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ANSWER KEY

Chapter 9 · Propositions and their Converses

Class IX · As per New Curriculum **Ganita Manjari**

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Topic 1: Propositions, Converses and Counterexamples

1. (i) Proposition, true ($4k = 2 \times 2k$)

(ii) Exclamation — not a proposition

(iii) Proposition, false: $91 = 7 \times 13$

(iv) Command — not a proposition

(v) Proposition, false: a 4 cm × 2 cm rectangle is not a square

Answer: (i), (iii), (v) are propositions: (i) true, (iii) false, (v) false

2. (i) If a number ends in 0, then it is divisible by 5. Hyp.: ends in 0; Concl.: divisible by 5

(ii) If all angles of a quadrilateral are right angles, then it is a rectangle

(iii) If all angles of a triangle are equal, then it is equilateral

Answer: Hypothesis = the "if" part; conclusion = the "then" part.

3. (i) If a number is a multiple of 5, then it is a multiple of 15 — false (10)

(ii) If two angles are equal, then they are vertically opposite — false (two base angles of an isosceles triangle)

(iii) If a triangle is isosceles, then it is equilateral — false (sides 5, 5, 6)

(iv) If a number is a multiple of 10, then it is a multiple of 100 — false (20)

Answer: All four converses are false.

4. (i) If the road is wet, then it has rained

(ii) The road can be wet for another reason, e.g., a water tanker spilt water *[a counterexample]*

(iii) e.g., "If a person lives in Delhi, then she lives in India" (converse false: a person in Mumbai); "If it is a mango, then it is a fruit" (converse false: an apple).

Answer: The converse need not be true even when the proposition is true.

5. (i) 9 is odd but $9 = 3 \times 3$

(ii) 12 is a multiple of 4 but not of 8

(iii) a trapezium with exactly one pair of parallel sides

(iv) $\frac{1}{2}$: $(\frac{1}{2})^2 = \frac{1}{4} < \frac{1}{2}$ (also 1: $1^2 = 1$)

Answer: (i) 9 (ii) 12 (iii) a trapezium (iv) $\frac{1}{2}$

6. (i) True: $6k = 2(3k)$. Converse "divisible by 2 ⇒ by 6": false (4)

(ii) False (7). Converse "multiple of 14 ⇒ multiple of 7": true, $14k = 7(2k)$

(iii) True: 10k = 5(2k). Converse "divisible by 5 ⇒ ends in 0": false (15)

Answer: (i) T, F (ii) F, T (iii) T, F

7. 20 is a multiple of 4 (20 = 4 × 5) but not of 8 *[8, 16, 24 are multiples of 8]*

Answer: 20 — two examples cannot justify a statement about all multiples of 4.

8. (i) Both true: AB = AC ⇔ ∠B = ∠C; n is divisible by 2 and 5 ⇔ by 10

(ii) Only P true: square ⇒ rectangle; multiple of 6 ⇒ multiple of 3

(iii) Only converse true: rectangle ⇒ square (converse: square ⇒ rectangle); multiple of 3 ⇒ of 6

(iv) Both false: If n is prime, then n is odd (2); if n is odd, then n is prime (9)

Answer: Many answers are possible; one valid example for each case is enough.

9. Converse: If two lines do not meet, then they are parallel

Converse of the converse: If two lines are parallel, then they do not meet *[the original]*

Answer: Taking the converse twice gives back the original proposition.

10. Converse: If the diagonals of a quadrilateral are equal, then it is a square — false (a rectangle)

Negation-type: If a quadrilateral is not a square, then its diagonals are not equal — false (a rectangle again)

The converse swaps hypothesis and conclusion; the negation-type statement denies both.

Answer: Different statements; here both are false while the original is true.

11. n = 40: 1600 + 40 + 41 = 1681 = 41^2

n = 41: 41^2 + 41 + 41 = 41 × 43

Answer: No — ten examples do not prove it; n = 40 and n = 41 are counterexamples.

12. A counterexample satisfies the hypothesis and violates the conclusion, so the statement "for all" fails at once.
Examples check only finitely many cases; a later case may fail. Fermat's numbers were prime for n = 0 to 4, yet n = 5 gives 641 × 6700417.

