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TOPIC-WISE ASSIGNMENT

Chapter 13 · Two Variables, One Line

Class IX · As per New Curriculum **Ganita Manjari**

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TOPIC 1

1 Linear Equations in Two Variables and Their Solutions

Standard form ax + by + c = 0 | Solution as an ordered pair | Infinitely many solutions | Graph is a line | Lines through the origin

QUICK RECAP (from earlier classes)

- A linear equation in one variable (like 3x − 5 = 7) has exactly one solution.
- A point (x, y) is plotted using its coordinates; the first number is the x-value.
- y = mx + d is a linear polynomial; its graph is a straight line.

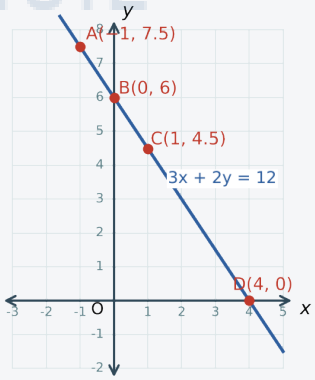
01 ✦ Concept Zone: Theorems & Results

Standard Form

- An equation that can be written as **ax + by + c = 0**, where a, b, c are real numbers and a, b are **not both zero**, is a **linear equation in two variables**. a, b are the coefficients of x, y and c is the constant term.
- Radha buys x kg of mangoes at ₹60/kg and y kg of bananas at ₹50/kg for ₹280: 60x + 50y = 280, i.e., 60x + 50y − 280 = 0.
- Fractions can be cleared: $\frac{x}{4} + \frac{3y}{5} - \frac{1}{2} = 0$ becomes 5x + 12y − 10 = 0 (multiply by 20).
- ax + c = 0 is also one: 4x + 7 = 0 is 4x + 0·y + 7 = 0 (a = 4, b = 0, c = 7).

Solutions and the Graph

- A **solution** is an **ordered pair** (x, y) that satisfies the equation: (2, 3) and (4, 0) satisfy 3x + 2y = 12, but (1, 3) does not.
- Choose any value of x, solve for y: so there are **infinitely many** solutions.
- Every solution is a point on the graph (a straight line), and every point on the line is a solution.
- Easy points: put x = 0 (the y-intercept) and y = 0 (the x-intercept). If c = 0, (0, 0) satisfies ax + by = 0, so the line passes through the origin.



RESULT 1

ax + by = c and kax + kby = kc (k ≠ 0) have exactly the same solutions.

Given: k ≠ 0 and an ordered pair (p, q).
To prove: (p, q) satisfies one equation ⟺ it satisfies the other.

Proof:

Suppose ap + bq = c

Then kap + kbq = k(ap + bq) = kc *[multiply both sides by k]*

Conversely, if kap + kbq = kc, divide by k $[k \neq 0]$

∴ ap + bq = c, so both equations have the same solutions

Hence proved.

So 3x + 4y = 7 and 6x + 8y = 14 have the same graph.

KEY NOTES

- ✔ ax + by + c = 0 with a, b not both zero.
- ✔ Solution = ordered pair; order matters.
- ✔ Infinitely many solutions, all on one straight line.
- ✔ c = 0 ⟺ the line passes through the origin.

NOTE x = 3 is a linear equation in two variables too: x + 0·y − 3 = 0. Its graph is a vertical line.

02 ✎ Solved Examples

EXAMPLE 1

Write in the form ax + by + c = 0 and give a, b, c: (i) 5x + 2y = 2.7, (ii) $\frac{y}{3} - \frac{x}{2} = 1$, (iii) 4 = 1.5x − 1.3y, (iv) 3y = 5.

Solution

- (i) 5x + 2y − 2.7 = 0: a = 5, b = 2, c = −2.7.
- (ii) Multiply by 6: −3x + 2y − 6 = 0: a = −3, b = 2, c = −6.
- (iii) 1.5x − 1.3y − 4 = 0: a = 1.5, b = −1.3, c = −4.
- (iv) 0·x + 3y − 5 = 0: a = 0, b = 3, c = −5.

∴ As above (any non-zero multiple is also correct)

EXAMPLE 2

Is (i) (2, −1), (ii) (1, 1) a solution of 3x + 4y = 2?

Solution

- (i) 3(2) + 4(−1) = 6 − 4 = 2 ✓
- (ii) 3(1) + 4(1) = 7 ≠ 2 ✗

∴ (2, −1) is a solution; (1, 1) is not

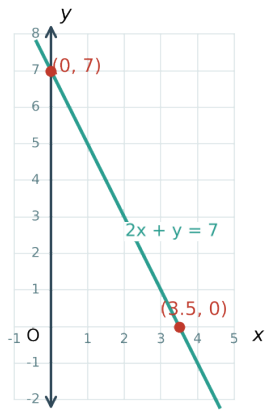
EXAMPLE 3

Find two solutions of 2x + y = 7, draw its graph and read four more solutions.

TOPIC 1

Solution

- x = 0 ⇒ y = 7: (0, 7). y = 0 ⇒ x = 3.5: (3.5, 0).
- Join the points; read (1, 5), (2, 3), (3, 1), (4, −1) from the line.



∴ (0, 7), (3.5, 0), (1, 5), (2, 3), (3, 1), (4, −1)

EXAMPLE 4

(−3, 4) is a solution of both 3ax + 4y = −2 and 2x + by = 14. Find a and b.

