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TOPIC-WISE ASSIGNMENT

Chapter 4 · Exploring Algebraic Identities

Class IX · As per New Curriculum **Ganita Manjari**

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TOPIC 1

1 Identities for Squares

Identity vs equation | $(a + b)^2$ | $(a - b)^2$ | $a^2 - b^2$ | $(a + b + c)^2$ | Quick squares

QUICK RECAP (from earlier classes)

- Distributive law: $a(b + c) = ab + ac$.
- Area of a square = side^2; area of a rectangle = length × breadth.
- $(-a)^2 = a^2$; $2ab$ is negative when a and b have opposite signs.

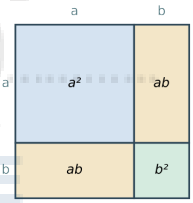
MUST KNOW · Śrīdharācārya's Rule for Squares

In about 750 CE, **Śrīdharācārya** gave a quick method for squaring: $a^2 = (a + b)(a - b) + b^2$, choosing b to make one factor a round number. For 97^2: $(97 + 3)(97 - 3) + 3^2 = 100 \times 94 + 9 = 9409$. The same mathematician is remembered for his work on the quadratic formula and on mixtures (weighted means).

01 ✦ Concept Zone: Theorems & Results

RESULT 1

$(a + b)^2 = a^2 + 2ab + b^2$ for all numbers a and b.



Given: Any numbers a, b (and, for the picture, a square of side a + b).

To prove: $(a + b)^2 = a^2 + 2ab + b^2$.

Proof:

$(a + b)^2 = (a + b)(a + b) = a(a + b) + b(a + b)$ *[distributive law]*

$= a^2 + ab + ba + b^2$

$\therefore (a + b)^2 = a^2 + 2ab + b^2$ *[$ab = ba$]*

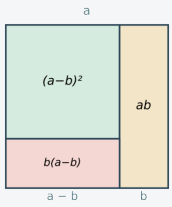
Area model: the square of side a + b is made of a square a^2, a square b^2 and two rectangles ab.

Hence proved.

An **identity** is true for **all** values of its variables; an **equation** such as $x^2 - 1 = 24$ is true only for some ($x = \pm 5$).

Related Square Identities

- Replace b by −b: **$(a - b)^2 = a^2 - 2ab + b^2$** (figure: remove two strips from the a × a square).
- **$a^2 - b^2 = (a + b)(a - b)$**. Śrīdharācārya (c. 750 CE): $a^2 = (a + b)(a - b) + b^2$, e.g., $55^2 = 60 \times 50 + 25 = 3025$.
- $(a + b)^2 \neq a^2 + b^2$ in general: they differ by $2ab$, so $(a + b)^2 > a^2 + b^2$ exactly when $ab > 0$.



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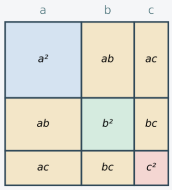
Square of a Trinomial

- **$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$** — put $d = b + c$ in $(a + d)^2$, or split a square of side $a + b + c$ into 9 pieces.
- $119^2 = (100 + 10 + 9)^2 = 10000 + 100 + 81 + 2000 + 1800 + 180 = 14161$.



KEY NOTES

- ✔ $(a \pm b)^2 = a^2 \pm 2ab + b^2$.
- ✔ $a^2 - b^2 = (a + b)(a - b)$.
- ✔ $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$.
- ✔ $(a + b)^2 - (a - b)^2 = 4ab$.

NOTE Keep the sign with each term: in $(3x - 2y + 4z)^2$ take $a = 3x$, $b = -2y$, $c = 4z$, so $2ab = -12xy$.

02 ✎ Solved Examples

EXAMPLE 1

Take three consecutive squares. Add the smallest and largest and subtract twice the middle one. Show that the answer is always 2.

Solution

› Squares $(n - 1)^2$, n^2, $(n + 1)^2$.

› $(n^2 - 2n + 1) + (n^2 + 2n + 1) - 2n^2 = 2$.

∴ Always 2

EXAMPLE 2

Find (i) $(5x + 2y)^2$, (ii) 43^2, (iii) 29^2.

Solution

› (i) $25x^2 + 2(5x)(2y) + 4y^2 = 25x^2 + 20xy + 4y^2$.

› (ii) $(40 + 3)^2 = 1600 + 240 + 9 = 1849$.

› (iii) $(30 - 1)^2 = 900 - 60 + 1 = 841$.

∴ $25x^2 + 20xy + 4y^2$; 1849; 841

EXAMPLE 3

Use Śrīdharācārya's rule to find 55^2 and 105^2.

Solution

› $55^2 = (55 + 5)(55 - 5) + 5^2 = 60 \times 50 + 25 = 3025$.

› $105^2 = 110 \times 100 + 25 = 11025$.

∴ 3025 and 11025

EXAMPLE 4

Expand $(3x - 2y + 4z)^2$.

TOPIC 1

Solution

- a = 3x, b = −2y, c = 4z.
- $9x^2 + 4y^2 + 16z^2 + 2(3x)(-2y) + 2(-2y)(4z) + 2(4z)(3x)$.



∴ $9x^2 + 4y^2 + 16z^2 - 12xy - 16yz + 24zx$

EXAMPLE 5

If a + b = 9 and ab = 14, find (i) $a^2 + b^2$, (ii) $(a - b)^2$.