Answer: Disproof needs one case; proof needs an argument valid for every case.

Topic 2: Propositions in Number Theory and Algebra

1.



(i) 49: (1, 49), (7, 7) ⇒ 1, 7, 49 — 3 factors

(ii) 48: (1, 48), (2, 24), (3, 16), (4, 12), (6, 8) — 10 factors

(iii) 100: (1, 100), (2, 50), (4, 25), (5, 20), (10, 10) — 9 factors

Answer: 49 and 100 (perfect squares) have an odd number of factors; 48 has an even number.

2.

(i) It proves: If n has an odd number of factors, then n is a perfect square *[it starts from the odd count]*

(ii) It never starts from “n is a perfect square”, so it says nothing about that direction

(iii) If n = m × m, the pair (m, m) gives one factor; every other factor f ≠ m has a different partner

Answer: So factors = 2 × (other pairs) + 1, which is odd.

3.

(i) Converse “by 2 and 9 ⇒ by 18”: true (2, 9 co-prime)

(ii) “by 6 and 12 ⇒ by 36”: false (12)

(iii) “by 9 and 5 ⇒ by 45”: true (co-prime)

(iv) “by 2 and 4 ⇒ by 8”: false (4)

Answer: The converse is true exactly when the two divisors have no common factor other than 1.

4.

If n is divisible by 9, then its digit sum is a multiple of 9; if the digit sum is a multiple of 9, then n is divisible by 9

(i) 4 + 5 + 8 + 1 = 18 and 4581 = 9 × 509 ✓

(ii) 7 + 2 + 5 + 4 = 18 and 7254 = 9 × 806 ✓

Answer: Because both directions are true, checking the digit sum decides divisibility by 9.

5.

(i) $a = p^2$, $b = q^2 \Rightarrow ab = (pq)^2$

(ii) Converse: If ab is a perfect square, then a and b are perfect squares — false: 2 × 8 = 16

(iii) True if a and b have no common factor other than 1 *[each prime factor of ab then belongs wholly to a or to b]*

Answer: (i) proved (ii) false (a = 2, b = 8) (iii) a, b co-prime

6.

(i) n = 4: 65 = 5 × 13

(ii) n = 3: 15

(iii) n = 11: 2047 = 23 × 89

(iv) n = 4: 9

(v) n = 4: 2n + 1 = 9 is not prime; take n = 9: 19 is prime but 9 is not

Answer: (i) 4 (ii) 3 (iii) 11 (iv) 4 (v) 9

7. (i) $n = 2k \Rightarrow n^2 = 4k^2 = 2(2k^2)$, even

(ii) Converse: If n^2 is even, then n is even. True: if n were odd, $n = 2k + 1$, $n^2 = 2(2k^2 + 2k) + 1$ would be odd

(iii) $n = 2$: $n^2 = 4$ is a multiple of 4, but 2 is not

Answer: (i) proved (ii) true (iii) n = 2 (also 6, 10, ...)

8. (i) 15: 1, 3, 5, 15; 35: 1, 5, 7, 35 — 4 each ✓

(ii) $8 = 2^3$: 1, 2, 4, 8; 27: 1, 3, 9, 27 — 4 divisors but not products of two unequal primes

Answer: P true; converse false (8, 27).

9. (i) n = 4, 7: no common factor, 4 not a multiple of 3 ✓; n = 5, 8 ✓; n = 6, 9: common factor 3

(ii) If $n = 3k$, then $n + 3 = 3(k + 1)$, so 3 is a common factor. Hence no common factor ⇒ n not a multiple of 3

(iii) Converse: If n is not a multiple of 3, then n and n + 3 have no common factor other than 1

Any common factor d of n and n + 3 divides 3, so d = 1 or 3; if $3 \nmid n$ then d = 1. The converse is also true.

Answer: P true; converse true as well.

