 34$, $DA^2 = 25 + 9 = 34$

Diagonals:

$AC^2 = 4 + 64 = 68$, $BD^2 = 64 + 4 = 68$

All four sides are equal and the diagonals are equal.

∴ ABCD is a square.

---

14. Find the coordinates of the points of trisection of the line segment joining (4, −1) and (−2, −3).

Solution : Let A(4, −1) and B(−2, −3). The points P and Q divide AB in the ratios 1 : 2 and 2 : 1.

$$P = \left(\frac{1(-2) + 2(4)}{3}, \frac{1(-3) + 2(-1)}{3}\right) = \left(2, -\frac{5}{3}\right)$$

$$Q = \left(\frac{2(-2) + 1(4)}{3}, \frac{2(-3) + 1(-1)}{3}\right) = \left(0, -\frac{7}{3}\right)$$

∴ Points of trisection: $\left(2, -\frac{5}{3}\right)$ and $\left(0, -\frac{7}{3}\right)$.

---

15. Find the coordinates of the points which divide the line segment joining A(−2, 2) and B(2, 8) into four equal parts.

Solution : The middle point Q is the mid-point of AB:

$Q = (0, 5)$

P is the mid-point of AQ:

$$P = \left(\frac{-2 + 0}{2}, \frac{2 + 5}{2}\right) = \left(-1, \frac{7}{2}\right)$$

R is the mid-point of QB:

$$R = \left(\frac{0 + 2}{2}, \frac{5 + 8}{2}\right) = \left(1, \frac{13}{2}\right)$$

∴ The points are $\left(-1, \frac{7}{2}\right)$, (0, 5) and $\left(1, \frac{13}{2}\right)$.

---

16. If A(1, 2), B(4, y), C(x, 6) and D(3, 5) are the vertices of a parallelogram taken in order, find x and y.

Solution : Diagonals of a parallelogram bisect each other, so the mid-points of AC and BD coincide.

$$\left(\frac{1 + x}{2}, \frac{2 + 6}{2}\right) = \left(\frac{4 + 3}{2}, \frac{y + 5}{2}\right)$$

$$\frac{1+x}{2} = \frac{7}{2} \Rightarrow x = 6$$

$$4 = \frac{y+5}{2} \Rightarrow y = 3$$

$\therefore x = 6, y = 3.$

---

17. Find the values of y for which the distance between the points P(2, −3) and Q(10, y) is 10 units.

Solution :

$PQ^2 = (10 - 2)^2 + (y + 3)^2 = 100$

$64 + (y + 3)^2 = 100 \Rightarrow (y + 3)^2 = 36$

$y + 3 = \pm 6$

$\therefore y = 3$ or $y = -9$.

---

18. Find a relation between x and y such that the point (x, y) is equidistant from the points (3, 6) and (−3, 4).

Solution : Equal squared distances:

$(x - 3)^2 + (y - 6)^2 = (x + 3)^2 + (y - 4)^2$

$x^2 - 6x + 9 + y^2 - 12y + 36 = x^2 + 6x + 9 + y^2 - 8y + 16$

$-12x - 4y + 20 = 0$

$\therefore 3x + y - 5 = 0.$

---

19. Determine whether the points (1, 5), (2, 3) and (−2, −11) are collinear.

Solution : Let A(1, 5), B(2, 3), C(−2, −11).

$AB = \sqrt{1+4} = \sqrt{5} \approx 2.24$

$BC = \sqrt{16+196} = \sqrt{212} \approx 14.56$

$AC = \sqrt{9+256} = \sqrt{265} \approx 16.28$

For collinear points, the sum of the two smaller distances equals the largest.

$AB + BC \approx 16.80 \neq 16.28 = AC$

$\therefore$ The points are not collinear.

---

20. The vertices of ΔABC are A(4, 6), B(1, 5) and C(7, 2). A line is drawn to intersect sides AB and AC at D and E respectively, such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$. Find the coordinates of D and E.

Solution : $\frac{AD}{AB} = \frac{1}{4} \Rightarrow AD : DB = 1 : 3$. Similarly $AE : EC = 1 : 3$.

$$D = \left(\frac{1\times1 + 3\times4}{4}, \frac{1\times5 + 3\times6}{4}\right) = \left(\frac{13}{4}, \frac{23}{4}\right)$$

$$E = \left(\frac{1\times7 + 3\times4}{4}, \frac{1\times2 + 3\times6}{4}\right) = \left(\frac{19}{4}, 5\right)$$

$\therefore D\left(\frac{13}{4}, \frac{23}{4}\right)$ and $E\left(\frac{19}{4}, 5\right)$.

Chapter 8 – Introduction to Trigonometry

1. If $\sin(A - B) = \frac{1}{2}$ and $\tan(A + B) = \sqrt{3}$, find the values of A and B.

Solution :

$$\sin(A - B) = \frac{1}{2} = \sin 30^\circ \Rightarrow A - B = 30^\circ \ \ldots(1)$$

$$\tan(A + B) = \sqrt{3} = \tan 60^\circ \Rightarrow A + B = 60^\circ \ \ldots(2)$$

Adding (1) and (2):

$$2A = 90^\circ \Rightarrow A = 45^\circ$$

$$B = 60^\circ - 45^\circ = 15^\circ$$

$\therefore A = 45^\circ, B = 15^\circ$.

---

2. Prove that $\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = \tan\theta + \cot\theta$.

Solution : Simplify the expression under the root:

$$\sec^2\theta + \operatorname{cosec}^2\theta = \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\ \cos^2\theta}$$

$$= \frac{1}{\sin^2\theta\ \cos^2\theta} \text{ (using } \sin^2\theta + \cos^2\theta = 1)$$

So

$$\text{LHS} = \frac{1}{\sin\theta\ \cos\theta}$$

Now simplify the RHS:

$$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\ \cos\theta} = \frac{1}{\sin\theta\ \cos\theta}$$

$\therefore$ LHS = RHS.

---

3. If $\sin\theta + \cos\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$, prove that $q(p^2 - 1) = 2p$.

Solution : Find $p^2 - 1$:

$$p^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 1 + 2\sin\theta\cos\theta$$

$$p^2 - 1 = 2\sin\theta\cos\theta$$

Write q in terms of sin and cos:

$$q = \frac{1}{\cos\theta} + \frac{1}{\sin\theta} = \frac{\sin\theta + \cos\theta}{\sin\theta\ \cos\theta} = \frac{p}{\sin\theta\ \cos\theta}$$

Multiply:

$$q(p^2 - 1) = [\frac{p}{\sin\theta\ \cos\theta}] \times 2\ \sin\theta\ \cos\theta = 2p$$

$\therefore q(p^2 - 1) = 2p.$

---

4. If $\sin\theta + \cos\theta = \sqrt{3}$, prove that $\tan\theta + \cot\theta = 1$.

Solution : Square both sides:

$\sin^2\theta + \cos^2\theta + 2\ \sin\theta\ \cos\theta = 3$

$1 + 2\ \sin\theta\ \cos\theta = 3 \Rightarrow \sin\theta\ \cos\theta = 1$

Now

$$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\ \cos\theta} = \frac{1}{\sin\theta\ \cos\theta} = \frac{1}{1}$$

$\therefore \tan\theta + \cot\theta = 1.$

---

5. Prove that $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$.

Solution : Take common factors:

$$LHS = \sin\theta \frac{1 - 2\sin^2\theta}{\cos\theta(2\cos^2\theta - 1)}$$

Use $\sin^2\theta = 1 - \cos^2\theta$ in the numerator bracket:

$1 - 2\sin^2\theta = 1 - 2(1 - \cos^2\theta) = 2\cos^2\theta - 1$

So the brackets cancel:

$$LHS = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$\therefore$ LHS = RHS.

---

6. Prove that $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$.

Solution : Write everything in terms of sin and cos:

$$1 + \cot A - \operatorname{cosec} A = \frac{\sin A + \cos A - 1}{\sin A}$$

$$1 + \tan A + \sec A = \frac{\cos A + \sin A + 1}{\cos A}$$

Multiply:

$$LHS = \frac{(\sin A + \cos A)^2 - 1}{\sin A\ \cos A}$$

$$= \frac{\sin^2 A + \cos^2 A + 2\ \sin A\ \cos A - 1}{\sin A\ \cos A}$$

$$= 2\ \sin A\ \frac{\cos A}{\sin A\ \cos A} = 2$$

$\therefore$ LHS = RHS.

---

7. Prove that $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$.

Solution : Expand both squares:

$(\sin A + \operatorname{cosec} A)^2 = \sin^2 A + \operatorname{cosec}^2 A + 2\ \sin A\ \operatorname{cosec} A = \sin^2 A + \operatorname{cosec}^2 A + 2$

$(\cos A + \sec A)^2 = \cos^2 A + \sec^2 A + 2$

Add:

$\text{LHS} = (\sin^2 A + \cos^2 A) + \operatorname{cosec}^2 A + \sec^2 A + 4$

$= 1 + (1 + \cot^2 A) + (1 + \tan^2 A) + 4$

$= 7 + \tan^2 A + \cot^2 A$

$\therefore$ LHS = RHS.

---

8. Prove that $(\operatorname{cosec}\theta - \cot\theta)^2 = \dfrac{1 - \cos\theta}{1 + \cos\theta}$.

Solution : Write in terms of sin and cos:

$$\operatorname{cosec}\theta - \cot\theta = \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} = \frac{1 - \cos\theta}{\sin\theta}$$

Square:

$$\text{LHS} = \frac{(1 - \cos\theta)^2}{\sin^2\theta} = \frac{(1 - \cos\theta)^2}{1 - \cos^2\theta}$$

$$= \frac{(1 - \cos\theta)^2}{(1 - \cos\theta)(1 + \cos\theta)}$$

$$= \frac{1 - \cos\theta}{1 + \cos\theta}$$

$\therefore$ LHS = RHS.

---

9. Prove that $\dfrac{\tan\theta}{1 - \cot\theta} + \dfrac{\cot\theta}{1 - \tan\theta} = 1 + \sec\theta\ \operatorname{cosec}\theta$.

Solution : Write in terms of sin and cos:

$$\frac{\tan\theta}{1 - \cot\theta} = \left(\frac{\sin\theta}{\cos\theta}\right) \times \frac{\sin\theta}{\sin\theta - \cos\theta} = \frac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)}$$

$$\frac{\cot\theta}{1 - \tan\theta} = \left(\frac{\cos\theta}{\sin\theta}\right) \times \frac{\cos\theta}{\cos\theta - \sin\theta} = -\frac{\cos^2\theta}{\sin\theta(\sin\theta - \cos\theta)}$$

Add over the common denominator $\sin\theta\ \cos\theta(\sin\theta - \cos\theta)$:

$$\text{LHS} = \frac{\sin^3\theta - \cos^3\theta}{\sin\theta\ \cos\theta(\sin\theta - \cos\theta)}$$

Use $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$:

$$= \frac{\sin^2\theta + \sin\theta\ \cos\theta + \cos^2\theta}{\sin\theta\ \cos\theta} = \frac{1 + \sin\theta\ \cos\theta}{\sin\theta\ \cos\theta}$$

$$= \frac{1}{\sin\theta\ \cos\theta} + 1 = 1 + \sec\theta\ \text{cosec}\theta$$

$\therefore$ LHS = RHS.

---

10. Prove that $\dfrac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \dfrac{1}{\sec\theta - \tan\theta}$.

Solution : Divide the numerator and denominator by $\cos\theta$:

$$\text{LHS} = \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$$

Replace 1 in the numerator by $\sec^2\theta - \tan^2\theta$:

$$\text{Numerator} = (\tan\theta + \sec\theta) - (\sec^2\theta - \tan^2\theta)$$

$$= (\tan\theta + \sec\theta) - (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)$$

$$= (\sec\theta + \tan\theta)(1 - \sec\theta + \tan\theta)$$

So

$$\text{LHS} = (\sec\theta + \tan\theta)\frac{1 - \sec\theta + \tan\theta}{\tan\theta - \sec\theta + 1} = \sec\theta + \tan\theta$$

Since $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$:

$$\sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}$$

$\therefore$ LHS = RHS.

---

11. Prove that $\sqrt{\dfrac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$.

Solution : Multiply the numerator and denominator inside the root by $(1 + \sin A)$:

$$\sqrt{\frac{(1 + \sin A)^2}{1 - \sin^2 A}} = \sqrt{\frac{(1 + \sin A)^2}{\cos^2 A}}$$

$$= \frac{1 + \sin A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A$$

$\therefore$ LHS = RHS.

---

12. If $\sec\theta + \tan\theta = p$, show that $\sin\theta = \dfrac{p^2 - 1}{p^2 + 1}$.

Solution : Work with the RHS, substituting $p = \sec\theta + \tan\theta$:

$p^2 = \sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta$

$p^2 - 1 = (\sec^2\theta - 1) + \tan^2\theta + 2\sec\theta\tan\theta = 2\tan^2\theta + 2\sec\theta\tan\theta = 2\tan\theta(\tan\theta + \sec\theta)$

$p^2 + 1 = \sec^2\theta + (1 + \tan^2\theta) + 2\sec\theta\tan\theta = 2\sec^2\theta + 2\sec\theta\tan\theta = 2\sec\theta(\sec\theta + \tan\theta)$

Divide:

$$\frac{p^2 - 1}{p^2 + 1} = \frac{\tan\theta}{\sec\theta} = \left(\frac{\sin\theta}{\cos\theta}\right) \times \cos\theta = \sin\theta$$

$\therefore \sin\theta = \dfrac{p^2 - 1}{p^2 + 1}$.

---

13. If $\cot\theta = \dfrac{7}{8}$, evaluate $\dfrac{(1 + \sin\theta)(1 - \sin\theta)}{(1 + \cos\theta)(1 - \cos\theta)}$.

Solution : Use $(a + b)(a - b) = a^2 - b^2$:

Numerator $= 1 - \sin^2\theta = \cos^2\theta$

Denominator $= 1 - \cos^2\theta = \sin^2\theta$

$$\text{Expression} = \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta = \left(\frac{7}{8}\right)^2$$

$\therefore$ Value $= \dfrac{49}{64}$.

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14. If $3\cot A = 4$, check whether $\dfrac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$.

Solution :

$$\cot A = \frac{4}{3} \Rightarrow \tan A = \frac{3}{4}$$

Take a right triangle with base 4 and perpendicular 3:

$$\text{hypotenuse} = \sqrt{9 + 16} = 5 \Rightarrow \sin A = \frac{3}{5},\ \cos A = \frac{4}{5}$$

LHS:

$$\frac{1 - \frac{9}{16}}{1 + \frac{9}{16}} = \frac{\frac{7}{16}}{\frac{25}{16}} = \frac{7}{25}$$

RHS:

$$\frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$\therefore$ Yes, LHS = RHS = $\frac{7}{25}$.

---

15. In ΔABC, right-angled at B, if $\tan A = \frac{1}{\sqrt{3}}$, find the value of $\sin A \cos C + \cos A \sin C$.

Solution :

$$\tan A = \frac{1}{\sqrt{3}} = \tan 30^\circ \Rightarrow \angle A = 30^\circ$$

$$\angle C = 180^\circ - 90^\circ - 30^\circ = 60^\circ$$

Substitute the standard values:

$$\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ = (\frac{1}{2})(\frac{1}{2}) + (\frac{\sqrt{3}}{2})(\frac{\sqrt{3}}{2})$$

$$= \frac{1}{4} + \frac{3}{4} = 1$$

$\therefore$ Value = 1.