Solution

- (i) $a^2 + b^2 = (a + b)^2 - 2ab = 81 - 28 = 53$.
- (ii) $(a - b)^2 = (a + b)^2 - 4ab = 81 - 56 = 25$.

∴ 53 and 25

03 ⚡ Practice Zone

Attempt in your notebook. Draw a neat figure wherever needed. ★ = higher-order thinking

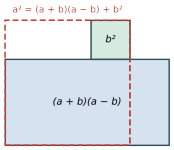
1. Expand: (i) $(7x + 4y)^2$, (ii) $(\frac{7}{5}x + \frac{3}{2}y)^2$, (iii) $(2.5p + 1.5q)^2$, (iv) $(\frac{3}{4}s + 8t)^2$, (v) $(\frac{1}{2}x + 2y)^2$, (vi) $(\frac{1}{x} + \frac{1}{y})^2$.

2. Find using identities: (i) 64^2, (ii) 105^2, (iii) 205^2, (iv) 79^2, (v) 193^2, (vi) 299^2.

3. Expand: (i) $(p + 3q + 7r)^2$, (ii) $(2a - b - 3c)^2$, (iii) $(-3m + 4k - l)^2$.

4. Choose the most convenient identity and find: (i) 117^2, (ii) 78^2, (iii) 198^2, (iv) 214^2, (v) 1104^2, (vi) 1120^2.

5. Use $a^2 = (a + b)(a - b) + b^2$ to find 35^2, 65^2, 85^2 and 95^2. What pattern do you see for squares of numbers ending in 5?

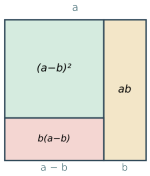


6. Find using $a^2 - b^2 = (a + b)(a - b)$: (i) 104 × 96, (ii) 23 × 17, (iii) 24 × 16, (iv) 17 × 21, (v) 0.98 × 1.02.

7. (i) If a + b = 7 and ab = 10, find $a^2 + b^2$ and $(a - b)^2$. (ii) If $x - \frac{1}{x} = 3$, find $x^2 + \frac{1}{x^2}$.

8. Is $(a + b - c)^2 + (a - b + c)^2 + (a - b - c)^2 = 2a^2 + 2b^2 + 2c^2$ an identity? Check with a = 1, b = c = 0 and then by expanding.

9. Draw a figure showing $(a - b)^2 = a^2 - 2ab + b^2$ and explain why b^2 is added back.



10. For four consecutive squares n^2, $(n + 1)^2$, $(n + 2)^2$, $(n + 3)^2$, show that $(n + 3)^2 - 3(n + 2)^2 + 3(n + 1)^2 - n^2$ is always 0. ★

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11 When is $(a + b)^2$ less than, greater than or equal to $a^2 + b^2$? Give an example of each. ★

MY NOTES ✎

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TOPIC 2

2 Factorisation Using Identities and Algebra Tiles

Perfect squares | Difference of squares | Algebra tiles | (x + a)(x + b) | Splitting the middle term | Common factors first

QUICK RECAP (from earlier classes)

- Highest common factor of terms: $50p^2$, $60pq$, $18q^2$ have HCF 2.
- Factor pairs: $30 = 1 \times 30 = 2 \times 15 = 3 \times 10 = 5 \times 6$.
- Product of two negatives is positive; sum of two negatives is negative.

01 ✦ Concept Zone: Theorems & Results

Factorising with Identities

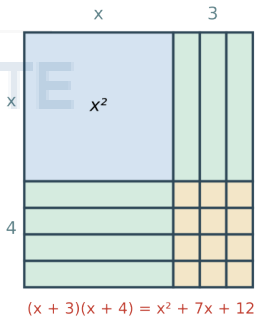
- Spot $a^2 + 2ab + b^2$: $x^2 + 4x + 4 = x^2 + 2(x)(2) + 2^2 = (x + 2)^2$; $36x^2 + 12x + 1 = (6x + 1)^2$.
- Take out common factors first: $50p^2 + 60pq + 18q^2 = 2(25p^2 + 30pq + 9q^2) = 2(5p + 3q)^2$.
- Difference of squares: $36s^2 - 49t^2 = (6s + 7t)(6s - 7t)$.
- $(a + b + c)^2$ backwards: $9a^2 + b^2 + 4c^2 - 6ab + 12ac - 4bc = (3a - b + 2c)^2$.

RESULT 2

$(x + a)(x + b) = x^2 + (a + b)x + ab$ and, more generally, $(px + a)(qx + b) = pqx^2 + (pb + qa)x + ab$.

Given: Linear factors $x + a$ and $x + b$ (algebra tiles: an x^2-tile, x-tiles and unit tiles).

To prove: $(x + a)(x + b) = x^2 + (a + b)x + ab$.



$(x + 3)(x + 4) = x^2 + 7x + 12$

Proof:

$(x + a)(x + b) = x(x + b) + a(x + b)$ *[distributive law]*

$= x^2 + bx + ax + ab$

$\therefore (x + a)(x + b) = x^2 + (a + b)x + ab$

$(px + a)(qx + b) = pqx^2 + pbx + qax + ab$ *[same method]*

$\therefore (px + a)(qx + b) = pqx^2 + (pb + qa)x + ab$

Hence proved.

Splitting the middle term: to factor $x^2 + 11x + 30$, find a, b with $a + b = 11$ and $ab = 30$: 5 and 6 $\Rightarrow (x + 5)(x + 6)$. For $px^2 + mx + n$, find two numbers with sum m and product pn.

