INFINITY THINK BEYOND
Mathematics Learning Initiative

MIND CURVE · STUDENT–TEACHER COMMUNITY

CLASS XII · CBSE MATHEMATICS

FORMULA GUIDE

2026–27

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CBSE Curriculum 2026–27 | Mathematics (041)

0 Foundation Recap (Class XI)

CHAPTER 0

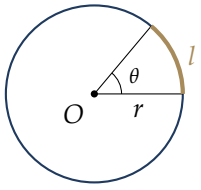
❖ What this chapter covers

- Only those Class XI formulas that are used again in Class XII, with the XII chapter where each one is needed.

Class XI topic	Used in Class XII
Trigonometric functions	Inverse trig, Differentiation, Integration, Differential equations
Sets	Relations & Functions, Probability
Permutations & Combinations	Counting functions, Probability
Straight lines	Linear Programming, Area under curves, Rate of change
Conic sections	Application of Integrals (area)
Linear inequalities & sign method	LPP, Increasing/Decreasing, Domains

❖ A. Trigonometric Functions — Angle Measure

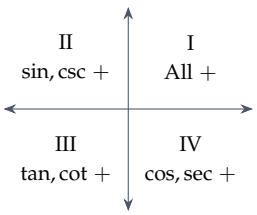
- π radian $= 180^\circ$
  - ↪ 1 radian $= \dfrac{180^\circ}{\pi} \approx 57^\circ 16'$; $1^\circ = \dfrac{\pi}{180}$ radian ≈ 0.01746 radian.
- Degree → radian: multiply by $\dfrac{\pi}{180}$; radian → degree: multiply by $\dfrac{180}{\pi}$.
- Arc length: $l = r\theta$ (θ in radians).
- $1^\circ = 60'$ (minutes), $1' = 60''$ (seconds).



$l = r\theta$

❖ Signs, Domains, Ranges and Periods

Function	Domain	Range	Period
$\sin x$	$\mathbb{R}$	$[-1, 1]$	2π
$\cos x$	$\mathbb{R}$	$[-1, 1]$	2π
$\tan x$	$\mathbb{R} - \{(2n+1)\frac{\pi}{2}\}$	$\mathbb{R}$	π
$\cot x$	$\mathbb{R} - \{n\pi\}$	$\mathbb{R}$	π
$\sec x$	$\mathbb{R} - \{(2n+1)\frac{\pi}{2}\}$	$\mathbb{R} - (-1, 1)$	2π
$\csc x$	$\mathbb{R} - \{n\pi\}$	$\mathbb{R} - (-1, 1)$	2π



memory aid: *All Silver Tea Cups*

↪ $\sin^2 x + \cos^2 x = 1$, $1 + \tan^2 x = \sec^2 x$, $1 + \cot^2 x = \csc^2 x$ hold for all x in the domain.

❖ Values at Standard Angles

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	N.D.	0	N.D.	0

N.D. = not defined

❖ Allied Angles

Step 1: Look at the angle: for $\frac{\pi}{2} \pm \theta$ or $\frac{3\pi}{2} \pm \theta$ the ratio **changes** (sin ↔ cos, tan ↔ cot, sec ↔ csc); for $\pi \pm \theta$ or $2\pi \pm \theta$ it stays the **same**.

Step 2: Find the quadrant of the angle (taking θ acute) and put the sign of the **original** ratio there.

	$-\theta$	$\frac{\pi}{2}-\theta$	$\frac{\pi}{2}+\theta$	$\pi-\theta$	$\pi+\theta$	$2\pi-\theta$	$2\pi+\theta$
sin	$-\sin\theta$	$\cos\theta$	$\cos\theta$	$\sin\theta$	$-\sin\theta$	$-\sin\theta$	$\sin\theta$
cos	$\cos\theta$	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$\cos\theta$	$\cos\theta$
tan	$-\tan\theta$	$\cot\theta$	$-\cot\theta$	$-\tan\theta$	$\tan\theta$	$-\tan\theta$	$\tan\theta$

Example: $\sin 120^\circ = \sin(90^\circ + 30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$; $\cos 210^\circ = \cos(180^\circ + 30^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$.