Solution

- −9a + 16 = −2 ⇒ a = 2.
- −6 + 4b = 14 ⇒ b = 5.

∴ a = 2, b = 5

EXAMPLE 5

A notebook costs twice as much as a pen. With notebook ₹t and pen ₹p, Charlie writes t = 2p and Meera writes p = 2t. Who is right?

Solution

- 'Notebook is twice the pen' means t = 2 × p.
- Check: pen ₹10 ⇒ notebook ₹20 ⇒ t = 2p.

∴ Charlie (t = 2p)

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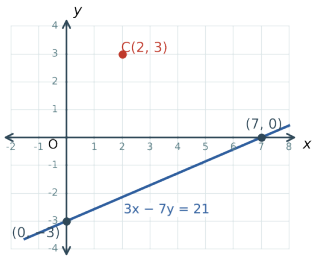
03 ⚡ Practice Zone

Attempt in your notebook. Draw a neat figure wherever needed. ★ = higher-order thinking

1. (i) Write a linear equation with a = 3, b = 0, c = −1. (ii) Write in standard form and give a, b, c: y − 7.5 = 2x; 3y − 2x = 0; 5x = 3y; $\frac{x}{3} + \frac{y}{4} = 1$.
2. Verify whether the point is a solution: (i) (4, 3) for 5x − 6y = 2, (ii) (1, 3) for 3x + 2y = 12, (iii) (−2, 5) for 4x + 3y = 7.
3. Find two solutions of each and draw the graphs: (i) 7x − 3y = 21, (ii) 2x + 3y = 5, (iii) x − 4y = 0.
4. (2, −1) is a solution of both 3mx + 2y = 10 and 5x + ny = 7. Find m and n.
5. For each equation find two solutions lying in different quadrants and name the quadrants: (i) 5x + 3y = 7, (ii) 5x − 3y = 7, (iii) −5x + 3y = 7, (iv) −5x − 3y = 7.

6 The figure shows the graph of $3x - 7y = 21$. (i) Read its intercepts and verify them. (ii) Does C(2, 3) lie on the line? Does it satisfy the equation? (iii) Can a point not on the line satisfy the equation?



7 True or false? Justify. (i) A linear equation in two variables has only one solution. (ii) Its graph always passes through the origin. (iii) It can never have rational solutions. (iv) $x = 3$ is a linear equation in two variables. (v) $2x + 3y = 7$ has infinitely many solutions. (vi) (1, 2) is a solution of $2x + 3y = 7$.

8 (i) Show that $3x + 4y = 7$ and $6x + 8y = 14$ have the same solutions. (ii) Prove Result 1 for $ax + by = c$ and $kax + kby = kc$.

9 Ranju says: 'If $c = 0$, the line $ax + by + c = 0$ passes through the origin.' Do you agree? Is the converse true? Explain with $2x + 3y = 0$, $5x = 0$ and $7y = 0$. ★

10 Radha pays ₹280 for x kg of mangoes (₹60/kg) and y kg of bananas (₹50/kg). (i) Find all possibilities if she buys whole kilograms. (ii) Find the possibilities if each amount is a whole number of half-kilograms. ★

MY NOTES ✎

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TOPIC 1

TOPIC 2

2 Slope and the Slope–Intercept Form

Rise and run | Slope formula | y = mx + d | Positive, negative, zero and undefined slope | Ramps and gradients

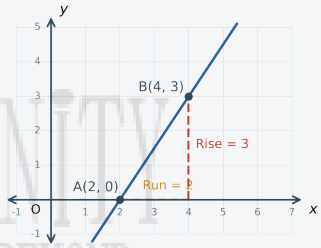
QUICK RECAP (from earlier classes)

- Moving right is a positive run; moving up is a positive rise.
- y = mx + d meets the y-axis at (0, d).
- Division by 0 is not defined.

01 ✦ Concept Zone: Theorems & Results

Slope of a Line

- The **slope** (gradient) measures steepness: slope = $\frac{Rise}{Run}$ = $\frac{y_2 - y_1}{x_2 - x_1}$ for points (x_1, y_1), (x_2, y_2).
- A(2, 0), B(4, 3): rise 3, run 2 ⇒ slope $\frac{3}{2}$. The order of the points does not matter: (−4, 5), (−1, 2) give $\frac{-3}{3} = \frac{3}{-3} = -1$.
- Positive slope: the line goes up from left to right; negative slope: it goes down.



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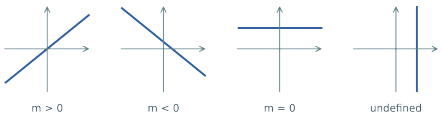
RESULT 2

For the line y = mx + d, the slope between any two of its points is m, and d is the y-intercept.

Given: Points (x_1, y_1) and (x_2, y_2) on y = mx + d, $x_1 \neq x_2$.
To prove: Slope = m.



Proof:

$y_1 = mx_1 + d$ and $y_2 = mx_2 + d$ *[points lie on the line]*

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(mx_2 + d) - (mx_1 + d)}{x_2 - x_1}$$

$$= \frac{m(x_2 - x_1)}{x_2 - x_1}$$

∴ slope = m, whichever two points are chosen

Putting x = 0 gives y = d, so the line meets the y-axis at (0, d).
Hence proved.

TOPIC 2

$ax + by + c = 0$ ($b \neq 0$) becomes $y = -\frac{a}{b}x - \frac{c}{b}$: slope $-\frac{a}{b}$, y-intercept $-\frac{c}{b}$. $m = 0$: horizontal line $y = d$. A vertical line $x = k$ has **undefined** slope (run = 0).