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KEY NOTES

- ✔ Common factor first, then identities.
- ✔ $x^2 + (a + b)x + ab = (x + a)(x + b)$.
- ✔ $px^2 + mx + n$: split m into two numbers with product pn.
- ✔ Check by multiplying the factors back.

NOTE Signs: for $x^2 - 5x + 6$ both numbers are negative (−2, −3); for $r^2 - r - 42$ they have opposite signs (−7, 6).

02 ✎ Solved Examples

EXAMPLE 1

Factorise (i) $x^2 + 4x + 4$, (ii) $36x^2 + 12x + 1$, (iii) $50p^2 + 60pq + 18q^2$.

Solution

- (i) $(x + 2)^2$. (ii) $(6x)^2 + 2(6x)(1) + 1^2 = (6x + 1)^2$.
- (iii) $2(25p^2 + 30pq + 9q^2) = 2(5p + 3q)^2$.

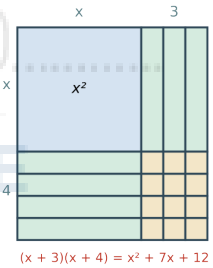
∴ $(x + 2)^2$; $(6x + 1)^2$; $2(5p + 3q)^2$

EXAMPLE 2

Use algebra tiles or splitting to factorise (i) $x^2 + 7x + 12$, (ii) $x^2 + 11x + 30$, (iii) $x^2 - 5x + 6$.

Solution

- (i) $3 + 4 = 7$, $3 \times 4 = 12 \Rightarrow (x + 3)(x + 4)$.
- (ii) $5 + 6 = 11$, $5 \times 6 = 30 \Rightarrow (x + 5)(x + 6)$.
- (iii) $(-2) + (-3) = -5$, $(-2)(-3) = 6 \Rightarrow (x - 2)(x - 3)$.



∴ $(x + 3)(x + 4)$; $(x + 5)(x + 6)$; $(x - 2)(x - 3)$

EXAMPLE 3

Factorise $10x^2 - 11x - 6$.

Solution

- pn = 10 × (−6) = −60; numbers with sum −11 and product −60: −15 and 4.
- $10x^2 - 15x + 4x - 6 = 5x(2x - 3) + 2(2x - 3)$.

∴ $(2x - 3)(5x + 2)$

EXAMPLE 4

Saira joins one x × x square, 8 strips x × 1 and 15 unit squares into a rectangle. Find its sides.

Solution

- Area $= x^2 + 8x + 15$.
- $3 + 5 = 8$ and $3 \times 5 = 15 \Rightarrow (x + 3)(x + 5)$.

∴ x + 3 and x + 5

TOPIC 2

EXAMPLE 5

Find $(2x + 3)(3x + 1)$ using $(px + a)(qx + b)$, and check with $x = 1$.

Solution

- $pq = 6$, $pb + qa = 2 \times 1 + 3 \times 3 = 11$, $ab = 3 \Rightarrow 6x^2 + 11x + 3$.
- $x = 1$: $5 \times 4 = 20$ and $6 + 11 + 3 = 20$ ✓

$\therefore 6x^2 + 11x + 3$

03 ⚡ Practice Zone

Attempt in your notebook. Draw a neat figure wherever needed. ★ = higher-order thinking

1. Factorise completely: (i) $9x^2 + 24xy + 16y^2$, (ii) $4s^2 + 20st + 25t^2$, (iii) $49x^2 + 28xy + 4y^2$, (iv) $16y^2 - 24y + 9$, (v) $64p^2 + \frac{32}{3}pq + \frac{4}{9}q^2$.

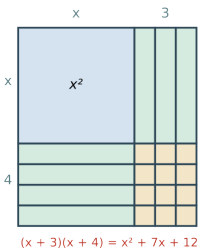
2. Factorise by first taking out a common factor: (i) $3a^2 + 4ab + \frac{4}{3}b^2$, (ii) $\frac{9}{5}s^2 + 6sv + 5v^2$, (iii) $2x^2 - 50$.

3. Factorise: (i) $36s^2 - 49t^2$, (ii) $4x^2 - 25$, (iii) $p^4 - 16$, (iv) $9m^2 - (m - 2)^2$.

4. Factorise by splitting the middle term: (i) $s^2 - 11s + 24$, (ii) $r^2 - r - 42$, (iii) $3x^2 - 4x - 7$, (iv) $6x^2 + 7x + 2$, (v) $2x^2 + 7x + 3$.

5. Factorise: (i) $9a^2 + b^2 + 4c^2 - 6ab + 12ac - 4bc$, (ii) $64u^2 + 121v^2 + 4w^2 - 176uv - 32uw + 44vw$.

6. (i) Draw algebra tiles for $(x + 2)(x + 3)$. (ii) Arrange tiles for $x^2 + 11x + 30$ into a rectangle. (iii) If $7x$ in $x^2 + 7x + 12$ is split as $2x + 5x$, can a rectangle be formed?



7. (i) Complete: $(px + a)(qx + b) = (__)x^2 + (__)x + __$. (ii) Use it to expand $(3x - 2)(4x + 5)$. (iii) Fill in: $10x^2 - 11x - 6 = (2x - __)(__ + 2)$.

