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# TOPIC-WISE ASSIGNMENT

## Chapter 9 · Propositions and their Converses

Class IX · As per New Curriculum **Ganita Manjari**

**Reviewed by Deepika Bhati, Maths Passionate Since 2009**
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TOPIC 1

# 1 Propositions, Converses and Counterexamples

Propositions | If X then Y | X implies Y / Y when X | Converse | Counterexamples | Four combinations

**QUICK RECAP** (from earlier classes)

- A **multiple** of 6 is a number 6k (k a natural number); every multiple of 6 is even.
- A **prime** number has exactly two factors, 1 and itself; 1 is neither prime nor composite.
- Every square is a rectangle and every rectangle is a parallelogram, but not the other way round.

**MUST KNOW · Fermat, Euler and a Famous Counterexample**

In 1640, Pierre de Fermat believed that every number $2^{2^n} + 1$ is prime: 3, 5, 17, 257, 65537 all are. About 90 years later, Leonhard Euler showed that the next one, $2^{32} + 1 = 4294967297$, equals $641 \times 6700417$. A single counterexample settled the question for ever.

## 01 ✦ Concept Zone: Theorems & Results

### What is a Proposition?

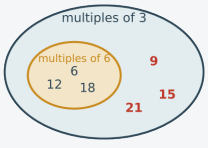
- A **proposition** is a statement that is either **true** or **false** (but not both). “17 is prime” and “every rectangle is a square” are propositions (one true, one false).
- Questions (“Is 7 prime?”), commands (“Draw a circle.”) and exclamations (“What a pattern!”) are **not** propositions.
- Many propositions have the form **If X then Y**. X is the **hypothesis** and Y is the **conclusion**.
- The same meaning can be written as **X implies Y**, or **Y when X**. “A number is even when it is a multiple of 4” = “If a number is a multiple of 4, then it is even.”

### The Converse

- The **converse** of “If X then Y” is “**If Y then X**”: hypothesis and conclusion change places.
- **P:** If it rains, then the road is wet (true). **Converse:** If the road is wet, then it has rained (not always true: a tanker may have spilt water).
- So the truth of a proposition tells us **nothing** about the truth of its converse. Each must be checked separately.
- The converse of the converse is the original proposition.

### Counterexamples

- A **counterexample** is a single example in which the hypothesis is true but the conclusion is false. **One** counterexample is enough to prove a proposition false.
- But examples, however many, can never prove an “if … then …” statement true for all cases; that needs a proof.
- **P:** If n is a multiple of 6, then n is a multiple of 3 (true: $6k = 3(2k)$). **Converse:** If n is a multiple of 3, then it is a multiple of 6 — false; counterexample n = 9.



### Four Possible Combinations

- **Both true:** If a triangle has two equal sides, then it has two equal angles (and conversely).
- **Proposition true, converse false:** If n is a multiple of 6, then it is a multiple of 3.
- **Proposition false, converse true:** If n is a multiple of 3, then it is a multiple of 6.
- **Both false:** If n is prime, then n is odd (2 is prime) / If n is odd, then n is prime (9 is odd).

### KEY NOTES

- ✔ Proposition: a statement that is true or false.
- ✔ If X then Y = X implies Y = Y when X; converse: If Y then X.
- ✔ One counterexample disproves a statement; examples alone never prove it.
- ✔ A proposition and its converse can be true/false in all four combinations.

**NOTE** The converse is **not** the same as the negation. The converse of "If X then Y" is "If Y then X", not "If not X then not Y".

## 02 ✎ Solved Examples

**EXAMPLE 1**

Which of these are propositions? For each proposition, say whether it is true or false. (i) 119 is a prime number. (ii) Bisect the given angle. (iii) The sum of two odd numbers is even. (iv) Is 0 an even number?

Solution

- (i) A proposition; false, since $119 = 7 \times 17$.
- (ii) A command, not a proposition.
- (iii) A proposition; true: $(2a + 1) + (2b + 1) = 2(a + b + 1)$.
- (iv) A question, not a proposition.

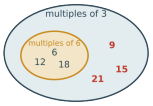
**∴ (i) and (iii) are propositions: (i) false, (iii) true.**

**EXAMPLE 2**

P: If a number is a multiple of 6, then it is a multiple of 3. Write the converse Q. Justify P and decide whether Q is true.

Solution

- Q: If a number is a multiple of 3, then it is a multiple of 6.
- P is true: a multiple of 6 is $6k = 3 \times 2k$, a multiple of 3.
- Q is false: 9 is a multiple of 3 but not of 6 (counterexample).




**∴ P true, Q false (counterexample 9).**

**EXAMPLE 3**

Rewrite in "If … then …" form and write the converse: "A triangle is isosceles when two of its angles are equal."

Solution

- X: two angles of a triangle are equal; Y: the triangle is isosceles.
- If two angles of a triangle are equal, then it is isosceles.
- Converse: If a triangle is isosceles, then two of its angles are equal.

TOPIC 1

∴ **Both statements are true (Topic 3).**

**EXAMPLE 4**

Fermat claimed that all numbers $2^{2^n} + 1$ are prime. Check n = 0, 1, 2, 3, 4. How did Euler disprove the claim?

Solution

- n = 0, 1, 2, 3, 4 give 3, 5, 17, 257, 65537 — all prime.
- Euler showed that for n = 5, $2^{32} + 1 = 4294967297 = 641 \times 6700417$ is composite.