❖ Sum and Difference Formulae

- $\sin(A+B) = \sin A \cos B + \cos A \sin B$
- $\sin(A-B) = \sin A \cos B - \cos A \sin B$
- $\cos(A+B) = \cos A \cos B - \sin A \sin B$
- $\cos(A-B) = \cos A \cos B + \sin A \sin B$
- $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
- $\cot(A \pm B) = \dfrac{\cot A \cot B \mp 1}{\cot B \pm \cot A}$
- $\tan\left(\dfrac{\pi}{4} + A\right) = \dfrac{1+\tan A}{1-\tan A}$
- $\tan\left(\dfrac{\pi}{4} - A\right) = \dfrac{1-\tan A}{1+\tan A}$
- $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$
- $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$

❖ Multiple and Sub-multiple Angles

- $\sin 2x = 2\sin x \cos x = \dfrac{2\tan x}{1+\tan^2 x}$
- $\cos 2x = \cos^2 x - \sin^2 x$
- $\cos 2x = 1 - 2\sin^2 x = 2\cos^2 x - 1$
- $\cos 2x = \dfrac{1-\tan^2 x}{1+\tan^2 x}$
- $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$
- $\sin 3x = 3\sin x - 4\sin^3 x$
- $\cos 3x = 4\cos^3 x - 3\cos x$
- $\tan 3x = \dfrac{3\tan x - \tan^3 x}{1-3\tan^2 x}$
- $1 - \cos 2x = 2\sin^2 x$, $1 + \cos 2x = 2\cos^2 x$
- $\sin x = 2\sin\dfrac{x}{2}\cos\dfrac{x}{2}$, $\cos x = 2\cos^2\dfrac{x}{2} - 1$

↪ These are used again and again in inverse trig, differentiation and integration.

❖ Product ↔ Sum Formulae

Product to sum

- $2\sin A\cos B = \sin(A+B)+\sin(A-B)$
- $2\cos A\sin B = \sin(A+B)-\sin(A-B)$
- $2\cos A\cos B = \cos(A+B)+\cos(A-B)$
- $2\sin A\sin B = \cos(A-B)-\cos(A+B)$

Sum to product

- $\sin C+\sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$
- $\sin C-\sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$
- $\cos C+\cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$
- $\cos C-\cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

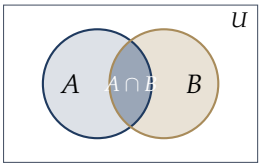
☞ Tips to Remember

- Useful values: $\sin 15^\circ = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$, $\cos 15^\circ = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$, $\tan 15^\circ = 2-\sqrt{3}$, $\tan 75^\circ = 2+\sqrt{3}$.
- $\sin 18^\circ = \dfrac{\sqrt{5}-1}{4}$, $\cos 36^\circ = \dfrac{\sqrt{5}+1}{4}$.

❖ B. Sets — Operations and Laws

- $A\cup B$: in A or B; $A\cap B$: in both; $A-B$: in A but not in B; $A' = U-A$.
- **Disjoint sets:** $A\cap B=\phi$.
- **De Morgan's laws:** $(A\cup B)' = A'\cap B'$, $(A\cap B)' = A'\cup B'$.
- $n(A\cup B) = n(A)+n(B)-n(A\cap B)$
- Number of subsets of a set with n elements $= 2^n$.

↪ The same idea gives $P(A\cup B) = P(A)+P(B)-P(A\cap B)$ in Probability.