8. Find using $(x + a)(x + b)$: (i) 103×104, (ii) 18×29 (as $(2x - 2)(3x - 1)$ with $x = 10$), (iii) 97×95.

9. The area of a rectangle is (i) $25a^2 - 30ab + 9b^2$, (ii) $36s^2 - 49t^2$. Find possible length and breadth in each case. ★

10. Factorise $x^2 + 13x + 42$ and $x^2 + 13x + 40$. Why do both have the same middle term but different factors? ★

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TOPIC 3

3 Identities for Cubes

(a + b)³ with a cube model | (a − b)³ | x³ − y³ and x³ + y³ | x³ + y³ + z³ − 3xyz | Sum zero ⇒ a³ + b³ + c³ = 3abc

QUICK RECAP (from earlier classes)

- $(a + b)^2 = a^2 + 2ab + b^2$.
- Volume of a cube = edge³; of a cuboid = l × w × h.
- $(-b)^3 = -b^3$.

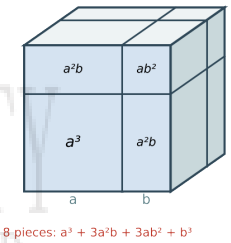
01 ✦ Concept Zone: Theorems & Results

RESULT 3

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ and $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$.

Given: Any numbers a, b; a cube of edge a + b cut into 8 pieces.

To prove: $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.



Proof:

$(a + b)^3 = (a + b)(a^2 + 2ab + b^2)$ *[Result 1]*

$= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3$

$\therefore (a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Cube model: 1 cube a^3, 3 cuboids a × a × b, 3 cuboids a × b × b and 1 cube b^3.

Putting $-b$ for b: $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Hence proved.

Replace b by $-b$ to get $(a - b)^3$: the signs then alternate +, −, +, −.

Sum and Difference of Cubes

- **$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$**; **$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$** — check by multiplying out.
- x − y is a factor of $x^2 - y^2$, $x^3 - y^3$, $x^4 - y^4 = (x^2 - y^2)(x^2 + y^2)$, …
- **$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$**.
- Hence, if x + y + z = 0, then **$x^3 + y^3 + z^3 = 3xyz$**.

KEY NOTES

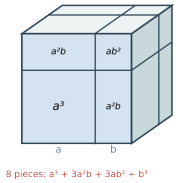
- ✔ $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$.
- ✔ $x^3 \pm y^3 = (x \pm y)(x^2 \mp xy + y^2)$.
- ✔ $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$.
- ✔ $x + y + z = 0 \Rightarrow x^3 + y^3 + z^3 = 3xyz$.

NOTE In $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$ the second factor has a **plus** xy; it is not $(x - y)^2$.

02 ✎ Solved Examples

EXAMPLE 1

A cube has volume $p^3 + 6p^2q + 12pq^2 + 8q^3$. Find its edge.



8 pieces: $a^3 + 3a^2b + 3ab^2 + b^3$

Solution

› $= p^3 + 3(p)^2(2q) + 3(p)(2q)^2 + (2q)^3 = (p + 2q)^3$.

∴ p + 2q

EXAMPLE 2

Write $8n^3 - 60n^2m + 150nm^2 - 125m^3$ as $(a - b)^3$.

Solution

› $(2n)^3 - 3(2n)^2(5m) + 3(2n)(5m)^2 - (5m)^3$.

∴ $(2n - 5m)^3$

EXAMPLE 3

Find (i) 104^3, (ii) 99^3 using identities.

Solution

› (i) $(100 + 4)^3 = 1\,000\,000 + 120\,000 + 4\,800 + 64 = 1\,124\,864$.

› (ii) $(100 - 1)^3 = 1\,000\,000 - 30\,000 + 300 - 1 = 970\,299$.

∴ 1 124 864 and 970 299

EXAMPLE 4

Three numbers have sum 10, product 25 and sum of squares 38. Find the sum of their cubes.

Solution

› $(x + y + z)^2 = 38 + 2(xy + yz + zx) \Rightarrow 100 = 38 + 2(xy + yz + zx) \Rightarrow xy + yz + zx = 31$.

› $x^3 + y^3 + z^3 - 75 = 10(38 - 31) = 70$.

∴ 145

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TOPIC 3

EXAMPLE 5

Without cubing, find $(-12)^3 + 7^3 + 5^3$.

Solution

› $-12 + 7 + 5 = 0 \Rightarrow$ sum of cubes $= 3 \times (-12) \times 7 \times 5$.

$\therefore -1260$

03 ⚡ Practice Zone

Attempt in your notebook. Draw a neat figure wherever needed. ★ = higher-order thinking

1. Expand: (i) $(2x + 3)^3$, (ii) $(x - 2y)^3$, (iii) $(\frac{1}{2}x + y)^3$, (iv) $(3a - \frac{1}{3})^3$.

2. Find using identities: (i) 147^3, (ii) 199^3, (iii) 127^3, (iv) $(-107)^3$, (v) $(-299)^3$.

3. Factorise: (i) $x^3 + 6x^2y + 12xy^2 + 8y^3$, (ii) $8p^3 - 36p^2 + 54p - 27$, (iii) $27u^3 - \frac{27}{5}u^2 + \frac{9}{25}u - \frac{1}{125}$.

4. Factorise: (i) $27b^3 - 64$, (ii) $8x^3 + 125$, (iii) $64y^3 + \frac{1}{125}z^3$, (iv) $a^3 - 27b^3$.

5. Factorise: (i) $p^3 + 27q^3 + r^3 - 9pqr$, (ii) $8x^3 + y^3 + 27z^3 - 18xyz$.

6. If $a + b + c = 5$ and $ab + bc + ca = 10$, prove that $a^3 + b^3 + c^3 - 3abc = -25$.

7. Without cubing, find (i) $(-12)^3 + 7^3 + 5^3$ (check), (ii) $28^3 + (-15)^3 + (-13)^3$, (iii) $(0.2)^3 - (0.3)^3 + (0.1)^3$.