∴ **One counterexample (n = 5) is enough to show the claim is false.**

**EXAMPLE 5**

Give a proposition such that both it and its converse are false.

Solution

- P: If n is a prime number, then n is odd. False: 2 is prime but even.
- Q: If n is odd, then n is prime. False: 9 is odd but not prime.

∴ **Both P and Q are false.**

## 03 ⚡ Practice Zone

Attempt in your notebook. Draw a neat figure wherever needed. ★ = higher-order thinking

1. State whether each sentence is a proposition. If it is, state whether it is true or false. (i) Every multiple of 4 is a multiple of 2. (ii) How beautiful this pattern is! (iii) 91 is a prime number. (iv) Draw a line segment of length 5 cm. (v) Every rectangle is a square.
2. Write each in "If … then …" form and identify the hypothesis and the conclusion. (i) X implies Y, where X: a number ends in 0, Y: it is divisible by 5. (ii) A quadrilateral is a rectangle when all its angles are right angles. (iii) A triangle is equilateral when all its angles are equal.
3. Write the converse of each proposition and decide whether the converse is true. (i) If a number is a multiple of 15, then it is a multiple of 5. (ii) If two angles are vertically opposite, then they are equal. (iii) If a triangle is equilateral, then it is isosceles. (iv) If a number is a multiple of 100, then it is a multiple of 10.
4. P: If it rains, then the road is wet. (i) Write the converse. (ii) Explain why the converse may be false. (iii) Give two real-life propositions of your own that are true but whose converses are false, with counterexamples.
5. Give a counterexample to each claim. (i) Every odd number is prime. (ii) Every multiple of 4 is a multiple of 8. (iii) Every quadrilateral with a pair of parallel sides is a parallelogram. (iv) The square of every number is greater than the number.
6. For each, say whether the proposition and its converse are true; give counterexamples where needed. (i) If a number is divisible by 6, then it is divisible by 2. (ii) If a number is a multiple of 7, then it is a multiple of 14. (iii) If a number ends in 0, then it is divisible by 5.
7. A student says: "8 is a multiple of 4 and of 8; so every multiple of 4 is a multiple of 8." Which number among 8, 16, 20, 24 shows the conclusion is wrong? Explain.
8. Give one example from geometry and one from numbers for each of the four combinations: (i) both true, (ii) only the proposition true, (iii) only the converse true, (iv) both false.

9 Write the converse of the converse of “If two lines are parallel, then they do not meet.” What do you notice?

10 Explain the difference between the converse and the negation of “If a quadrilateral is a square, then its diagonals are equal.” Write both and decide whether each is true.

11 A student checks the statement “$n^2 + n + 41$ is prime” for n = 1, 2, …, 10 and finds a prime each time. Has she proved the statement? Test n = 40 and n = 41. ★

12 Why can a single counterexample prove a statement false, while a thousand examples cannot prove it true? Illustrate with Fermat’s numbers $2^{2^n} + 1$. ★

MY NOTES ✎

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TOPIC 2

# 2 Propositions in Number Theory and Algebra

Factor pairs | Statements that imply each other | Divisibility statements | Prime-generating claims | Equal numbers, squares and cubes

**QUICK RECAP** (from earlier classes)

- Factors of a number come in **pairs** whose product is the number: 12 = 1 × 12 = 2 × 6 = 3 × 4.
- Divisibility tests: by 3 (or 9) — digit sum divisible by 3 (or 9); by 4 — last two digits; by 8 — last three digits.
- Two numbers are **co-prime** if their only common factor is 1, e.g., 4 and 9.

## 01 ✦ Concept Zone: Theorems & Results

**RESULT 1**

**For a positive integer n: n is a perfect square ⇔ n has an odd number of factors. (Each statement implies the other.)**

Given: A positive integer n; its factors paired as (f, $\frac{n}{f}$).




To prove: (i) odd number of factors ⇒ perfect square; (ii) perfect square ⇒ odd number of factors.

Proof:

**(i) Odd number of factors ⇒ perfect square.** Pair every factor f with its partner $\frac{n}{f}$.

If f ≠ $\frac{n}{f}$ for every factor, the factors split into pairs

then the number of factors would be even *[ contradiction ]*

**∴ some factor is its own partner: f = $\frac{n}{f}$ ⇒ n = f ×**

**f, a perfect square**

**(ii) Perfect square ⇒ odd number of factors.** Let n = m × m.

m is its own partner, so the pair (m, m) contributes only **one** factor

No other factor f can satisfy f × f = n, since f < m ⇒ $f^2$ < n and f > m ⇒ $f^2$ > n *[ only one such pair ]*

**∴ number of factors = 2 × (number of other pairs) + 1, which is odd**

Hence proved.

36 = 6 × 6 has 9 factors (6 is its own partner); 12 has 6 factors (no factor is its own partner).

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### Divisibility Statements and their Converses

- **If n is divisible by ab, then n is divisible by both a and b** — always true (ab = a × b).
- Converse: "If n is divisible by both a and b, then it is divisible by ab" is true when a and b are **co-prime**: by 5 and 12 ⇒ by 60 ✓.
- It is **false** when a and b share a factor: 12 is divisible by 4 and 6 but not by 24; 4 is divisible by 2 and 4 but not by 8.
- Tests like "digit sum divisible by 3" work in **both directions**: a number is divisible by 3 ⇔ its digit sum is divisible by 3.