KEY NOTES

- ✔ Slope = rise ÷ run, the same for any two points of a line.
- ✔ $y = mx + d$: slope m, y-intercept d.
- ✔ $m > 0$ up, $m < 0$ down, $m = 0$ horizontal, vertical undefined.
- ✔ In $y = -5x + 3$, y falls 5 units for each unit step right.

NOTE Slope is a ratio: a ramp rising 1 cm for every 12 cm has slope $\frac{1}{12}$, whatever its length.

02 ✎ Solved Examples

EXAMPLE 1

Find the slope of the line through A(−1, 0), B(1, 2) and C(4, 5) using two different pairs of points.

Solution

› A, B: $\frac{2-0}{1-(-1)} = 1$.

› B, C: $\frac{5-2}{4-1} = 1$.

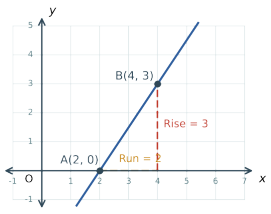
∴ Slope 1

EXAMPLE 2

The line through A(2, 0) and B(4, 3) is extended. Find (i) y when $x = 8$, (ii) x when $y = 6$, (iii) its equation.

Solution

› Slope $\frac{3}{2}$: from A, a run of 6 gives a rise of 9 ⇒ (8, 9).



› Rise 6 needs run 4 ⇒ (6, 6).

› $y = \frac{3}{2}x + d$, (2, 0) on it ⇒ $d = -3$.

∴ (8, 9); (6, 6); $y = \frac{3}{2}x - 3$

EXAMPLE 3

Write $5x + y = 3$ in slope–intercept form. Interpret m and d and find three points.

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TOPIC 2

Solution

- $y = -5x + 3$: $m = -5$, $d = 3$.
- Each unit right, y drops 5; the line cuts the y-axis at (0, 3).
- Points (0, 3), (1, −2), (−1, 8).

∴ m = −5, d = 3

EXAMPLE 4

A ramp at a loading dock rises 2.5 m over a run of 4 m. Find its slope.

Solution

- Slope $= \frac{2.5}{4} = \frac{5}{8}$.

∴ $\frac{5}{8}$ = 0.625

EXAMPLE 5

A wheelchair ramp must rise 1 cm for every 12 cm. How long (horizontally) must it be for stairs 18 cm high?

Solution

- $\frac{18}{run} = \frac{1}{12} \Rightarrow$ run $= 18 \times 12$.



∴ 216 cm

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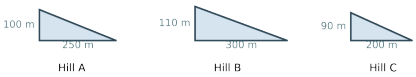
03 ⚡ Practice Zone

Attempt in your notebook. Draw a neat figure wherever needed. ★ = higher-order thinking

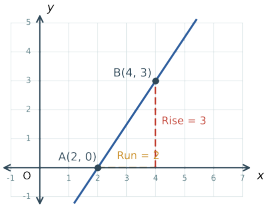
1 Rank the ski hills from least steep to most steep, giving the slope of each.



2 Find the slope of the line through: (i) (1, 2), (4, 8); (ii) (−2, 3), (2, −5); (iii) (3, 4), (7, 4); (iv) (5, 1), (5, 6); (v) (0, 0), (6, −2). Say whether each line rises, falls, is horizontal or vertical.

3 Write in slope–intercept form and give m and d: (i) $2x + y = 7$, (ii) $3x - 4y = 12$, (iii) $4x + 3y = 12$, (iv) $y - 5 = 0$. Which line is the steepest?

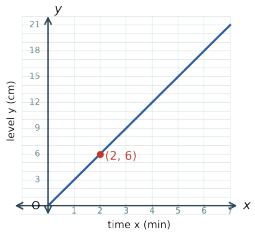
4 Using the figure (A(2, 0), B(4, 3)): (i) find the slope; (ii) find the point on the line with $x = 10$; (iii) find the point with $y = -3$; (iv) write the equation of the line.



5 (i) A wheelchair ramp (slope 1 : 12) is needed for stairs 45 cm high. Find its horizontal length. (ii) A road rises 30 m over a horizontal distance of 750 m. Find its slope as a fraction and as a percentage.

TOPIC 2

6 Water fills a tank so that the level y (cm) after x minutes follows the graph. (i) Find the slope and explain its meaning. (ii) What is the y-intercept? (iii) Find the level after 5 minutes and the time to reach 21 cm. (iv) Does (4, 10) lie on the line? (v) Where does it meet x = 3?



7 Give a formula for the x-intercept of $y = mx + c$ ($m \neq 0$). Use it for $y = 2x - 6$ and $y = -\frac{1}{2}x + 4$.

8 Show, using slopes, that A(1, 2), B(3, 6) and C(5, 10) lie on one line. Does D(4, 9) lie on it?

9 How many lines (i) have a given slope, (ii) have a given slope and pass through a given point? Explain. ★

10 Prove that the lines $y = mx + d_1$ and $y = mx + d_2$ ($d_1 \neq d_2$) are parallel. ★

MY NOTES ✎

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TOPIC 3

3 Pairs of Equations: Substitution and Elimination

Framing a pair of equations | Common solution | Substitution method | Elimination method | Word problems

QUICK RECAP (from earlier classes)

- Solving one equation in one variable.
- Expanded form of a two-digit number: 10x + y.
- Ratio a : b ⇒ quantities ak and bk.

