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# TOPIC-WISE ASSIGNMENT

## Chapter 12 · Quadrilaterals

Class IX · As per New Curriculum **Ganita Manjari**

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TOPIC 1

# 1 What Exactly is a Quadrilateral?

Definition of a quadrilateral | Vertices, sides, diagonals | Adjacent and opposite | Convex and non-convex | Self-intersecting and non-planar 4-gons

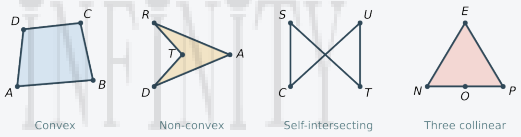
**QUICK RECAP** (from earlier classes)

- Three non-collinear points form a triangle; collinear points give only a segment.
- The angles of a quadrilateral add up to 360°.
- A reflex angle is greater than 180°.

## 01 ✦ Concept Zone: Theorems & Results

### Definition of a Quadrilateral

- **Definition 1.** Let A, B, C, D be four distinct points in a **plane**. The points on segments AB, BC, CD, DA form **quadrilateral ABCD** if every such point other than A, B, C, D lies on **exactly one** of the four segments.
- This rules out three collinear vertices (like NOPE — a triangle or overlapping sides), and self-intersecting 4-gons (like CUTS). Non-planar 4-gons (like BENT) are excluded by 'in a plane'.
- ABCD may also be named BCDA, CDAB, DABC, DCBA, ADCB, BADC or CBAD — but by no other order of the letters.

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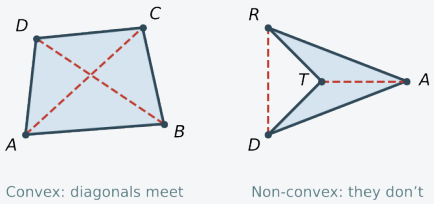
### Parts, Convexity and Kinds of 4-gons

- Vertices A, B, C, D; sides AB, BC, CD, DA; diagonals AC, BD. Sides with a common endpoint are **adjacent**; sides with no common endpoint are **opposite**. Likewise for angles.
- A quadrilateral is **convex** if all its internal angles are less than 180°; otherwise **non-convex** (it has a 'dent', like DART with ∠D > 180°).
- Test: the diagonals of a convex quadrilateral intersect; those of a non-convex one do not.
- 4-gons: planar or non-planar (skew); planar ones are self-intersecting or not; non-self-intersecting ones (our 'quadrilaterals') are convex or non-convex. Linkages in machines take all these shapes.




TOPIC 1

**KEY NOTES**

- ✔ No three vertices collinear; all four in one plane; sides meet only at vertices.
- ✔ Each side has 2 adjacent sides and 1 opposite side.
- ✔ Convex ⟺ every angle < 180° ⟺ diagonals intersect.
- ✔ 'Quadrilateral' means planar and non-self-intersecting unless said otherwise.

**NOTE** The angle sum 360° holds for non-convex quadrilaterals too (one angle may be reflex).

## 02 ✎ Solved Examples

**EXAMPLE 1**

Which of these name the quadrilateral ABCD: (i) DCBA, (ii) ACBD, (iii) BADC, (iv) CABD?

Solution

- Valid names go round the boundary in either direction, starting anywhere.
- DCBA and BADC do; ACBD and CABD use a diagonal as a side.

**∴ (i) and (iii)**

**EXAMPLE 2**

In quadrilateral PQRS, list (i) sides adjacent to PQ, (ii) the side opposite PQ, (iii) angles adjacent to ∠P, (iv) the angle opposite ∠P.

Solution

- PQ shares P with SP and Q with QR.
- RS has no common endpoint with PQ.
- ∠Q and ∠S are at adjacent vertices; ∠R is not.

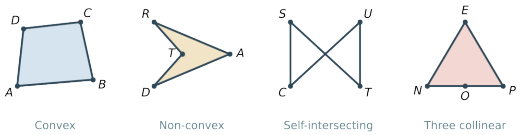
**∴ (i) SP, QR (ii) RS (iii) ∠Q, ∠S (iv) ∠R**

**EXAMPLE 3**

Explain why NOPE (N, O, P collinear) and CUTS are not quadrilaterals by Definition 1.

Solution

- NOPE: NO and OP lie on one line — the figure is the triangle NPE with O on a side.
- CUTS: sides CU and TS cross at a point that is not a vertex and lies on two sides.



Convex · Non-convex · Self-intersecting · Three collinear

**∴ Both fail Definition 1**

**EXAMPLE 4**

In a quadrilateral, AB ∥ DC. Show that ABCD is convex.

Solution

- A, B lie on one line and C, D on a parallel line.
- At each vertex one side runs along its line and the other goes across to the other line, so each angle is less than 180°.

TOPIC 1

**∴ ABCD is convex**

**EXAMPLE 5**

The angles of a quadrilateral are x, 2x, 3x and 4x. Find them. Is it convex?

Solution

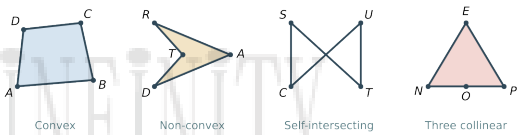
- x + 2x + 3x + 4x = 360° ⇒ x = 36°.
- Angles 36°, 72°, 108°, 144° — all less than 180°.

**∴ 36°, 72°, 108°, 144°; convex**

## 03 ⚡ Practice Zone

Attempt in your notebook. Draw a neat figure wherever needed. ★ = higher-order thinking

1. Let ABCD be a quadrilateral. (i) List the sides adjacent to AB and the side opposite AB. (ii) List the angles adjacent to ∠A and the angle opposite ∠A. (iii) Define opposite sides and opposite angles without naming vertices.
2. Write all 8 names of quadrilateral WXYZ. Which of WYXZ, ZYXW, XWZY are valid names?
3. For each figure, say whether it is a quadrilateral by Definition 1; if yes, whether it is convex. Give reasons.