### Propositions in Algebra and Prime Claims

- $x = y \Rightarrow x^2 = y^2$ (true); converse false: $2^2 = (-2)^2$ but $2 \neq -2$.
- $x = y \Leftrightarrow x^3 = y^3$ (both true); $x = y \Leftrightarrow x + a = y + a$ (both true — this is why we may add the same number to both sides of an equation).
- No simple expression gives only primes: $4n^2 + 1$ (n = 4: 65), $n^2 + n + 11$ (n = 10: 121), $4n + 3$ (n = 3: 15), $2^n - 1$ with n prime (n = 11: 2047 = 23 × 89).

### KEY NOTES

- ✔ n is a perfect square ⇔ n has an odd number of factors.
- ✔ Divisible by ab ⇒ divisible by a and b; converse true only if a, b are co-prime.
- ✔ $x = y \Rightarrow x^2 = y^2$ but not conversely; cubes work both ways.
- ✔ p prime ⇒ $p^2$ has exactly 3 factors (1, p, $p^2$) and conversely.

**NOTE** An argument that starts from Y and reaches X proves "If Y then X" — not "If X then Y". Always check which direction your argument actually goes.

## 02 ✎ Solved Examples

**EXAMPLE 1**

P: If n is divisible by 24, then n is divisible by both 4 and 6. Write the converse and decide whether each is true.

Solution

› P is true: n = 24k = 4(6k) = 6(4k).
› Converse: If n is divisible by both 4 and 6, then it is divisible by 24.
› False: 12 is divisible by 4 and 6 but not by 24 (4 and 6 are not co-prime).

**∴ P true; converse false (12).**

**EXAMPLE 2**

P: If n is divisible by 60, then n is divisible by both 5 and 12. Is the converse true?

Solution

› Converse: If n is divisible by both 5 and 12, then it is divisible by 60.
› n = 12k and 5 divides 12k. As 5 and 12 have no common factor, 5 divides k: k = 5m.
› So n = 60m.

**∴ Both P and its converse are true.**

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**EXAMPLE 3**

Show that $p^2$ has exactly 3 factors when p is prime. Is the converse "If n has exactly 3 factors, then n is the square of a prime" true?

Solution

- Factors of $p^2$: 1, p, $p^2$ only (p has no other factors).
- If n has exactly 3 factors, the number of factors is odd, so $n = f^2$ (Result 1).
- If f had a factor d other than 1 and f, then 1, d, f, $f^2$ would be 4 factors. So f is prime.

∴ **Both statements are true.**

**EXAMPLE 4**

For real numbers x and y, examine (i) If $x = y$ then $x^2 = y^2$, (ii) If $x = y$ then $x^3 = y^3$, with their converses.

Solution

- (i) True; converse false: $x = 3$, $y = -3$ give $x^2 = y^2 = 9$.
- (ii) True; converse true: a cube has exactly one real cube root, and $(-3)^3 = -27 \neq 27$.

∴ **(i) only the proposition is true; (ii) both are true.**

**EXAMPLE 5**

Find a counterexample to: "All numbers of the form $n^2 + n + 11$ are prime."

Solution

- n = 1, 2, 3, ... give 13, 17, 23, 31, 41, 53, 67, 83, 101 — primes.
- n = 10: 100 + 10 + 11 = 121 = 11 × 11.

∴ **n = 10 is a counterexample (also n = 11: 143 = 11 × 13).**

## 03 ⚡ Practice Zone

Attempt in your notebook. Draw a neat figure wherever needed. ★ = higher-order thinking

1. List the factors and pair them for (i) 49, (ii) 48, (iii) 100. Which have an odd number of factors? Verify Result 1.




2. A student argues: "Suppose n has an odd number of factors. Pair each factor with its partner ... so some pair is (f, f) and $n = f \times f$." (i) Which statement does this prove? (ii) Why does it not prove the other one? (iii) Supply the missing argument.

3. Write the converse and decide whether it is true: (i) If n is divisible by 18, then by both 2 and 9. (ii) If n is divisible by 36, then by both 6 and 12. (iii) If n is divisible by 45, then by both 9 and 5. (iv) If n is divisible by 8, then by both 2 and 4. State the general rule.

4. Write the relationship between "a number is divisible by 9" and "the sum of its digits is a multiple of 9" as two "If ... then ..." sentences. Verify both for (i) 4581, (ii) 7254. Why can it be used as a test?

5. For positive integers a and b: P: If a and b are perfect squares, then ab is a perfect square. (i) Prove P. (ii) Write the converse and give a counterexample. (iii) Give an extra condition under which the converse becomes true.

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6. Find a counterexample to each: (i) all numbers $4n^2 + 1$ are prime, (ii) all numbers $4n + 3$ are prime, (iii) if n is prime then $2^n - 1$ is prime, (iv) if n is even then $2n + 1$ is prime, (v) if $2n + 1$ is prime then n is prime.

7. Let n be a positive integer. (i) Prove: If n is even, then $n^2$ is even. (ii) Is the converse true? Justify. (iii) Give a counterexample to "If $n^2$ is a multiple of 4, then n is a multiple of 4".

8. P: If n is a product of two unequal primes, then it has exactly 4 divisors. (i) Verify P for 15 and 35. (ii) Is the converse true? Test $n = 8$ and $n = 27$.

9. P: If n and $n + 3$ have no common factor other than 1, then n is not a multiple of 3. (i) Test P for $n = 4, 5, 6$. (ii) Prove P. (iii) Write and examine the converse.