❖ **MUST KNOW** · Solving Equations Through the Ages

Systems of linear equations are among the oldest problems in mathematics. Traces appear on **Babylonian** clay tablets (c. 2000 BCE). The Chinese **Nine Chapters on the Mathematical Art** (about 2000 years ago) has a chapter, *fangcheng*, solving them by arranging coefficients in columns of counting rods and combining columns to cancel variables — essentially elimination, eighteen centuries before Gauss. In India, **Āryabhata** (499 CE) and **Brahmagupta** (628 CE) solved systems in several unknowns, and **Mahāvīrācārya** (c. 850 CE) posed the citron and wood-apple puzzle.

01 ✦ Concept Zone: Theorems & Results

A Pair of Linear Equations

- $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ (each with its a, b not both zero). A **solution** of the pair is an ordered pair satisfying **both**.
- Two numbers with sum 25 and difference 11: x + y = 25, x − y = 11.
- Tickets: 2 adults + 3 children ₹600, 3 adults + 2 children ₹700: 2x + 3y = 600, 3x + 2y = 700.
- Idea of both methods: reduce the pair to **one** equation in **one** variable.

Substitution and Elimination

- **Substitution:** (1) from one equation express one variable in terms of the other; (2) substitute it in the other equation and solve; (3) substitute back to find the second variable; (4) check in both.
- **Elimination:** (1) multiply the equations by suitable numbers so that the coefficients of one variable become equal; (2) subtract (or add) to eliminate it; (3) solve and substitute back.
- Special trick: when coefficients are interchanged (9x + 7y, 7x + 9y), **add** and **subtract** the equations to get x + y and x − y.

KEY NOTES

- ✔ Always check the solution in both original equations.
- ✔ Choose the variable with coefficient 1 to substitute.
- ✔ Make coefficients equal, then subtract (same signs) or add (opposite signs).
- ✔ Define the variables clearly in word problems.

NOTE If both variables disappear during elimination, the pair has either no solution or infinitely many — see the next topic.

02 ✎ Solved Examples

EXAMPLE 1

Solve $7x - 15y = 2$ and $x + 2y = 3$ by substitution.

Solution

- $x = 3 - 2y$.
- $7(3 - 2y) - 15y = 2 \Rightarrow 21 - 29y = 2 \Rightarrow y = \frac{19}{29}$.
- $x = 3 - \frac{38}{29} = \frac{49}{29}$.

$\therefore x = \frac{49}{29}, y = \frac{19}{29}$

EXAMPLE 2

Two angles of a triangle are $x°$ and $y°$ with $y = 4x$, and the third angle is $50°$. Find the angles.

Solution

- $x + y + 50 = 180$ and $y = 4x$.
- $5x = 130 \Rightarrow x = 26, y = 104$.

$\therefore$ 26°, 104°, 50°

EXAMPLE 3

The incomes of two persons are in the ratio 9 : 7 and their expenditures 4 : 3. Each saves ₹2000 a month. Find their incomes.

Solution

- $9x - 4y = 2000$ and $7x - 3y = 2000$.
- $\times 3$ and $\times 4$: $27x - 12y = 6000$, $28x - 12y = 8000 \Rightarrow x = 2000$.
- $y = 4000$.

$\therefore$ ₹18,000 and ₹14,000

EXAMPLE 4

The sum of a two-digit number and the number with its digits reversed is 66; the digits differ by 2. Find the number(s).

Solution

- $(10x + y) + (10y + x) = 66 \Rightarrow x + y = 6$.
- $x - y = 2 \Rightarrow x = 4, y = 2$.

$\therefore$ 42 (or 24 if the units digit is larger)

EXAMPLE 5

2 adult and 3 child tickets cost ₹600; 3 adult and 2 child tickets cost ₹700. Find each price.

Solution

TOPIC 3

- Add: 5x + 5y = 1300 ⇒ x + y = 260.
- Subtract: x − y = 100.
- x = 180, y = 80.

∴ Adult ₹180, child ₹80

03 ⚡ Practice Zone

Attempt in your notebook. Draw a neat figure wherever needed. ★ = higher-order thinking

1. Frame a pair of equations and solve: (i) 2 movie tickets and 3 snack boxes cost ₹850; 4 tickets and 1 box cost ₹1100. (ii) A taxi trip of 6 km costs ₹122 and one of 8 km costs ₹160 (fixed charge + rate per km). (iii) 200 people attend a meet; adult tickets ₹150, child tickets ₹100; collection ₹25,000.

2. Solve by substitution: (i) x + y = 14, x − y = 4; (ii) s − t = 3, $\frac{s}{3} + \frac{t}{2} = 6$; (iii) 0.2x + 0.3y = 1.3, 0.4x + 0.5y = 2.3.

3. Solve by elimination: (i) 3x + 4y = 10, 2x − 2y = 2; (ii) $\frac{x}{2} + \frac{2y}{3} = -1$, $x - \frac{y}{3} = 3$; (iii) a coach buys 7 bats and 6 balls for ₹8880, and 3 bats and 5 balls for ₹4000 — find the price of each.

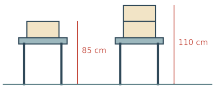
4. (i) The sum of two numbers is 52 and their difference is 14. (ii) The difference of two numbers is 26 and one is three times the other. (iii) A taxi charges ₹190 for 12 km and ₹302 for 20 km; find the fixed charge, the rate per km and the fare for 30 km. (iv) The digits of a two-digit number add up to 9, and nine times the number is twice the reversed number.

