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ANSWER KEY

Chapter 12 · Quadrilaterals

Class IX · As per New Curriculum **Ganita Manjari**

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Topic 1: What Exactly is a Quadrilateral?

1. (i) Adjacent to AB: BC and DA; opposite: CD

(ii) Adjacent to ∠A: ∠B and ∠D; opposite: ∠C

(iii) Two sides with no common endpoint are opposite sides; two angles whose vertices are not the ends of one side are opposite angles.

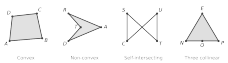
Answer: As above

2. WXYZ, XYZW, YZWX, ZWXY, WZYX, ZYXW, YXWZ, XWZY *[start anywhere, go either way]*

WYXZ uses diagonal WY as a side ⇒ invalid

Answer: ZYXW and XWZY are valid; WYXZ is not

3. ABCD: quadrilateral, convex (all angles < 180°)



DART: quadrilateral, non-convex (reflex angle at T)

CUTS: not a quadrilateral — two sides cross at a non-vertex point *[Definition 1]*

NOPE: not a quadrilateral — N, O, P are collinear

Answer: As above

4. (ii) The diagonals do not intersect: the diagonal from the reflex vertex lies inside, the other lies outside.

(iii) No: two reflex angles would already exceed 360° *[angle sum = 360°]*

Answer: As above

5. (i) No: A, B on one line and C, D on a parallel line ⇒ every angle < 180°

(ii) Yes: A(0, 0), B(4, 0), C(2, 5), D(2, 1) gives AB = CD = 4 with a reflex angle at D *[D inside ΔABC]*

(iii) Yes: a dart symmetric about BD has ∠A = ∠C and a reflex angle at D

Answer: No; yes; yes

6. (i) 12 parts = 360° ⇒ 60°, 90°, 90°, 120°

(ii) (360° − 210°) ÷ 3 = 50° each

(iii) 360° − (85° + 95° + 70°) = 110°

Answer: 60°, 90°, 90°, 120°; 50°; 110°

7. If two sides cross, the two lines containing them meet, so they lie in one plane — and all four vertices lie on these two lines.

Answer: No: a self-intersecting 4-gon is always planar

8. (i) 3 four-gons: ABCD, ABDC, ACBD *[each uses a different pair as diagonals]*

(ii) D inside ΔABC: all 3 are non-convex quadrilaterals; none convex, none self-intersecting

If A, B, C, D are in convex position: 1 convex, 2 self-intersecting

Answer: As above

9. Internal angle at A = ∠DAB (the angle less than 180° between AB and AD), unless A lies inside ΔBCD (formed by the other three vertices, including the opposite vertex C); in that case it is 360° − ∠DAB.
This gives a reflex angle exactly at the 'dent' vertex.

Answer: As above

Topic 2: Parallelograms and Their Converses

1. (i) True: ∠A = 90° ⇒ ∠B = 90° (co-interior), ∠C = ∠A, ∠D = ∠B

(ii) False: every rhombus has perpendicular diagonals, e.g., a rhombus with a 60° angle

(iii) True: ΔABC ≅ ΔDCB (SSS) ⇒ ∠B = ∠C; ∠B + ∠C = 180° ⇒ each 90°

Answer: True, false, true

2. (i) 3a + 2 = 5a − 6 ⇒ a = 4, AB = 14 *[opposite sides equal]*

(ii) 2y + 15° = 3y − 20° ⇒ y = 35°, ∠A = 85°, ∠B = 95° *[opposite angles equal; co-interior]*

(iii) p + 7 = 3p − 1 ⇒ p = 4: AO = 11, AC = 22, BO = 9 *[diagonals bisect each other]*

Answer: 14; 95°; AC = 22, BO = 9

3. (i) Yes: AC bisects ∠A, ∠C ⇒ ΔABC ≅ ΔADC (ASA) ⇒ AB = AD, CB = CD; BD likewise ⇒ BA = BC

(ii) Yes: a parallelogram (Theorem 4) with ΔAOB ≅ ΔAOD (SAS) ⇒ AB = AD

(iii) No (e.g., an isosceles trapezium). Add: 'and the diagonals bisect each other'

Answer: Yes; yes; no

4. Theorem 2: AB = CD, BC = DA, AC common ⇒ ΔABC ≅ ΔCDA *[SSS]*

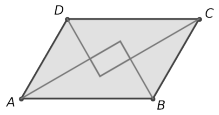
∠BAC = ∠DCA ⇒ AB ∥ DC; ∠BCA = ∠DAC ⇒ BC ∥ AD *[alternate angles]*

Theorem 4: OA = OC, OB = OD, ∠AOD = ∠COB ⇒ ΔAOD ≅ ΔCOB *[SAS]*

∠OAD = ∠OCB ⇒ AD ∥ BC; similarly AB ∥ DC *[alternate angles]*

Hence proved.