10. For real numbers x, y and a, examine each proposition and its converse: (i) If $x = y$ then $5x - 2 = 5y - 2$. (ii) If $x = y$ then $x + a = y + a$. (iii) If $x = y$ then $x^2 = y^2$. Explain how (ii) is used to solve $3x - 7 = 11$.

11. P: If n is a perfect square, then its units digit is 0, 1, 4, 5, 6 or 9. Write the converse and give a counterexample to it.

12. P: If a number is divisible by 8, then it is divisible by both 2 and 4. (i) Prove P. (ii) Show that the converse is false. (iii) Explain why "divisible by 3 and 4" is a valid test for 12 but "divisible by 2 and 4" is not a test for 8. ★

13. Show that if n is a perfect square, it has an odd number of factors, by explaining why a number cannot have two different factor pairs of the form (f, f). ★

MY NOTES ✎

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TOPIC 3

# 3 Propositions in Geometry

Isosceles triangle theorem | Its converse | Converse of Baudhāyana–Pythagoras | Parallel lines | Area and congruence | Quadrilaterals

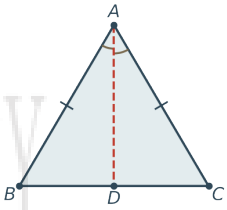
**QUICK RECAP** (from earlier classes)

- Congruence criteria: SSS, SAS, ASA, AAS and RHS.
- Baudhāyana–Pythagoras: in a right triangle with legs a, b and hypotenuse c, $a^2 + b^2 = c^2$.
- The angles of a triangle add up to 180°; a linear pair adds up to 180°.

## 01 ✦ Concept Zone: Theorems & Results

**THEOREM 1**

**If two sides of a triangle are equal, then the angles opposite them are equal.**




Given: ΔABC with AB = AC.

To prove: ∠B = ∠C.

Construction: Draw the bisector AD of ∠A meeting BC at D.

Proof:

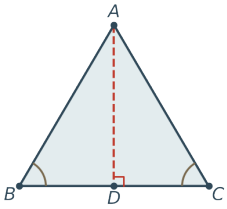
In ΔABD and ΔACD:

| | |
|---|---|
| AB = AC | *[ given ]* |
| ∠BAD = ∠CAD | *[ AD bisects ∠A ]* |
| AD = AD | *[ common ]* |
| ΔABD ≅ ΔACD | *[ SAS ]* |
| **∴ ∠B = ∠C** | *[ CPCT ]* |

Hence proved.

**THEOREM 2**

**(Converse) If two angles of a triangle are equal, then the sides opposite them are equal.**




Given: ΔABC with ∠B = ∠C.

To prove: AB = AC.

Construction: Draw the altitude AD ⊥ BC.

Proof:

In ΔABD and ΔACD:

| | |
|---|---|
| ∠B = ∠C | *[ given ]* |

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$\angle ADB = \angle ADC = 90°$ *[ AD ⊥ BC ]*

$AD = AD$ *[ common ]*

$\Delta ABD \cong \Delta ACD$ *[ AAS ]*

**$\therefore$ AB = AC** *[ CPCT ]*

Hence proved.

Theorems 1 and 2 together: in a triangle, two sides are equal ⇔ the angles opposite them are equal.

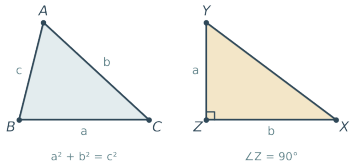
**THEOREM 3**

**(Converse of Baudhāyana-Pythagoras) If the sides of $\Delta ABC$ satisfy $a^2 + b^2 = c^2$, then the triangle is right-angled (the angle opposite c is 90°).**

Given: $\Delta ABC$ with $BC = a$, $CA = b$, $AB = c$ and $a^2 + b^2 = c^2$.

To prove: $\angle C = 90°$.

Construction: Draw $\Delta XYZ$ with $\angle Z = 90°$, $YZ = a$ and $ZX = b$.




Proof:

$XY^2 = YZ^2 + ZX^2 = a^2 + b^2$ *[ Baudhāyana–Pythagoras in $\Delta XYZ$ ]*

$a^2 + b^2 = c^2 \Rightarrow XY^2 = c^2 \Rightarrow XY = c$ *[ given ]*

In $\Delta ABC$ and $\Delta XYZ$: $BC = YZ = a$, $CA = ZX = b$, $AB = XY = c$.

$\Delta ABC \cong \Delta XYZ$ (A ↔ X, B ↔ Y, C ↔ Z) *[ SSS ]*

**$\therefore \angle C = \angle Z = 90°$** *[ CPCT ]*

Hence proved.

## More Propositions from Geometry

- **Parallel lines:** If two lines are parallel, then corresponding angles made by a transversal are equal; the converse is also true. We use the proposition to **find angles** and the converse to **prove lines parallel**.
- **Congruence and area:** If two triangles are congruent, then they have equal areas (true). Converse false: right triangles with legs 6, 4 and 8, 3 both have area 12 $cm^2$.
- **Quadrilaterals:** If a quadrilateral is a square, then all its angles are equal (true); converse false — a rectangle. If it is a rectangle, its diagonals are equal; converse false — an isosceles trapezium.
- A statement that is true **in both directions** (e.g., diagonals bisect each other ⇔ parallelogram) can be used to **identify** a figure: it is a **characterisation**.




TOPIC 3

**KEY NOTES**

- ✔ Equal sides ⇔ equal opposite angles (Theorems 1 and 2).
- ✔ $a^2 + b^2 = c^2$ ⇔ the angle opposite c is a right angle.
- ✔ Parallel lines ⇔ equal corresponding angles (or equal alternate angles).
- ✔ Congruent ⇒ equal area, but equal area ⇏ congruent.