10. (i), (ii) True both ways: multiply/divide by 5 and add/subtract 2; add/subtract a

(iii) True; converse false (x = 4, y = −4)

$3x - 7 = 11 \Leftrightarrow 3x = 18 \Leftrightarrow x = 6$

[add 7, divide by 3 — each step works both ways]

Answer: (i) both true (ii) both true (iii) only P true; x = 6

11. Converse: If the units digit of n is 0, 1, 4, 5, 6 or 9, then n is a perfect square

14 ends in 4 but is not a square

Answer: Counterexample: 14 (or 11, 19, 20, ...)

12. (i) $n = 8k = 2(4k) = 4(2k)$

(ii) 4 (or 12, 20) is divisible by 2 and 4 but not by 8

(iii) 3 and 4 are co-prime, so divisible by 3 and 4 ⇒ by 12; 2 and 4 share the factor 2

Answer: A two-part test works only when the parts are co-prime.

13. If (f, f) and (g, g) were both factor pairs, then $f^2 = g^2 = n$

f, g positive ⇒ f = g

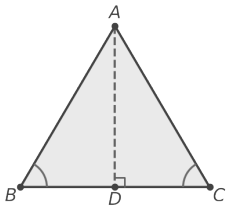
∴ only one repeated pair, so the count of factors is odd

Hence proved.

Topic 3: Propositions in Geometry

1. In ΔABD, ΔACD: ∠B = ∠C (given), ∠ADB = ∠ADC = 90°, AD common

ΔABD ≅ ΔACD (AAS) ⇒ AB = AC *[CPCT]*



Theorem 1 assumes AB = AC — exactly what we must prove — so using it would be circular.

Hence proved.

2. (i) If alternate interior angles are equal, then the lines are parallel — true

(ii) If the diagonals are perpendicular, then it is a rhombus — false (a kite)

(iii) If opposite angles are equal, then it is a parallelogram — true

(iv) If corresponding angles are equal, then the triangles are congruent — false (equilateral triangles of sides 2 and 5)

Answer: (i) T (ii) F (iii) T (iv) F

3. (i) $49 + 576 = 625 = 25^2$ ✓

(ii) $400 + 441 = 841 = 29^2$ ✓

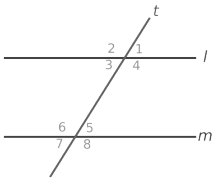
(iii) $36 + 49 = 85 \neq 81$

(iv) $2.25 + 4 = 6.25 = 2.5^2$ ✓

Answer: (i), (ii), (iv) are right-angled; (iii) is not.

4. (i) ∠8 = ∠4 = 75° (corresponding); ∠6 = ∠8 = 75° (vertically opposite); ∠7 = 180° − 75° = 105°

(ii) ∠3 and ∠5 are alternate interior angles; 110° ≠ 70°

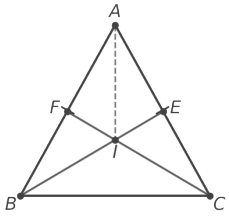


Answer: (i) 75°, 75°, 105° (ii) not parallel — by the converse (equal alternate angles ⇔ parallel)

5. (i) ∠B = ∠C ⇒ ∠IBC = ∠ICB ⇒ IB = IC *[Theorems 1 and 2]*

(ii) ΔIEC and ΔIFB: ∠EIC = ∠FIB (vert. opp.), IC = IB, ∠ICE = ∠IBF (half equal angles) ⇒ ≅ (ASA) ⇒ IE = IF

(iii) Converse: If IE = IF, then AB = AC



Answer: IB = IC, IE = IF; converse: IE = IF ⇒ AB = AC

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6. (i) Converse: equal diagonals ⇒ square — false (rectangle)



(ii) P false (a rhombus); converse: square ⇒ four equal sides — true

(iii) P false (3-4-5 and 4-4-4 have perimeter 12); converse: congruent ⇒ equal perimeters — true

Answer: (i) T, F (ii) F, T (iii) F, T

7. (i) An isosceles trapezium (or a kite with equal diagonals)

(ii) Sides 5, 5 with included angles 30° and 90° give different triangles (SSA is not a criterion)

(iii) A 3-4-5 triangle has no obtuse angle; converse "obtuse ⇒ right-angled" is false too

Answer: Each needs only one figure as a counterexample.