---

16. Evaluate: $\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$.

Solution : Standard values:

$$\cos 60^\circ = \frac{1}{2}, \sec 30^\circ = \frac{2}{\sqrt{3}}, \tan 45^\circ = 1$$

Numerator:

$$5(\frac{1}{4}) + 4(\frac{4}{3}) - 1 = \frac{5}{4} + \frac{16}{3} - 1 = \frac{15 + 64 - 12}{12} = \frac{67}{12}$$

Denominator:

$$\sin^2 30^\circ + \cos^2 30^\circ = 1$$

$\therefore$ Value = $\frac{67}{12}$.

---

17. Prove that $\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{2\sin^2 A - 1}$.

Solution : Take the LCM:

$$\text{LHS} = \frac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{\sin^2 A - \cos^2 A}$$

Expand the numerator:

$(\sin^2A + \cos^2A + 2\sin A\cos A) + (\sin^2A + \cos^2A - 2\sin A\cos A) = 2$

Denominator:

$\sin^2A - \cos^2A = \sin^2A - (1 - \sin^2A) = 2\sin^2A - 1$

$\therefore$ LHS $= \dfrac{2}{2\sin^2A - 1}$ = RHS.

---

18. If $x = a\sin\theta$ and $y = b\tan\theta$, prove that $\dfrac{a^2}{x^2} - \dfrac{b^2}{y^2} = 1$.

Solution :

$\dfrac{a}{x} = \dfrac{1}{\sin\theta} = \operatorname{cosec}\theta \Rightarrow \dfrac{a^2}{x^2} = \operatorname{cosec}^2\theta$

$\dfrac{b}{y} = \dfrac{1}{\tan\theta} = \cot\theta \Rightarrow \dfrac{b^2}{y^2} = \cot^2\theta$

Subtract:

$\dfrac{a^2}{x^2} - \dfrac{b^2}{y^2} = \operatorname{cosec}^2\theta - \cot^2\theta = 1$ (identity $1 + \cot^2\theta = \operatorname{cosec}^2\theta$)

$\therefore \dfrac{a^2}{x^2} - \dfrac{b^2}{y^2} = 1.$

---

19. Prove that $\dfrac{1 + \tan^2A}{1 + \cot^2A} = \left[\dfrac{1 - \tan A}{1 - \cot A}\right]^2 = \tan^2A.$

Solution : First expression:

$\dfrac{1 + \tan^2A}{1 + \cot^2A} = \dfrac{\sec^2A}{\operatorname{cosec}^2A} = \left(\dfrac{1}{\cos^2A}\right) \times \sin^2A = \tan^2A$

Second expression: write $\cot A = \dfrac{1}{\tan A}$:

$1 - \cot A = \dfrac{\tan A - 1}{\tan A}$

$\dfrac{1 - \tan A}{1 - \cot A} = (1 - \tan A) \times \dfrac{\tan A}{\tan A - 1} = -\tan A$

$\left[\dfrac{1 - \tan A}{1 - \cot A}\right]^2 = \tan^2A$

$\therefore$ Both expressions equal $\tan^2A$.

---

20. Prove that $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0.$

Solution : Let $s = \sin^2\theta$, $c = \cos^2\theta$, so $s + c = 1$.

$\sin^6\theta + \cos^6\theta = s^3 + c^3 = (s + c)^3 - 3sc(s + c) = 1 - 3sc$

$\sin^4\theta + \cos^4\theta = s^2 + c^2 = (s + c)^2 - 2sc = 1 - 2sc$

Substitute:

$2(1 - 3sc) - 3(1 - 2sc) + 1 = 2 - 6sc - 3 + 6sc + 1 = 0$

$\therefore$ LHS = 0 = RHS.

---

21. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

Solution : From the given relation:

$\sin\theta = \sqrt{2}\cos\theta - \cos\theta = (\sqrt{2} - 1)\cos\theta$

Multiply both sides by $(\sqrt{2} + 1)$:

$(\sqrt{2} + 1)\sin\theta = (2 - 1)\cos\theta = \cos\theta$

$\cos\theta = \sqrt{2}\sin\theta + \sin\theta$

$\therefore \cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

---

22. If $\sin\theta = \dfrac{a^2 - b^2}{a^2 + b^2}$, find the values of $\cos\theta$ and $\tan\theta$.

Solution : Take a right triangle with perpendicular = $a^2 - b^2$ and hypotenuse = $a^2 + b^2$.

Base by Pythagoras theorem:

$\text{base}^2 = (a^2 + b^2)^2 - (a^2 - b^2)^2 = 4a^2b^2 \Rightarrow \text{base} = 2ab$

Hence:

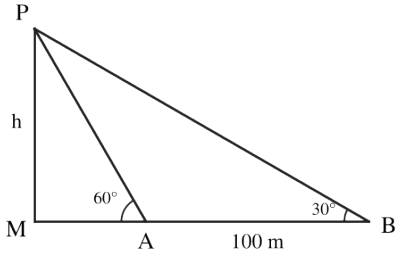
$$\cos\theta = \frac{2ab}{a^2 + b^2}$$

$$\tan\theta = \frac{a^2 - b^2}{2ab}$$

$$\therefore \cos\theta = \frac{2ab}{a^2 + b^2}, \tan\theta = \frac{a^2 - b^2}{2ab}.$$

Chapter 9 – Some Applications of Trigonometry

1. One observer estimates the angle of elevation of the basket of a hot-air balloon to be 60°, while another observer 100 m away (further from the balloon) estimates it to be 30°. Find (a) the height of the basket from the ground, (b) the distance of the basket from the first observer's eye, (c) the horizontal distance of the second observer from the basket.



Solution : Let the basket be at P with foot M on the ground, first observer at A and second at B, with AB = 100 m, PM = h and AM = x.

In right ΔPMA:

$$\tan 60° = \frac{h}{x} \Rightarrow h = \sqrt{3}\,x \ldots(1)$$

In right ΔPMB:

$$\tan 30° = \frac{h}{x + 100} \Rightarrow \sqrt{3}\,h = x + 100 \ldots(2)$$

Substitute (1) in (2):

$3x = x + 100 \Rightarrow x = 50$ m

(a) $h = 50\sqrt{3} = 50 \times 1.732 = 86.60$ m

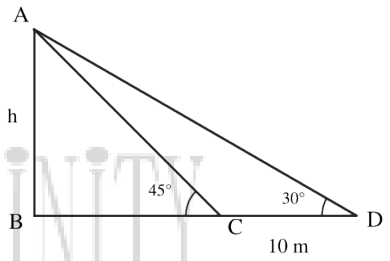
(b) In ΔPMA, $AP = \frac{AM}{\cos 60°} = \frac{50}{\frac{1}{2}} = 100$ m

(c) $BM = x + 100 = 150$ m

∴ (a) 86.60 m (b) 100 m (c) 150 m.

---

2. The shadow of a vertical tower on level ground increases by 10 m when the altitude of the sun changes from 45° to 30°. Find the height of the tower, correct to two decimal places.



Solution : Let the tower be AB = h; the shadow is BC when the sun's altitude is 45° and BD when it is 30°, with CD = 10 m.

In ΔABC:

$$\tan 45° = \frac{h}{BC} \Rightarrow BC = h$$

In ΔABD:

$$\tan 30° = \frac{h}{BD} \Rightarrow BD = \sqrt{3}\,h$$

Given BD − BC = 10:

$$\sqrt{3}\,h - h = 10 \Rightarrow h = \frac{10}{\sqrt{3} - 1}$$

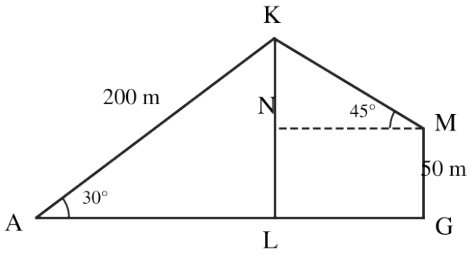
Rationalise:

$$h = \frac{10(\sqrt{3} + 1)}{2} = 5(\sqrt{3} + 1) = 5 \times 2.732$$

∴ Height of the tower = 13.66 m.

---

3. Manish is standing on level ground and observes a kite flying 200 m away from him (length of string) at an angle of elevation of 30°. Mahesh, standing on the roof of a 50 m high building on the opposite side of the kite, observes the same kite at an angle of elevation of 45°. Find the distance between the kite and Mahesh.



Solution : Let K be the kite, with the string from Manish (A) of length AK = 200 m.

Height of the kite above the ground:

$$KL = AK \sin 30° = 200 \times \frac{1}{2} = 100 \text{ m}$$

Mahesh (M) is on a 50 m roof, so the height of the kite above his eye level:

$$KN = 100 - 50 = 50 \text{ m}$$

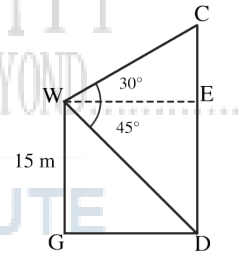
In right ΔKNM:

$$\sin 45° = \frac{KN}{KM} \Rightarrow KM = \frac{50}{\frac{1}{\sqrt{2}}} = 50\sqrt{2}$$

$\therefore$ Distance between the kite and Mahesh = $50\sqrt{2}$ m (≈ 70.71 m).

---

4. From a window 15 m high above the ground in a street, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are 30° and 45° respectively. Find the height of the opposite house. (Use $\sqrt{3} = 1.732$)



Solution : Let the window be W at height 15 m, the opposite house be CD, and WE be horizontal to the house.

Angle of depression of the foot D is 45°:

$$\tan 45° = \frac{ED}{WE} = \frac{15}{WE} \Rightarrow WE = 15 \text{ m (width of the street)}$$

Angle of elevation of the top C is 30°:

$$\tan 30° = \frac{CE}{WE} \Rightarrow CE = \frac{15}{\sqrt{3}} = 5\sqrt{3} = 8.66 \text{ m}$$

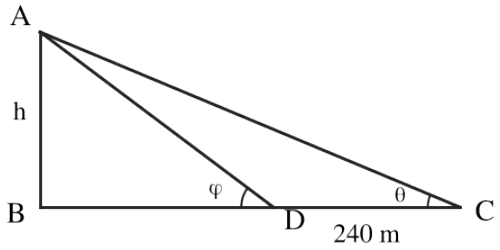
Height of the house:

$$CD = CE + ED = 8.66 + 15$$

$\therefore$ Height of the opposite house = 23.66 m.

---

5. The angle of elevation θ of the top of a lighthouse, as seen by a person on the ground, is such that $\tan\theta = \frac{5}{12}$. When the person moves a distance of 240 m towards the lighthouse, the angle of elevation becomes φ, such that $\tan\varphi = \frac{3}{4}$. Find the height of the lighthouse.



Solution : Let the lighthouse be AB = h, first position C, second position D, CD = 240 m, BD = x.

In ΔABC:

$$\tan\theta = \frac{h}{x+240} = \frac{5}{12} \Rightarrow x + 240 = \frac{12h}{5} \quad \ldots(1)$$

In ΔABD:

$$\tan\varphi = \frac{h}{x} = \frac{3}{4} \Rightarrow x = \frac{4h}{3} \quad \ldots(2)$$

Subtract (2) from (1):

$$240 = \frac{12h}{5} - \frac{4h}{3} = \frac{36h - 20h}{15} = \frac{16h}{15}$$

$$h = 240 \times \frac{15}{16} = 225$$

∴ Height of the lighthouse = 225 m.

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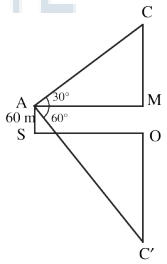
6. The angle of elevation of a cloud from a point 60 m above a lake is 30° and the angle of depression of the reflection of the cloud in the lake is 60°. Find the height of the cloud from the surface of the lake.



Solution : Let the cloud C be H m above the lake, A the point 60 m above the lake, C′ the reflection (H m below the surface), and AM the horizontal distance x.

Angle of elevation of C:

$$\tan 30° = \frac{H - 60}{x} \Rightarrow x = \sqrt{3}(H - 60) \quad \ldots(1)$$

Angle of depression of C′:

$$\tan 60° = \frac{H + 60}{x} \Rightarrow x = \frac{H + 60}{\sqrt{3}} \quad \ldots(2)$$

Equate (1) and (2):

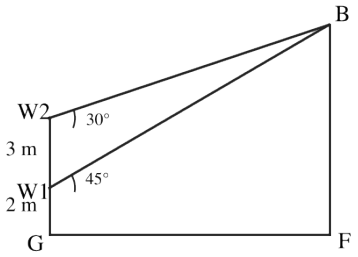
$$3(H - 60) = H + 60$$

$2H = 240 \Rightarrow H = 120$

$\therefore$ Height of the cloud = 120 m.

---

7. There are two windows in a house. One window is at a height of 2 m above the ground and the other is 3 m vertically above the lower window. Ram and Shyam are sitting inside the two windows. At an instant, the angles of elevation of a balloon from these windows are observed as 45° and 30° respectively. Find the height of the balloon from the ground. ($\sqrt{3}$ = 1.732)



Solution : Let the balloon be H m high and its horizontal distance from the house be x m.

Lower window (2 m), angle 45°:

$$\tan 45° = \frac{H - 2}{x} \Rightarrow x = H - 2 \ldots(1)$$

Upper window (5 m), angle 30°:

$$\tan 30° = \frac{H - 5}{x} \Rightarrow x = \sqrt{3}(H - 5) \ldots(2)$$

Equate (1) and (2):

$$H - 2 = \sqrt{3}H - 5\sqrt{3}$$

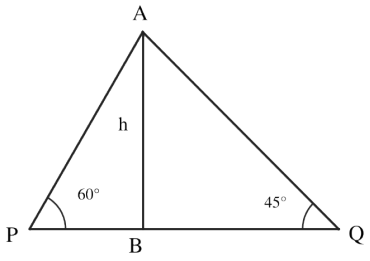
$$H(\sqrt{3} - 1) = 5\sqrt{3} - 2$$

$$H = \frac{5\sqrt{3} - 2}{\sqrt{3} - 1} = \frac{8.66 - 2}{0.732} \approx 9.098$$

$\therefore$ Height of the balloon $\approx$ 9.098 m.

---

8. Two ships are sailing in the sea on either side of a lighthouse. The angles of depression of the two ships as observed from the top of the lighthouse are 60° and 45° respectively. If the distance between the ships is $\frac{100(\sqrt{3}+1)}{\sqrt{3}}$ m, find the height of the lighthouse.



Solution : Let the lighthouse be AB = h, with ships P and Q on either side of the foot B.

Angle of depression = angle of elevation from the ship (alternate angles).

In ΔABP: $\tan 60° = \frac{h}{BP} \Rightarrow BP = \frac{h}{\sqrt{3}}$

In ΔABQ: $\tan 45° = \frac{h}{BQ} \Rightarrow BQ = h$

Distance between the ships:

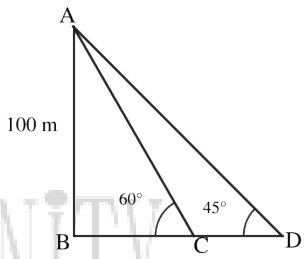
$$BP + BQ = \frac{h}{\sqrt{3}} + h = h\frac{1+\sqrt{3}}{\sqrt{3}}$$

$$h\frac{\sqrt{3}+1}{\sqrt{3}} = \frac{100(\sqrt{3}+1)}{\sqrt{3}}$$

$\therefore$ Height of the lighthouse = 100 m.

---

9. A man in a boat rowing away from a lighthouse 100 m high takes 2 minutes to change the angle of elevation of the top of the lighthouse from 60° to 45°. Find the speed of the boat.



Solution : Let the lighthouse be AB = 100 m; the boat moves from C to D.