4. (i) Draw a non-convex quadrilateral and mark its reflex angle. (ii) Draw its diagonals. What do you notice? (iii) Can a non-convex quadrilateral have two reflex angles? Why?
5. In quadrilateral ABCD, can ABCD be non-convex if (i) AB ∥ DC, (ii) AB = CD, (iii) ∠A = ∠C? Draw figures to support your answers.
6. Find the unknown angles: (i) angles in the ratio 2 : 3 : 3 : 4; (ii) a non-convex quadrilateral with a reflex angle of 210° and the other three angles equal; (iii) three angles 85°, 95°, 70°.
7. Can a 4-gon be both self-intersecting and non-planar? Explain.
8. Four points A, B, C, D, no three collinear, are given. (i) How many different 4-gons (self-intersecting allowed) do they form? (ii) How many are convex and how many self-intersecting if D lies inside ΔABC? If ABCD is convex? ★
9. Precisely define the internal angle of a quadrilateral at a vertex, so that the definition works for a non-convex quadrilateral too. (Hint: use the opposite vertex.) ★

**MY NOTES** ✎

TOPIC 1

TOPIC 2

# 2 Parallelograms and Their Converses

Properties of a parallelogram | Converse tests | Equal parallel sides test | Rectangles, rhombuses and squares

**QUICK RECAP** (from earlier classes)

- Congruence tests: SSS, SAS, ASA, AAS (and RHS).
- Alternate angles are equal and co-interior angles add up to 180° for parallel lines; conversely they give parallel lines.
- A parallelogram has both pairs of opposite sides parallel.

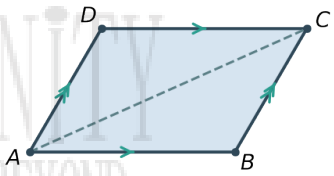
## 01 ✦ Concept Zone: Theorems & Results

**THEOREM 1**

**In a parallelogram (a) opposite sides are equal, (b) opposite angles are equal, (c) the diagonals bisect each other.**

Given: Parallelogram ABCD (AB ∥ DC, AD ∥ BC), diagonals meeting at E.

To prove: AB = DC, AD = BC; ∠A = ∠C, ∠B = ∠D; EA = EC, EB = ED.




Proof:

**(a)** AC is a transversal for AB ∥ DC and for AD ∥ BC.

∠BAC = ∠DCA and ∠BCA = ∠DAC *[ alternate angles ]*

ΔACD ≅ ΔCAB *[ ASA, AC common ]*

**∴ AB = DC and AD = BC** *[ CPCT ]*

**(b)** ∠A + ∠B = 180°, ∠B + ∠C = 180° *[ co-interior angles ]*

**∴ ∠A = ∠C; similarly ∠B = ∠D**

**(c)** ΔAED ≅ ΔCEB *[ ASA/AAS using AD = CB ]*

**∴ EA = EC and EB = ED** *[ CPCT ]*

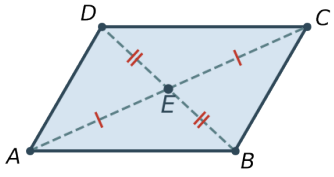
Hence proved.

**THEOREMS 2–4**

**A quadrilateral is a parallelogram if (2) both pairs of opposite sides are equal, or (3) both pairs of opposite angles are equal, or (4) its diagonals bisect each other.**

Given: Quadrilateral ABCD with the stated property.

To prove: AB ∥ DC and AD ∥ BC.




TOPIC 2

Proof:

**(2)** AB = CD, AD = CB, AC common ⇒ ΔACD ≅ ΔCAB — *[ SSS ]*

**∴ ∠BAC = ∠DCA ⇒ AB ∥ DC; ∠DAC = ∠BCA ⇒ AD ∥ BC** — *[ alternate angles ]*

**(3)** 360° = ∠A + ∠B + ∠C + ∠D = 2(∠A + ∠B) = 2(∠B + ∠C) — *[ ∠A = ∠C, ∠B = ∠D ]*

**∴ ∠A + ∠B = 180° ⇒ BC ∥ AD; ∠B + ∠C = 180° ⇒ AB ∥ DC** — *[ co-interior angles ]*

**(4)** EA = EC, EB = ED, ∠AED = ∠CEB ⇒ ΔAED ≅ ΔCEB — *[ SAS ]*

**∴ ∠DAE = ∠BCE ⇒ AD ∥ BC; similarly ΔEAB ≅ ΔECD ⇒ AB ∥ DC** — *[ alternate angles ]*

Hence proved.

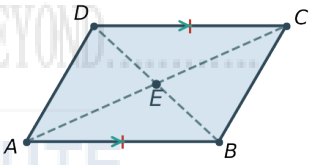
Strategy: run the proof of Theorem 1 **backwards**, justifying each step afresh. Theorem 3 can also be read: 'If each pair of adjacent angles adds up to 180°, then ABCD is a parallelogram.'

**THEOREM 5**

**Equal parallel sides test: a quadrilateral with one pair of opposite sides equal and parallel is a parallelogram.**

Given: AB ∥ DC and AB = DC; diagonals meet at E.

To prove: ABCD is a parallelogram.

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Proof:

∠EAB = ∠ECD and ∠EBA = ∠EDC — *[ alternate angles, AB ∥ DC ]*

ΔEAB ≅ ΔECD — *[ ASA, AB = CD ]*

EA = EC and EB = ED — *[ CPCT ]*

**∴ the diagonals bisect each other, so ABCD is a parallelogram** — *[ Theorem 4 ]*

Hence proved.

## Special Parallelograms

- A parallelogram with one right angle is a **rectangle** (adjacent angles add to 180°, opposite angles are equal); its diagonals are equal. Conversely, a parallelogram with equal diagonals is a rectangle.
- A parallelogram with a pair of equal adjacent sides is a **rhombus**; its diagonals are perpendicular and bisect its angles.
- A **square** is both a rectangle and a rhombus. In ancient Indian study, quadrilaterals were first classified by whether their diagonals were equal.

**KEY NOTES**

- ✔ Opposite sides equal ⇔ parallelogram ⇔ opposite angles equal ⇔ diagonals bisect each other.
- ✔ One pair of sides equal and parallel is enough.
- ✔ Adjacent angles of a parallelogram add up to 180°.
- ✔ The truth of a statement and of its converse are separate questions.