8. (i) $\angle A = 180° - 100° = 80°$

(ii) $\angle B = \angle C \Rightarrow AC = AB = 7$ cm *[Theorem 2]*

(iii) $\Delta ABD \cong \Delta ACD$ (SSS) $\Rightarrow \angle ADB = \angle ADC$, and they add to 180°

Answer: (i) 80° (ii) 7 cm (iii) 90°

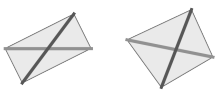
9. (i) $5^2 + 12^2 = 169 = 13^2 \Rightarrow$ the angle opposite PQ, i.e., $\angle R = 90°$ *[converse]*

(ii) $\frac{1}{2} \times 5 \times 12 = 30$ cm^2

(iii) $\frac{1}{2} \times 13 \times h = 30 \Rightarrow h = \frac{60}{13}$ cm

Answer: (i) $\angle R$ (ii) 30 cm^2 (iii) $\frac{60}{13} \approx 4.62$ cm

10. (i) Any two sticks crossing at their mid-points: diagonals bisect ⇒ parallelogram (converse true)



(ii) Equal sticks crossing at mid-points ⇒ rectangle

(iii) Sticks crossing at mid-points at right angles ⇒ rhombus

(iv) Equal sticks crossing at mid-points at right angles ⇒ square

Answer: Each property works in both directions, so it can be used to build the figure.

11. Asha is faulty: she uses the converse (congruent ⇒ equal area) to support the proposition.

Bharat: right triangles with legs 6, 4 and 8, 3 both have area 12 cm^2 but are not congruent

Chetna: congruent triangles coincide when placed on each other, so they cover equal regions

Answer: Asha is wrong; Bharat and Chetna are right.

12. See Theorem 3: construct ΔXYZ with ∠Z = 90°, YZ = a, ZX = b.

$XY^2 = a^2 + b^2 = c^2 \Rightarrow XY = c$

ΔABC ≅ ΔXYZ *[SSS]*

∴ ∠C = ∠Z = 90° *[CPCT]*

Hence proved.

13. (i) ΔABC, ΔADC: AB = AD, CB = CD, AC common ⇒ ≅ (SSS) ⇒ ∠BAC = ∠DAC

(ii) If AC bisects ∠A and ∠C: ∠BAC = ∠DAC, AC common, ∠BCA = ∠DCA ⇒ ≅ (ASA)

Answer: (i) proved (ii) yes, AB = AD (and CB = CD)

Miscellaneous Questions

1. (i) $\sqrt{2} + (-\sqrt{2}) = 0$, rational

(ii) Converse: if x + y is irrational, then x and y are irrational — false: $1 + \sqrt{2}$

(iii) $\sqrt{2} \times \sqrt{2} = 2$ is a counterexample; converse 'if xy is irrational then both are irrational' is false: $2 \times \sqrt{3}$

Answer: P false; converses false

2. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$

Converse: if $a^3 + b^3 + c^3 = 3abc$ then a + b + c = 0. a = b = c = 1: 3 = 3 but sum 3 ≠ 0

Answer: P true; converse false

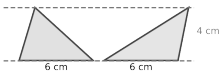
3. (i) Converse: opposite angles supplementary ⇒ cyclic — both true *[Ch 5 theorems]*

(ii) Converse: a rectangle is a cyclic parallelogram — both true

(iii) Converse: the longest chord passes through the centre — both true (it is a diameter)

Answer: All three pairs are true

4. (i) True — congruent figures coincide



(ii) Same base 6 cm and height 4 cm ⇒ both areas 12 cm^2, but the triangles are not congruent