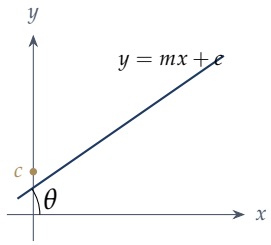
❖ C. Permutations and Combinations

- **Fundamental principle:** if one job can be done in m ways and then another in n ways, both together in $m\times n$ ways.
- $n! = 1\times 2\times\cdots\times n$; $0! = 1$.
- **Permutations** (arrangements, order matters): $^nP_r = \dfrac{n!}{(n-r)!}$
- **Combinations** (selections, order does not matter): $^nC_r = \dfrac{n!}{r!\,(n-r)!}$
  - $^nC_r = {}^nC_{n-r}$, $^nC_0 = {}^nC_n = 1$, $^nP_r = r!\cdot {}^nC_r$.

↪ Used for counting one-one/onto functions and for counting favourable cases in Probability (e.g. choosing 2 cards from 52: $^{52}C_2$).

❖ D. Straight Lines — Slope and Equations

- Slope: $m = \tan\theta = \dfrac{y_2 - y_1}{x_2 - x_1}$ (θ = angle with positive x-axis).
- Parallel lines: $m_1 = m_2$; perpendicular lines: $m_1 m_2 = -1$.
- Forms of the equation:
  - point–slope: $y - y_1 = m(x - x_1)$
  - slope–intercept: $y = mx + c$
  - two-point: $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
  - intercept form: $\dfrac{x}{a} + \dfrac{y}{b} = 1$
- Slope of $ax + by + c = 0$ is $-\dfrac{a}{b}$.
- Distance of (x_1, y_1) from $ax + by + c = 0$: $\dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.



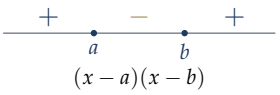
❖ E. Conic Sections — Circle, Parabola, Ellipse

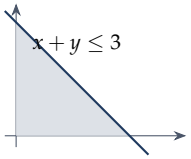
Curve	Equation	Key facts	Shape
Circle	$(x-h)^2 + (y-k)^2 = r^2$	centre (h,k), radius r; $x^2 + y^2 = r^2$ has centre at origin	
Parabola	$y^2 = 4ax$	vertex $(0,0)$, focus $(a,0)$, axis = x-axis, opens right; $x^2 = 4ay$ opens up	
Ellipse	$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$	centre origin; meets axes at $(\pm a, 0)$, $(0, \pm b)$; symmetric about both axes	

↪ General circle $x^2 + y^2 + 2gx + 2fy + c = 0$: centre $(-g, -f)$, radius $\sqrt{g^2 + f^2 - c}$.

❖ F. Linear Inequalities and Sign Method

- Multiplying/dividing an inequality by a **negative** number reverses the sign.
- Graph of $ax + by \le c$: draw the line $ax + by = c$ (solid for $\le, \ge$; dashed for $<, >$), test $(0,0)$, shade the side that satisfies it.
- $|x| < a \iff -a < x < a$; $\quad |x| > a \iff x < -a$ or $x > a$.





Sign method for $(x-a)(x-b)\ldots$

Step 1: Find the zeros and mark them on the number line.

Step 2: Starting from the right, signs are $+, -, +, \ldots$ (for simple factors).

Step 3: Choose the intervals with the required sign.

1 Relations and Functions

CHAPTER 1

❖ Ordered Pairs, Cartesian Product and Relations

- **Ordered pair** (a, b): a is the first element, b the second; $(a, b) \neq (b, a)$ unless $a = b$.
- **Cartesian product:** $A \times B = \{(a, b) : a \in A,\ b \in B\}$; if $n(A) = p$, $n(B) = q$ then $n(A \times B) = pq$
- **Relation from A to B:** any subset of $A \times B$. A **relation on** A is a subset of $A \times A$.
  - Domain = set of first elements; Range = set of second elements; Co-domain = B.
- **Empty relation:** $R = \phi$. **Universal relation:** $R = A \times A$. (These two are **trivial relations**.)
- **Identity relation** on A: $I_A = \{(a, a) : a \in A\}$.