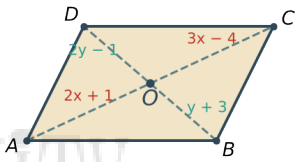
**NOTE** To prove a converse, try retracing the original proof backwards — but the same congruence may need a different test (e.g., ASA in Theorem 1(a), SSS in Theorem 2).

## 02 ✎ Solved Examples

**EXAMPLE 1**

In parallelogram ABCD the diagonals meet at O with OA = 2x + 1, OC = 3x − 4, OB = y + 3, OD = 2y − 1. Find x, y, AC and BD.

Solution



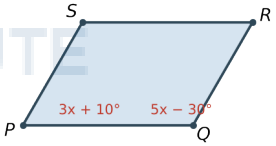

- Diagonals bisect each other: 2x + 1 = 3x − 4 ⇒ x = 5.
- y + 3 = 2y − 1 ⇒ y = 4.
- AC = 2 × 11 = 22, BD = 2 × 7 = 14.

**∴ x = 5, y = 4, AC = 22, BD = 14**

**EXAMPLE 2**

In parallelogram PQRS, ∠P = 3x + 10° and ∠Q = 5x − 30°. Find all the angles.

Solution




- ∠P + ∠Q = 180° (co-interior) ⇒ 8x − 20° = 180° ⇒ x = 25°.
- ∠P = 85°, ∠Q = 95°; ∠R = ∠P, ∠S = ∠Q.

**∴ 85°, 95°, 85°, 95°**

**EXAMPLE 3**

The diagonal AC of parallelogram ABCD bisects ∠A. Show that it bisects ∠C and that ABCD is a rhombus.

Solution

- ∠DAC = ∠BCA and ∠BAC = ∠DCA (alternate angles). Since ∠DAC = ∠BAC, ∠BCA = ∠DCA.
- In ΔABC, ∠BAC = ∠BCA ⇒ AB = BC.

**∴ AC bisects ∠C; AB = BC, so ABCD is a rhombus**

**EXAMPLE 4**

Show that a parallelogram with a right angle is a rectangle.

Solution

- ∠A = 90° ⇒ ∠B = 180° − 90° = 90° (adjacent angles).
- ∠C = ∠A = 90°, ∠D = ∠B = 90° (opposite angles).

TOPIC 2

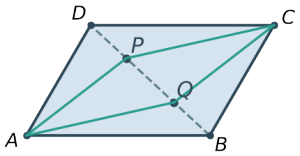
**∴ All angles 90° — a rectangle**

**EXAMPLE 5**

In parallelogram ABCD, P and Q are on BD with DP = BQ. Show that APCQ is a parallelogram.

Solution

- Let the diagonals meet at O: OA = OC and OB = OD.
- OP = OD − DP = OB − BQ = OQ.
- Diagonals AC and PQ of APCQ bisect each other (Theorem 4).




**∴ APCQ is a parallelogram**

## 03 ⚡ Practice Zone

Attempt in your notebook. Draw a neat figure wherever needed. ★ = higher-order thinking

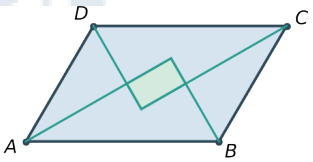
1. True or false? Justify. (i) A parallelogram with a right angle is a rectangle. (ii) A rhombus with perpendicular diagonals is a square. (iii) If the diagonals of a parallelogram are equal, it is a rectangle.

2. Find the unknowns in parallelogram ABCD: (i) AB = 3a + 2, DC = 5a − 6; (ii) ∠A = 2y + 15°, ∠C = 3y − 20°; find ∠B; (iii) diagonals meet at O with AO = p + 7, OC = 3p − 1, BD = 18.

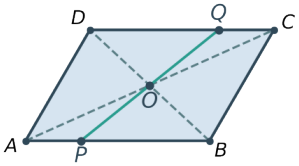
3. Answer Yes or No; if No, add a condition that makes it Yes. (i) If the diagonals of ABCD bisect its angles, must it be a rhombus? (ii) If they bisect each other at right angles? (iii) If they are equal, must it be a rectangle?

4. Prove Theorem 2 (opposite sides equal ⇒ parallelogram) and Theorem 4 (diagonals bisect each other ⇒ parallelogram).

5. ABCD is a parallelogram with AB ≠ BC. Show that the four angle bisectors meet pairwise in the vertices of a rectangle. Why is AB ≠ BC assumed?




6. The diagonals of parallelogram ABCD meet at O. A line through O meets AB at P and CD at Q. Show that O is the midpoint of PQ.




7. In quadrilateral ABCD, ∠A = 70°, ∠B = 110°, ∠C = 70°. (i) Find ∠D. (ii) Is ABCD a parallelogram? Which theorem do you use?

8. ABCD is a quadrilateral with AB ∥ DC and AB = DC = 7 cm, AD = 5 cm. Find the perimeter and state the theorem used.

9. If x = y then $x^2 = y^2$. Why can this not be reversed? Give another true statement whose converse is false, and one whose converse is true. ★

10. In quadrilateral ABCD, AB ∥ DC and ∠A = ∠C. Prove that ABCD is a parallelogram. ★

TOPIC 2

TOPIC 3

# 3 The Midpoint Theorem and Its Converse

Midpoint Theorem | Auxiliary construction | Converse of the Midpoint Theorem | Four congruent triangles | Trapezium midline

**QUICK RECAP** (from earlier classes)

- Equal parallel sides test (Theorem 5).
- Vertically opposite angles are equal.
- Through a point there is exactly one line parallel to a given line.

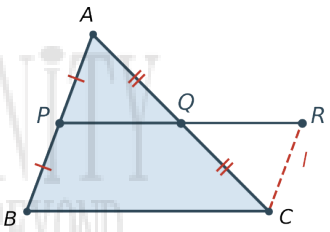
## 01 ✦ Concept Zone: Theorems & Results

**THEOREM 6**

**Midpoint Theorem: the segment joining the midpoints of two sides of a triangle is parallel to the third side and half of it.**

Given: In ΔABC, P and Q are the midpoints of AB and AC.

To prove: PQ ∥ BC and $PQ = \frac{1}{2}BC$.




Construction: Through C draw line l ∥ BA, meeting PQ produced at R.