(iii) Right triangles with legs 3, 8 and 4, 6 both have area 12

Answer: Converse false

5. (i) b − a = c − b ⇒ 2b = a + c

(ii) 2b = a + c ⇒ b − a = c − b ⇒ AP: true

(iii) $b^2 = ac$ with a, b, c ≠ 0 ⇒ $\frac{b}{a} = \frac{c}{b}$: true

Answer: Both reversible

6. (i) x < 0, y > 0 ⇒ xy < 0: true

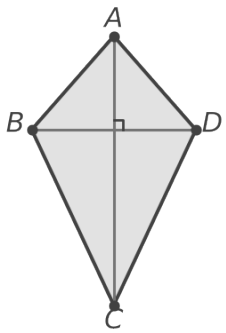
(ii) Converse: xy < 0 ⇒ Quadrant II. False: (2, −3)

Answer: (iii) e.g., If (x, y) is in Quadrant III, then x + y < 0 and xy > 0

7. (i) The kite ABCD has perpendicular diagonals but AB ≠ BC

(ii) '…and bisect each other' makes it a rhombus

(iii) Converse: a rhombus has perpendicular diagonals — true



Answer: P false, converse true

Competency Based Question Bank

1. It is a statement that is true.

Answer: (c) Every multiple of 4 is even.

2. A command is not a proposition; "3 + 5 = 9" is a (false) proposition.

Answer: (c) Bisect the given angle.

3. Swap hypothesis and conclusion.

Answer: (b) If a multiple of 5, then a multiple of 10.

4. "X implies Y" = "If X then Y".

Answer: (b) If X then Y

5. One counterexample disproves a statement.

Answer: (c) prove it false

6. "Y when X": X = diagonals bisect, Y = parallelogram.

Answer: (b) If diagonals bisect each other, then parallelogram

7. 9k = 3(3k); converse false: 6.

Answer: (b) proposition true, converse false

8. $20 = 4 \times 5$ is not a multiple of 8.

Answer: (c) 20

9. True; converse false: $2^2 = (-2)^2$.

Answer: (d) true; converse false

10. $2^{32} + 1 = 641 \times 6700417$.

Answer: (b) $2^{32} + 1$

11. Converse false: a rectangle.

Answer: (d) true, false

12. Hypothesis $a^2 + b^2 = c^2$.

Answer: (b) $a^2 + b^2 = c^2 \Rightarrow$ right-angled

13. Result 1.

Answer: (a) imply each other

14. $100 + 10 + 11 = 121 = 11^2$

Answer: (c) n = 10

15. 3 and 4 are co-prime, so the converse is true.

Answer: (a) is true

16. 20 is divisible by 2 and 4 but not 8.

Answer: (d) 20

17. Both false is possible (prime ⇒ odd).

Answer: (c) both are false

18. 2 is prime but even; 9 is odd but not prime.

Answer: (b) n prime ⇒ n odd

19. A tanker may spill water.

Answer: (d) the road may be wet for another reason

20. Converse false: $2 \times 8 = 16$.

Answer: (a) true, false

21. 3 factors ⇒ odd ⇒ square f^2; f must be prime.

Answer: (b) is true

22. It starts from the odd count.

Answer: (c) odd factors ⇒ perfect square

23. $2^{11} - 1 = 2047 = 23 \times 89$

Answer: (d) n = 11

24. Theorem 2.

Answer: (b) true: sides opposite equal angles are equal

25. One non-rectangle with equal diagonals, e.g., an isosceles trapezium.

Answer: (d) one non-rectangle with equal diagonals

26. Any common factor divides 3.

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Answer: (c) is true

27. Both areas are 12 cm^2

Answer: (c) right triangles with legs 6, 4 and 8, 3

28. $20^2 + 21^2 = 841 = 29^2$

Answer: (c) 20, 21, 29

29. $180° - 118° = 62°$

Answer: (b) 62°

30. Swapping twice returns the original.

Answer: (b) the proposition itself

31. 5 and 12 are co-prime.

Answer: (a) true, as 5 and 12 are co-prime

32. $x^2 = y^2$ does not give $x = y$.