In ΔABC: $\tan 60° = \frac{100}{BC} \Rightarrow BC = \frac{100}{\sqrt{3}}$

In ΔABD: $\tan 45° = \frac{100}{BD} \Rightarrow BD = 100$

Distance travelled:

$$CD = 100 - \frac{100}{\sqrt{3}} = \frac{100(\sqrt{3}-1)}{\sqrt{3}}$$

Speed = distance ÷ time (2 min):

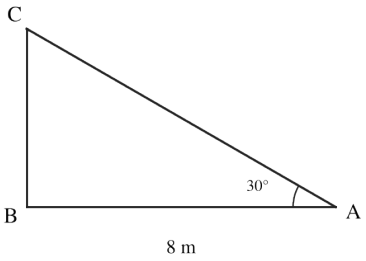
$$= \frac{50(\sqrt{3}-1)}{\sqrt{3}} = \frac{50(3-\sqrt{3})}{3} \text{ m/min}$$

$$\approx 50 \times \frac{1.268}{3} \approx 21.13 \text{ m/min}$$

$\therefore$ Speed of the boat = $(\frac{50}{3})(3-\sqrt{3})$ m/min $\approx$ 21.13 m/min.

---

10. A tree breaks due to a storm and the broken part bends so that the top of the tree touches the ground, making an angle of 30° with it. The distance between the foot of the tree and the point where the top touches the ground is 8 m. Find the height of the tree.



Solution : Let BC be the standing part and CA the broken part touching the ground at A, with AB = 8 m and ∠BAC = 30°.

In right ΔABC:

$$\tan 30° = \frac{BC}{AB} \Rightarrow BC = \frac{8}{\sqrt{3}} \text{ m}$$

$$\cos 30° = \frac{AB}{AC} \Rightarrow AC = \frac{8}{\frac{\sqrt{3}}{2}} = \frac{16}{\sqrt{3}} \text{ m}$$

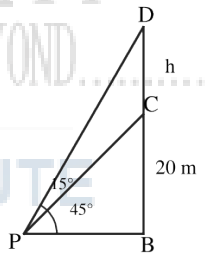
Height of the tree:

$$BC + AC = \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = \frac{24}{\sqrt{3}} = 8\sqrt{3} \text{ m}$$

∴ Height of the tree = $8\sqrt{3}$ m ≈ 13.86 m.

---

11. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.



Solution : Let the building be BC = 20 m, the tower CD = h, and the point P at distance x from B.

$$\tan 45° = \frac{BC}{PB} \Rightarrow x = 20 \text{ m}$$

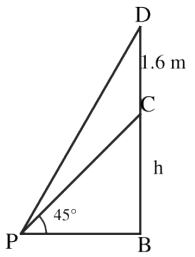
$$\tan 60° = \frac{BD}{PB} \Rightarrow 20 + h = 20\sqrt{3}$$

$$h = 20(\sqrt{3} - 1) = 20 \times 0.732$$

∴ Height of the tower = 14.64 m.

---

12. A statue 1.6 m tall stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.



Solution : Let the pedestal be BC = h, the statue CD = 1.6 m, and the point P at distance x from B.

$$\tan 45° = \frac{h}{x} \Rightarrow x = h$$

$$\tan 60° = \frac{h + 1.6}{x} \Rightarrow h + 1.6 = \sqrt{3}\,h$$

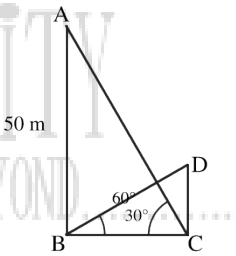
$$h(\sqrt{3} - 1) = 1.6$$

$$h = \frac{1.6}{\sqrt{3} - 1} = 1.\frac{6(\sqrt{3} + 1)}{2} = 0.8(\sqrt{3} + 1)$$

$\therefore$ Height of the pedestal = $0.8(\sqrt{3} + 1)$ m $\approx$ 2.19 m.

---

13. The angle of elevation of the top of a building from the foot of a tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.



Solution : Let the tower be AB = 50 m, the building CD = h, and BC = d (distance between their feet).

From the foot C of the building:

$$\tan 60° = \frac{AB}{BC} \Rightarrow d = \frac{50}{\sqrt{3}} \text{ m}$$

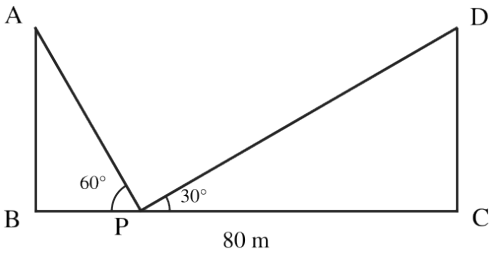
From the foot B of the tower:

$$\tan 30° = \frac{CD}{BC} \Rightarrow h = \frac{d}{\sqrt{3}} = \frac{50}{3}$$

$\therefore$ Height of the building = $\frac{50}{3}$ m = 16⅔ m.

---

14. Two poles of equal heights are standing opposite each other on either side of a road 80 m wide. From a point between them on the road, the angles of elevation of the tops of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles.



Solution : Let each pole be h m and the point be x m from the first pole (80 − x from the second).

$$\tan 60° = \frac{h}{x} \Rightarrow h = \sqrt{3}\,x$$

$$\tan 30° = \frac{h}{80 - x} \Rightarrow h = \frac{80 - x}{\sqrt{3}}$$

Equate:

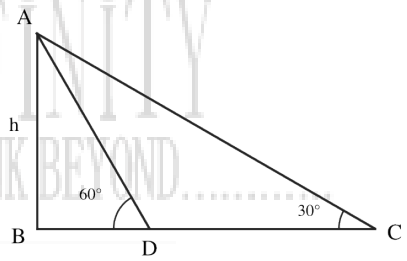
$$3x = 80 - x \Rightarrow x = 20 \text{ m}$$

$$h = 20\sqrt{3} \approx 34.64 \text{ m}$$

$\therefore$ Height = $20\sqrt{3}$ m ≈ 34.64 m; distances 20 m and 60 m.

---

15. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.



Solution : Let the tower be AB = h; the car moves from C to D.

$$BC = \frac{h}{\tan 30°} = \sqrt{3}\,h, \quad BD = \frac{h}{\tan 60°} = \frac{h}{\sqrt{3}}$$

Distance covered in 6 s:

$$CD = \sqrt{3}\,h - \frac{h}{\sqrt{3}} = \frac{2h}{\sqrt{3}}$$

$$\text{Speed} = \left(\frac{2h}{\sqrt{3}}\right) \div 6 = \frac{h}{3\sqrt{3}}$$

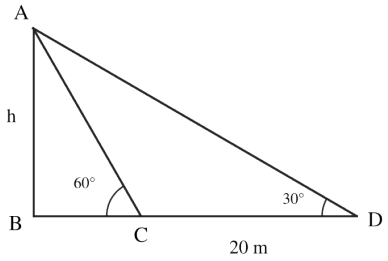
Time for the remaining distance BD:

$$\left(\frac{h}{\sqrt{3}}\right) \div \left(\frac{h}{3\sqrt{3}}\right) = 3 \text{ s}$$

$\therefore$ The car takes 3 more seconds.

---

16. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation is 30°. Find the height of the tower and the width of the canal.



Solution : Let the tower be AB = h and the width of the canal BC = w; D is 20 m beyond C.

$$\tan 60° = \frac{h}{w} \Rightarrow h = \sqrt{3}\,w$$

$$\tan 30° = \frac{h}{w + 20} \Rightarrow \sqrt{3}\,h = w + 20$$

Substitute:

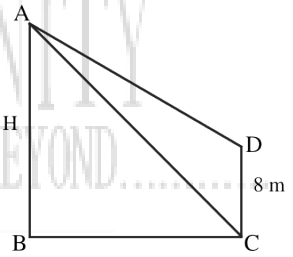
$3w = w + 20 \Rightarrow w = 10$ m

$h = 10\sqrt{3} \approx 17.32$ m

$\therefore$ Height = $10\sqrt{3}$ m ≈ 17.32 m; width of the canal = 10 m.

---

17. The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storeyed building are 30° and 45° respectively. Find the height of the multi-storeyed building and the distance between the two buildings.



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Solution : Let the multi-storeyed building be AB = H and the distance between the buildings be d.

Angle of depression of the bottom is 45°:

$$\tan 45° = \frac{H}{d} \Rightarrow d = H$$

Angle of depression of the top (8 m building) is 30°:

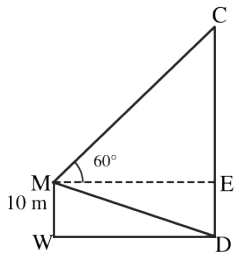
$$\tan 30° = \frac{H - 8}{d} \Rightarrow H - 8 = \frac{H}{\sqrt{3}}$$

$$H\left(1 - \frac{1}{\sqrt{3}}\right) = 8 \Rightarrow H = \frac{8\sqrt{3}}{\sqrt{3} - 1} = 4\sqrt{3}\,(\sqrt{3} + 1) = 12 + 4\sqrt{3}$$

$\therefore$ Height = distance = $(12 + 4\sqrt{3})$ m ≈ 18.93 m.

---

18. A man on the deck of a ship is 10 m above the water level. He observes that the angle of elevation of the top of a hill is 60° and the angle of depression of the base of the hill is 30°. Find the distance of the hill from the ship and the height of the hill.



Solution : Let the man be at M (10 m above water) and the hill be CD; ME is horizontal to the hill.

Angle of depression of the base D is 30°:

$$\tan 30° = \frac{10}{ME} \Rightarrow ME = 10\sqrt{3} \text{ m} \approx 17.32 \text{ m}$$

Angle of elevation of the top C is 60°:

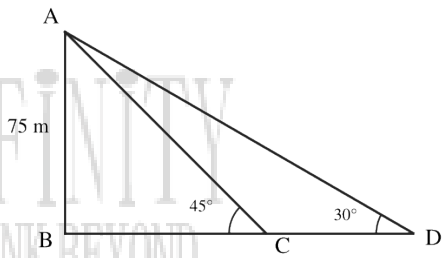
$CE = ME \tan 60° = 10\sqrt{3} \times \sqrt{3} = 30$ m

Height of the hill = CE + ED = 30 + 10 = 40 m

∴ Distance = $10\sqrt{3}$ m ≈ 17.32 m; height of the hill = 40 m.

---

19. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.



Solution : Let the lighthouse be AB = 75 m, with ships C (nearer) and D (farther).

$$\tan 45° = \frac{75}{BC} \Rightarrow BC = 75 \text{ m}$$

$$\tan 30° = \frac{75}{BD} \Rightarrow BD = 75\sqrt{3} \text{ m}$$

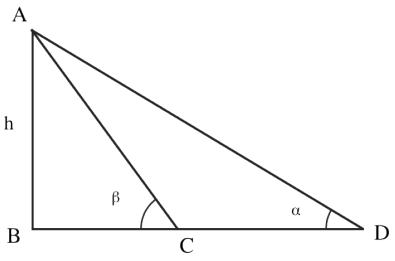
Distance between the ships:

$CD = 75\sqrt{3} - 75 = 75(\sqrt{3} - 1)$

∴ Distance = $75(\sqrt{3} - 1)$ m ≈ 54.9 m.

---

20. From the top of a tower h m high, the angles of depression of two objects, which are in line with the foot of the tower, are α and β (β > α). Find the distance between the two objects.



Solution : Let the tower be AB = h, with objects C (nearer, angle β) and D (farther, angle α).

$$\tan \beta = \frac{h}{BC} \Rightarrow BC = h \cot \beta$$

$$\tan\alpha = \frac{h}{BD} \Rightarrow BD = h\cot\alpha$$

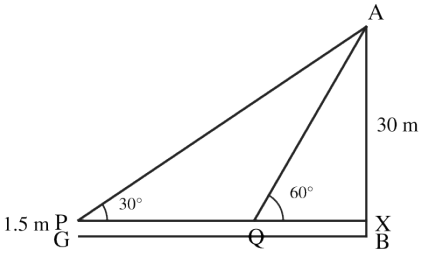
Distance between the objects:

$$CD = BD - BC = h\cot\alpha - h\cot\beta$$

$\therefore$ Distance = $h(\cot\alpha - \cot\beta)$.

---

21. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.



Solution : Height of the building above the boy's eyes:

$$30 - 1.5 = 28.5 \text{ m}$$

Horizontal distances from the building:

at 30°: $\frac{28.5}{\tan 30^\circ} = 28.5\sqrt{3}$ m

at 60°: $\frac{28.5}{\tan 60^\circ} = \frac{28.5}{\sqrt{3}}$ m

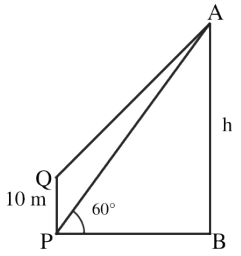
Distance walked:

$$28.5\sqrt{3} - \frac{28.5}{\sqrt{3}} = 28.5 \times \frac{2}{\sqrt{3}} = \frac{57}{\sqrt{3}} = 19\sqrt{3}$$

$\therefore$ Distance walked = $19\sqrt{3}$ m ≈ 32.91 m.

---

22. The angle of elevation of the top of a vertical tower from a point on the ground is 60°. From another point 10 m vertically above the first, its angle of elevation is 45°. Find the height of the tower.



Solution : Let the tower be AB = h at horizontal distance x from the point P; Q is 10 m above P.

From P: $\tan 60^\circ = \frac{h}{x} \Rightarrow h = \sqrt{3}\,x$

From Q: $\tan 45^\circ = \frac{h-10}{x} \Rightarrow h - 10 = x$

Substitute:

$$\sqrt{3}\,x - 10 = x \Rightarrow x = \frac{10}{\sqrt{3}-1} = 5(\sqrt{3}+1)$$

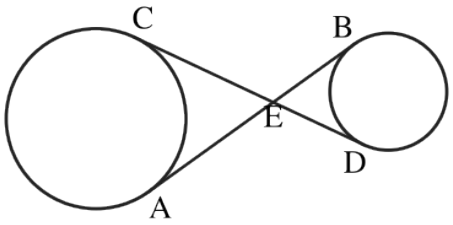
$h = 5\sqrt{3}(\sqrt{3} + 1) = 15 + 5\sqrt{3}$

$\therefore$ Height of the tower = $(15 + 5\sqrt{3})$ m $\approx$ 23.66 m.

Note: Case-study questions are recommended from this chapter for **CBSE Board 2026-27**.
Practise them here: **Class X Case Studies (click here)**

Chapter 10 – Circles

1. In the figure, common tangents AB and CD to two circles intersect at E. Prove that AB = CD.



Solution : Lengths of tangents drawn from an external point to a circle are equal.

From E to the first circle:

EA = EC …(1)

From E to the second circle:

EB = ED …(2)

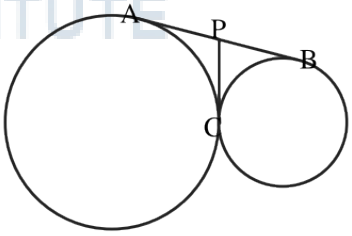
Adding (1) and (2):

EA + EB = EC + ED

$\therefore$ AB = CD.

---

2. Two circles touch each other externally at C. Prove that the common tangent at C bisects the other two (direct) common tangents.



Solution : Let AB be a direct common tangent touching the circles at A and B, and let the common tangent at C meet AB at P.

Tangents from P to the first circle:

PA = PC …(1)

Tangents from P to the second circle:

PB = PC …(2)

From (1) and (2):

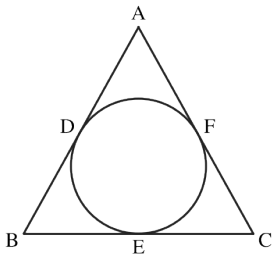
PA = PB, i.e. P is the mid-point of AB.