Proof:

| | |
|---|---|
| In ΔAPQ and ΔCRQ: AQ = CQ, ∠AQP = ∠CQR | *[ Q midpoint; vertically opposite ]* |
| ∠APQ = ∠CRQ | *[ alternate angles, AP ∥ RC ]* |
| ΔAPQ ≅ ΔCRQ | *[ AAS ]* |
| $PQ = QR = \frac{PR}{2}$ and CR = AP = BP | *[ CPCT; P midpoint ]* |
| CR = BP and CR ∥ BP ⇒ BCRP is a parallelogram | *[ Theorem 5 ]* |
| **∴ PQ ∥ BC and $PQ = \frac{PR}{2} = \frac{BC}{2}$** | *[ opposite sides of BCRP ]* |

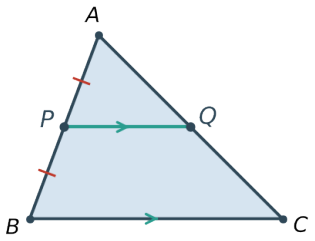
Hence proved.

The new line l is a classic **auxiliary construction**; such ideas come with experience.

TOPIC 3

**THEOREM 7**

**Converse of the Midpoint Theorem: the line through the midpoint of one side of a triangle, parallel to another side, bisects the third side (and the segment is half the parallel side).**




Given: P is the midpoint of AB; the line through P parallel to BC meets AC at Q.

To prove: Q is the midpoint of AC.

Proof:

Let M be the midpoint of AC.

PM ∥ BC *[ Midpoint Theorem in ΔABC ]*

Line PM = line PQ *[ only one line through P parallel to BC ]*

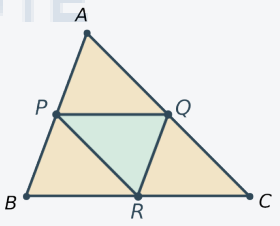
**∴ both meet AC at the same point: M = Q, so Q is the midpoint of AC**

**and PQ = PM =** $\frac{BC}{2}$ *[ Midpoint Theorem ]*

Hence proved.

## Using the Midpoint Theorem

- Joining the three midpoints cuts a triangle into **4 congruent triangles**, each a half-size copy of it.
- **Trapezium:** if AB ∥ DC and E is the midpoint of AD, the line through E parallel to AB bisects BC at F, and EF = $\frac{1}{2}$(AB + DC).
- **Equally spaced parallel lines** cut any transversal into equal parts — this finds midpoints without measuring.




## KEY NOTES

- ✔ PQ ∥ BC and PQ = $\frac{BC}{2}$.
- ✔ Midpoint + parallel ⇒ midpoint (converse).
- ✔ Medial triangle: perimeter is half, and it is congruent to the 3 corner triangles.
- ✔ Trapezium midline = half the sum of the parallel sides.

**NOTE** A theorem can sometimes be used to prove its own converse — here with the extra fact that only one line through P is parallel to BC.

TOPIC 3

## 02 ✎ Solved Examples

**EXAMPLE 1**

The sides of ΔABC are 6 cm, 8 cm and 10 cm. Find the perimeter of the triangle formed by joining the midpoints of its sides.

Solution

- Each side of the midpoint triangle is half the parallel side: 3, 4, 5.
- Perimeter = 3 + 4 + 5.

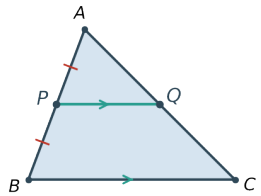
**∴ 12 cm**

**EXAMPLE 2**

In ΔABC, P is the midpoint of AB and PQ ∥ BC meets AC at Q. If AQ = 2x + 1, QC = x + 4 and BC = 13 cm, find x and PQ.

Solution

- Q is the midpoint of AC (converse): 2x + 1 = x + 4 ⇒ x = 3.
- PQ = $\frac{1}{2}$ BC.



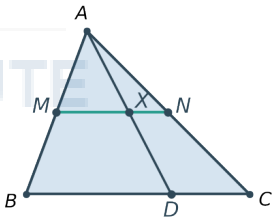

**∴ x = 3, PQ = 6.5 cm**

**EXAMPLE 3**

M, N are the midpoints of AB, AC and D is any point on BC. Show that MN bisects AD.

Solution

- MN ∥ BC (Midpoint Theorem). Let MN meet AD at X.
- In ΔABD, M is the midpoint of AB and MX ∥ BD ⇒ X is the midpoint of AD (converse).



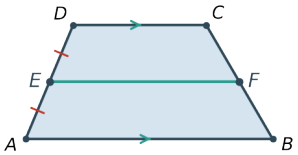

**∴ AX = XD**

**EXAMPLE 4**

ABCD is a trapezium with AB ∥ DC, AB = 14 cm, DC = 8 cm. E is the midpoint of AD and EF ∥ AB meets BC at F. Find EF.

Solution

- Join AC, meeting EF at G. In ΔADC: G is the midpoint of AC, EG = $\frac{DC}{2}$ = 4.
- In ΔCAB: GF = $\frac{AB}{2}$ = 7.




**∴ EF = 11 cm**

TOPIC 3

**EXAMPLE 5**

Prove that the four triangles formed by joining the midpoints P, Q, R of AB, AC, BC are congruent.

Solution

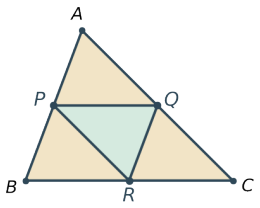
- PQ = BR = RC, QR = AP = PB, PR = AQ = QC (Midpoint Theorem).
- So ΔAPQ, ΔPBR, ΔQRC and ΔRQP all have sides $\frac{a}{2}, \frac{b}{2}, \frac{c}{2}$ ⇒ congruent by SSS.

**∴ All four are congruent**

## 03 ⚡ Practice Zone

Attempt in your notebook. Draw a neat figure wherever needed. ★ = higher-order thinking

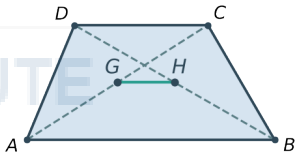
**1** P, Q, R are the midpoints of AB, AC, BC of ΔABC. (i) Show that ΔPQR ≅ ΔQPA and identify two other triangles congruent to it. (ii) If ΔABC is erased, how can you reconstruct it from ΔPQR?