8. Find the value of (i) $x^3 + y^3 - 12xy + 64$ when $x + y = -4$, (ii) $x^3 - 8y^3 - 36xy - 216$ when $x = 2y + 6$.

9. (i) Show that $x - y$ is a factor of $x^4 - y^4$. (ii) Multiply $(x - y)(x^4 + x^3y + x^2y^2 + xy^3 + y^4)$. What do you get?

10. James writes $(a - b)^2(a + b) = (a^2 - 2ab + b^2)(a + b)$; Reshma writes $(a - b)(a^2 - b^2)$. Who is correct? Expand both. ★

11. By factorising $n^3 - n$, show that it is divisible by 6 for every natural number n. ★

MY NOTES ✎

TOPIC 3

TOPIC 4

4 Rational Expressions and Applications

Rational algebraic expressions | Cancelling non-zero common factors | Dimensions from area/volume | Quadratic word problems

QUICK RECAP (from earlier classes)

- A fraction is simplified by dividing numerator and denominator by a common factor.
- $a - b = -(b - a)$.
- If $PQ = 0$ then $P = 0$ or $Q = 0$.

01 ✦ Concept Zone: Theorems & Results

Simplifying Rational Expressions

- A **rational expression** is a quotient of two polynomials, e.g., $\frac{x^2 - 7x + 12}{5x^2 + 5x - 100}$ (denominator $\neq 0$).
- Factorise numerator and denominator completely, then cancel common factors (which are non-zero because the denominator is non-zero).
- $\frac{x^2 - 7x + 12}{5x^2 + 5x - 100} = \frac{(x-3)(x-4)}{5(x-4)(x+5)} = \frac{x-3}{5(x+5)}$.

Using Identities in Problems

- **Dimensions:** a rectangle of area $x^2 + 8x + 15$ can have sides $x + 3$ and $x + 5$; a cuboid of volume $3ps^2 - 15ps + 12p = 3p(s - 1)(s - 4)$.
- **Word problems:** a pool 4 m narrower than long with area 96 m^2: $x(x - 4) = 96 \Rightarrow (x - 12)(x + 8) = 0 \Rightarrow x = 12$ (lengths are positive).
- **Paths:** a square of side 40 m with a path of width s around it: path area $= (40 + 2s)^2 - 40^2 = 160s + 4s^2$.



KEY NOTES

- ✔ Factorise fully before cancelling.
- ✔ Cancel factors, never terms: $\frac{x+3}{x+5}$ cannot be simplified.
- ✔ Reject negative lengths in word problems.
- ✔ Area/volume expression → factors → possible dimensions.

NOTE $(5z - 2y)$ and $(2y - 5z)$ differ only in sign: $(2y - 5z)^2 = (5z - 2y)^2$, which helps in cancelling.

02 ✎ Solved Examples

EXAMPLE 1

Simplify $\frac{x^2 - 7x + 12}{5x^2 + 5x - 100}$.

Solution

- Numerator = $(x - 3)(x - 4)$; denominator = $5(x^2 + x - 20) = 5(x - 4)(x + 5)$.
- Cancel $(x - 4) \neq 0$.

$\therefore \frac{x - 3}{5(x + 5)}$

EXAMPLE 2

Simplify $\frac{36s^2 - 12st + t^2}{t^2 + 2ts - 48s^2}$.

Solution

- Numerator = $(6s - t)^2$; denominator = $(t + 8s)(t - 6s) = -(t + 8s)(6s - t)$.
- Cancel $(6s - t)$.

$\therefore \frac{t - 6s}{t + 8s}$

EXAMPLE 3

A rectangular pool is 4 m less in breadth than in length and its area is 96 m². Find its dimensions.

Solution

- $x(x - 4) = 96 \Rightarrow x^2 - 4x - 96 = 0 \Rightarrow (x - 12)(x + 8) = 0$.
- $x = 12$ ($x = -8$ is impossible).

$\therefore$ 12 m × 8 m

EXAMPLE 4

A number plus its reciprocal is $\frac{10}{3}$. Find the number.

Solution

- $x + \frac{1}{x} = \frac{10}{3} \Rightarrow 3x^2 - 10x + 3 = 0$.
- $(3x - 1)(x - 3) = 0$.

$\therefore$ 3 or $\frac{1}{3}$

TOPIC 4

EXAMPLE 5

A square playground of side 40 m has a path of width s m around it (outside). Find the area of the path.

Solution



- Outer square side = 40 + 2s.
- $(40 + 2s)^2 - 1600 = 160s + 4s^2$.

∴ **$(4s^2 + 160s)$ m²**

03 ⚡ Practice Zone

Attempt in your notebook. Draw a neat figure wherever needed. ★ = higher-order thinking

1. Simplify: (i) $\frac{3p^2 - 3pq - 18q^2}{p^2 - 9q^2}$, (ii) $\frac{n^3 - 3n^2m + 3nm^2 - m^3}{5m^2 - 10mn + 5n^2}$, (iii) $\frac{4y^2 - 20yz + 25z^2}{25z^2 - 4y^2}$.