The same argument applies to the other direct common tangent.

$\therefore$ The common tangent at C bisects both direct common tangents.

---

3. A circle is inscribed in ΔABC, touching AB at D, BC at E and AC at F. If AB = AC, prove that BE = CE.



Solution : Tangents from an external point are equal:

AD = AF, BD = BE, CE = CF

Given AB = AC:

AD + DB = AF + FC

Since AD = AF:

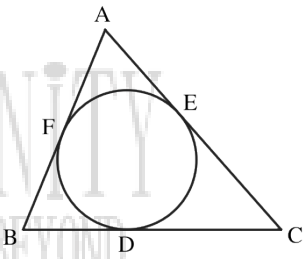
DB = FC

Replace DB by BE and FC by CE:

∴ BE = CE.

---

4. The incircle of ΔABC touches BC, CA and AB at D, E and F respectively. Show that AF + BD + CE = AE + BF + CD = ½ (perimeter of ΔABC).



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Solution : Tangents from an external point are equal:

AF = AE, BD = BF, CE = CD …(1)

Adding these:

AF + BD + CE = AE + BF + CD …(2)

Perimeter of ΔABC:

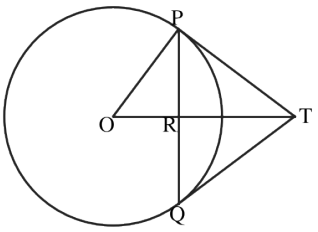
AB + BC + CA = (AF + BF) + (BD + CD) + (CE + AE)

= (AF + BD + CE) + (AE + BF + CD) = 2(AF + BD + CE) [using (2)]

∴ AF + BD + CE = AE + BF + CD = ½ (perimeter of ΔABC).

---

5. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at T. Find the length of TP.



Solution : OT is the perpendicular bisector of PQ; let it meet PQ at R.

PR = 4 cm, OP = 5 cm

In right ΔORP:

$OR = \sqrt{5^2 - 4^2} = 3$ cm

Let TR = y and TP = x. In right ΔTRP and right ΔTPO ($\angle OPT = 90°$):

$x^2 = y^2 + 16$ …(1)

$OT^2 = OP^2 + TP^2 \Rightarrow (y + 3)^2 = 25 + x^2$ …(2)

Substitute (1) in (2):

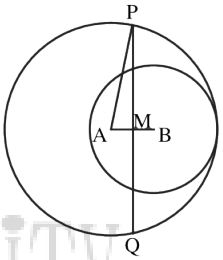
$$y^2 + 6y + 9 = 25 + y^2 + 16 \Rightarrow 6y = 32 \Rightarrow y = \frac{16}{3}$$

$$x^2 = \frac{256}{9} + 16 = \frac{400}{9} \Rightarrow x = \frac{20}{3}$$

$\therefore TP = \frac{20}{3}$ cm.

---

6. Two circles with centres A and B and radii 5 cm and 3 cm touch each other internally. If the perpendicular bisector of AB meets the bigger circle at P and Q, find PQ.



Solution : For circles touching internally, distance between centres = difference of radii:

AB = 5 − 3 = 2 cm

Let the perpendicular bisector meet AB at M:

AM = 1 cm, AP = 5 cm (radius of the bigger circle)

In right ΔAMP:

$PM = \sqrt{5^2 - 1^2} = \sqrt{24} = 2\sqrt{6}$ cm

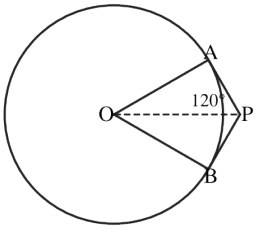
PQ is a chord bisected by AB at M:

$PQ = 2 \times PM = 4\sqrt{6}$ cm

$\therefore PQ = 4\sqrt{6}$ cm.

---

7. Two tangents making an angle of 120° with each other are drawn to a circle of radius 6 cm. Show that the length of each tangent is $2\sqrt{3}$ cm.



Solution : Let PA and PB be the tangents from P with $\angle APB = 120°$, and O the centre.

OP bisects $\angle APB$:

$\angle APO = 60°$

In right ΔOAP ($\angle OAP = 90°$, radius $\perp$ tangent):

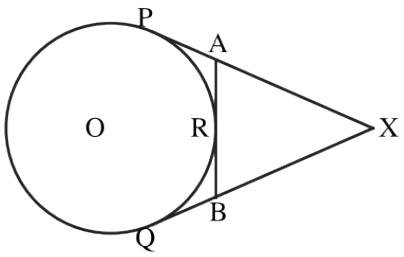
$$\tan 60° = \frac{OA}{PA} \Rightarrow \sqrt{3} = \frac{6}{PA}$$

$$PA = \frac{6}{\sqrt{3}} = 2\sqrt{3}$$

$\therefore$ Length of each tangent = $2\sqrt{3}$ cm.

---

8. XP and XQ are tangents from X to the circle with centre O. R is a point on the circle and AB is the tangent at R, with A on XP and B on XQ. Prove that XA + AR = XB + BR.



Solution : Tangents from an external point are equal:

XP = XQ, AP = AR, BQ = BR

Now

XA + AR = XA + AP = XP

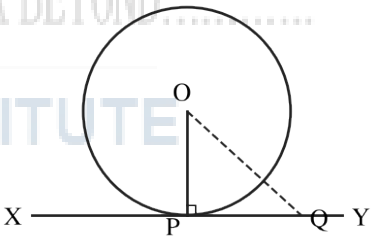
XB + BR = XB + BQ = XQ

Since XP = XQ:

$\therefore$ XA + AR = XB + BR.

---

9. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.



Solution : Given: A circle with centre O and a tangent XY at the point P.

To prove: OP $\perp$ XY.

Take any point Q on XY other than P and join OQ.

Q lies outside the circle (a tangent meets the circle at only one point), so

OQ > OP (OP is a radius)

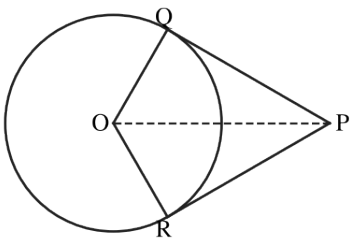
This is true for every point Q on XY other than P, so OP is the shortest distance from O to XY.

The shortest segment from a point to a line is the perpendicular.

$\therefore$ OP $\perp$ XY.

---

10. Prove that the lengths of the tangents drawn from an external point to a circle are equal.



Solution : Given: PQ and PR are tangents from P to a circle with centre O.

To prove: PQ = PR.

Join OP, OQ, OR. In right triangles OQP and ORP:

$\angle OQP = \angle ORP = 90°$ (radius $\perp$ tangent)

OQ = OR (radii)

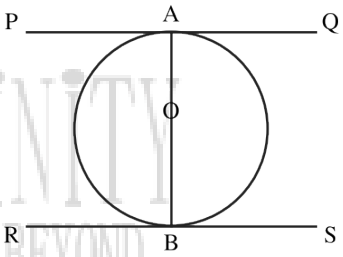
OP = OP (common hypotenuse)

$\Delta OQP \cong \Delta ORP$ (RHS congruence)

$\therefore$ PQ = PR (CPCT).

---

11. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.



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Solution : Let AB be a diameter of a circle with centre O, and PQ, RS be the tangents at A and B.

Radius $\perp$ tangent at the point of contact:

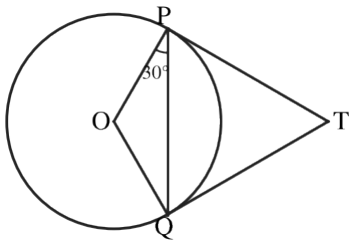
$\angle OAQ = 90°$ and $\angle OBR = 90°$

These are alternate angles formed by the transversal AB with PQ and RS, and they are equal.

$\therefore$ PQ $\parallel$ RS.

---

12. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.



Solution : Let PA and PB be tangents from P to a circle with centre O.

In quadrilateral OAPB:

$\angle OAP = \angle OBP = 90°$ (radius $\perp$ tangent)

Angle sum of a quadrilateral:

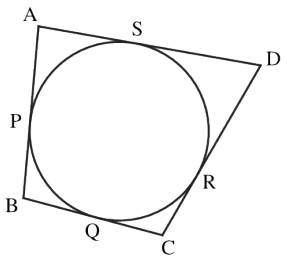
$\angle OAP + \angle APB + \angle OBP + \angle AOB = 360°$

$90° + \angle APB + 90° + \angle AOB = 360°$

$\therefore \angle APB + \angle AOB = 180°$.

---

13. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC.



Solution : Let the circle touch AB, BC, CD, DA at P, Q, R, S.

Tangents from an external point are equal:

$AP = AS, BP = BQ, CR = CQ, DR = DS$

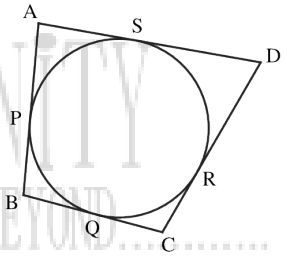
Add:

$(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$

$\therefore AB + CD = AD + BC$.

---

14. Prove that the parallelogram circumscribing a circle is a rhombus.



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Solution : Let ABCD be a parallelogram circumscribing a circle.

For a quadrilateral circumscribing a circle (equal tangents):

$AB + CD = AD + BC$

In a parallelogram, AB = CD and AD = BC:

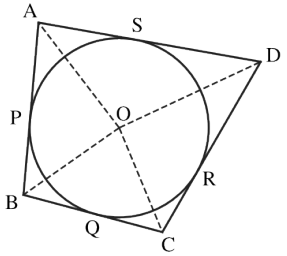
$2AB = 2BC \Rightarrow AB = BC$

So all four sides are equal: AB = BC = CD = DA.

$\therefore$ ABCD is a rhombus.

---

15. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.



Solution : Let the circle with centre O touch AB, BC, CD, DA at P, Q, R, S.

$\Delta OAP \cong \Delta OAS$ (RHS), so $\angle 1 = \angle 2$. Similarly, at each vertex the angles at O pair up:

$\angle AOP = \angle AOS = \angle 1, \angle BOP = \angle BOQ = \angle 2$

$\angle COQ = \angle COR = \angle 3, \angle DOR = \angle DOS = \angle 4$

Sum of angles around O:

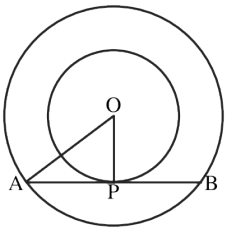
$2(\angle 1 + \angle 2 + \angle 3 + \angle 4) = 360° \Rightarrow (\angle 1 + \angle 2) + (\angle 3 + \angle 4) = 180°$

But $\angle AOB = \angle 1 + \angle 2$ and $\angle COD = \angle 3 + \angle 4$.

$\therefore \angle AOB + \angle COD = 180°$ (similarly $\angle BOC + \angle AOD = 180°$).

---

16. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.



Solution : Let AB be the chord of the larger circle touching the smaller circle at P, with centre O.

$OP \perp AB$ (radius $\perp$ tangent), and OP bisects AB

In right ΔOPA:

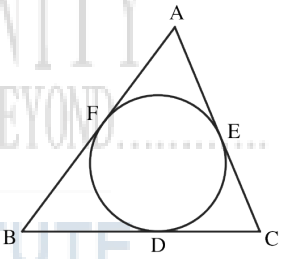
$AP = \sqrt{OA^2 - OP^2} = \sqrt{25 - 9} = 4$ cm

$AB = 2 \times AP = 8$ cm

$\therefore$ Length of the chord = 8 cm.

---

17. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively. Find the sides AB and AC.



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Solution : Let the circle touch AB at F and AC at E, and let $AF = AE = x$ (equal tangents).

$BF = BD = 8$, $CE = CD = 6$

$AB = x + 8$, $AC = x + 6$, $BC = 14$, $s = x + 14$

Area by Heron's formula:

$\sqrt{(x + 14)(x)(6)(8)} = \sqrt{48x(x + 14)}$

Area as the sum of three triangles with height r = 4:

$\frac{1}{2} \times 4 \times (AB + BC + CA) = 4s = 4(x + 14)$

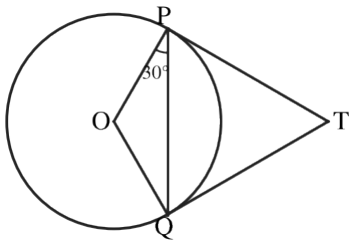
Equate and square:

$48x(x + 14) = 16(x + 14)^2 \Rightarrow 3x = x + 14 \Rightarrow x = 7$

$\therefore$ AB = 15 cm, AC = 13 cm.

---

18. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∠PTQ = 2∠OPQ.



Solution : Let ∠PTQ = θ. TP = TQ, so ΔTPQ is isosceles:

$$\angle TPQ = \angle TQP = \frac{180° - \theta}{2} = 90° - \frac{\theta}{2}$$

Radius ⊥ tangent:

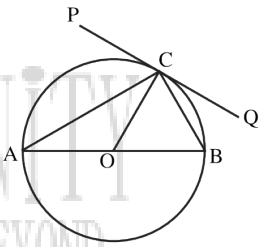
$$\angle OPT = 90°$$

$$\angle OPQ = \angle OPT - \angle TPQ = 90° - (90° - \frac{\theta}{2}) = \frac{\theta}{2}$$

∴ ∠PTQ = 2∠OPQ.

---

19. In the figure, PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and ∠CAB = 30°, find ∠PCA.



Solution : OA = OC (radii), so ΔOAC is isosceles:

$$\angle OCA = \angle OAC = 30°$$

Radius ⊥ tangent at C:

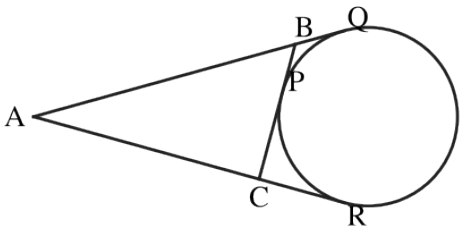
$$\angle OCP = 90°$$

$$\angle PCA = \angle OCP - \angle OCA = 90° - 30° = 60°$$

∴ ∠PCA = 60°.

---

20. A circle touches the side BC of ΔABC at P and touches AB and AC produced at Q and R respectively. Show that AQ = ½ (perimeter of ΔABC).



Solution : Tangents from an external point are equal:

AQ = AR, BQ = BP, CP = CR

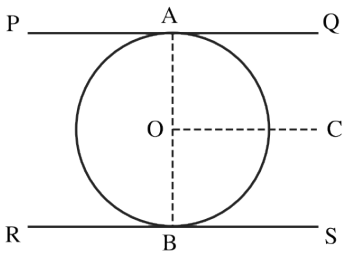
Perimeter of ΔABC:

AB + BC + CA = AB + (BP + PC) + CA

= (AB + BQ) + (CR + CA) = AQ + AR = 2AQ

∴ AQ = ½ (perimeter of ΔABC).

21. Prove that the line segment joining the points of contact of two parallel tangents to a circle passes through its centre.



Solution : Let PQ and RS be parallel tangents touching the circle at A and B; O is the centre. Draw OC ∥ PQ through O.

Radius ⊥ tangent:

$\angle PAO = 90° \Rightarrow \angle AOC = 90°$ (co-interior angles, PQ ∥ OC)

$\angle RBO = 90° \Rightarrow \angle BOC = 90°$ (co-interior angles, RS ∥ OC)

$\angle AOC + \angle BOC = 180°$

So AO and OB lie on one straight line.

∴ AB passes through the centre O.