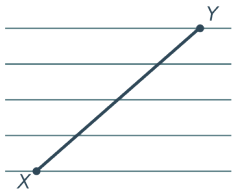

**2** Find the unknowns (P, Q midpoints of AB, AC): (i) BC = 9.4 cm, find PQ; (ii) PQ = 3x − 1, BC = 4x + 6, find x and BC; (iii) ∠ABC = 62°, find ∠APQ.

**3** In ΔABC, D, E, F are the midpoints of BC, CA, AB. If AB = 8 cm, BC = 11 cm and CA = 9 cm, find the perimeters of ΔDEF and of quadrilateral BDEF.

**4** In quadrilateral ABCD, AB ∥ DC and AB ≠ CD. G and H are the midpoints of AC and BD. Prove that GH ∥ AB. Why is AB ≠ CD assumed?




**5** The horizontal lines in the figure are equally spaced. Mark the midpoint of XY and justify your answer.




**6** A right-triangle shaped paper ABC (right angle at B) is folded so that A falls on B. Show that the crease passes through the midpoints of AB and AC.

**7** ABCD is a trapezium with AB ∥ DC. E is the midpoint of AD. (i) If EF ∥ AB meets BC at F, show F is the midpoint of BC and EF = $\frac{1}{2}$(AB + DC). (ii) Find EF if AB = 18 cm and DC = 11 cm.

**8** Give another proof of the Midpoint Theorem: extend PQ beyond Q to S with QS = PQ, show ΔAPQ ≅ ΔCSQ and complete the proof. ★

**9** ABCD is a trapezium with parallel sides AD = 3 cm and BC = 5 cm. E, F are the midpoints of the non-parallel sides. Find the ratio of the areas of AEFD and EBCF. ★

TOPIC 3

TOPIC 4

# 4 Centroid, Varignon Parallelogram and Tiling

Medians and centroid | 2 : 1 division | Midpoint Theorem for quadrilaterals | Tiling with any quadrilateral | Why regular pentagons fail

**QUICK RECAP** (from earlier classes)

- A median joins a vertex to the midpoint of the opposite side.
- The angles around a point add up to 360°.
- Only one point divides a segment in a given ratio.

❖ **MUST KNOW · Tilings that Never Repeat**

Mathematicians have found ways to tile the plane with small sets of shapes that never repeat — **aperiodic tilings**. They turned out to describe the atoms of **quasicrystals**, discovered by Dan Shechtman, who won the 2011 Nobel Prize in Chemistry. In **2023** four mathematicians (Smith, Myers, Kaplan and Goodman-Strauss) found the first single shape, the 'hat', that tiles the plane endlessly without ever repeating. And in 2014 Lee Sallows, an amateur, found how to cut a triangle along its medians and reassemble it into 3 congruent triangles.

## 01 ✦ Concept Zone: Theorems & Results

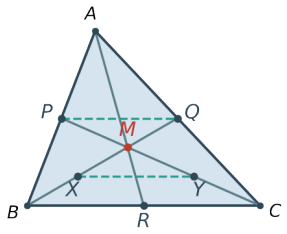
**THEOREM 8**

**Centroid Theorem: the three medians of a triangle pass through a common point (the centroid), which divides each median in the ratio 2 : 1, the longer part towards the vertex.**

Given: ΔABC with midpoints P, Q, R of AB, AC, BC; medians CP and BQ meet at M.

To prove: CM : MP = BM : MQ = 2 : 1, and AR also passes through M.

Construction: Take X, Y the midpoints of BM and CM.




Proof:

PQ ∥ BC, $PQ = \frac{BC}{2}$; XY ∥ BC, $XY = \frac{BC}{2}$ *[ Midpoint Theorem in ΔABC and ΔMBC ]*

PQ ∥ XY and PQ = XY

ΔMPQ ≅ ΔMYX *[ ASA, alternate angles ]*

MP = MY = YC and MQ = MX = XB *[ CPCT; X, Y midpoints ]*

**∴ CM : MP = 2 : 1 and BM : MQ = 2 : 1**

Repeating with medians AR and BQ meeting at N gives BN : NQ = 2 : 1. Only one point divides BQ in this ratio, so N = M: all three medians pass through M.

TOPIC 4

Hence proved.

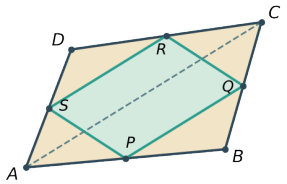
**THEOREM 9**

**Midpoint Theorem for Quadrilaterals (Varignon): the midpoints of the four sides of any quadrilateral are the vertices of a parallelogram.**

Given: P, Q, R, S are the midpoints of AB, BC, CD, DA.

To prove: PQRS is a parallelogram.

Construction: Join diagonals AC and BD.




Proof:

PQ ∥ AC and $PQ = \frac{AC}{2}$ *[ Midpoint Theorem in ΔABC ]*

SR ∥ AC and $SR = \frac{AC}{2}$ *[ Midpoint Theorem in ΔADC ]*

PQ ∥ SR; similarly QR ∥ PS *[ ΔBCD and ΔBAD ]*

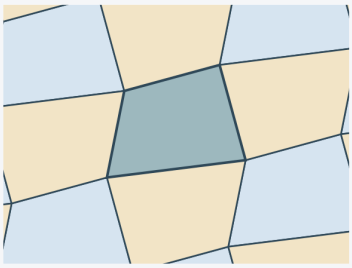
**∴ PQRS is a parallelogram**

Hence proved.

Also, $PQ = \frac{1}{2}$ AC and $QR = \frac{1}{2}$ BD, so the perimeter of PQRS = AC + BD.

## Tiling the Plane with Any Quadrilateral

- At every vertex of a tiling the angles add up to 360°. The four angles of a quadrilateral add up to 360°, so 4 copies can fit around a point, each angle used once.
- **Method 1:** rotate a copy by 180° about the midpoint of a side; repeat at every edge of every copy.
- **Method 2:** draw the grid of Varignon parallelograms of the quadrilateral and place copies to match it; the gaps fill with more copies automatically.
- Any triangle tiles (two copies form a parallelogram). A regular pentagon cannot: 108° × 3 = 324° < 360° < 432°.