**NOTE** To prove a converse, start from **its** hypothesis. Using the original theorem in the proof of its own converse is a common mistake (circular reasoning).

## 02 ✎ Solved Examples

**EXAMPLE 1**

Use the converse of the Baudhāyana–Pythagoras theorem to check which triangles are right-angled: (i) 9 cm, 40 cm, 41 cm, (ii) 8 cm, 15 cm, 16 cm.

Solution

- (i) $9^2 + 40^2 = 81 + 1600 = 1681 = 41^2$ ⇒ right-angled (opposite 41 cm).
- (ii) $8^2 + 15^2 = 64 + 225 = 289 \neq 256 = 16^2$ ⇒ not right-angled.

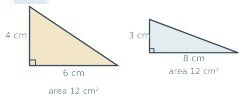
**∴ (i) right-angled (ii) not right-angled**

**EXAMPLE 2**

P: If two triangles have the same area, then they are congruent. Q: If two triangles are congruent, then they have the same area. Decide which is true.

Solution

- Q is true: congruent triangles can be placed exactly on each other, so they cover the same region.
- P is false: right triangles with legs 6 cm, 4 cm and 8 cm, 3 cm both have area $\frac{1}{2} \times 6 \times 4 = \frac{1}{2} \times 8 \times 3 = 12$ cm², but their sides differ.



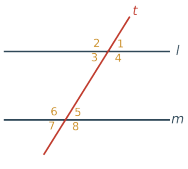

**∴ Q true; P false (counterexample shown).**

**EXAMPLE 3**

Lines l and m are cut by transversal t. (i) If l ∥ m and ∠1 = 118°, find ∠5, ∠3 and ∠8. (ii) If ∠2 = 62° and ∠6 = 62° are measured, what can you conclude?

Solution

- (i) ∠5 = ∠1 = 118° (corresponding angles, l ∥ m); ∠3 = ∠1 = 118° (vertically opposite); ∠8 = 180° − 118° = 62° (linear pair with ∠5).
- (ii) ∠2 and ∠6 are corresponding angles and are equal, so l ∥ m (converse).



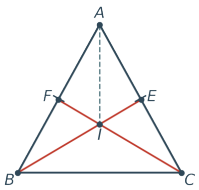

**∴ (i) 118°, 118°, 62° (ii) l ∥ m**

TOPIC 3

**EXAMPLE 4**

In ΔABC, AB = AC. The bisectors of ∠B and ∠C meet AC at E and AB at F. Prove that BE = CF.

Solution



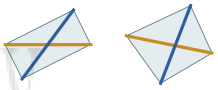

- ∠B = ∠C (Theorem 1), so $\angle EBC = \frac{1}{2}\angle B = \frac{1}{2}\angle C = \angle FCB$.
- In ΔEBC and ΔFCB: ∠EBC = ∠FCB, BC = CB (common), ∠ECB = ∠FBC (= ∠C = ∠B).
- ΔEBC ≅ ΔFCB (ASA).

**∴ BE = CF (CPCT).**

**EXAMPLE 5**

Two thin sticks are placed as the diagonals of a quadrilateral of type Q. (i) If "type Q ⇒ equal diagonals", must the sticks be equal? (ii) If "equal diagonals ⇒ type Q", how should the sticks be chosen?

Solution



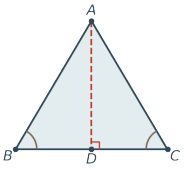
- (i) Yes: every type-Q quadrilateral has equal diagonals, so the sticks must be equal; this alone may not be enough to get type Q, so the way they are put together may matter.
- (ii) Now any two equal sticks, placed in any way, give a type-Q quadrilateral; equal length is enough.

**∴ (i) equal sticks needed, placing may matter (ii) equal sticks, placed in any way**

## 03 ⚡ Practice Zone

Attempt in your notebook. Draw a neat figure wherever needed. ★ = higher-order thinking

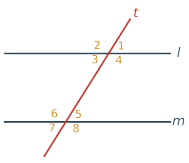
**1** Prove Theorem 2 using the altitude AD. Why can we not simply say "by Theorem 1"?




**2** Write the converse of each and decide whether it is true. (i) If two lines are parallel, then alternate interior angles are equal. (ii) If a quadrilateral is a rhombus, then its diagonals are perpendicular. (iii) If a quadrilateral is a parallelogram, then its opposite angles are equal. (iv) If two triangles are congruent, then their corresponding angles are equal.

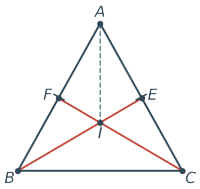
**3** Using the converse of the Baudhāyana–Pythagoras theorem, check whether triangles with these sides are right-angled: (i) 7, 24, 25, (ii) 20, 21, 29, (iii) 6, 7, 9, (iv) 1.5, 2, 2.5.

**4** Lines l and m are cut by transversal t with angles numbered 1 to 8. (i) If l ∥ m and ∠4 = 75°, find ∠8, ∠6 and ∠7. (ii) If ∠3 = 110° and ∠5 = 70°, are l and m parallel? Which statement do you use?




TOPIC 3

**5** In ΔABC, AB = AC and the bisectors of ∠B and ∠C meet at I and the opposite sides at E and F. (i) Prove that IB = IC. (ii) Prove that IE = IF. (iii) Write the converse of "If AB = AC, then IE = IF".