Note: Case-study questions are recommended from this chapter for **CBSE Board 2026-27**.
Practise them here: **Class X Case Studies (click here)**

Chapter 11 – Areas Related to Circles

1. The short and long hands of a clock are 6 cm and 8 cm long respectively. Find the sum of the distances travelled by their tips in 1 day. (Take $\pi = \frac{22}{7}$)

Solution : In 1 day, the hour hand makes 2 rounds and the minute hand makes 24 rounds.

Distance travelled by the tip of the short hand (r = 6 cm):

$2 \times 2\pi r = 2 \times 2\pi \times 6 = 24\pi$ cm

Distance travelled by the tip of the long hand (r = 8 cm):

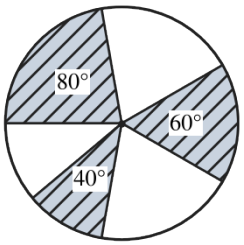
$24 \times 2\pi \times 8 = 384\pi$ cm

Total distance:

$$24\pi + 384\pi = 408\pi = 408 \times \frac{22}{7} = \frac{8976}{7}$$

∴ Total distance = $1282\frac{2}{7}$ cm ≈ 1282.29 cm.

2. In the given figure, three sectors of a circle of radius 7 cm, making angles of 60°, 80° and 40° at the centre, are shown. Find the area (in cm²) of the shaded region.



Solution : Area of a sector = $(\frac{\theta}{360^\circ}) \times \pi r^2$.

Sum of the angles of the shaded sectors:

$60^\circ + 80^\circ + 40^\circ = 180^\circ$

Total shaded area:

$(\frac{180^\circ}{360^\circ}) \times (\frac{22}{7}) \times 7 \times 7 = ½ \times 154$

∴ Area of the shaded region = 77 cm².

---

3. The radius of the wheel of a bus is 25 cm. If the speed of the bus is 33 km/h, how many revolutions will the wheel make in 1 minute?

Solution : Distance covered in 1 minute:

$33 \text{ km/h} = 33 \times \frac{100000}{60} \text{ cm/min} = 55000 \text{ cm/min}$

Distance covered in one revolution = circumference:

$2\pi r = 2 \times \frac{22}{7} \times 25 = \frac{1100}{7} \text{ cm}$

Number of revolutions:

$55000 \div (\frac{1100}{7}) = 55000 \times \frac{7}{1100} = 350$

∴ The wheel makes 350 revolutions per minute.

---

4. The radii of two circles are 4 cm and 3 cm. Find the radius of the circle whose area is equal to the sum of the areas of the two circles. Also find the circumference of this circle.

Solution : Let the radius of the required circle be R.

$\pi R^2 = \pi(4)^2 + \pi(3)^2$

$R^2 = 16 + 9 = 25 \Rightarrow R = 5 \text{ cm}$

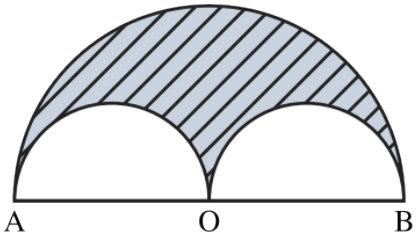
Circumference:

$2\pi R = 2 \times \frac{22}{7} \times 5 = \frac{220}{7} \approx 31.43 \text{ cm}$

∴ Radius = 5 cm; circumference ≈ 31.43 cm.

---

5. In the given figure, a semicircle is drawn with O as centre and AB as diameter. Semicircles are drawn with AO and OB as diameters. If AB = 28 m, find the perimeter of the shaded region. ($\pi = \frac{22}{7}$)



Solution : The boundary of the shaded region consists of three semicircular arcs.

Arc of the big semicircle (r = 14 m):

$$\pi r = \frac{22}{7} \times 14 = 44 \text{ m}$$

Arcs of the two small semicircles (r = 7 m each):

$$\pi r = \frac{22}{7} \times 7 = 22 \text{ m each}$$

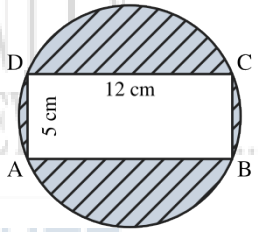
Perimeter:

$44 + 22 + 22 = 88$

$\therefore$ Perimeter of the shaded region = 88 m.

---

6. Find the area of the shaded region, where ABCD is a rectangle with DC = 12 cm and AD = 5 cm inscribed in the circle. (Take $\pi = 3.14$)



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Solution : The diagonal of the inscribed rectangle is a diameter of the circle.

$AC = \sqrt{12^2 + 5^2} = \sqrt{169} = 13 \text{ cm} \Rightarrow r = 6.5 \text{ cm}$

Area of the circle:

$\pi r^2 = 3.14 \times 6.5 \times 6.5 = 132.665 \text{ cm}^2$

Area of the rectangle:

$12 \times 5 = 60 \text{ cm}^2$

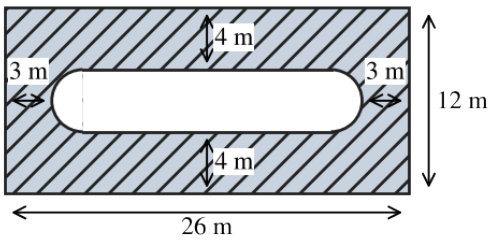
Shaded area = circle − rectangle:

$132.665 - 60 = 72.665$

$\therefore$ Area of the shaded region = 72.665 cm^2.

---

7. Find the area of the shaded region in the figure.



Solution : Area of the outer rectangle:

$26 \times 12 = 312 \text{ m}^2$

The unshaded track: width = 12 − 4 − 4 = 4 m, so each end is a semicircle of radius 2 m.

Length of the straight part:

$26 - 3 - 3 - 2 - 2 = 16 \text{ m}$

Area of the unshaded track:

rectangle $16 \times 4 = 64 \text{ m}^2$

two semicircles = one circle = $\pi(2)^2 = 4\pi \text{ m}^2$

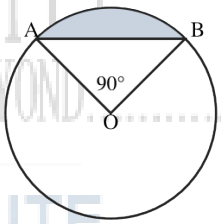
Shaded area:

$312 - (64 + 4\pi) = 248 - 4\pi$

∴ Area of the shaded region = $(248 - 4\pi) \text{ m}^2 \approx 235.43 \text{ m}^2$ ($\pi = 3.14$).

---

8. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the areas of the minor and major segments. (Use $\pi = 3.14$)



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Solution : Area of the sector ($\theta = 90°$):

$$\left(\frac{90}{360}\right) \times 3.14 \times 10 \times 10 = 78.5 \text{ cm}^2$$

Area of right ΔOAB:

$\frac{1}{2} \times 10 \times 10 = 50 \text{ cm}^2$

Minor segment = sector − triangle:

$78.5 - 50 = 28.5 \text{ cm}^2$

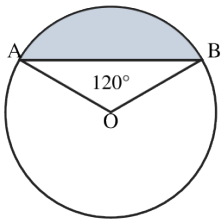
Major segment = circle − minor segment:

$3.14 \times 100 - 28.5 = 314 - 28.5 = 285.5 \text{ cm}^2$

∴ Minor segment = 28.5 cm^2, major segment = 285.5 cm^2.

---

9. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use π = 3.14 and $\sqrt{3}$ = 1.73)



Solution : Area of the sector:

$$(\frac{120}{360}) \times 3.14 \times 144 = 150.72 \text{ cm}^2$$

Area of ΔOAB (two sides 12 cm, included angle 120°): draw OM ⊥ AB.

OM = 12 cos 60° = 6 cm, AM = 12 sin 60° = $6\sqrt{3}$ cm ⇒ AB = $12\sqrt{3}$ cm

Area = ½ × $12\sqrt{3}$ × 6 = $36\sqrt{3}$ = 36 × 1.73 = 62.28 cm²

Area of the segment:

150.72 − 62.28 = 88.44 cm²

∴ Area of the segment = 88.44 cm².

---

10. A horse is tied to a peg at one corner of a square-shaped grass field of side 15 m by means of a 5 m long rope. Find (i) the area of that part of the field in which the horse can graze; (ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use π = 3.14)

Solution : The horse grazes a quadrant (90° sector) with radius = length of the rope.

(i) r = 5 m:

¼ × 3.14 × 25 = 19.625 m²

(ii) r = 10 m:

¼ × 3.14 × 100 = 78.5 m²

Increase = 78.5 − 19.625 = 58.875 m²

∴ (i) 19.625 m² (ii) 58.875 m².

---

11. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors. Find (i) the total length of the silver wire required (ii) the area of each sector of the brooch.

Solution :

$$r = \frac{35}{2} \text{ mm}$$

(i) Wire = circumference + 5 diameters:

$$\pi d + 5d = (\frac{22}{7})(35) + 5 \times 35 = 110 + 175 = 285 \text{ mm}$$

(ii) Each sector is $\frac{1}{10}$ of the circle:

$$(\frac{1}{10}) \times (\frac{22}{7}) \times (\frac{35}{2})^2 = (\frac{1}{10}) \times (\frac{22}{7}) \times (\frac{1225}{4}) = \frac{385}{4}$$

$\therefore$ (i) 285 mm (ii) $\frac{385}{4}$ mm^2 = 96.25 mm^2.

---

12. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Solution : Area swept by one wiper = sector of radius 25 cm and angle 115°:

$$(\frac{115}{360}) \times (\frac{22}{7}) \times 25 \times 25$$

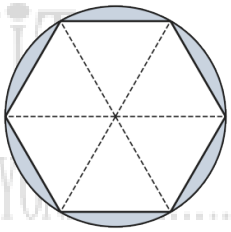
Total for two wipers:

$$2 \times (\frac{115}{360}) \times (\frac{22}{7}) \times 625 = \frac{158125}{126}$$

$\therefore$ Total area cleaned = $\frac{158125}{126}$ cm^2 $\approx$ 1254.96 cm^2.

---

13. A round table cover has six equal designs as shown in the figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹ 0.35 per cm^2. (Use $\sqrt{3}$ = 1.7)



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Solution : Each design is a segment whose chord subtends $\frac{360°}{6}$ = 60° at the centre, so each triangle is equilateral (side 28 cm).

Area of one sector:

$$(\frac{60}{360}) \times (\frac{22}{7}) \times 28 \times 28 = \frac{1232}{3} \text{ cm}^2$$

Area of one equilateral triangle:

$$(\frac{\sqrt{3}}{4}) \times 28^2 = 1.7 \times 196 = 333.2 \text{ cm}^2$$

Area of one design:

$$\frac{1232}{3} - 333.2 \approx 410.67 - 333.2 = 77.47 \text{ cm}^2$$

Six designs and cost:

$6 \times 77.47 \approx 464.8$ cm^2

Cost = $464.8 \times 0.35 \approx$ ₹ 162.68

$\therefore$ Cost of making the designs $\approx$ ₹ 162.68.

---

14. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find (i) the length of the arc (ii) the area of the sector formed by the arc (iii) the area of the segment formed by the corresponding chord. (Use $\sqrt{3}$ = 1.732)

Solution : (i) Length of the arc:

$$(\frac{60}{360}) \times 2 \times (\frac{22}{7}) \times 21 = 22 \text{ cm}$$

(ii) Area of the sector:

$$(\frac{60}{360}) \times (\frac{22}{7}) \times 21 \times 21 = 231 \text{ cm}^2$$

(iii) The triangle is equilateral with side 21 cm:

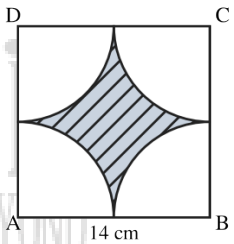
$$\text{Area} = (\frac{\sqrt{3}}{4}) \times 441 = 0.433 \times 441 = 190.95 \text{ cm}^2$$

$$\text{Segment} = 231 - 190.95 = 40.05 \text{ cm}^2$$

∴ (i) 22 cm (ii) 231 cm^2 (iii) 40.05 cm^2.

---

15. ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touches externally two of the remaining three circles. Find the area of the shaded region.



Solution : Each circle has radius 7 cm (half the side). Inside the square, each circle covers a quadrant.

Area of the square:

$$14 \times 14 = 196 \text{ cm}^2$$

Area of 4 quadrants = area of one full circle:

$$(\frac{22}{7}) \times 7 \times 7 = 154 \text{ cm}^2$$

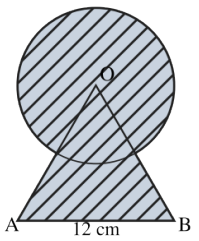
Shaded area:

$$196 - 154 = 42 \text{ cm}^2$$

∴ Area of the shaded region = 42 cm^2.

---

16. In the figure, a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre. Find the area of the shaded region.



Solution : The shaded region = circle of radius 6 cm (except the 60° sector inside the triangle) + triangle.

Area of the circle excluding the 60° sector:

$$(\frac{300}{360}) \times (\frac{22}{7}) \times 36 = \frac{660}{7} \approx 94.29 \text{ cm}^2$$

Area of the equilateral triangle:

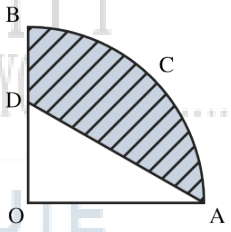
$$(\frac{\sqrt{3}}{4}) \times 144 = 36\sqrt{3} \approx 62.35 \text{ cm}^2$$

Total:

$94.29 + 62.35 = 156.64 \text{ cm}^2$

∴ Area of the shaded region ≈ 156.64 cm².

17. In the figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of (i) the quadrant OACB (ii) the shaded region.



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Solution : (i) Area of the quadrant:

$$¼ \times (\frac{22}{7}) \times 3.5 \times 3.5 = \frac{77}{8} = 9.625 \text{ cm}^2$$

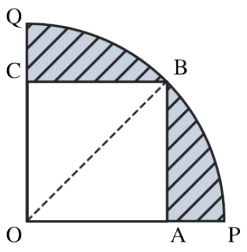
(ii) The unshaded ΔOAD is right-angled at O with OA = 3.5 cm, OD = 2 cm:

Area = $½ \times 3.5 \times 2 = 3.5 \text{ cm}^2$

Shaded area = $9.625 - 3.5 = 6.125 \text{ cm}^2$

∴ (i) 9.625 cm² (ii) 6.125 cm².

18. In the figure, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. (Use π = 3.14)



Solution : The radius of the quadrant is the diagonal OB of the square:

$OB = \sqrt{20^2 + 20^2} = 20\sqrt{2}$ cm

Area of the quadrant:

$\frac{1}{4} \times 3.14 \times (20\sqrt{2})^2 = \frac{1}{4} \times 3.14 \times 800 = 628$ cm^2

Area of the square:

$20 \times 20 = 400$ cm^2

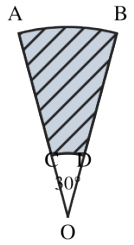
Shaded area:

$628 - 400 = 228$ cm^2

$\therefore$ Area of the shaded region = 228 cm^2.

---

19. AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm with centre O. If ∠AOB = 30°, find the area of the shaded region.



Solution : Shaded area = sector of radius 21 cm − sector of radius 7 cm (same angle 30°):

$$\left(\frac{30}{360}\right) \times \left(\frac{22}{7}\right) \times (21^2 - 7^2)$$

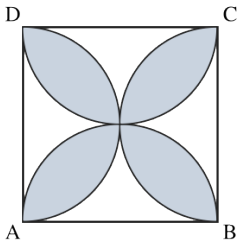
$$= \left(\frac{1}{12}\right) \times \left(\frac{22}{7}\right) \times (441 - 49) = \left(\frac{1}{12}\right) \times \left(\frac{22}{7}\right) \times 392$$

$$= \left(\frac{1}{12}\right) \times 1232 = \frac{308}{3}$$

$\therefore$ Area of the shaded region = $\frac{308}{3}$ cm$^2 \approx 102.67$ cm^2.