5.



$\angle A + \angle B = 180° \Rightarrow \frac{1}{2}\angle A + \frac{1}{2}\angle B = 90°$ *[co-interior]*

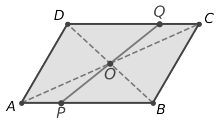
So the bisectors of ∠A and ∠B meet at 90°; likewise at the other three vertices *[angle sum of a triangle]*

∴ the inner quadrilateral has four right angles — a rectangle

If AB = BC the parallelogram is a rhombus: the bisectors lie along the diagonals and all meet at one point, so no rectangle is formed.

Hence proved.

6.



OA = OC *[diagonals bisect each other]*

∠OAP = ∠OCQ, ∠AOP = ∠COQ *[alternate; vertically opposite]*

ΔOAP ≅ ΔOCQ *[ASA]*

∴ OP = OQ *[CPCT]*

Hence proved.

7.

(i) ∠D = 360° − 250° = 110°

(ii) ∠A = ∠C = 70°, ∠B = ∠D = 110°

Answer: Yes — Theorem 3 (opposite angles equal)

8.

AB ∥ DC and AB = DC ⇒ parallelogram *[Theorem 5]*

BC = AD = 5 cm *[opposite sides]*

Answer: Perimeter = 2(7 + 5) = 24 cm

9.

$x^2 = y^2 \Rightarrow x = \pm y$, e.g., x = 3, y = −3

Converse false: ‘If n is divisible by 4 then n is even’ (6 is even but not divisible by 4).
Converse true: ‘If a quadrilateral is a parallelogram, its diagonals bisect each other’ (Theorem 4).

Answer: As above

10.

∠A + ∠D = 180° and ∠B + ∠C = 180° *[co-interior, AB ∥ DC]*

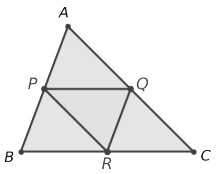
∠B = 180° − ∠C = 180° − ∠A = ∠D *[∠A = ∠C]*

∴ both pairs of opposite angles are equal ⇒ parallelogram *[Theorem 3]*

Hence proved.

Topic 3: The Midpoint Theorem and Its Converse

1.



(i) PQ common, QR = AP = $\frac{AB}{2}$, PR = AQ = $\frac{AC}{2}$ ⇒ ΔPQR ≅ ΔQPA *[SSS, Midpoint Theorem]*

Also ΔPQR ≅ ΔBRP ≅ ΔRCQ

(ii) Through each vertex of ΔPQR draw a line parallel to the opposite side; the three lines form ΔABC.

Answer: As above

2.

(i) PQ = 4.7 cm

(ii) 2(3x − 1) = 4x + 6 ⇒ x = 4, BC = 22 *[BC = 2PQ]*

(iii) ∠APQ = ∠ABC = 62° *[corresponding angles, PQ ∥ BC]*

Answer: 4.7 cm; x = 4, BC = 22; 62°

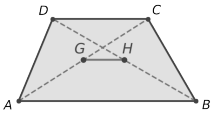
3.

ΔDEF: $\frac{8+11+9}{2}$ = 14 cm

BDEF: BD = 5.5, DE = 4, EF = 5.5, FB = 4 *[half the parallel sides]*

Answer: 14 cm and 19 cm

4.



Let E be the midpoint of AD. In ΔADC: EG ∥ DC; in ΔDAB: EH ∥ AB *[Midpoint Theorem]*

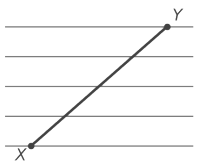
AB ∥ DC ⇒ EG and EH are lines through E parallel to AB

∴ E, G, H are collinear and GH ∥ AB *[unique parallel through E]*

If AB = CD, ABCD is a parallelogram and G = H (diagonals bisect each other), so there is no segment GH.

Hence proved.

5.



X lies on line 0 and Y on line 4; the midpoint M is where XY crosses line 2.

Through X draw a line perpendicular to the ruled lines meeting line 4 at Z; equal spacing makes line 2 pass through the midpoint N of XZ.

In ΔXZY: N is the midpoint of XZ and NM ∥ ZY ⇒ M is the midpoint of XY *[Theorem 7]*

Hence proved.