5. A fraction becomes $\frac{2}{3}$ when 1 is added to both its numerator and denominator, and $\frac{3}{4}$ when 4 is added to both. Find the fraction.

6. Six years ago Asha was four times as old as her son. Four years from now she will be twice as old as him. Find their present ages.

7. (i) Meena withdraws ₹3500 in ₹50 and ₹100 notes, 40 notes in all. How many of each? (ii) A library charges a fixed amount for the first three days and a fixed amount per extra day. Saritha pays ₹38 for 8 days and Susy ₹26 for 5 days. Find both charges.

8. Find the height of the stool and of each block from the figure.



9. Mahāvīrācārya (Gaṇita-sāra-saṅgraha, c. 850 CE): 9 citrons and 7 wood-apples cost 107; 7 citrons and 9 wood-apples cost 101. Find each price without direct substitution or elimination. Use the same trick for: 5 pencils and 7 pens cost ₹50; 7 pencils and 5 pens cost ₹46. ★

10. Jacob's watch reads a minutes after two o'clock; fifteen minutes later it reads b minutes after three o'clock, and a = 6b. What time was it at the second look? ★

TOPIC 3

TOPIC 4

4 Nature of Solutions and the Graphical Method

Intersecting, parallel, coincident lines | Ratio tests | Solving graphically | Finding unknown coefficients | Comparing plans

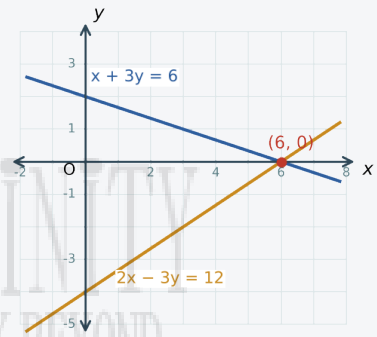
QUICK RECAP (from earlier classes)

- A solution of a pair is a point on both lines.
- Lines with equal slopes and different intercepts are parallel.
- kax + kby = kc has the same graph as ax + by = c.

01 ✦ Concept Zone: Theorems & Results

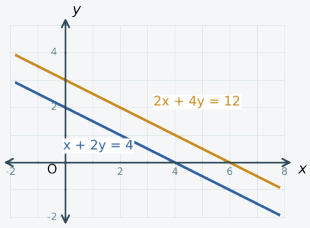
Graphical Method

- Draw both lines (two points each). Points common to both lines are the solutions.
- **Intersecting lines:** exactly one (unique) solution — the point of intersection.
- **Parallel lines:** no solution. **Coincident lines:** infinitely many solutions.
- x + 3y = 6 and 2x − 3y = 12 meet at (6, 0).



Ratio Tests

- For $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$:
- $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow$ **unique solution** (lines intersect).
- $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow$ **infinitely many** (one equation is a multiple of the other; coincident lines).
- $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow$ **no solution** (parallel lines): x − y = 10 and 10x − 10y = 101 give 100 = 101 after elimination.
- Why: if both variables vanish in elimination, a true statement (0 = 0) means infinitely many solutions and a false one means none.



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TOPIC 4

KEY NOTES

- ✔ Unique: $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.
- ✔ None: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.
- ✔ Infinitely many: all three ratios equal.
- ✔ A pair never has exactly two solutions.

NOTE Read graphical answers carefully; check them in both equations, since a graph gives only approximate values.

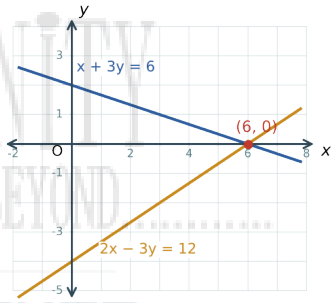
02 ✎ Solved Examples

EXAMPLE 1

Solve $x + 3y = 6$ and $2x - 3y = 12$ graphically.

Solution

- $x + 3y = 6$: (0, 2), (6, 0). $2x - 3y = 12$: (0, −4), (3, −2).
- The lines meet at (6, 0); check: $6 + 0 = 6$, $12 - 0 = 12$.



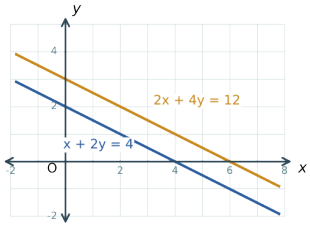
∴ x = 6, y = 0 (unique)

EXAMPLE 2

Do $x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$ intersect, coincide or run parallel?

Solution

- $\frac{1}{2} = \frac{2}{4}$ but $\frac{-4}{-12} = \frac{1}{3} \neq \frac{1}{2}$.
- Graph: (0, 2), (4, 0) and (0, 3), (6, 0) — the lines never meet.



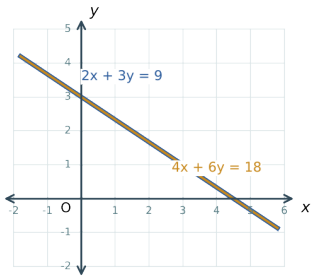
∴ Parallel — no solution

EXAMPLE 3

Romila buys 2 erasers and 3 sheets for ₹9; Sonali buys 4 erasers and 6 sheets for ₹18. Represent graphically. Can the prices be found?

TOPIC 4

Solution



- $2x + 3y = 9$ and $4x + 6y = 18$: all ratios equal $\frac{1}{2}$.
- The lines coincide.