---

20. Find the area of the shaded design, where ABCD is a square of side 10 cm and semicircles are drawn with each side of the square as diameter. (Use π = 3.14)



Solution : The four semicircles together cover the square, and each petal (shaded) is covered twice.

Sum of the areas of the 4 semicircles:

$4 \times \frac{1}{2} \times 3.14 \times 5^2 = 157$ cm^2

Area of the square:

$10 \times 10 = 100$ cm^2

Shaded petals = (sum of semicircles) − square:

$157 - 100 = 57$ cm^2

$\therefore$ Area of the shaded design = 57 cm^2.

---

21. The perimeter of a sector of a circle of radius 5.2 cm is 16.4 cm. Find the area of the sector.

Solution : Perimeter of the sector = 2r + length of the arc:

$16.4 = 2 \times 5.2 + l \Rightarrow l = 6$ cm

Area of a sector in terms of its arc:

Area = $\frac{1}{2} \times l \times r = \frac{1}{2} \times 6 \times 5.2 = 15.6$

$\therefore$ Area of the sector = 15.6 cm^2.

Chapter 12 – Surface Areas and Volumes

1. A hollow cube of internal edge 22 cm is filled with spherical marbles of diameter 0.5 cm, and it is assumed that $\frac{1}{8}$ of the space of the cube remains unfilled. Find the number of marbles that the cube can accommodate.

Solution : Volume of the cube:

$22^3 = 10648$ cm^3

Space filled by marbles ($\frac{7}{8}$ of the cube):

$\frac{7}{8} \times 10648 = 9317$ cm^3

Volume of one marble (r = 0.25 cm):

$(\frac{4}{3})\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (\frac{1}{4})^3 = \frac{11}{168}$ cm^3

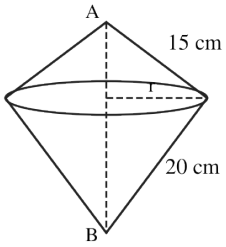
Number of marbles:

$9317 \div \frac{11}{168} = 9317 \times \frac{168}{11} = 847 \times 168 = 142296$

$\therefore$ Number of marbles = 142296.

---

2. A right-angled triangle whose sides other than the hypotenuse are 15 cm and 20 cm is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. ($\pi = 3.14$)



Solution : Hypotenuse:

$\sqrt{15^2 + 20^2} = 25$ cm

The common radius is the altitude to the hypotenuse:

$r = \frac{15 \times 20}{25} = 12$ cm

The heights of the two cones add up to the hypotenuse, $h_1 + h_2 = 25$ cm.

Volume of the double cone:

$$(\frac{1}{3})\pi r^2(h_1 + h_2) = \frac{1}{3} \times 3.14 \times 144 \times 25 = 3768 \text{ cm}^3$$

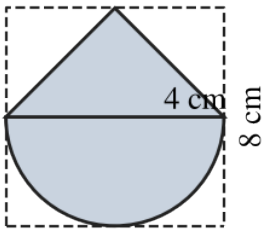
Surface area (slant heights 15 cm and 20 cm):

$$\pi r(l_1 + l_2) = 3.14 \times 12 \times 35 = 1318.8 \text{ cm}^2$$

$\therefore$ Volume = 3768 cm^3; surface area = 1318.8 cm^2.

---

3. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 4 cm and the diameter of the base is 8 cm. If a right circular cylinder circumscribes the solid, find how much more space it will cover.



Solution :

r = 4 cm, height of cone = 4 cm

The cylinder has radius 4 cm and height = 4 + 4 = 8 cm.

Volume of cylinder = $\pi r^2 h = \pi \times 16 \times 8 = 128\pi$ cm^3

Volume of the toy:

$$\text{hemisphere} = (\frac{2}{3})\pi r^3 = \frac{128\pi}{3} \text{ cm}^3$$

$$\text{cone} = (\frac{1}{3})\pi r^2 h = \frac{64\pi}{3} \text{ cm}^3$$

$$\text{toy} = \frac{192\pi}{3} = 64\pi \text{ cm}^3$$

Extra space:

$128\pi - 64\pi = 64\pi$ $cm^3 \approx 201.14$ cm^3

$\therefore$ The cylinder covers 64π cm^3 more space.

---

4. A tent is in the shape of a cylinder surmounted by a conical top. The height and radius of the cylindrical part are 3 m and 14 m respectively, and the total height of the tent is 13.5 m. Find the area of the canvas required for making the tent, keeping a provision of 26 m^2 of canvas for stitching and wastage. Also find the cost of the canvas at ₹ 250 per m^2.

Solution :

Height of the conical part = 13.5 − 3 = 10.5 m

Slant height $l = \sqrt{14^2 + 10.5^2} = \sqrt{306.25} = 17.5$ m

Curved surface area of the cylinder:

$2\pi rh = 2 \times \frac{22}{7} \times 14 \times 3 = 264 \text{ m}^2$

Curved surface area of the cone:

$\pi rl = \frac{22}{7} \times 14 \times 17.5 = 770 \text{ m}^2$

Canvas required:

$264 + 770 + 26 = 1060 \text{ m}^2$

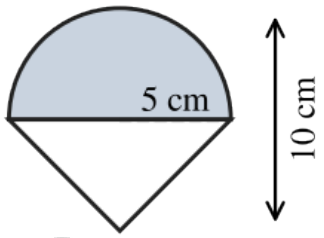
Cost:

$1060 \times 250 = ₹\ 2{,}65{,}000$

$\therefore$ Canvas = 1060 m^2; cost = ₹ 2,65,000.

---

5. An ice-cream cone full of ice-cream has radius 5 cm and total height 10 cm, as shown in the figure. Calculate the volume of the ice-cream, provided that $\frac{1}{6}$ part is left unfilled with ice-cream.



Solution : The solid is a cone with a hemisphere on top, both of radius 5 cm.

Height of the cone = 10 − 5 = 5 cm

Volume of the cone:

$(\frac{1}{3})\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 25 \times 5 = \frac{2750}{21} \text{ cm}^3$

Volume of the hemisphere:

$(\frac{2}{3})\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 125 = \frac{5500}{21} \text{ cm}^3$

Total volume:

$\frac{8250}{21} \approx 392.86 \text{ cm}^3$

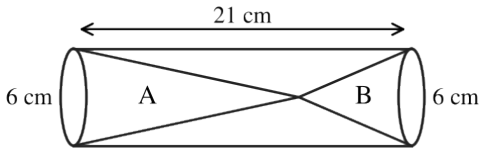
Ice-cream fills $\frac{5}{6}$ of it:

$\frac{5}{6} \times \frac{8250}{21} = \frac{6875}{21} \approx 327.4 \text{ cm}^3$

$\therefore$ Volume of ice-cream $\approx$ 327.4 cm^3.

---

6. Two solid cones A and B are placed in a cylindrical tube as shown in the figure. The ratio of their capacities is 2 : 1. Find the heights and capacities of the cones. Also find the volume of the remaining portion of the cylinder.



Solution : Radius of each cone = radius of the tube = 3 cm. Cones with equal radii have volumes in the ratio of their heights:

$h_1 : h_2 = 2 : 1$ and $h_1 + h_2 = 21$ cm $\Rightarrow h_1 = 14$ cm, $h_2 = 7$ cm

Capacities:

$$\text{Cone A} = \frac{1}{3} \times \frac{22}{7} \times 9 \times 14 = 132 \text{ cm}^3$$

$$\text{Cone B} = \frac{1}{3} \times \frac{22}{7} \times 9 \times 7 = 66 \text{ cm}^3$$

Volume of the cylinder:

$$\pi r^2 h = \frac{22}{7} \times 9 \times 21 = 594 \text{ cm}^3$$

Remaining volume:

$594 - (132 + 66) = 396 \text{ cm}^3$

$\therefore$ Heights 14 cm and 7 cm; capacities 132 cm^3 and 66 cm^3; remaining volume 396 cm^3.

---

7. Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively offered to provide place and the canvas for 1500 tents, and decided to share the whole expenditure equally. The lower part of each tent is cylindrical with base radius 2.8 m and height 3.5 m, with a conical upper part of the same base radius but of height 2.1 m. If the canvas used to make the tents costs ₹ 120 per m^2, find the amount shared by each school to set up the tents.

Solution : Slant height of the conical part:

$$l = \sqrt{2.8^2 + 2.1^2} = \sqrt{12.25} = 3.5 \text{ m}$$

Canvas for one tent = CSA of cylinder + CSA of cone:

$$2\pi rh = 2 \times \frac{22}{7} \times 2.8 \times 3.5 = 61.6 \text{ m}^2$$

$$\pi rl = \frac{22}{7} \times 2.8 \times 3.5 = 30.8 \text{ m}^2$$

Total = 92.4 m^2

Canvas for 1500 tents and its cost:

$92.4 \times 1500 = 138600 \text{ m}^2$

Cost = $138600 \times 120 =$ ₹ 1,66,32,000

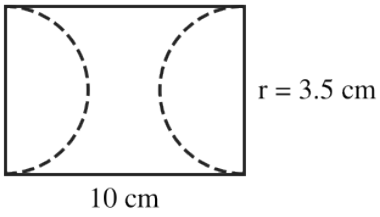
Share of each school:

16632000 ÷ 50 = ₹ 3,32,640

∴ Each school pays ₹ 3,32,640.

---

8. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder. If the height of the cylinder is 10 cm and its base radius is 3.5 cm, find the total surface area of the article.



Solution : Surfaces: curved surface of the cylinder + inner surfaces of two hemispheres.

CSA of cylinder = $2\pi rh = 2 \times (\frac{22}{7}) \times 3.5 \times 10 = 220$ cm^2

CSA of two hemispheres = $2 \times 2\pi r^2 = 4 \times (\frac{22}{7}) \times 12.25 = 154$ cm^2

Total = 220 + 154 = 374 cm^2

∴ Total surface area = 374 cm^2.

---

9. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.



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Solution :

r = 2.5 mm, length of the cylindrical part = 14 – 2 × 2.5 = 9 mm

CSA of cylinder = $2\pi rh = 2 \times (\frac{22}{7}) \times 2.5 \times 9 = \frac{990}{7}$ mm^2

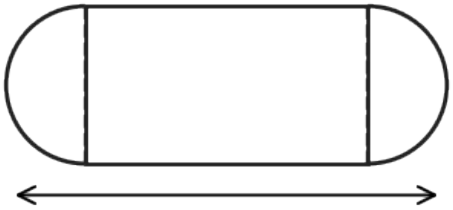
Surface of two hemispheres = $4\pi r^2 = 4 \times (\frac{22}{7}) \times 6.25 = \frac{550}{7}$ mm^2

Total = $\frac{1540}{7} = 220$ mm^2

∴ Surface area of the capsule = 220 mm^2.

---

10. Gulab jamuns contain sugar syrup up to about 30% of their volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends, with length 5 cm and diameter 2.8 cm.



Solution :

r = 1.4 cm, cylinder length = 5 − 2.8 = 2.2 cm

Volume of cylinder = $(\frac{22}{7}) \times 1.96 \times 2.2 = 13.552$ cm^3

Volume of two hemispheres = $(\frac{4}{3}) \times (\frac{22}{7}) \times 1.4^3 = 11.499$ cm^3

One gulab jamun ≈ 25.05 cm^3

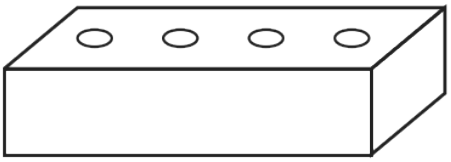
Syrup in 45 gulab jamuns:

$0.3 \times 45 \times 25.05 \approx 338.2$ cm^3

∴ Syrup ≈ 338 cm^3.

---

11. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The cuboid is 15 cm × 10 cm × 3.5 cm. The radius of each depression is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand.



Solution :

Volume of cuboid = $15 \times 10 \times 3.5 = 525$ cm^3

Volume of one cone = $(\frac{1}{3}) \times (\frac{22}{7}) \times 0.25 \times 1.4 = \frac{11}{30}$ cm^3

Four cones = $\frac{44}{30} \approx 1.47$ cm^3

Volume of wood:

$525 - 1.47 = 523.53$ cm^3

∴ Volume of wood ≈ 523.53 cm^3.

---

12. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top is 5 cm. It is filled with water up to the brim. When lead shots, each a sphere of radius 0.5 cm, are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped.

Solution :

Volume of cone = $(\frac{1}{3})\pi \times 25 \times 8 = \frac{200\pi}{3}$ cm^3

Water that flows out = volume of the shots:

$¼ \times \frac{200\pi}{3} = \frac{50\pi}{3}$ cm^3

Volume of one shot = $(\frac{4}{3})\pi \times 0.125 = \frac{\pi}{6}$ cm^3

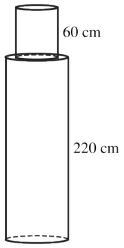
Number of shots:

$$(\frac{50\pi}{3}) \div (\frac{\pi}{6}) = 100$$

$\therefore$ 100 lead shots were dropped.

---

13. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass. (Use $\pi = 3.14$)



Solution :

Lower cylinder: $\pi \times 12^2 \times 220 = 31680\pi$ cm^3

Upper cylinder: $\pi \times 8^2 \times 60 = 3840\pi$ cm^3

Total = $35520\pi = 3.14 \times 35520 = 111532.8$ cm^3

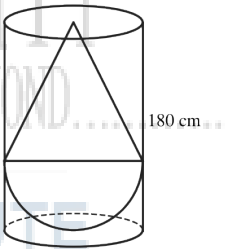
Mass:

$111532.8 \times 8 = 892262.4$ g ≈ 892.26 kg

$\therefore$ Mass of the pole ≈ 892.26 kg.

---

14. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.



Solution :

Volume of cylinder = $\pi \times 60^2 \times 180 = 648000\pi$ cm^3

Volume of cone = $(\frac{1}{3})\pi \times 60^2 \times 120 = 144000\pi$ cm^3

Volume of hemisphere = $(\frac{2}{3})\pi \times 60^3 = 144000\pi$ cm^3

Water left = cylinder − solid:

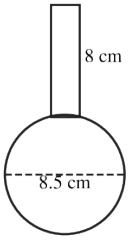
$$648000\pi - 288000\pi = 360000\pi = 360000 \times \frac{22}{7} \text{ cm}^3$$

≈ 1131428.57 $cm^3 \approx 1.131$ m^3

$\therefore$ Water left ≈ 1.131 m^3.

---

15. A spherical glass vessel has a cylindrical neck 8 cm long and 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm³. Check whether she is correct, taking the above as inside measurements and π = 3.14.



Solution :

Volume of the sphere = $(\frac{4}{3}) \times 3.14 \times 4.25^3 = 321.39$ cm³

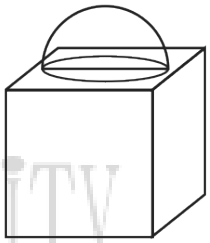
Volume of the neck = $3.14 \times 1^2 \times 8 = 25.12$ cm³

Total = 346.51 cm³

∴ She is not correct; the correct volume is 346.51 cm³.

---

16. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.



Solution : The greatest diameter equals the side of the cube = 7 cm, so r = 3.5 cm.

Surface area = cube − circular base covered + curved surface of hemisphere:

$6 \times 7^2 - \pi r^2 + 2\pi r^2 = 294 + \pi r^2$

$= 294 + (\frac{22}{7}) \times 12.25 = 294 + 38.5 = 332.5$ cm²

∴ Greatest diameter = 7 cm; surface area = 332.5 cm².