6. Folding A onto B makes the crease the perpendicular bisector of AB: it passes through the midpoint of AB and is perpendicular to AB.

BC ⊥ AB ⇒ crease ∥ BC

∴ the crease bisects AC *[Theorem 7 in ΔABC]*

Hence proved.

7.

(i) Join AC, meeting EF at G. In ΔADC: E midpoint, EG ∥ DC ⇒ G midpoint of AC, EG = $\frac{DC}{2}$ *[Theorem 7]*

In ΔCAB: G midpoint, GF ∥ AB ⇒ F midpoint of BC, GF = $\frac{AB}{2}$ *[Theorem 7]*

∴ EF = EG + GF = $\frac{1}{2}$(AB + DC)

(ii) EF = $\frac{18+11}{2}$

Answer: 14.5 cm

8.

QS = QP, QA = QC, ∠AQP = ∠CQS ⇒ ΔAPQ ≅ ΔCSQ *[SAS]*

CS = AP = PB and ∠QAP = ∠QCS ⇒ CS ∥ PB *[CPCT; alternate angles]*

PBCS is a parallelogram *[Theorem 5]*

∴ PQ ∥ BC and PQ = $\frac{PS}{2}$ = $\frac{BC}{2}$

Hence proved.

9.

EF = $\frac{3+5}{2}$ = 4 cm, EF ∥ AD ∥ BC *[trapezium midline]*

Both parts have height $\frac{h}{2}$ *[E, F are midpoints]*

Areas: $\frac{1}{2}(3+4)\frac{h}{2} : \frac{1}{2}(4+5)\frac{h}{2}$

Answer: 7 : 9

Topic 4: Centroid, Varignon Parallelogram and Tiling

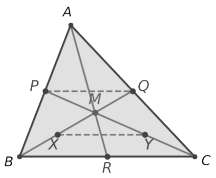
1. (i) AM = $\frac{2}{3} \times 15 = 10$ cm

(ii) $3y + 1 = 2(y + 2) \Rightarrow y = 3$; BM = 10, MQ = 5

(iii) CM = $2 \times 2.4 = 4.8 \Rightarrow$ CP = 7.2 cm

Answer: 10 cm; y = 3, BQ = 15; 7.2 cm

2. PQ ∥ BC, XY ∥ BC and PQ = XY = $\frac{BC}{2}$ *[Midpoint Theorem in ΔABC, ΔMBC]*



∠MPQ = ∠MYX, ∠MQP = ∠MXY *[alternate angles, PQ ∥ XY]*

ΔMPQ ≅ ΔMYX ⇒ MP = MY = YC, MQ = MX = XB *[ASA; CPCT]*

∴ CM : MP = 2 : 1

AR and BQ meet at N with BN : NQ = 2 : 1 (same argument). Only one point divides BQ in this ratio, so N = M.

Hence proved.

3. (i) PQRS is a parallelogram; PR and QS are its diagonals *[Theorem 9]*

∴ they bisect each other *[Theorem 1]*

(ii) AC = BD ⇒ PQ = $\frac{AC}{2} = \frac{BD}{2}$ = QR ⇒ PQRS is a rhombus ⇒ PR ⊥ QS

Converse: PR ⊥ QS makes the parallelogram PQRS a rhombus, so PQ = QR, i.e., AC = BD. True.

Hence proved.

4. (i) PQ = QR ⇒ AC = BD; PQ ⊥ QR with PQ ∥ AC, QR ∥ BD ⇒ AC ⊥ BD

(ii) AC = BD ⇒ PQ = QR; AC ⊥ BD ⇒ ∠PQR = 90°

∴ PQRS is a square

Hence proved.

5. MB = DN and MB ∥ DN ⇒ MBND is a parallelogram ⇒ DM ∥ NB *[Theorem 5]*

Let DM, BN meet AC at X, Y.

In ΔABY: M midpoint of AB, MX ∥ BY ⇒ AX = XY *[Theorem 7]*

In ΔCDX: N midpoint of CD, NY ∥ DX ⇒ CY = YX *[Theorem 7]*

∴ AX = XY = YC

To trisect PQ: draw any parallelogram with diagonal PQ, join each vertex to the midpoint of the opposite side as above.

Hence proved.

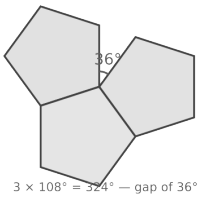
6. Rotate a copy of the triangle by 180° about the midpoint of one side; the two copies form a parallelogram (diagonals bisect each other).