## KEY NOTES

- ✔ Centroid divides each median 2 : 1 from the vertex.
- ✔ Varignon figure: PQ ∥ AC ∥ SR, QR ∥ BD ∥ PS.
- ✔ Any quadrilateral — even non-convex — tiles the plane.
- ✔ Angles at a tiling vertex sum to 360°.

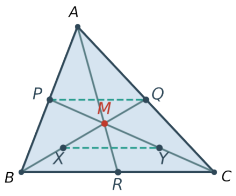
TOPIC 4

**NOTE** Varignon's figure is a rectangle when AC ⊥ BD, a rhombus when AC = BD, and a square when both hold.

## 02 ✎ Solved Examples

**EXAMPLE 1**

The medians of ΔABC meet at M. If AR = 12 cm and BM = 8 cm, find AM, MR and MQ.




Solution

- AM : MR = 2 : 1 ⇒ AM = 8 cm, MR = 4 cm.
- BM : MQ = 2 : 1 ⇒ MQ = 4 cm.

**∴ 8 cm, 4 cm, 4 cm**

**EXAMPLE 2**

M is the centroid of ΔABC and CP is a median with CM = 3x − 2 and MP = x + 1. Find x and CP.

Solution

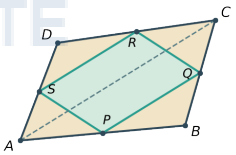
- CM = 2MP ⇒ 3x − 2 = 2x + 2 ⇒ x = 4.
- CM = 10, MP = 5.

**∴ x = 4, CP = 15**

**EXAMPLE 3**

The diagonals of quadrilateral ABCD are 10 cm and 8 cm. Find the perimeter of its Varignon parallelogram.

Solution




- PQ = SR = $\frac{1}{2}$ AC = 5, QR = PS = $\frac{1}{2}$ BD = 4.
- Perimeter = 2(5 + 4).

**∴ 18 cm**

**EXAMPLE 4**

Show that if AC = BD, the Varignon parallelogram PQRS is a rhombus and PR ⊥ QS.

Solution

- PQ = $\frac{AC}{2}$, QR = $\frac{BD}{2}$ ⇒ PQ = QR.
- A parallelogram with equal adjacent sides is a rhombus; its diagonals PR, QS are perpendicular.

**∴ Rhombus; PR ⊥ QS**

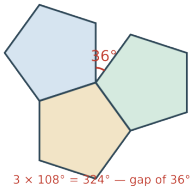
**EXAMPLE 5**

Can the plane be tiled with (i) regular hexagons, (ii) regular pentagons?

TOPIC 4

**Solution**

- (i) Each angle 120°; 3 × 120° = 360° — yes.
- (ii) Each angle 108°; 3 copies give 324°, 4 give 432° — never exactly 360°.



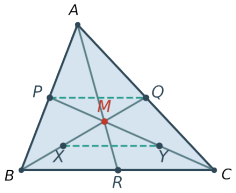

**∴ Hexagons yes; pentagons no**

## 03 ⚡ Practice Zone

Attempt in your notebook. Draw a neat figure wherever needed. ★ = higher-order thinking

1. Find the unknowns (M is the centroid): (i) median AR = 15 cm, find AM; (ii) BM = 3y + 1, MQ = y + 2, find y and BQ; (iii) MP = 2.4 cm, find CP.

2. Complete the proof of the Centroid Theorem: show ΔMPQ ≅ ΔMYX, and deduce CM : MP = 2 : 1. Then explain why AR also passes through M.



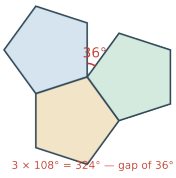

3. P, Q, R, S are the midpoints of AB, BC, CD, DA of quadrilateral ABCD. (i) Show that PR and QS bisect each other. (ii) Show that if AC = BD then PR ⊥ QS. Is the converse true?

4. (i) Show that if PQRS (the Varignon parallelogram) is a square, then AC and BD are equal and perpendicular. (ii) Prove the converse.

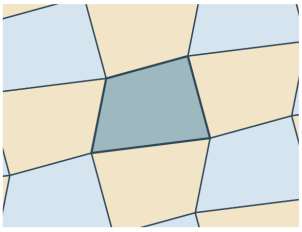
5. ABCD is a parallelogram; M, N are the midpoints of AB and CD. Show that DM and BN trisect the diagonal AC. Use this to trisect a given segment.

6. Using a fact about parallelograms, show how to tile the plane with any given triangle. Draw a sketch.

7. Justify why a regular pentagon cannot tile the plane, but a regular hexagon and any quadrilateral can.




8. Draw a non-convex quadrilateral and show how copies of it tile the plane using Method 1. Which angle sum ensures the copies fit around each vertex?



9. Medians AP, BQ, CR of ΔABC meet at M. Cutting along them gives 6 triangles. Show how to reassemble them into 3 congruent triangles (hint: join ΔMPB and ΔMPC along PB = PC). ★

10. Is Theorem 9 true for a non-convex quadrilateral? For a self-intersecting one? When does the 'Varignon parallelogram' shrink to a segment? ★

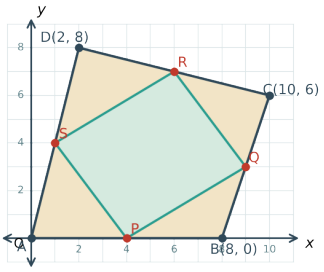
# Miscellaneous Questions

7 mixed questions from all topics of this chapter and earlier Ganita Manjari chapters

Attempt in your notebook. Draw a neat figure wherever needed. ★ = higher-order thinking

**1** A(1, 2), B(5, 3), C(7, 7), D(3, 6). (i) Find the midpoints of AC and BD. What do you conclude? (ii) Is ABCD a rhombus? (iii) Is it a rectangle?

**2** ABCD has A(0, 0), B(8, 0), C(10, 6), D(2, 8) and P, Q, R, S are the midpoints of its sides. (i) Find P, Q, R, S. (ii) Show that PQRS is a parallelogram using midpoints of its diagonals. (iii) Verify $PQ = \frac{1}{2}AC$.