2. Simplify: (i) $\frac{(x^2 + x - 6)(x^2 - 7x + 12)}{(x^2 - 6x + 8)(x^2 - 9)}$, (ii) $\frac{4x^2 + 4x + 1}{4x^2 - 1}$.

3. Simplify: (i) $\frac{3a^3 - 24b^3}{9a^2 - 36b^2}$, (ii) $\frac{s^3 + 125t^3}{s^2 - 2st - 35t^2}$, (iii) $\frac{p^4 - 16}{p^2 - 4p + 4}$.

4. Find possible length and breadth of rectangles with areas (i) $25a^2 - 30ab + 9b^2$, (ii) $36s^2 - 49t^2$, (iii) $2x^2 + 7x + 3$.

5. Find possible dimensions of cuboids with volumes (i) $6a^2 - 24b^2$, (ii) $3ps^2 - 15ps + 12p$, (iii) $x^3 - x$.

6. A rectangular pool has area $2x^2 + 7x + 3$ square hastas and width $2x + 1$ hastas. Find its length. Find the area when $x = 4$.

7. The length of a garden is 3 m more than its breadth and its area is 108 m². Find its dimensions.

8. (i) A number plus its reciprocal is $\frac{26}{5}$. Find it. (ii) The product of two consecutive positive integers is 132. Find them.

9. A square ground of side 40 m has a path of width s m running along the inside of its edges. (i) Find the area of the path. (ii) Find s if the path has area 156 m².



10. Show that $\frac{(a+b)^2}{a^2 - b^2} \div \frac{a+b}{a-b} = 1$ (a ≠ ±b). ★

TOPIC 4

11 A student cancels x in $\frac{x+6}{x+3}$ to get 2. Show with x = 1 that this is wrong, and explain the mistake. ★

MY NOTES ✎

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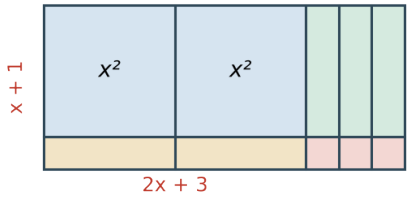
+ Miscellaneous Questions

11 mixed questions from all topics of this chapter and earlier Ganita Manjari chapters

Attempt in your notebook. Draw a neat figure wherever needed. ★ = higher-order thinking

1 The rectangle is made of algebra tiles. (i) Write its area as a product and as a sum of tile areas. (ii) Verify using (px + a)(qx + b). (iii) Find its perimeter. (iv) Draw tiles for (x + 2)(3x + 1).



2 Rational or irrational? Use identities. (i) $(\sqrt{3} + \sqrt{2})^2$, (ii) $(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})$, (iii) $(2 + \sqrt{3})(2 - \sqrt{3})$, (iv) $(\sqrt{7} + \sqrt{3})^2 - (\sqrt{7} - \sqrt{3})^2$.

3 For whole numbers m > n, the point P($m^2 - n^2$, 2mn) is plotted. (i) Show that OP = $m^2 + n^2$. (ii) Find P and OP for (m, n) = (2, 1), (3, 2), (4, 1). (iii) What triples do you get?

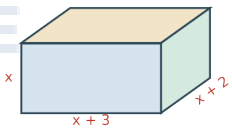
4 Show that each is a linear polynomial and find its zero: (i) $p(x) = (x + 3)^2 - (x - 3)^2$, (ii) $q(x) = (x + 1)^3 - x^3 - 3x^2$, (iii) $r(x) = (x + 2)(x + 5) - (x + 1)(x + 4)$.

5 Using identities find: (i) 1003^2, (ii) 997×1003, (iii) 5.02×4.98, (iv) 101^3, (v) 98^3.

6 If $x + \frac{1}{x} = 5$, find (i) $x^2 + \frac{1}{x^2}$, (ii) $x^3 + \frac{1}{x^3}$. If instead $x - \frac{1}{x} = 2$, find (iii) $x^2 + \frac{1}{x^2}$, (iv) $x^3 - \frac{1}{x^3}$.

7 Factorise: (i) $x^4 - 81$, (ii) $2x^3 - 16$, (iii) $a^2 + 2ab + b^2 - 9c^2$, (iv) $4x^2 - 12x + 9 - y^2$.

8 A cuboid has edges x, x + 2 and x + 3 (in cm). (i) Write its volume as a polynomial. (ii) Find the volume when x = 4. (iii) Find its total surface area as a polynomial.



9 If a + b + c = 0, prove that $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3$. Check with a = 1, b = 2, c = −3.

10 Show that $(x - y)^3 + (y - z)^3 + (z - x)^3 = 3(x - y)(y - z)(z - x)$. Verify with x = 5, y = 2, z = 1.

11 Show that the product of four consecutive integers plus 1 is a perfect square: $n(n + 1)(n + 2)(n + 3) + 1 = (n^2 + 3n + 1)^2$. Check for n = 1, 2, 5. ★