Parallelograms tile the plane in a grid of two sets of parallel lines; so the triangle tiles too.

Answer: As above

7. Regular pentagon: $3 \times 108° = 324° < 360°$, $4 \times 108° = 432° > 360°$

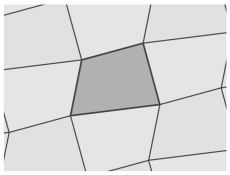
Regular hexagon: $3 \times 120° = 360°$



Quadrilateral: its four angles add up to 360°, so four copies (each angle once) fit around a point; Method 1 continues the tiling.

Answer: As above

8. Rotate copies by 180° about the midpoints of the sides again and again (Method 1).



Around every vertex the four different angles of the quadrilateral meet once each; they add up to 360°, so the copies fit without gaps.

Answer: Angle sum 360°

9. PB = PC, ∠MPB + ∠MPC = 180° ⇒ joining along PB = PC gives a triangle with base 2MP

2MP = AM *[Centroid Theorem]*

So ΔMPB + ΔMPC form a triangle with sides AM, BM, CM. The other two pairs give the same sides.

∴ three congruent triangles *[SSS]*

Hence proved.

10. Yes for both: the proof only applies the Midpoint Theorem to ΔABC, ΔADC, ΔBCD, ΔBAD, which exist in every case.

PQ ∥ AC and QR ∥ BD; if AC ∥ BD then P, Q, R, S are collinear

Answer: It becomes a segment when AC ∥ BD (possible for a self-intersecting 4-gon)

Miscellaneous Questions

1. Midpoint AC = (4, 4.5) = midpoint BD

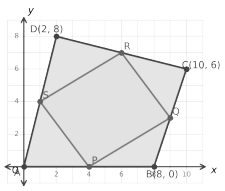
∴ diagonals bisect each other ⇒ parallelogram *[Theorem 4]*

AB = $\sqrt{17}$, BC = $\sqrt{20}$ ⇒ not a rhombus

AC = $\sqrt{61}$, BD = $\sqrt{13}$ ⇒ not a rectangle

Answer: Parallelogram only

2. P(4, 0), Q(9, 3), R(6, 7), S(1, 4)



Midpoint PR = (5, 3.5) = midpoint QS ⇒ parallelogram

PQ = $\sqrt{25+9} = \sqrt{34}$; AC = $\sqrt{100+36} = 2\sqrt{34}$

Answer: PQ = $\frac{1}{2}$ AC ✓

3. The midpoint triangle and the three corner triangles are congruent (SSS, Midpoint Theorem), so each is $\frac{1}{4}$ of ΔABC.

$64 \times \frac{1}{4} \times \frac{1}{4} = 4$ cm²

Answer: 4 cm²

4. R(3, 0); G = (3, 3) *[B(3, 9) to R(3, 0): two-thirds of the way]*

BG = 6, GR = 3 ⇒ BG = 2GR

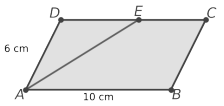
Answer: G(3, 3)

5. EF = $\frac{14+8}{2} = 11$ cm

Area = $\frac{1}{2}$(AB + DC) × h = EF × h

Answer: 66 cm²

6. ∠DEA = ∠EAB (alternate) = ∠DAE *[AE bisects ∠A]*



⇒ DE = DA = 6 cm, EC = 10 − 6 = 4 cm

∠AEC = 180° − ∠DEA = 180° − 35° = 145°

Answer: DE = 6 cm, EC = 4 cm; 145°

7. (i) Each angle 135°; 2 × 135 = 270, 3 × 135 = 405 ⇒ no

(ii) 135° + 135° + 90° = 360° ⇒ yes

Answer: (iii) Equilateral triangles, squares and regular hexagons

Competency Based Question Bank

1. Equal sides are not required.

Answer: (c) all sides equal

2. Two sides lie on one line.

Answer: (c) not a quadrilateral

3. Go round the boundary.

Answer: (d) BADC

4. No common endpoint.

Answer: (a) CD

5. Definition of convex.

Answer: (b) all angles are less than 180°

6. One diagonal lies outside.

Answer: (c) do not intersect

7. $10x = 360°$.

Answer: (d) 36°

8. Vertices on two parallel lines.

Answer: (d) must be convex

9. $\Delta ACD \cong \Delta CAB$ by ASA.

Answer: (b) ASA

10. Congruence by SSS.

Answer: (c) SSS

11. $8x - 20° = 180°$.