**6** For each, write the converse and decide which are true, with counterexamples: (i) If a quadrilateral is a square, then its diagonals are equal. (ii) If a quadrilateral has four equal sides, then it is a square. (iii) If two triangles have equal perimeters, then they are congruent.




**7** Give a counterexample (with a figure) to: (i) "Every quadrilateral with equal diagonals is a rectangle." (ii) "If two triangles have two pairs of equal sides, they are congruent." (iii) "If a triangle is right-angled, it has an obtuse angle" and its converse.

**8** In ΔABC, ∠B = ∠C = 50°. (i) Find ∠A. (ii) If AB = 7 cm, find AC. (iii) D is the mid-point of BC; find ∠ADB.

**9** ΔPQR has PQ = 13 cm, QR = 5 cm, PR = 12 cm. (i) Which angle is a right angle? (ii) Find its area. (iii) Find the altitude from R to PQ.

**10** Using two sticks as diagonals, how would you place them to get (i) a parallelogram, (ii) a rectangle, (iii) a rhombus, (iv) a square? In each case, state the property of the diagonals as an "If … then …" statement and say whether its converse is true.



**11** Three students discuss "If two triangles have the same area, then they are congruent." Asha: "True, because congruent triangles have equal areas." Bharat: "False." Chetna: "Its converse is certainly true." Whose reasoning is faulty? Support Bharat and Chetna. ★

**12** Prove the converse of the Baudhāyana–Pythagoras theorem (Theorem 3) yourself, using a labelled figure and naming the congruence criterion. ★

**13** In a quadrilateral ABCD, AB = AD and CB = CD. (i) Prove that AC bisects ∠A. (ii) State the converse-type question: if AC bisects ∠A and ∠C, must AB = AD? Justify. ★

**MY NOTES** ✎

# + Miscellaneous Questions

7 mixed questions from all topics of this chapter and earlier Ganita Manjari chapters

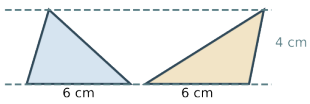
Attempt in your notebook. Draw a neat figure wherever needed. ★ = higher-order thinking

**1** P: If x and y are irrational, then x + y is irrational. (i) Give a counterexample to P. (ii) Write the converse of P and decide whether it is true. (iii) Repeat for 'If x and y are irrational, then xy is irrational.'

**2** P: If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$. (i) Is P true? Justify using an identity. (ii) Write its converse and test it with $a = b = c = 1$.

**3** Write the converse of each, and state whether the proposition and its converse are true: (i) If a quadrilateral is cyclic, its opposite angles add up to 180°. (ii) If a parallelogram is cyclic, it is a rectangle. (iii) If a chord passes through the centre, it is the longest chord.

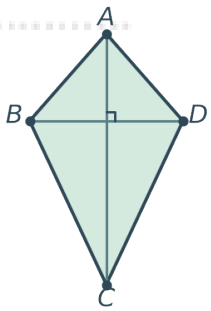
**4** P: If two triangles are congruent, then their areas are equal. (i) Is P true? (ii) Use the figure to show that its converse is false. (iii) Give a counterexample with right triangles.




**5** For an AP: (i) Prove: if a, b, c are in AP, then $2b = a + c$. (ii) Is the converse true? (iii) Is 'If a, b, c are in GP, then $b^2 = ac$' reversible for non-zero numbers?

**6** P: If the point (x, y) lies in Quadrant II, then $xy < 0$. (i) Is P true? (ii) Write its converse and give a counterexample. (iii) Write a true proposition about points in Quadrant III using x + y and xy.

**7** P: If the diagonals of a quadrilateral are perpendicular, then it is a rhombus. (i) Use the figure to show P is false. (ii) Add a condition to make it true. (iii) Write its converse. Is the converse true? ★

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MISCELLANEOUS

**MY NOTES** ✎

# ★ Competency Based Question Bank

32 MCQs and 6 Assertion–Reason questions from the whole chapter and linked topics

**1** Which sentence is a proposition?

(a) Draw a 5 cm segment.
(b) Is 7 prime?
(c) Every multiple of 4 is even.
(d) What a pattern!

**2** Which sentence is not a proposition?

(a) 17 is prime.
(b) Every rectangle is a square.
(c) Bisect the given angle.
(d) $3 + 5 = 9$

**3** The converse of "If a number is a multiple of 10, then it is a multiple of 5" is

(a) If not a multiple of 10, then not a multiple of 5.
(b) If a multiple of 5, then a multiple of 10.
(c) If not a multiple of 5, then not a multiple of 10.
(d) Every multiple of 10 is a multiple of 5.