❖ Types of Relations

Type	Condition	In words
Reflexive	$(a, a) \in R$ for every $a \in A$	every element is related to itself
Symmetric	$(a, b) \in R \Rightarrow (b, a) \in R$	if a is related to b, then b is related to a
Transitive	$(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$	a chain $a \to b \to c$ gives $a \to c$
Equivalence	reflexive **and** symmetric **and** transitive	all three together

- **Equivalence class** of a: $[a] = \{x \in A : (x, a) \in R\}$.
  - ↪ Equivalence classes are disjoint and together they make up the whole set A.

❖ How to Prove a Relation is an Equivalence Relation

Step 1: **Reflexive:** take any $a \in A$ and show $(a, a) \in R$.

Step 2: **Symmetric:** assume $(a, b) \in R$; use the rule to show $(b, a) \in R$.

Step 3: **Transitive:** assume $(a, b) \in R$ and $(b, c) \in R$; add/combine the two conditions to show $(a, c) \in R$.

Step 4: Conclude: R is reflexive, symmetric and transitive, hence an equivalence relation.

↪ To **disprove** a property, one counter-example is enough. E.g. $R = \{(a, b) : a \leq b\}$ is not symmetric: $(1, 2) \in R$ but $(2, 1) \notin R$.

❖ Counting Relations ($n(A) = n$)

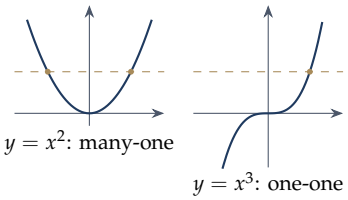
- Relations from A to B ($n(A) = p$, $n(B) = q$): 2^{pq} Total relations on A: 2^{n^2}
- Reflexive relations: 2^{n^2-n}; symmetric relations: $2^{n(n+1)/2}$.

☞ Tips to Remember

- Number of functions from A (m elements) to B (n elements) $= n^m$.
- One-one functions from A to B ($m \leq n$): $\dfrac{n!}{(n-m)!}$; bijections when $m = n$: $n!$.

❖ Types of Functions

A **function** $f : A \to B$ assigns to each $x \in A$ exactly one $f(x) \in B$. Domain $= A$, co-domain $= B$, range $= \{f(x) : x \in A\} \subseteq B$.



- **One-one (injective):** different inputs give different outputs.
  ↪ $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.
- **Many-one:** at least two different elements of A have the same image.
- **Onto (surjective):** every element of B has a pre-image in A.
  ↪ Range of f = co-domain B.
- **Into:** some element of B has no pre-image (range $\neq$ co-domain).
- **Bijective:** both one-one and onto.
- **Horizontal line test:** if every horizontal line meets the graph at most once, f is one-one.

❖ How to Prove One-one and Onto

One-one

Step 1: Take $x_1, x_2 \in A$ with $f(x_1) = f(x_2)$.

Step 2: Simplify to get $x_1 = x_2$. Hence f is one-one.

Onto

Step 1: Take any y in the co-domain and put $y = f(x)$.

Step 2: Solve for x in terms of y.

Step 3: Check this x belongs to A (domain) and $f(x) = y$. Hence f is onto.

- Onto functions from A to B ($m \geq n$): $n^m - {}^nC_1(n-1)^m + {}^nC_2(n-2)^m - \ldots$
  ↪ e.g. $m = 3,\ n = 2$: $2^3 - 2 \cdot 1^3 = 6$.
- For a finite set A, a function $f : A \to A$ is one-one $\iff$ it is onto.
- $f(x) = x^2$ on $\mathbb{R}$ is neither one-one nor onto; on $[0, \infty) \to [0, \infty)$ it is bijective. **Domain and co-domain matter.**