Answer: (d) 25°

12. Theorem 4.

Answer: (b) parallelogram

13. Co-interior angles ⇒ parallel sides.

Answer: (a) parallelogram

14. Theorem 5.

Answer: (c) parallelogram

15. SSS gives $\angle B = \angle C = 90°$.

Answer: (d) rectangle

16. Base angles equal ⇒ AB = BC.

Answer: (a) rhombus

17. $x = 5 \Rightarrow AC = 2 \times 11$.

Answer: (b) 22

18. Half of BC.

Answer: (c) 6.5 cm

19. Half of 24.

Answer: (d) 12

20. Medial triangle.

Answer: (b) 4 congruent triangles

21. Theorem 7.

Answer: (a) bisects AC

22. l ∥ BA through C.

Answer: (c) BA

23. (14 + 8) ÷ 2.

Answer: (c) 11 cm

24. Centroid Theorem.

Answer: (a) 2 : 1

25. $\frac{2}{3} \times 12$.

Answer: (c) 8 cm

26. x = 4: 10 + 5.

Answer: (c) 15

27. Theorem 9.

Answer: (a) parallelogram

28. AC + BD.

Answer: (a) 18 cm

29. PQ = QR.

Answer: (b) AC = BD

30. Angles cannot make 360°.

Answer: (c) 108° is not a factor of 360°

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31. Four angles fit around a point.

Answer: (d) its angles add up to 360°

32. Smith, Myers, Kaplan, Goodman-Strauss.

Answer: (c) 2023

33. A true; R is the step that gives the parallel sides.

Answer: (a)

34. A false (non-convex ⇒ diagonals don’t meet); R true.

Answer: (d)

35. A true; the parallelogram BCRP proves it.

Answer: (a)

36. R is exactly the argument of the Centroid Theorem.

Answer: (a)

37. A true; R false (polygon angle sums vary).

Answer: (c)

38. A false (angles need not be 90°); R true.

Answer: (d)

Story Time: Case Studies

Case 1 · Robot Arm Linkage



(i) ABCD.

(ii) DART.

(iii) CUTS.

(iv) Linkages pass through non-convex and self-intersecting shapes.

Case 2 · The Folding Gate

(i) Theorem 2 (opposite sides equal).

(ii) 116°, 64°, 116°.

(iii) When one angle is 90°.

(iv) They become equal.

Case 3 · Kite Frame Check

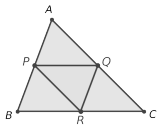
(i) At the midpoint of each: 30 cm and 20 cm from the ends.

(ii) A rhombus.

(iii) A rectangle.

(iv) No — e.g., an isosceles trapezium.

Case 4 · Triangular Park



(i) 60 m, 45 m, 75 m.

(ii) 180 m.

(iii) 4.

(iv) $\frac{1}{4}$.

Case 5 · Ladder Rungs

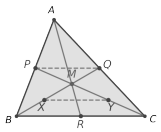
(i) (50 + 30) ÷ 2 = 40 cm.

(ii) Trapezium midline (Midpoint Theorem).

(iii) 50, 45, 40, 35, 30 cm.

(iv) 200 cm.

Case 6 · Balancing a Triangle



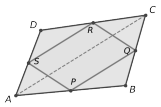
(i) AM = 12 cm, MR = 6 cm.

(ii) 10 cm.

(iii) Fold to find midpoints of two sides; the two medians cross at M.

(iv) Centroid Theorem.

Case 7 · Garden in a Garden



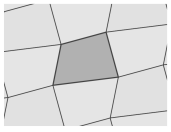
(i) A parallelogram.

(ii) 80 + 60 = 140 m.

(iii) A rectangle.

(iv) Each corner triangle is $\frac{1}{4}$ of a half of ABCD ⇒ PQRS is $\frac{1}{2}$ of the field.

Case 8 · Tile Designer



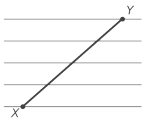
(i) 360°.

(ii) Its four angles add up to 360°.

(iii) Rotate copies by 180° about the midpoints of the sides.

(iv) No: 108° copies cannot make 360°.

Case 9 · Equal Spacing Trick



(i) Put X on one line and Y on the line 2 gaps away; the middle line marks the midpoint.

(ii) Span 4 gaps; the 3 lines in between divide XY into 4 parts.

(iii) Converse of the Midpoint Theorem (equal intercepts).

(iv) 6 equal parts.

Case 10 · The Folded Right Triangle

(i) The fold is the perpendicular bisector of AB.

(ii) Both the crease and BC are ⊥ AB.

(iii) The midpoint of AC.

(iv) 5 cm.

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