---

17. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Solution :

Radius of the hemisphere = $\frac{l}{2}$

Surface area = cube − circular top removed + inner hemispherical surface:

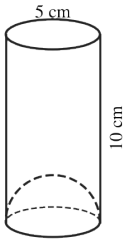
$6l^2 - \pi(\frac{l}{2})^2 + 2\pi(\frac{l}{2})^2$

$= 6l^2 + \frac{\pi l^2}{4}$

∴ Surface area = $(\frac{l^2}{4})(\pi + 24)$.

---

18. A juice seller serves his customers using glasses with inner diameter 5 cm. The bottom of each glass has a hemispherical raised portion which reduces its capacity. If the height of a glass is 10 cm, find the apparent capacity and the actual capacity of the glass. (Use $\pi = 3.14$)



Solution :

$r = 2.5$ cm

Apparent capacity $= \pi r^2 h = 3.14 \times 6.25 \times 10 = 196.25$ cm^3

Volume of the raised hemisphere $= (\frac{2}{3}) \times 3.14 \times 2.5^3 \approx 32.71$ cm^3

Actual capacity:

$196.25 - 32.71 = 163.54$ cm^3

$\therefore$ Apparent capacity $= 196.25$ cm^3; actual capacity ≈ 163.54 cm^3.

---

19. A building is in the form of a cylinder surmounted by a hemispherical vaulted dome and contains 41 $\frac{19}{21}$ m^3 of air. If the internal diameter of the dome is equal to its total height above the floor, find the height of the building.

Solution : Let the radius be r. Total height = 2r, so the cylinder has height 2r − r = r.

Volume $= \pi r^2 \times r + (\frac{2}{3})\pi r^3 = (\frac{5}{3})\pi r^3$

$(\frac{5}{3}) \times (\frac{22}{7}) \times r^3 = \frac{880}{21}$

$r^3 = (\frac{880}{21}) \times (\frac{21}{110}) = 8 \Rightarrow r = 2$ m

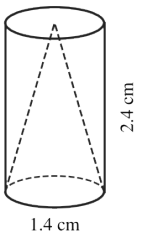
Height of the building:

$2r = 4$ m

$\therefore$ Height of the building = 4 m.

---

20. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm².



Solution :

$r = 0.7$ cm, $h = 2.4$ cm, slant height $l = \sqrt{2.4^2 + 0.7^2} = 2.5$ cm

Surfaces: curved surface of the cylinder + one base + inner surface of the cone.

$2\pi rh = 2 \times (\frac{22}{7}) \times 0.7 \times 2.4 = 10.56\ cm^2$

$\pi r^2 = (\frac{22}{7}) \times 0.49 = 1.54\ cm^2$

$\pi rl = (\frac{22}{7}) \times 0.7 \times 2.5 = 5.5\ cm^2$

Total = $17.6\ cm^2$

∴ Total surface area ≈ $18\ cm^2$.

---

21. Two cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

Solution :

Edge of each cube = $\sqrt[3]{64} = 4$ cm

The cuboid is 8 cm × 4 cm × 4 cm.

Surface area = $2(lb + bh + hl) = 2(32 + 16 + 32) = 160\ cm^2$

∴ Surface area = $160\ cm^2$.

Chapter 13 – Statistics

1. Find the mean of the following frequency distribution.

Class	25–30	30–35	35–40	40–45	45–50	50–55	55–60
Frequency	14	22	16	6	5	3	4

Solution : Take class marks x_i and use the direct method, mean = $\frac{\Sigma f_i x_i}{\Sigma f_i}$.

Class marks: 27.5, 32.5, 37.5, 42.5, 47.5, 52.5, 57.5

$f_i x_i$: 385, 715, 600, 255, 237.5, 157.5, 230

$\Sigma f_i = 70$, $\Sigma f_i x_i = 2580$

Mean:

$2580 \div 70 = 36.857\ldots$

∴ Mean ≈ 36.86.

---

2. If the mode of the following series is 54, find the value of f.

C.I.	0–15	15–30	30–45	45–60	60–75	75–90
Frequency	3	5	f	16	12	7

Solution : Mode = 54 lies in the class 45–60, so this is the modal class.

$l = 45$, $h = 15$, $f_1 = 16$, $f_0 = f$, $f_2 = 12$

Mode formula: Mode $= l + [\frac{f_1 - f_0}{2f_1 - f_0 - f_2}] \times h$

$54 = 45 + [\frac{16 - f}{32 - f - 12}] \times 15$

$9 = \frac{15(16 - f)}{20 - f}$

$9(20 - f) = 15(16 - f)$

$180 - 9f = 240 - 15f \Rightarrow 6f = 60$

$\therefore f = 10.$

---

3. The mean of the following frequency table is 50, but the frequencies f_1 and f_2 in classes 20–40 and 60–80 are missing. Find the missing frequencies.

C.I.	0–20	20–40	40–60	60–80	80–100	Total
Frequency	17	f_1	32	f_2	19	120

Solution : Total frequency:

$17 + f_1 + 32 + f_2 + 19 = 120 \Rightarrow f_1 + f_2 = 52$ …(1)

Class marks: 10, 30, 50, 70, 90.

$\Sigma f_i x_i = 170 + 30f_1 + 1600 + 70f_2 + 1710 = 3480 + 30f_1 + 70f_2$

Mean = 50:

$\frac{3480 + 30f_1 + 70f_2}{120} = 50$

$30f_1 + 70f_2 = 2520 \Rightarrow 3f_1 + 7f_2 = 252$ …(2)

From (1), $f_1 = 52 - f_2$. Substitute in (2):

$156 - 3f_2 + 7f_2 = 252 \Rightarrow 4f_2 = 96 \Rightarrow f_2 = 24$

$f_1 = 28$

$\therefore f_1 = 28, f_2 = 24.$

---

4. Find the values of the missing frequencies p and q in the following distribution of 100 observations. The median of the distribution is 47.

C.I.	30–35	35–40	40–45	45–50	50–55	55–60	60–65
Frequency	12	p	17	20	q	12	8

Solution : Total frequency:

$12 + p + 17 + 20 + q + 12 + 8 = 100 \Rightarrow p + q = 31$ …(1)

Median = 47 lies in the class 45–50 (median class).

$l = 45, h = 5, f = 20, \frac{N}{2} = 50$

cf (before the median class) $= 12 + p + 17 = 29 + p$

Median formula: Median $= l + [\frac{\frac{N}{2} - cf}{f}] \times h$

$$47 = 45 + [\frac{50 - 29 - p}{20}] \times 5$$

$$2 = \frac{21 - p}{4} \Rightarrow 21 - p = 8 \Rightarrow p = 13$$

From (1):

$q = 31 - 13 = 18$

$\therefore p = 13, q = 18.$

---

5. The following table shows the daily expenditure (in ₹) on food of 30 households in a locality. Find the mean daily expenditure on food by the assumed mean method.

Expenditure (₹)	100–150	150–200	200–250	250–300	300–350	350–400
No. of households	4	5	12	2	2	5

Solution : Use the assumed mean method with a = 225: mean = $a + \frac{\Sigma f_i d_i}{\Sigma f_i}$, where $d_i = x_i - a$.

Class marks x_i: 125, 175, 225, 275, 325, 375

$d_i = x_i - 225$: −100, −50, 0, 50, 100, 150

$f_i d_i$: −400, −250, 0, 100, 200, 750

$\Sigma f_i = 30$, $\Sigma f_i d_i = 400$

Mean:

$$225 + \frac{400}{30} = 225 + (13.33) = 238.33$$

$\therefore$ Mean = 238.33.

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6. Find the mean marks of the students from the following distribution using the step-deviation method.

Marks	0–20	20–40	40–60	60–80	80–100
No. of students	7	8	12	13	10

Solution : Use the step-deviation method with a = 50, h = 20: mean = $a + h \times \frac{\Sigma f_i u_i}{\Sigma f_i}$, where $u_i = \frac{x_i - a}{h}$.

Class marks x_i: 10, 30, 50, 70, 90

u_i: −2, −1, 0, 1, 2

$f_i u_i$: −14, −8, 0, 13, 20

$\Sigma f_i = 50$, $\Sigma f_i u_i = 11$

Mean:

$$50 + 20 \times \frac{11}{50} = 50 + (4.4) = 54.4$$

$\therefore$ Mean = 54.4.

---

7. Mangoes are packed in boxes. The number of mangoes in the boxes is given below. Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

No. of mangoes	50–52	53–55	56–58	59–61	62–64
No. of boxes	15	110	135	115	25

Solution : The class marks are equally spaced (step 3), so the step-deviation method is the most convenient.

Class marks x_i: 51, 54, 57, 60, 63; a = 57, h = 3

$u_i = \frac{x_i - 57}{3}$: −2, −1, 0, 1, 2

f_iu_i: −30, −110, 0, 115, 50

$\Sigma f_i = 400$, $\Sigma f_iu_i = 25$

Mean:

$$57 + 3 \times \frac{25}{400} = 57 + 0.1875 = 57.1875$$

∴ Mean ≈ 57.19 mangoes per box (step-deviation method).

---

8. The following table shows the ages of the patients admitted in a hospital during a year. Find the mode of the data.

Age (years)	5–15	15–25	25–35	35–45	45–55	55–65
No. of patients	6	11	21	23	14	5

Solution : The maximum frequency is 23, so the modal class is 35–45.

l = 35, h = 10, $f_1 = 23$, $f_0 = 21$, $f_2 = 14$

$$\text{Mode} = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

$$= 35 + \left[\frac{23 - 21}{46 - 21 - 14}\right] \times 10$$

$$= 35 + \left(\frac{2}{11}\right) \times 10 = 35 + 1.82$$

∴ Mode = 36.82.

---

9. The following distribution gives the lifetimes (in hours) of 196 electrical components. Find the modal lifetime of the components.

Lifetime	0–20	20–40	40–60	60–80	80–100
Frequency	10	35	52	61	38

Solution : The maximum frequency is 61, so the modal class is 60–80.

l = 60, h = 20, $f_1 = 61$, $f_0 = 52$, $f_2 = 38$

$$\text{Mode} = l + [\frac{f_1 - f_0}{2f_1 - f_0 - f_2}] \times h$$

$$= 60 + [\frac{61 - 52}{122 - 52 - 38}] \times 20$$

$$= 60 + (\frac{9}{32}) \times 20 = 60 + 5.63$$

$\therefore$ Mode = 65.63.

---

10. Find the median of the following distribution.

Class	0–10	10–20	20–30	30–40	40–50	50–60
Frequency	5	8	20	15	7	5

Solution : Cumulative frequencies: 5, 13, 33, 48, 55, 60

$$N = 60, \frac{N}{2} = 30$$

The cumulative frequency just greater than 30 is 33, so the median class is 20–30.

$l = 20, h = 10, f = 20, cf = 13$

$$\text{Median} = l + [\frac{\frac{N}{2} - cf}{f}] \times h$$

$$= 20 + [\frac{30 - 13}{20}] \times 10 = 20 + 8.5$$

$\therefore$ Median = 28.5.

---

11. The following distribution gives the heights (in cm) of 51 girls of a class. Find the median height.

Height (cm)	< 140	< 145	< 150	< 155	< 160	< 165
No. of girls	4	11	29	40	46	51

Solution : Convert the 'less than' table into class intervals with frequencies:

140–145: 11 − 4 = 7; 145–150: 29 − 11 = 18; 150–155: 40 − 29 = 11; 155–160: 46 − 40 = 6; 160–165: 51 − 46 = 5

(The first class, below 140, has frequency 4.)

$$N = 51, \frac{N}{2} = 25.5$$

The cumulative frequency just greater than 25.5 is 29, so the median class is 145–150.

$l = 145, h = 5, f = 18, cf = 11$

$$\text{Median} = l + [\frac{\frac{N}{2} - cf}{f}] \times h$$

$$= 145 + [\frac{25.5 - 11}{18}] \times 5 = 145 + \frac{72.5}{18} = 145 + 4.03$$

$\therefore$ Median height $\approx$ 149.03 cm.

---

12. Find the median of the following 'more than' type distribution of marks of 60 students.

Marks	≥ 0	≥ 10	≥ 20	≥ 30	≥ 40	≥ 50
No. of students	60	56	46	31	13	4

Solution : Convert the 'more than' table into class intervals with frequencies:

0–10: 60 – 56 = 4; 10–20: 56 – 46 = 10; 20–30: 46 – 31 = 15; 30–40: 31 – 13 = 18; 40–50: 13 – 4 = 9; 50–60: 4

Cumulative frequencies (less than type): 4, 14, 29, 47, 56, 60

$$N = 60, \frac{N}{2} = 30$$

The cumulative frequency just greater than 30 is 47, so the median class is 30–40.

$l = 30, h = 10, f = 18, cf = 29$

$$\text{Median} = l + [\frac{\frac{N}{2} - cf}{f}] \times h$$

$$= 30 + [\frac{30 - 29}{18}] \times 10 = 30 + 0.56$$

$\therefore$ Median $\approx$ 30.56 marks.

---

13. Find the mean, median and mode of the following distribution.

Class	0–10	10–20	20–30	30–40	40–50
Frequency	6	9	15	12	8

Solution : Mean:

Use the direct method: mean $= \frac{\Sigma f_i x_i}{\Sigma f_i}$, where x_i is the class mark.

Class marks x_i: 5, 15, 25, 35, 45

$f_i x_i$: 30, 135, 375, 420, 360

$\Sigma f_i = 50$, $\Sigma f_i x_i = 1320$

Mean:

$$\frac{1320}{50} = 26.4$$

Mean = 26.4.

Median:

Cumulative frequencies: 6, 15, 30, 42, 50

$$N = 50, \frac{N}{2} = 25$$

The cumulative frequency just greater than 25 is 30, so the median class is 20–30.

$l = 20, h = 10, f = 15, cf = 15$

$$\text{Median} = l + [\frac{\frac{N}{2} - cf}{f}] \times h$$

$$= 20 + [\frac{25 - 15}{15}] \times 10 = 20 + 6.67$$

Median = 26.67.

Mode:

The maximum frequency is 15, so the modal class is 20–30.

$l = 20, h = 10, f_1 = 15, f_0 = 9, f_2 = 12$

$$\text{Mode} = l + [\frac{f_1 - f_0}{2f_1 - f_0 - f_2}] \times h$$

$$= 20 + [\frac{15 - 9}{30 - 9 - 12}] \times 10$$

$$= 20 + (\frac{6}{9}) \times 10 = 20 + 6.67$$

Mode = 26.67.

$\therefore$ Mean = 26.4, Median = 26.67, Mode = 26.67.

---

14. For a moderately skewed distribution, the mean is 27.2 and the median is 26.4. Find the mode using the empirical relationship.

Solution : Empirical relationship between the three measures:

Mode = 3 Median – 2 Mean

$= 3 \times 26.4 - 2 \times 27.2$

$= 79.2 - 54.4 = 24.8$

$\therefore$ Mode = 24.8.

---

15. Find the sum of the lower limit of the modal class and the upper limit of the median class of the following distribution.

Class	0–10	10–20	20–30	30–40	40–50
Frequency	3	9	15	30	18

Solution : Modal class: the maximum frequency is 30, so the modal class is 30–40.

Lower limit of the modal class = 30

Median class:

Cumulative frequencies: 3, 12, 27, 57, 75; $\frac{N}{2} = 37.5$

The cumulative frequency just greater than 37.5 is 57, so the median class is 30–40.

Upper limit of the median class = 40

Required sum:

$30 + 40 = 70$

$\therefore$ Required sum = 70.