**4** "X implies Y" has the same meaning as

(a) If Y then X
(b) If X then Y
(c) X and Y are both true
(d) X and Y imply each other

**5** A single counterexample to a proposition is enough to

(a) prove it true
(b) prove its converse true
(c) prove it false
(d) prove both false

**6** "A quadrilateral is a parallelogram when its diagonals bisect each other" means

(a) If parallelogram, then diagonals bisect each other
(b) If diagonals bisect each other, then parallelogram
(c) If not parallelogram, diagonals do not bisect
(d) Diagonals of a parallelogram are equal

**7** For "If n is a multiple of 9, then n is a multiple of 3":

(a) both true
(b) proposition true, converse false
(c) proposition false, converse true
(d) both false

**8** The number that disproves "every multiple of 4 is a multiple of 8" is

(a) 8 (b) 16 (c) 20 (d) 24

**9** For real x, y, "If $x = y$ then $x^2 = y^2$" is

(a) false; converse true
(b) true; converse true
(c) false; converse false
(d) true; converse false

**10** Euler disproved Fermat's claim by showing that the number composite for n = 5 is

(a) $2^5 + 1$ (b) $2^{32} + 1$ (c) $2^{16} + 1$ (d) 257

**11** "If a quadrilateral is a square, then all its angles are equal" and its converse are

(a) false, true (b) false, false (c) true, true (d) true, false

QUESTION BANK

**12** The converse of the Baudhāyana–Pythagoras theorem says

(a) right-angled ⇒ $a^2 + b^2 = c^2$ (b) $a^2 + b^2 = c^2$ ⇒ right-angled
(c) c is the longest side (d) $a^2 + b^2 \neq c^2$ ⇒ not right-angled

**13** "n is a perfect square" and "n has an odd number of factors"

(a) imply each other (b) only the first implies the second
(c) only the second implies the first (d) neither implies the other

**14** $n^2 + n + 11$ is not prime for

(a) n = 1 (b) n = 5 (c) n = 10 (d) n = 2

**15** The converse of "If divisible by 12, then divisible by both 3 and 4"

(a) is true (b) is false, 6 (c) is false, 18 (d) is false, 24

**16** Which number shows that "divisible by 2 and 4 ⇒ divisible by 8" is false?

(a) 16 (b) 24 (c) 32 (d) 20

**17** For a proposition and its converse, it is possible that

(a) exactly one is true, never both (b) both true, never both false
(c) both are false (d) the converse is true only if the proposition is

**18** In which are both the proposition and converse false?

(a) multiple of 6 ⇒ multiple of 3 (b) n prime ⇒ n odd
(c) perfect square ⇒ odd factors (d) divisible by 15 ⇒ by 3 and 5

**19** The converse of "If it rains, the road is wet" is false because

(a) rain always wets the road (b) the road can be dry in rain
(c) they mean the same (d) the road may be wet for another reason

**20** "If a and b are perfect squares, then ab is a perfect square" and its converse are

(a) true, false (b) false, true (c) true, true (d) false, false

**21** The converse of "If n is the square of a prime, then n has exactly 3 factors"

(a) is false, n = 16 (b) is true
(c) is false, n = 12 (d) is false, n = 4

**22** An argument from "n has an odd number of factors" to "n is a perfect square" proves

(a) perfect square ⇒ odd factors (b) both statements
(c) odd factors ⇒ perfect square (d) neither

**23** A counterexample to "if n is prime, then $2^n - 1$ is prime" is

(a) n = 3 (b) n = 5 (c) n = 7 (d) n = 11

**24** The converse of "angles opposite equal sides are equal" is

(a) false: equilateral triangle (b) true: sides opposite equal angles are equal
(c) true only for equilateral triangles (d) true only for right triangles

**25** To disprove "every quadrilateral with equal diagonals is a rectangle", it is enough to show

(a) one rectangle with equal diagonals (b) a rectangle with unequal diagonals
(c) all quadrilaterals with unequal diagonals (d) one non-rectangle with equal diagonals

QUESTION BANK

**26** "If n and n + 3 have no common factor other than 1, then n is not a multiple of 3"
(a) false, n = 6 (b) false, n = 4 (c) is true (d) false, n = 9

**27** A counterexample to "if two triangles have equal area they are congruent" is
(a) two 3-4-5 triangles (b) two equilateral triangles of side 5
(c) right triangles with legs 6, 4 and 8, 3 (d) two 5-12-13 triangles

**28** Which triangle is right-angled?
(a) 8, 15, 16 (b) 6, 7, 9 (c) 20, 21, 29 (d) 4, 5, 6

**29** If $l \parallel m$ and a corresponding angle is 118°, the co-interior angle on the same side is
(a) 118° (b) 62° (c) 72° (d) 180°

**30** The converse of the converse of a proposition is
(a) its negation (b) the proposition itself
(c) always true (d) always false

**31** The converse of "If n is divisible by 60, then by both 5 and 12" is
(a) true, as 5 and 12 are co-prime (b) false, n = 30
(c) false, n = 120 (d) false, n = 12

**32** For real x, y, which converse is false?
(a) $x = y \Rightarrow x^3 = y^3$ (b) $x = y \Rightarrow x + 2 = y + 2$
(c) $x = y \Rightarrow x^2 = y^2$ (d) $x = y \Rightarrow 5x - 2 = 5y - 2$

## ⚖ Assertion – Reason

**Directions:** (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.

**33** **Assertion (A):** The converse of "If n is a multiple of 6, then n is a multiple of 3" is false.
**Reason (R):** 9 is a multiple of 3 but not a multiple of 6.

**34** **Assertion (A):** A triangle with sides 9 cm, 40 cm and 41 cm is right-angled.
**Reason (R):** If $a^2 + b^2 = c^2$ for the sides of a triangle, then the angle opposite c is a right angle.

**35** **Assertion (A):** Every number of the form $n^2 + n + 41$ is prime.
**Reason (R):** $41^2 + 41 + 41 = 41 \times 43$.

**36** **Assertion (A):** If two numbers are equal, their squares are equal.
**Reason (R):** If the squares of two numbers are equal, the numbers are equal.

**37** **Assertion (A):** A number has exactly three factors if and only if it is the square of a prime.
**Reason (R):** A number with an odd number of factors is a perfect square.