∴ Infinitely many solutions — prices cannot be fixed

EXAMPLE 4

For what values of p does $4x + py + 8 = 0$, $2x + 2y + 2 = 0$ have a unique solution?

Solution

- Need $\frac{4}{2} \neq \frac{p}{2}$.

∴ p ≠ 4

EXAMPLE 5

Find k so that $x + 2y = 3$ and $(k - 1)x + (k + 1)y = k + 3$ represent coincident lines.

Solution

- $\frac{1}{k-1} = \frac{2}{k+1} \Rightarrow k + 1 = 2k - 2 \Rightarrow k = 3$.
- Check $\frac{3}{k+3} = \frac{3}{6} = \frac{1}{2}$ ✓

∴ k = 3

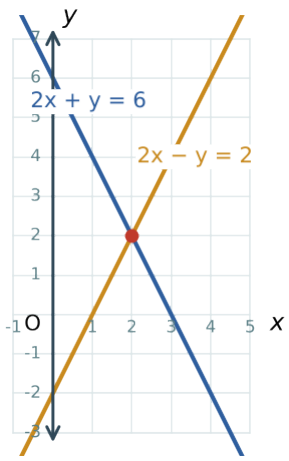
03 ⚡ Practice Zone

Attempt in your notebook. Draw a neat figure wherever needed. ★ = higher-order thinking

1. Using ratios, decide whether the lines intersect, are parallel or coincide: (i) $5x - 4y + 8 = 0$, $7x + 6y - 9 = 0$; (ii) $9x + 3y + 12 = 0$, $18x + 6y + 24 = 0$; (iii) $6x - 3y + 10 = 0$, $2x - y + 9 = 0$; (iv) $3x + 2y = 5$, $2x - 3y = 7$.
2. Which pairs have solutions? Solve those graphically: (i) $x + y = 5$, $2x + 2y = 10$; (ii) $x - y = 8$, $3x - 3y = 16$; (iii) $x + y = 7$, $x - y = 1$; (iv) $2x - 2y - 2 = 0$, $4x - 4y - 5 = 0$.

TOPIC 4

3 Solve graphically $2x + y = 6$ and $2x - y - 2 = 0$. Find also where each line meets the y-axis.



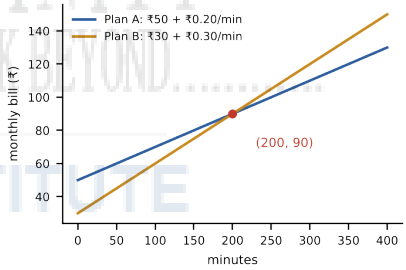
4 12 students took part in a quiz; there were 2 more girls than boys. Form the equations and find the numbers graphically.

5 Given $2x + 3y - 8 = 0$, write another linear equation so that the pair represents (i) intersecting lines, (ii) parallel lines, (iii) coincident lines.

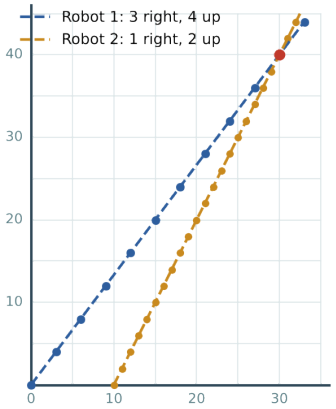
6 Half the perimeter of a rectangular garden is 40 m and its length is 6 m more than its width. Find its dimensions graphically and check algebraically.

7 (i) Find p for a unique solution: $3x + py = 5$, $6x + 8y = 10$. (ii) Find a, b for infinitely many solutions: $(a + b)x - 2by = 5a + 2b + 1$, $3x - y = 14$. (iii) Find k for no solution: $kx + 3y = k - 3$, $12x + ky = k$.

8 Plan A: ₹50 per month + ₹0.20 per minute; Plan B: ₹30 per month + ₹0.30 per minute. For how many minutes are both bills equal? When is each plan cheaper?



9 (i) Robot 1 starts at (0, 0) and repeatedly moves 3 right then 4 up; Robot 2 starts at (10, 0) and repeatedly moves 1 right then 2 up. Find where the lines through their corner points meet (see figure). Do the robots both pass that point? (ii) Robot 1 starts at (3, 0) moving 5 right, 5 up; Robot 2 starts at (7, 0) moving 5 right, 3 down. Do their paths meet? ★



10 Can a pair of linear equations have exactly two solutions? Use the graphs to explain. Are the converses of the three ratio tests true? ★

MY NOTES ✎

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TOPIC 4

Miscellaneous Questions

7 mixed questions from all topics of this chapter and earlier Ganita Manjari chapters

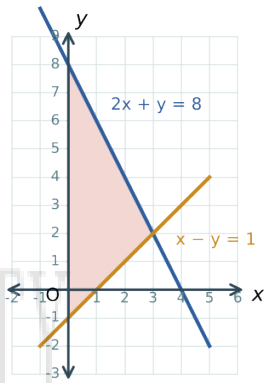
Attempt in your notebook. Draw a neat figure wherever needed. ★ = higher-order thinking

1. The 5th term of an AP is 23 and its 12th term is 51. Form a pair of equations in a and d and solve. Find the 20th term.

2. Alloys with 80% and 50% copper are melted together to make 30 kg of an alloy with 70% copper. How much of each is used? Check with the weighted mean.