---

16. The daily electricity consumption (in units) of 70 households is given below. Find the mean consumption using the step-deviation method.

Units	0–100	100–200	200–300	300–400	400–500
No. of households	8	16	24	15	7

Solution : Use the step-deviation method with a = 250, h = 100: mean = $a + h \times \frac{\Sigma f_i u_i}{\Sigma f_i}$, where $u_i = \frac{x_i - a}{h}$.

Class marks x_i: 50, 150, 250, 350, 450

u_i: −2, −1, 0, 1, 2

$f_i u_i$: −16, −16, 0, 15, 14

$\Sigma f_i = 70$, $\Sigma f_i u_i = -3$

Mean:

$$250 + 100 \times \frac{-3}{70} = 250 + (-4.29) = 245.71$$

$\therefore$ Mean = 245.71.

---

17. Find the modal marks of 40 students from the following distribution.

Marks	15–20	20–25	25–30	30–35	35–40	40–45
No. of students	2	3	8	14	10	3

Solution : The maximum frequency is 14, so the modal class is 30–35.

$l = 30$, $h = 5$, $f_1 = 14$, $f_0 = 8$, $f_2 = 10$

$$\text{Mode} = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

$$= 30 + \left[\frac{14 - 8}{28 - 8 - 10}\right] \times 5$$

$$= 30 + \left(\frac{6}{10}\right) \times 5 = 30 + 3$$

$\therefore$ Mode = 33.

---

18. Find the mean and median of the following distribution and hence estimate the mode using the empirical formula.

Class	10–20	20–30	30–40	40–50	50–60
Frequency	3	8	20	12	7

Solution : Mean:

Use the direct method: mean $= \frac{\Sigma f_i x_i}{\Sigma f_i}$, where x_i is the class mark.

Class marks x_i: 15, 25, 35, 45, 55

$f_i x_i$: 45, 200, 700, 540, 385

$\Sigma f_i = 50$, $\Sigma f_i x_i = 1870$

Mean:

$\frac{1870}{50} = 37.4$

Mean = 37.4.

Median:

Cumulative frequencies: 3, 11, 31, 43, 50

$N = 50$, $\frac{N}{2} = 25$

The cumulative frequency just greater than 25 is 31, so the median class is 30–40.

$l = 30$, $h = 10$, $f = 20$, $cf = 11$

$\text{Median} = l + [\frac{\frac{N}{2} - cf}{f}] \times h$

$= 30 + [\frac{25 - 11}{20}] \times 10 = 30 + 7$

Median = 37.

Use Mode = 3 Median − 2 Mean:

$3 \times 37 - 2 \times 37.4 = 36.2$

∴ Mean = 37.4, Median = 37, Mode ≈ 36.2.

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19. The mean of 20 observations is 15. On checking, it was found that two observations were wrongly copied as 3 and 6 instead of 8 and 4. Find the correct mean.

Solution :

Incorrect sum = $20 \times 15 = 300$

Correct the sum by removing the wrong values and adding the right ones:

Correct sum = $300 - (3 + 6) + (8 + 4) = 300 - 9 + 12 = 303$

Correct mean $= \frac{303}{20} = 15.15$

∴ Correct mean = 15.15.

Note: Case-study questions are recommended from this chapter for **CBSE Board 2026-27**.
Practise them here: **Class X Case Studies (click here)**

Chapter 14 – Probability

1. From a well-shuffled pack of 52 cards, a few cards of the same colour are missing. If P(red card) = $\frac{2}{3}$ and P(black card) = $\frac{1}{3}$, which colour cards are missing and how many?

Solution : P(red) = $\frac{2}{3}$ and P(black) = $\frac{1}{3}$, so in the remaining pack

red cards : black cards = 2 : 1

Only one colour is missing. If red cards were missing, there would be fewer red cards than black cards — not possible. So black cards are missing and all 26 red cards remain.

Black cards remaining = 26 ÷ 2 = 13

Black cards missing = 26 − 13 = 13

∴ 13 black cards are missing.

---

2. A bag contains 8 red balls and some blue balls. If the probability of drawing a blue ball is three times that of a red ball, find the number of blue balls in the bag.

Solution : Let the number of blue balls = x. Total balls = 8 + x.

$$P(\text{red}) = \frac{8}{8+x},\ P(\text{blue}) = \frac{x}{8+x}$$

Given P(blue) = 3 × P(red):

$$\frac{x}{8+x} = 3 \times \frac{8}{8+x}$$

$$x = 24$$

∴ There are 24 blue balls.

---

3. The probability of guessing the correct answer to a certain test question is $\frac{x}{12}$. If the probability of not guessing the correct answer is $\frac{5}{6}$, find the value of x.

Solution : P(E) + P(not E) = 1:

$$\frac{x}{12} + \frac{5}{6} = 1$$

$$\frac{x}{12} = \frac{1}{6}$$

∴ x = 2.

---

4. The probability of getting a defective toy in a carton of 200 toys is 0.24. Find the number of defective toys in the carton.

Solution : P(defective) = number of defective toys ÷ total toys:

$$0.24 = \frac{n}{200}$$

$n = 0.24 \times 200 = 48$

$\therefore$ Number of defective toys = 48.

---

5. Two dice are thrown at the same time. Find the probability of getting (i) a sum of 8 (ii) a prime number as the sum (iii) a doublet.

Solution : Total outcomes = $6 \times 6 = 36$.

(i) Sum 8: (2, 6), (3, 5), (4, 4), (5, 3), (6, 2) — 5 outcomes

$$P = \frac{5}{36}$$

(ii) Prime sums 2, 3, 5, 7, 11:

$1 + 2 + 4 + 6 + 2 = 15$ outcomes $\Rightarrow P = \frac{15}{36} = \frac{5}{12}$

(iii) Doublets (1, 1), (2, 2), …, (6, 6) — 6 outcomes

$$P = \frac{6}{36} = \frac{1}{6}$$

$\therefore$ (i) $\frac{5}{36}$ (ii) $\frac{5}{12}$ (iii) $\frac{1}{6}$.

---

6. Two dice are thrown simultaneously. Find the probability that (i) the product of the numbers is even (ii) the sum of the numbers is at least 10.

Solution : Total outcomes = 36.

(i) Product is odd only when both numbers are odd: $3 \times 3 = 9$ outcomes.

Even product = $36 - 9 = 27$ outcomes $\Rightarrow P = \frac{27}{36} = \frac{3}{4}$

(ii) Sum ≥ 10: (4, 6), (5, 5), (6, 4), (5, 6), (6, 5), (6, 6) — 6 outcomes

$$P = \frac{6}{36} = \frac{1}{6}$$

$\therefore$ (i) $\frac{3}{4}$ (ii) $\frac{1}{6}$.

---

7. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting (i) a face card (ii) a red king (iii) a black face card (iv) neither a heart nor a king.

Solution : Total outcomes = 52.

(i) Face cards (J, Q, K of 4 suits) = 12:

$$P = \frac{12}{52} = \frac{3}{13}$$

(ii) Red kings (hearts, diamonds) = 2:

$$P = \frac{2}{52} = \frac{1}{26}$$

(iii) Black face cards (spades, clubs) = 6:

$$P = \frac{6}{52} = \frac{3}{26}$$

(iv) Hearts = 13, other kings = 3, so 16 cards are a heart or a king:

$$P = \frac{52 - 16}{52} = \frac{36}{52} = \frac{9}{13}$$

$\therefore$ (i) $\frac{3}{13}$ (ii) $\frac{1}{26}$ (iii) $\frac{3}{26}$ (iv) $\frac{9}{13}$.

---

8. A box contains 90 discs numbered from 1 to 90. If one disc is drawn at random, find the probability that it bears (i) a two-digit number (ii) a perfect square number (iii) a number divisible by 5.

Solution : Total outcomes = 90.

(i) Two-digit numbers 10 to 90:

$$81 \text{ outcomes} \Rightarrow P = \frac{81}{90} = \frac{9}{10}$$

(ii) Perfect squares 1, 4, 9, …, 81:

$$9 \text{ outcomes} \Rightarrow P = \frac{9}{90} = \frac{1}{10}$$

(iii) Multiples of 5: 5, 10, …, 90:

$$18 \text{ outcomes} \Rightarrow P = \frac{18}{90} = \frac{1}{5}$$

$\therefore$ (i) $\frac{9}{10}$ (ii) $\frac{1}{10}$ (iii) $\frac{1}{5}$.

---

9. Three unbiased coins are tossed together. Find the probability of getting (i) at least two heads (ii) at most one tail (iii) exactly one head.

Solution : Sample space: HHH, HHT, HTH, THH, HTT, THT, TTH, TTT — 8 outcomes.

(i) At least two heads: HHH, HHT, HTH, THH

$$P = \frac{4}{8} = \frac{1}{2}$$

(ii) At most one tail is the same as at least two heads:

$$P = \frac{4}{8} = \frac{1}{2}$$

(iii) Exactly one head: HTT, THT, TTH

$$P = \frac{3}{8}$$

$\therefore$ (i) $\frac{1}{2}$ (ii) $\frac{1}{2}$ (iii) $\frac{3}{8}$.

---

10. Hanif tosses a coin twice. He wins if he gets the same result both times (both heads or both tails) and loses otherwise. Find the probability that Hanif will lose the game.

Solution : Sample space: HH, HT, TH, TT — 4 outcomes.

He loses on HT or TH:

$$P(\text{lose}) = \frac{2}{4} = \frac{1}{2}$$

$\therefore$ Probability that Hanif loses = $\frac{1}{2}$.

---

11. A bag contains 5 red, 8 white and 4 green balls. One ball is taken out at random. Find the probability that it is (i) red (ii) white (iii) not green.

Solution : Total balls = 5 + 8 + 4 = 17.

(i) $P(\text{red}) = \frac{5}{17}$

(ii) $P(\text{white}) = \frac{8}{17}$

(iii) $P(\text{not green}) = \frac{17-4}{17} = \frac{13}{17}$

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$\therefore$ (i) $\frac{5}{17}$ (ii) $\frac{8}{17}$ (iii) $\frac{13}{17}$.

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12. Find the probability that a leap year selected at random contains 53 Sundays. What is the probability for a non-leap year?

Solution : Leap year:

366 days = 52 weeks + 2 extra days

The 2 extra days can be (Sun, Mon), (Mon, Tue), (Tue, Wed), (Wed, Thu), (Thu, Fri), (Fri, Sat), (Sat, Sun) — 7 cases, 2 of which contain a Sunday.

$$P = \frac{2}{7}$$

Non-leap year:

365 days = 52 weeks + 1 extra day; the extra day is a Sunday in 1 of 7 cases

$$P = \frac{1}{7}$$

$\therefore$ Leap year: $\frac{2}{7}$; non-leap year: $\frac{1}{7}$.

---

13. A number x is selected at random from the numbers 1, 2, 3 and then a number y is selected at random from the numbers 1, 4, 9. Find the probability that the product xy is less than 9.

Solution : Total outcomes = $3 \times 3 = 9$.

List the products:

x = 1: 1, 4, 9

x = 2: 2, 8, 18

x = 3: 3, 12, 27

Products less than 9: 1, 4, 2, 8, 3 — 5 outcomes

$$P = \frac{5}{9}$$

$\therefore P(xy < 9) = \frac{5}{9}$.

---

14. A piggy bank contains hundred 50 p coins, fifty ₹ 1 coins, twenty ₹ 2 coins and ten ₹ 5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin (i) will be a 50 p coin (ii) will not be a ₹ 5 coin?

Solution : Total coins = 100 + 50 + 20 + 10 = 180.

(i) $P(50\text{ p coin}) = \frac{100}{180} = \frac{5}{9}$

(ii) $P(\text{not } ₹ 5 \text{ coin}) = \frac{180 - 10}{180} = \frac{170}{180} = \frac{17}{18}$

$\therefore$ (i) $\frac{5}{9}$ (ii) $\frac{17}{18}$.

---

15. A die is thrown twice. What is the probability that (i) 5 will not come up either time (ii) 5 will come up at least once?

Solution : Total outcomes = 36.

(i) 5 does not come up on either throw: 5 choices each time

$5 \times 5 = 25$ outcomes $\Rightarrow P = \frac{25}{36}$

(ii) Complementary event of (i):

$$P = 1 - \frac{25}{36} = \frac{11}{36}$$

$\therefore$ (i) $\frac{25}{36}$ (ii) $\frac{11}{36}$.

---

16. A box contains 12 balls, out of which x are black. If one ball is drawn at random, the probability of getting a black ball is P. If 6 more black balls are put in the box, the probability of drawing a black ball becomes double of P. Find x.

Solution :

$$P = \frac{x}{12}$$

After adding 6 black balls: x + 6 black out of 18 balls.

$$\frac{x+6}{18} = 2 \times \frac{x}{12} = \frac{x}{6}$$

$6(x + 6) = 18x$

$6x + 36 = 18x \Rightarrow 12x = 36 \Rightarrow x = 3$

$\therefore x = 3.$

---

17. A jar contains 24 marbles, some green and others blue. If a marble is drawn at random and the probability that it is green is $\frac{2}{3}$, find the number of blue marbles.

Solution :

Number of green marbles = $(\frac{2}{3}) \times 24 = 16$

Blue marbles = 24 − 16 = 8

$\therefore$ There are 8 blue marbles.

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18. A lot of 144 ball pens contains 20 defective pens and the rest are good. Nuri will buy a pen if it is good but will not buy it if it is defective. The shopkeeper draws one pen at random. What is the probability that (i) she will buy it (ii) she will not buy it?

Solution :

Good pens = 144 − 20 = 124

(i) $P(\text{buy}) = \frac{124}{144} = \frac{31}{36}$

(ii) $P(\text{not buy}) = \frac{20}{144} = \frac{5}{36}$

$\therefore$ (i) $\frac{31}{36}$ (ii) $\frac{5}{36}$.

---

19. A die has its six faces marked 0, 1, 1, 1, 6, 6. Two such dice are thrown together and the total score is recorded. (i) How many different scores are possible? (ii) What is the probability of getting a total of 7?

Solution : Total outcomes = 6 × 6 = 36.

Possible totals:

0 + 0 = 0, 0 + 1 = 1, 1 + 1 = 2, 0 + 6 = 6, 1 + 6 = 7, 6 + 6 = 12

(i) Different scores: 0, 1, 2, 6, 7, 12 — 6 scores

(ii) Total 7 needs one face 1 (3 ways) and one face 6 (2 ways), in either order:

$3 \times 2 \times 2 = 12$ outcomes $\Rightarrow P = \frac{12}{36} = \frac{1}{3}$

$\therefore$ (i) 6 different scores (ii) $\frac{1}{3}$.

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20. A child has a die whose six faces show the letters A, B, C, D, E and A. The die is thrown once. What is the probability of getting (i) A (ii) D?

Solution : Total outcomes = 6.

(i) A appears on 2 faces: $P(A) = \frac{2}{6} = \frac{1}{3}$

(ii) D appears on 1 face: $P(D) = \frac{1}{6}$

$\therefore$ (i) $\frac{1}{3}$ (ii) $\frac{1}{6}$.

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21. Cards marked with numbers 3, 4, 5, …, 50 are placed in a box and mixed thoroughly. One card is drawn at random. Find the probability that the number on the card is (i) divisible by 7 (ii) a perfect square.

Solution : Total cards = 50 − 3 + 1 = 48.

(i) Multiples of 7: 7, 14, 21, 28, 35, 42, 49 — 7

$P = \frac{7}{48}$

(ii) Perfect squares: 4, 9, 16, 25, 36, 49 — 6

$P = \frac{6}{48} = \frac{1}{8}$

$\therefore$ (i) $\frac{7}{48}$ (ii) $\frac{1}{8}$.

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