**38** **Assertion (A):** Two triangles with equal areas are congruent.
**Reason (R):** Congruent triangles have equal areas.

QUESTION BANK

# Story Time: Case Studies & Twisters

Real-life stories, linked chapters and tricky twists. Read carefully, think, then answer.

## Case 1 · The Proposition Board

BASICS

Class IX pins statements on a board: (A) 51 is a prime number. (B) Close the door. (C) Every square is a rhombus. (D) If a number is a multiple of 4, then it is even.

(i) Which of A–D are propositions?

(ii) Which propositions are true, and which false? Justify.

(iii) Write the converse of D and decide whether it is true.

(iv) Rewrite D in the form “Y when X”.

## Case 2 · The Detective’s Clue

REAL LIFE

A detective says: “If a person was at the scene, then there is mud on his shoes. Raj has mud on his shoes, so Raj was at the scene.”

(i) Write the hypothesis and conclusion of the detective’s rule.

(ii) Which statement did the detective actually use about Raj — the rule or its converse?

(iii) Give a counterexample showing the detective’s conclusion may be wrong.

(iv) Is the rule itself necessarily true? Discuss.

## Case 3 · Divisibility Shortcuts

NUMBER THEORY

Meena claims: “To check if a number is divisible by 8, check whether it is divisible by 2 and by 4.” Arjun claims: “To check divisibility by 12, check 3 and 4.”

(i) Write Meena’s test as an “If … then …” statement.

(ii) Give a counterexample to Meena’s test.

(iii) Why does Arjun’s test work?

(iv) Is 7128 divisible by 12? Use Arjun’s test.

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## Case 4 · Prime Machines

TWISTED

Three friends invent "prime machines": Ria uses $4n + 3$, Sam uses $n^2 + n + 11$ and Tara uses $2^n - 1$ with n prime (n is a positive integer).

(i) Find the smallest n for which Ria's machine fails.

(ii) Find n for which Sam's machine gives 121.

(iii) Show that Tara's machine gives $2047 = 23 \times 89$ for $n = 11$.

(iv) Can checking many values prove a machine always works? Explain.

## Case 5 · Locker Puzzle

FACTORS

100 lockers numbered 1–100 are closed. Student k toggles (opens/closes) every locker whose number is a multiple of k, for k = 1, 2, …, 100.




(i) How many times is locker 12 toggled? Is it open or closed at the end?

(ii) How many times is locker 36 toggled? Open or closed?

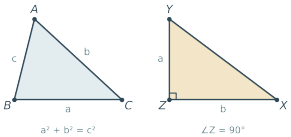
(iii) Which lockers are open at the end? Use Result 1.

(iv) How many lockers are open?

## Case 6 · The Carpenter's Check

PYTHAGORAS CONVERSE

A carpenter wants a right-angled corner. She marks 30 cm on one edge and 40 cm on the other and measures the diagonal.




(i) What diagonal length confirms a right angle?

(ii) Which theorem — Baudhāyana–Pythagoras or its converse — does she use?

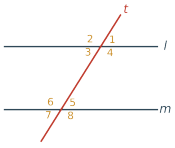
(iii) If the diagonal is 49 cm, is the corner acute or obtuse? Compare $30^2 + 40^2$ with $49^2$.

(iv) Suggest another set of whole-number marks she could use.

## Case 7 · Railway Tracks

PARALLEL LINES

Two rails l and m are crossed by a road t. An engineer measures corresponding angles ∠2 and ∠6.




(i) If $\angle 2 = \angle 6 = 64°$, what can she conclude? Which statement does she use?

STORY TIME

(ii) If the rails are parallel and $\angle 1 = 116°$, find $\angle 5$ and $\angle 8$.

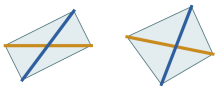
(iii) If $\angle 3 = 115°$ and $\angle 6 = 63°$, are the rails parallel?

(iv) State the proposition and its converse used here.

## Case 8 · The Kite Festival

QUADRILATERALS

Kites are made by tying two sticks as diagonals and wrapping paper around their ends.



(i) Two equal sticks cross at their mid-points. What shape is formed?

(ii) Two unequal sticks cross at right angles at their mid-points. What shape?

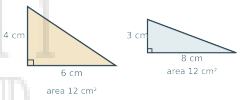
(iii) Write "If a quadrilateral is a rhombus, then its diagonals are perpendicular" and its converse. Is the converse true?

(iv) Can unequal sticks ever give a rectangle? Why?

## Case 9 · Equal Areas

TWISTED

Tina cuts two right-triangular cards: one with legs 6 cm and 4 cm, another with legs 8 cm and 3 cm.




(i) Find the area of each card.

(ii) Are the cards congruent? Why?

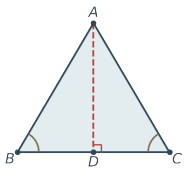
(iii) Which proposition does this disprove?

(iv) Write a true proposition linking congruence and area.

## Case 10 · The Isosceles Roof

GEOMETRY

A roof truss ABC has equal base angles $\angle B = \angle C = 37°$ (approximately) and the rafter AB = 5 m.




(i) Find $\angle A$.

(ii) Find AC and justify using a theorem.

(iii) The vertical post AD $\perp$ BC. Show that D is the mid-point of BC.

(iv) If BC = 8 m, find the height AD.

STORY TIME

MY NOTES ✎

STORY TIME

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