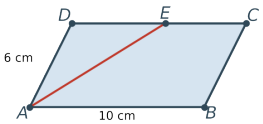



**3** (i) Show that the triangle formed by joining the midpoints of the sides of ΔABC has $\frac{1}{4}$ of its area. (ii) If ar(ABC) = 64 cm², find the area of the triangle formed by joining the midpoints of the midpoint triangle.

**4** O(0, 0), A(6, 0), B(3, 9) form a triangle. (i) Find the midpoint R of OA and the centroid G as the point dividing BR in the ratio 2 : 1. (ii) Check that BG = 2GR using the distance formula.

**5** ABCD is a trapezium with AB = 14 cm, DC = 8 cm and height 6 cm; EF joins the midpoints of AD and BC. (i) Find EF. (ii) Show that area = EF × height. (iii) Find the area.

**6** In parallelogram ABCD, AB = 10 cm, AD = 6 cm and the bisector of ∠A meets DC at E. (i) Show that ΔADE is isosceles. (ii) Find DE and EC. (iii) If ∠A = 70°, find ∠AEC.




**7** (i) Can regular octagons alone tile the plane? (ii) Show that regular octagons and squares together can fill the space around a point. (iii) Which regular polygons alone can tile the plane? ★

**MY NOTES** ✎

MISCELLANEOUS

# ★ Competency Based Question Bank

32 MCQs and 6 Assertion–Reason questions from the whole chapter and linked topics

**1** Which is NOT required in Definition 1 of a quadrilateral?

(a) four distinct points (b) all points in one plane
(c) all sides equal (d) sides meet only at vertices

**2** If three vertices of a 4-gon are collinear, the figure is

(a) self-intersecting (b) a square
(c) not a quadrilateral (d) always convex

**3** Which is a valid name of quadrilateral ABCD?

(a) CABD (b) ABDC (c) ACBD (d) BADC

**4** The side opposite to AB in ABCD is

(a) CD (b) DA (c) AC (d) BC

**5** A quadrilateral is convex if

(a) one angle is reflex (b) all angles are less than 180°
(c) its diagonals are equal (d) two sides are parallel

**6** The diagonals of a non-convex quadrilateral

(a) are perpendicular (b) bisect each other
(c) do not intersect (d) are equal

**7** Angles x, 2x, 3x, 4x of a quadrilateral give x =

(a) 40° (b) 45° (c) 30° (d) 36°

**8** If AB ∥ DC in quadrilateral ABCD, then ABCD

(a) must be non-convex (b) is self-intersecting
(c) is a square (d) must be convex

**9** Opposite sides of a parallelogram are equal is proved using

(a) SSS (b) ASA (c) RHS (d) SAS

**10** Its converse (opposite sides equal ⇒ parallelogram) uses

(a) RHS (b) ASA (c) SSS (d) AAS

**11** If ∠P = 3x + 10° and ∠Q = 5x − 30° are adjacent angles of a parallelogram, x =

(a) 30° (b) 35° (c) 20° (d) 25°

**12** A quadrilateral whose diagonals bisect each other is a

(a) kite (b) parallelogram
(c) rectangle always (d) trapezium

**13** If each pair of adjacent angles of ABCD adds to 180°, ABCD is a

(a) parallelogram (b) kite
(c) trapezium only (d) non-convex quadrilateral

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QUESTION BANK

**14** A quadrilateral with one pair of opposite sides equal and parallel is a
(a) rectangle (b) rhombus (c) parallelogram (d) trapezium only

**15** A parallelogram with equal diagonals is a
(a) kite (b) trapezium (c) rhombus (d) rectangle

**16** If the diagonal AC of parallelogram ABCD bisects $\angle A$, ABCD is a
(a) rhombus (b) kite (c) trapezium (d) rectangle

**17** Diagonals of a parallelogram meet at O, $OA = 2x + 1$, $OC = 3x - 4$. Then AC =
(a) 11 (b) 22 (c) 5 (d) 15

**18** P, Q are midpoints of AB, AC and BC = 13 cm. Then PQ =
(a) 4.3 cm (b) 26 cm (c) 6.5 cm (d) 13 cm

**19** The triangle formed by joining midpoints of a triangle of sides 6, 8, 10 has perimeter
(a) 6 (b) 48 (c) 24 (d) 12

**20** Joining the midpoints of the sides of a triangle divides it into
(a) 3 congruent triangles (b) 4 congruent triangles
(c) 4 unequal triangles (d) 2 congruent triangles

**21** The line through the midpoint of AB parallel to BC
(a) bisects AC (b) is perpendicular to AC
(c) passes through C (d) bisects BC

**22** In the proof of the Midpoint Theorem, the auxiliary line through C is drawn parallel to
(a) AC (b) BC (c) BA (d) PQ

**23** Trapezium with parallel sides 14 cm and 8 cm: the segment joining midpoints of the other sides is
(a) 6 cm (b) 3 cm (c) 11 cm (d) 22 cm

**24** The centroid divides each median in the ratio (from the vertex)
(a) 2 : 1 (b) 1 : 2 (c) 3 : 1 (d) 1 : 1

**25** If median AR = 12 cm, the distance of the centroid from A is
(a) 4 cm (b) 6 cm (c) 8 cm (d) 9 cm

**26** $CM = 3x - 2$ and $MP = x + 1$ for centroid M on median CP. Then CP =
(a) 18 (b) 10 (c) 15 (d) 12

**27** The midpoints of the sides of any quadrilateral form a
(a) parallelogram (b) square (c) rectangle (d) rhombus

**28** Diagonals 10 cm and 8 cm: perimeter of the Varignon parallelogram is
(a) 18 cm (b) 9 cm (c) 40 cm (d) 36 cm

**29** The Varignon parallelogram is a rhombus when
(a) $AC \perp BD$ (b) $AC = BD$ (c) $AB \parallel CD$ (d) ABCD is convex

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QUESTION BANK

**30** A regular pentagon cannot tile the plane because

(a) its diagonals are equal (b) it has 5 sides
(c) 108° is not a factor of 360° (d) it is not convex

**31** Any quadrilateral can tile the plane mainly because

(a) it is convex (b) its diagonals bisect
(c) its sides are equal (d) its angles add up to 360°

**32** The first single aperiodic 'hat' tile was discovered in

(a) 2011 (b) 2015 (c) 2023 (d) 1960

## ⚖ Assertion – Reason

**Directions:** (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.

**33** Assertion (A): A quadrilateral whose opposite angles are equal is a parallelogram.
Reason (R): If opposite angles are equal, adjacent angles add up to 180°.