MY NOTES ✎

MISCELLANEOUS

★ Competency Based Question Bank

32 MCQs and 6 Assertion–Reason questions from the whole chapter and linked topics

1 $(a + b)^2 - (a - b)^2$ equals

(a) $2ab$ (b) $4ab$ (c) $2a^2 + 2b^2$ (d) 0

2 Which is an identity?

(a) $x^2 = 4$ (b) $x^2 - 1 = 24$

(c) $(x + y)^2 = x^2 + 2xy + y^2$ (d) $2x + 3 = 7$

3 43^2 by $(40 + 3)^2$ is

(a) 1843 (b) 1869 (c) 1849 (d) 1609

4 $(3x - 2y)^2 =$

(a) $9x^2 - 12xy + 4y^2$ (b) $9x^2 + 12xy + 4y^2$

(c) $9x^2 - 6xy + 4y^2$ (d) $9x^2 - 4y^2$

5 $104 \times 96 =$

(a) 9984 (b) 9996 (c) 10016 (d) 9884

6 If $a + b = 7$ and $ab = 10$, then $a^2 + b^2 =$

(a) 69 (b) 29 (c) 49 (d) 39

7 $(a + b + c)^2$ contains the term

(a) $3ab$ (b) b^2c (c) abc (d) $2bc$

8 Śrīdhara's rule gives 65^2 as

(a) $60 \times 60 + 5$ (b) $65 \times 60 + 25$ (c) $70 \times 65 + 5$ (d) $70 \times 60 + 25$

9 $(a + b)^2 > a^2 + b^2$ exactly when

(a) $a > b$ (b) $ab > 0$ (c) $ab < 0$ (d) $a = b$

10 $x^2 + 4x + 4$ factorises as

(a) $(x - 2)^2$ (b) $(x + 4)^2$ (c) $(x + 2)^2$ (d) $(x + 1)(x + 4)$

11 $36s^2 - 49t^2 =$

(a) $(36s - 49t)$ (b) $(6s + 7t)^2$

(c) $(6s - 7t)^2$ (d) $(6s + 7t)(6s - 7t)$

12 $x^2 + 11x + 30 =$

(a) $(x + 3)(x + 10)$ (b) $(x + 2)(x + 15)$

(c) $(x + 1)(x + 30)$ (d) $(x + 5)(x + 6)$

13 $x^2 - 5x + 6 =$

(a) $(x + 2)(x + 3)$ (b) $(x - 2)(x - 3)$ (c) $(x - 1)(x - 6)$ (d) $(x + 1)(x - 6)$

14 $r^2 - r - 42 =$

(a) $(r - 21)(r + 2)$ (b) $(r - 7)(r + 6)$

(c) $(r + 7)(r - 6)$ (d) $(r - 14)(r + 3)$

QUESTION BANK

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15 $10x^2 - 11x - 6 =$

(a) $(10x + 1)(x - 6)$ (b) $(5x - 3)(2x + 2)$

(c) $(2x - 3)(5x + 2)$ (d) $(2x + 3)(5x - 2)$

16 $(2x + 3)(3x + 1) =$

(a) $6x^2 + 9x + 3$ (b) $5x^2 + 11x + 3$ (c) $6x^2 + 3$ (d) $6x^2 + 11x + 3$

17 $50p^2 + 60pq + 18q^2 =$

(a) $2(5p + 3q)^2$ (b) $(5p + 3q)^2$ (c) $2(5p - 3q)^2$ (d) $(10p + 3q)^2$

18 $(a + b)^3 =$

(a) $a^3 + 2a^2b + 2ab^2 + b^3$ (b) $a^3 + b^3$

(c) $a^3 + 3a^2b + 3ab^2 + b^3$ (d) $a^3 + 3ab + b^3$

19 The edge of a cube of volume $p^3 + 6p^2q + 12pq^2 + 8q^3$ is

(a) $2p + q$ (b) $p + 4q$ (c) $p + 2q$ (d) $p + q$

20 $x^3 - y^3 =$

(a) $(x - y)(x^2 - xy + y^2)$ (b) $(x - y)^3$

(c) $(x + y)(x^2 - xy + y^2)$ (d) $(x - y)(x^2 + xy + y^2)$

21 $8x^3 + 125 =$

(a) $(2x + 5)(4x^2 - 10x + 25)$ (b) $(2x + 5)(4x^2 + 10x + 25)$

(c) $(2x + 5)^3$ (d) $(8x + 5)(x^2 + 25)$

22 If $x + y + z = 0$ then $x^3 + y^3 + z^3 =$

(a) $x + y + z$ (b) 0 (c) xyz (d) $3xyz$

23 $(-12)^3 + 7^3 + 5^3 =$

(a) -420 (b) 0 (c) -1260 (d) 1260

24 $99^3 =$

(a) 970 301 (b) 980 299 (c) 970 029 (d) 970 299

25 $x - y$ is a factor of

(a) $x^2 + y^2$ (b) $x^4 - y^4$ (c) $x^3 + y^3$ (d) $x^2 + xy + y^2$

26 If $x + y = -4$, then $x^3 + y^3 - 12xy + 64 =$

(a) 12 (b) 64 (c) 0 (d) -64

27 $\frac{x^2 - 7x + 12}{x^2 - 9}$ simplifies to

(a) $\frac{x-3}{x+3}$ (b) $x - 4$ (c) $\frac{x-4}{x+3}$ (d) $\frac{x+4}{x-3}$

QUESTION BANK

28 $\frac{4x^2+4x+1}{4x^2-1} =$

(a) $\frac{2x-1}{2x+1}$ (b) 1 (c) 2x + 1 (d) $\frac{2x+1}{2x-1}$

29 A pool with breadth 4 less than length and area 96 m^2 has length

(a) 8 m (b) 12 m (c) 16 m (d) 24 m

30 The area of a path of width s around a 40 m square is

(a) $1600 + 4s^2$ (b) $160s$ (c) $4s^2$ (d) $160s + 4s^2$

31 $n^3 - n$ is always divisible by

(a) 6 (b) 9 (c) 4 (d) 5

32 If $a + b + c = 5$ and $ab + bc + ca = 10$, then $a^3 + b^3 + c^3 - 3abc =$

(a) −25 (b) 50 (c) 0 (d) 25

⚖ Assertion – Reason

Directions: (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.