Answer: (c) $x = y \Rightarrow x^2 = y^2$

33. A: true.
R: true and is a counterexample that explains A.

Answer: (a)

34. A: $81 + 1600 = 1681 = 41^2$, true.
R: the converse of Baudhāyana–Pythagoras; it explains A.

Answer: (a)

35. A: false, e.g., $n = 41$ gives 41×43.
R: true.

Answer: (d)

36. A: true.
R: false: $3^2 = (-3)^2$.

Answer: (c)

37. A: true (Topic 2).
R: true, but it is only one step of the reason for A.

Answer: (b)

38. A: false (legs 6, 4 and 8, 3).
R: true.

Answer: (d)

Story Time: Case Studies

Case 1 · The Proposition Board

(i) A, C and D (B is a command).

(ii) A false ($51 = 3 \times 17$); C true (four equal sides); D true ($4k = 2 \times 2k$).

(iii) If a number is even, then it is a multiple of 4 — false (6).

(iv) A number is even when it is a multiple of 4.

Case 2 · The Detective's Clue

(i) Hypothesis: the person was at the scene; conclusion: mud on the shoes.

(ii) The converse (mud ⇒ at the scene).

(iii) Raj walked through a muddy field elsewhere.

(iv) Not necessarily: a person at the scene may have cleaned his shoes.

Case 3 · Divisibility Shortcuts

(i) If n is divisible by 2 and by 4, then it is divisible by 8.

(ii) 12 (or 4, 20).

(iii) 3 and 4 are co-prime, so divisible by both ⇒ divisible by 12.

(iv) 7 + 1 + 2 + 8 = 18 (div. by 3); 28 div. by 4 ⇒ 7128 is divisible by 12.

Case 4 · Prime Machines

(i) n = 3: 15.

(ii) n = 10: 100 + 10 + 11 = 121.

(iii) $2^{11} - 1 = 2048 - 1 = 2047 = 23 \times 89$.

(iv) No — examples never prove a statement for all n.

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Case 5 · Locker Puzzle



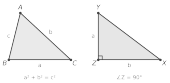
(i) 12 has 6 factors ⇒ 6 toggles ⇒ closed.

(ii) 36 has 9 factors ⇒ 9 toggles ⇒ open.

(iii) Open ⇔ odd number of toggles ⇔ odd number of factors ⇔ perfect square.

(iv) 1, 4, 9, ..., 100: 10 lockers.

Case 6 · The Carpenter's Check



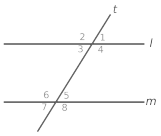
(i) $30^2 + 40^2 = 2500 = 50^2 \Rightarrow 50$ cm.

(ii) The converse (sides ⇒ right angle).

(iii) $49^2 = 2401 < 2500 \Rightarrow$ the angle is less than 90° (acute).

(iv) e.g., 60 cm, 80 cm, 100 cm or 5, 12, 13.

Case 7 · Railway Tracks



(i) l ∥ m, by the converse (equal corresponding angles ⇒ parallel).

(ii) ∠5 = 116°, ∠8 = 64°.

(iii) ∠3 and ∠6 are co-interior: 115° + 63° = 178° ≠ 180° ⇒ not parallel.

(iv) Parallel ⇒ equal corresponding angles; converse: equal corresponding angles ⇒ parallel.

Case 8 · The Kite Festival



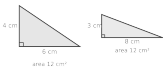
(i) A rectangle.

(ii) A rhombus.

(iii) If the diagonals are perpendicular, then it is a rhombus — false (a kite).

(iv) No: a rectangle has equal diagonals.

Case 9 · Equal Areas



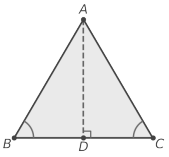
(i) $\frac{1}{2} \times 6 \times 4 = 12$ cm² and $\frac{1}{2} \times 8 \times 3 = 12$ cm².

(ii) No: their sides are different.

(iii) If two triangles have equal areas, then they are congruent.

(iv) If two triangles are congruent, then they have equal areas.

Case 10 · The Isosceles Roof



(i) 180° − 74° = 106°.

(ii) AC = AB = 5 m (sides opposite equal angles).

(iii) ΔABD ≅ ΔACD (AAS) ⇒ BD = DC.

(iv) BD = 4 m ⇒ AD = $\sqrt{5^2 - 4^2}$ = 3 m.

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