**34** Assertion (A): The diagonals of DART ($\angle D > 180°$) intersect.
Reason (R): A quadrilateral is convex when all its angles are less than 180°.

**35** Assertion (A): If P, Q are midpoints of AB, AC, then $PQ = \frac{1}{2}BC$.

Reason (R): BCRP is a parallelogram in the proof, where R is on PQ produced with QR = PQ.

**36** Assertion (A): The medians of a triangle are concurrent.
Reason (R): Any two medians meet at the point dividing each in the ratio 2 : 1, and only one point divides a segment in a given ratio.

**37** Assertion (A): A regular hexagon tiles the plane.
Reason (R): The angles of every polygon add up to 360°.

**38** Assertion (A): Every rhombus is a square.
Reason (R): A rhombus is a parallelogram with equal adjacent sides.

MY NOTES ✎

QUESTION BANK

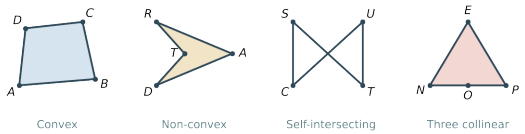
# Story Time: Case Studies & Twisters

Real-life stories, linked chapters and tricky twists. Read carefully, think, then answer.

## Case 1 · Robot Arm Linkage

KINDS OF 4-GONS

A 4-bar linkage in a factory moves through shapes like those in the figure as its free ends rotate.




(i) Which shape is a convex quadrilateral?

(ii) Which has a reflex angle?

(iii) Which is self-intersecting?

(iv) Why must engineers study all these shapes?

## Case 2 · The Folding Gate

PARALLELOGRAM

A folding gate is made of rods pinned at their ends so that each panel ABCD always keeps AB = DC and AD = BC.

(i) Which theorem shows each panel stays a parallelogram?

(ii) If ∠A = 64°, find the other angles.

(iii) When is the panel a rectangle?

(iv) What happens to the diagonals when it becomes a rectangle?

## Case 3 · Kite Frame Check

CONVERSES

Riya fixes two sticks AC = 60 cm and BD = 40 cm crossing at O for a frame ABCD.

(i) Where must O be on each stick for ABCD to be a parallelogram?

(ii) If they also cross at 90°, what is ABCD?

(iii) If AC = BD and they bisect each other, what is ABCD?

(iv) Is equal diagonals alone enough for a rectangle?

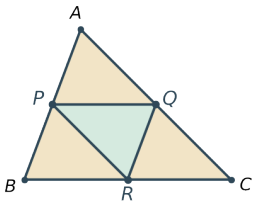
STORY TIME

INFINITY THINK BEYOND..... AN EDUCATIONAL INSTITUTE

## Case 4 · Triangular Park

MIDPOINT THEOREM

A triangular park has sides 120 m, 90 m and 150 m. Paths join the midpoints of the sides.




(i) Find the lengths of the paths.

(ii) Find the total length of the paths.

(iii) Into how many congruent parts do they divide the park?

(iv) What fraction of the park is the middle part?

## Case 5 · Ladder Rungs

TRAPEZIUM

A tapering ladder has its bottom rung 50 cm and top rung 30 cm long; the rungs are equally spaced.

(i) Find the length of the middle rung if there are 3 rungs.

(ii) Which theorem justifies this?

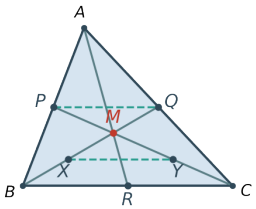
(iii) If there are 5 equally spaced rungs, find their lengths.

(iv) What is the sum of all 5 rungs?

## Case 6 · Balancing a Triangle

CENTROID

A triangular cardboard balances on a pencil tip at the centroid M. Median AR = 18 cm.




(i) Find AM and MR.

(ii) If BQ = 15 cm, find BM.

(iii) How can you locate the balance point by folding?

(iv) Why do all three medians pass through M?

STORY TIME

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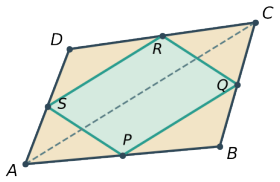
## Case 7 · Garden in a Garden

VARIGNON

A quadrilateral field ABCD has diagonals 80 m and 60 m. Fences join the midpoints of its sides.




(i) What shape do the fences form?

(ii) Find the total length of fencing.

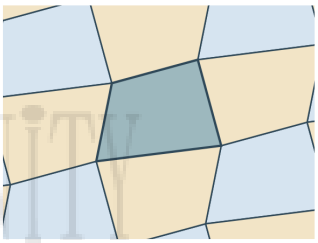
(iii) If the diagonals are perpendicular, what shape is it?

(iv) What fraction of the field's area does it enclose? (Use halving triangles.)

## Case 8 · Tile Designer

TILING

Asha wants to cover her floor with copies of one irregular quadrilateral tile.



(i) What must the angles at each vertex add up to?

(ii) Why can four copies fit around a point?

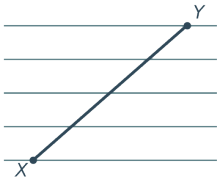
(iii) Describe Method 1 to continue the tiling.

(iv) Could she use regular pentagons instead? Why?

## Case 9 · Equal Spacing Trick

TWISTED

Sahil has ruled notebook paper with equally spaced lines but no ruler with markings.




(i) How can he find the midpoint of a stick XY?

(ii) How can he divide XY into 4 equal parts?

(iii) Which theorem justifies it?

(iv) Into how many equal parts can he divide XY using 6 gaps?

STORY TIME

### Case 10 · The Folded Right Triangle

CONVERSE

A right triangle ABC (∠B = 90°) of paper is folded so that A falls on B.

(i) Why is the crease perpendicular to AB?

(ii) Why is it parallel to BC?

(iii) Which point of AC does the crease pass through?

(iv) If BC = 10 cm, find the length of the crease inside the triangle.

MY NOTES ✎

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STORY TIME

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