33 Assertion (A): $(x + y)^2 = x^2 + 2xy + y^2$ is an identity.
Reason (R): It is true for all values of x and y.

34 Assertion (A): $(a + b)^2 = a^2 + b^2$ for all a, b.
Reason (R): $(a + b)^2 - (a^2 + b^2) = 2ab$.

35 Assertion (A): $28^3 - 15^3 - 13^3 = 3 \times 28 \times 15 \times 13$.
Reason (R): If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$.

36 Assertion (A): $x^2 + 7x + 12 = (x + 3)(x + 4)$.
Reason (R): $3 + 4 = 7$ and $3 \times 4 = 12$.

37 Assertion (A): $\frac{x+6}{x+3} = 2$.

Reason (R): Only common factors may be cancelled in a rational expression.

38 Assertion (A): $8n^3 - 60n^2m + 150nm^2 - 125m^3 = (2n - 5m)^3$.
Reason (R): $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$.

MY NOTES ✎

QUESTION BANK

Story Time: Case Studies & Twisters

Real-life stories, linked chapters and tricky twists. Read carefully, think, then answer.

Case 1 · Mental Maths Club

SQUARES

The club squares numbers mentally using identities.

(i) Find 103^2 using $(a + b)^2$.

(ii) Find 97^2 using $(a - b)^2$.

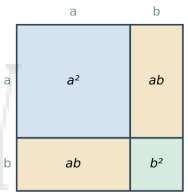
(iii) Find 45^2 using Śrīdhara's rule.

(iv) Find 51×49 using $a^2 - b^2$.

Case 2 · Tiling a Courtyard

AREA MODEL

A square courtyard of side $(a + b)$ m has a square lawn of side a m in one corner; the rest is paved.



(i) Write the area of the courtyard.

(ii) Write the paved area.

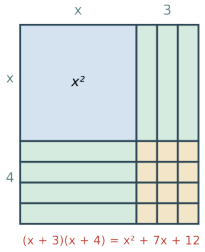
(iii) If $a = 8$ and $b = 2$, find the paved area.

(iv) Show that the paved area is $b(2a + b)$.

Case 3 · Saira's Tiles

ALGEBRA TILES

Saira has one $x \times x$ tile, some $x \times 1$ strips and some unit squares and arranges them into rectangles.



(i) With 7 strips and 12 unit squares, find the sides.

(ii) With 8 strips and 15 units, find the sides.

(iii) Can 5 strips and 7 units make a rectangle? Why?

(iv) Find the sides when $x = 10$ cm in the first case.

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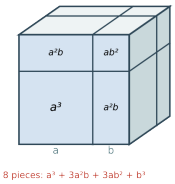
Case 4 · The Cube Puzzle

CUBES

A wooden cube of edge (a + b) cm is cut into 8 pieces by planes parallel to its faces.



8 pieces: $a^3 + 3a^2b + 3ab^2 + b^3$

(i) How many pieces are cubes? Write their volumes.

(ii) Write the volumes of the cuboids.

(iii) Write the identity shown.

(iv) Find the total volume when a = 3, b = 1 both ways.

Case 5 · The Swimming Pool

QUADRATIC

A rectangular pool is 4 m less in breadth than in length; its area is 96 m^2. A path 1 m wide runs around it.

(i) Form the equation for the length x.

(ii) Find the length and breadth.

(iii) Find the area of the path.

(iv) Find the perimeter of the outer edge of the path.

Case 6 · Packing Boxes

VOLUME

A cuboidal box has volume $x^3 + 7x^2 + 12x$ cm^3.

(i) Factorise the volume.

(ii) Suggest possible dimensions.

(iii) Find the volume when x = 5.

(iv) If x = 5, which dimension is the smallest?

Case 7 · The Sum of Cubes Trick

TWISTED

Asha says: "I can find $25^3 + (-17)^3 + (-8)^3$ in my head."

(i) What is $25 + (-17) + (-8)$?

(ii) Which identity does Asha use?

(iii) Find the value.

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(iv) Find $1^3 + 2^3 + (-3)^3$ the same way.

Case 8 · Three Numbers

IDENTITIES

Three numbers have sum 9, sum of squares 29 and product 24.

(i) Find the sum of their pairwise products.

(ii) Find the sum of their cubes.

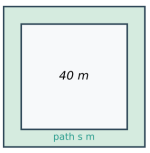
(iii) Guess the numbers and check.

(iv) Verify $(x + y + z)^2$ for your numbers.

Case 9 · Garden Path

AREA

A square garden of side 40 m has a path of width s m around the outside.



(i) Write the outer side length.

(ii) Find the path area as a polynomial.

(iii) Find the path area for s = 2.

(iv) Find s if the path area is 704 m^2.

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Case 10 · Divisibility Detective

FACTORISATION

Ravi notices $2^3 - 2 = 6$, $3^3 - 3 = 24$, $4^3 - 4 = 60$.

(i) Factorise $n^3 - n$.

(ii) Why is it always even?

(iii) Why is it always divisible by 3?

(iv) Is it always divisible by 6? By 12?

STORY TIME

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