3. The perimeter of a rectangle is 50 cm and its length is 1 cm more than twice its breadth. Find its dimensions and area.

4. The lines $2x + y = 8$ and $x - y = 1$ meet each other and the y-axis. (i) Solve the pair. (ii) Find where each line meets the y-axis. (iii) Find the area of the triangle formed.



5. A(1, 1), B(5, 2), C(6, 5) are three vertices of parallelogram ABCD. (i) Find D. (ii) Find the equation of the line AC. (iii) Find where the diagonals meet.

6. (i) Solve $x + y = 7$, $x^2 - y^2 = 21$ using an identity. (ii) Solve $x - y = 2$, $x^2 - y^2 = 32$. (iii) Are the solutions of $x + 2y = 3$, $3x - y = 1$ terminating decimals?

7. For $2x + ky = 4$ and $kx + 8y = 8$, find k so that the pair has (i) infinitely many solutions, (ii) no solution, (iii) exactly one solution. ★

MY NOTES ✎

MISCELLANEOUS

★ Competency Based Question Bank

32 MCQs and 6 Assertion–Reason questions from the whole chapter and linked topics

1 Which is a linear equation in two variables?

(a) $x^2 + y = 4$ (b) $xy = 6$ (c) $3x - 2y = 7$ (d) $\frac{1}{x} + y = 2$

2 In $4x + 7 = 0$ written as $ax + by + c = 0$, $b =$

(a) 1 (b) 4 (c) 0 (d) 7

3 (2, 3) is a solution of

(a) $3x - 2y = 12$ (b) $x + y = 6$ (c) $3x + 2y = 12$ (d) $2x + 3y = 12$

4 A linear equation in two variables has

(a) two solutions (b) no solution
(c) infinitely many solutions (d) one solution

5 The graph of $2x + 5y = 0$ passes through

(a) (0, 0) (b) (5, 2) (c) (0, 5) (d) (2, 0)

6 If $(-3, 4)$ satisfies $3ax + 4y = -2$, then $a =$

(a) -6 (b) -2 (c) 2 (d) 6

7 The x-intercept of $4x + 3y = 12$ is at

(a) (4, 0) (b) (0, 3) (c) (0, 4) (d) (3, 0)

8 $x = 3$ as a linear equation in two variables has graph

(a) parallel to the y-axis (b) through the origin
(c) a point (d) parallel to the x-axis

9 Slope of the line through (2, 0) and (4, 3) is

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 3 (d) $-\frac{3}{2}$

10 Slope of the line through $(-4, 5)$ and $(-1, 2)$ is

(a) -3 (b) 1 (c) -1 (d) 3

11 For $y = -5x + 3$, the slope and y-intercept are

(a) 3, -5 (b) -5, -3 (c) 5, 3 (d) -5, 3

12 The slope of $3x - 4y = 12$ is

(a) $-\frac{3}{4}$ (b) $\frac{4}{3}$ (c) -3 (d) $\frac{3}{4}$

13 A horizontal line has slope

(a) 1 (b) 0 (c) undefined (d) -1

QUESTION BANK

14 The slope of $x = 2$ is

(a) $\frac{1}{2}$ (b) 0 (c) 2 (d) undefined

15 A ramp rising 2.5 m over a run of 4 m has slope

(a) 6.5 (b) 10 (c) 1.6 (d) 0.625

16 Slope 1 : 12 for 18 cm stairs needs a run of

(a) 216 cm (b) 150 cm (c) 1.5 cm (d) 12 cm

17 The x-intercept of $y = 2x - 6$ is

(a) -6 (b) 3 (c) -3 (d) 6

18 Water level $y = 3x$ (cm after x min) reaches 21 cm after

(a) 18 min (b) 3 min (c) 7 min (d) 63 min

19 In substitution for $x + 2y = 3$, $7x - 15y = 2$, first write

(a) $x = 2 + 15y$ (b) $y = 3 + x$ (c) $x = 3 - 2y$ (d) $y = 7x - 2$

20 Angles $x°$, $4x°$, $50°$ of a triangle give $x =$

(a) 32.5 (b) 104 (c) 20 (d) 26

21 Incomes 9 : 7, expenditures 4 : 3, savings ₹2000 each: the first income is

(a) ₹14,000 (b) ₹18,000 (c) ₹16,000 (d) ₹9,000

22 If $9x + 7y = 107$ and $7x + 9y = 101$, then $x + y =$

(a) 16 (b) 13 (c) 3 (d) 26

23 Adult and child tickets: $2x + 3y = 600$, $3x + 2y = 700$. An adult ticket costs

(a) ₹260 (b) ₹100 (c) ₹80 (d) ₹180

24 Digits sum 6 and differ by 2; sum with reverse is 66. The number with the larger tens digit is

(a) 42 (b) 33 (c) 51 (d) 24

25 $x + y = 14$, $x - y = 4$ gives

(a) (9, 5) (b) (5, 9) (c) (10, 4) (d) (7, 7)

26 Lines that intersect at one point give

(a) two solutions (b) no solution
(c) a unique solution (d) infinitely many solutions

27 $9x + 3y + 12 = 0$ and $18x + 6y + 24 = 0$ are

(a) coincident (b) perpendicular (c) intersecting (d) parallel

28 $6x - 3y + 10 = 0$ and $2x - y + 9 = 0$ have

(a) no solution (b) infinitely many
(c) exactly two (d) a unique solution

29 $4x + py + 8 = 0$ and $2x + 2y + 2 = 0$ have a unique solution when

(a) $p = 4$ (b) $p \neq 4$ (c) $p = 2$ (d) $p \neq 2$

QUESTION BANK

30 $x + 2y = 3$ and $(k - 1)x + (k + 1)y = k + 3$ coincide for $k =$
(a) -1 (b) 1 (c) 2 (d) 3

31 Plans ₹50 + ₹0.20/min and ₹30 + ₹0.30/min cost the same at
(a) 300 min (b) 20 min (c) 100 min (d) 200 min

32 $2x + y = 6$ and $2x - y = 2$ meet at
(a) (1, 4) (b) (3, 0) (c) (2, 1) (d) (2, 2)

⚖ Assertion – Reason

Directions: (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.

33 Assertion (A): The line $2x + 3y = 0$ passes through the origin.
Reason (R): (0, 0) satisfies every equation $ax + by = 0$.

34 Assertion (A): The slope of $y = 4x - 7$ is -7.
Reason (R): In $y = mx + d$, m is the slope and d is the y-intercept.

35 Assertion (A): $x - y = 10$ and $10x - 10y = 101$ have no solution.
Reason (R): For them $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

36 Assertion (A): A linear equation in two variables has infinitely many solutions.
Reason (R): For every value of x we can solve for y.

37 Assertion (A): A vertical line has slope 0.
Reason (R): On a vertical line the run between two points is 0.

38 Assertion (A): $2x + 3y = 9$ and $4x + 6y = 18$ have a unique solution.
Reason (R): Their lines coincide.

MY NOTES ✎

QUESTION BANK

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Story Time: Case Studies & Twisters

Real-life stories, linked chapters and tricky twists. Read carefully, think, then answer.

Case 1 · Radha at the Fruit Cart

ONE EQUATION

Mangoes cost ₹60/kg and bananas ₹50/kg. Radha pays ₹280 in all for x kg of mangoes and y kg of bananas.

(i) Write the equation.

(ii) Find the amount of bananas if she buys 3 kg of mangoes.

(iii) If she buys only bananas, how much does she get?

(iv) Can she buy 4 kg of mangoes? Explain.

Case 2 · Paratha and Dahi

ELIMINATION

One paratha and 2 bowls of dahi cost ₹90; 2 parathas and 3 bowls of dahi cost ₹155.

(i) Frame the pair of equations.

(ii) Find the cost of a bowl of dahi.

(iii) Find the cost of a paratha.

(iv) Find the bill for 3 parathas and 4 bowls.

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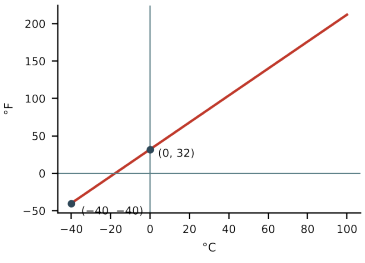
Case 3 · Celsius and Fahrenheit

SLOPE

Temperatures are converted using $F = \frac{9}{5}C + 32$.



(i) Convert 30 °C to °F.

(ii) Convert 95 °F to °C.

(iii) What do the slope and the intercept mean?

(iv) Which temperature has the same number in both scales?

STORY TIME

Case 4 · The Train Puzzle

FRAMING

A train covers a distance d at speed v in time t. At v + 6 km/h it takes 6 hours less; at v − 4 km/h it takes 6 hours more.

(i) Show that t − v = 6 from the first condition.

(ii) Show that 3v − 2t = 12 from the second.

(iii) Find v and t.

(iv) Find the distance.

Case 5 · Father and Four Children

AGES

A father's age equals the sum of the ages of his four children. After 20 years, the sum of the children's ages will be twice the father's age.

(i) By how much does the children's total age grow in 20 years?

(ii) Form an equation.

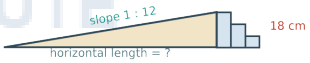
(iii) Find the father's present age.

(iv) Find his age when the children's total is twice his age.

Case 6 · The Accessible School

RAMPS

A school builds ramps with slope 1 : 12 beside its stairs.



(i) Stairs 24 cm high need a ramp of what horizontal length?

(ii) A ramp has run 3.6 m; what height can it climb?

(iii) Write the equation y = mx for the ramp.

(iv) Is a ramp rising 10 cm over 100 cm acceptable?

Case 7 · Two Taxi Companies

GRAPHICAL

Company P charges ₹40 plus ₹12 per km; Company Q charges ₹20 plus ₹14 per km.

(i) Write the fare equations.

(ii) For what distance are the fares equal?

(iii) Which is cheaper for 15 km?

(iv) What do the slopes represent?

STORY TIME

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Case 8 · Shape Puzzle

TWISTED

In a puzzle, triangle + triangle + square = 20 and triangle + square + square = 22; also circle + triangle + square = 19.

(i) Write the equations in T, R (square) and C.

(ii) Add the first two. What do you get?

(iii) Find T and R.

(iv) Find C.

Case 9 · Digit Swap

NUMBERS

The digits of a two-digit number add up to 15. Interchanging the digits gives a number 9 more.

(i) Form the equations.

(ii) Find the digits.

(iii) Find the number.

(iv) Check the condition.

Case 10 · Coin Purse

WORD PROBLEM

Neha has 22 coins of ₹5 and ₹2 worth ₹77 in all.

(i) Frame the pair of equations.

(ii) Solve by substitution.

(iii) Solve by elimination.

(iv) Which method was quicker here?

STORY TIME

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