✎ **My Notes**

2 Inverse Trigonometric Functions

CHAPTER 2

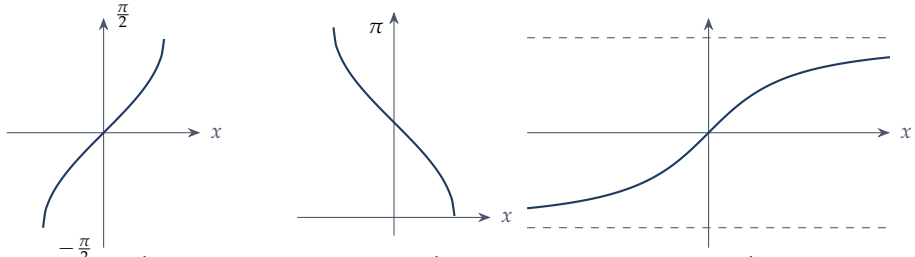
❖ Domain and Principal Value Branch

Function	Domain	Range (principal value branch)
$\sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1} x$	$[-1, 1]$	$[0,\ \pi]$
$\tan^{-1} x$	$\mathbb{R}$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\csc^{-1} x$	$\mathbb{R} - (-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$\sec^{-1} x$	$\mathbb{R} - (-1, 1)$	$[0,\ \pi] - \left\{\frac{\pi}{2}\right\}$
$\cot^{-1} x$	$\mathbb{R}$	$(0,\ \pi)$

↪ $\sin^{-1} x$ means "the angle whose sine is x". It is **not** $\dfrac{1}{\sin x}$.

↪ Why a branch? $\sin x$ is one-one and onto $[-1, 1]$ only on a restricted interval. Other intervals such as $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ give other branches; $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is the **principal** one. Similarly $[0, \pi]$ for $\cos^{-1}$.

❖ Graphs (principal branches)



$y = \sin^{-1} x$ $y = \cos^{-1} x$ $y = \tan^{-1} x$

❖ Finding the Principal Value

Step 1: Let $\sin^{-1} x = \theta$ (or $\cos^{-1} x = \theta, \ldots$). Then $\sin\theta = x$.

Step 2: Find a standard angle with that value.

Step 3: Check that the angle lies in the principal range; if not, choose the equivalent angle that does.

► Useful for negative values:

- $\sin^{-1}(-x) = -\sin^{-1} x,\ \tan^{-1}(-x) = -\tan^{-1} x,\ \csc^{-1}(-x) = -\csc^{-1} x$
- $\cos^{-1}(-x) = \pi - \cos^{-1} x,\ \sec^{-1}(-x) = \pi - \sec^{-1} x,\ \cot^{-1}(-x) = \pi - \cot^{-1} x$

Example: $\cos^{-1}\left(-\frac{1}{2}\right) = \pi - \cos^{-1}\frac{1}{2} = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ (lies in $[0, \pi]$ ✓).

✘ Common Mistakes

► $\sin^{-1}(\sin\theta) = \theta$ **only** when $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

❖ Useful Results on Inverse Trigonometric Functions

- $\sin(\sin^{-1} x) = x,\ x \in [-1, 1]$
- $\sin^{-1}(\sin x) = x,\ x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
- $\sin^{-1} \frac{1}{x} = \csc^{-1} x,\ |x| \geq 1$
- $\cos^{-1} \frac{1}{x} = \sec^{-1} x,\ |x| \geq 1$
- $\tan^{-1} \frac{1}{x} = \cot^{-1} x,\ x > 0$
- $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$
- $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$
- $\sec^{-1} x + \csc^{-1} x = \frac{\pi}{2}$
- $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy},\ xy < 1$
- $\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy},\ xy > -1$
- $2\tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2},\ |x| < 1$
- $2\tan^{-1} x = \sin^{-1} \frac{2x}{1+x^2},\ |x| \leq 1$
- $2\tan^{-1} x = \cos^{-1} \frac{1-x^2}{1+x^2},\ x \geq 0$

↪ $\sin^{-1}(\sin \frac{2\pi}{3}) = \sin^{-1}(\sin \frac{\pi}{3}) = \frac{\pi}{3}$, not $\frac{2\pi}{3}$.

- Similarly $\cos^{-1}(\cos\theta) = \theta$ only for $\theta \in [0, \pi]$; $\tan^{-1}(\tan\theta) = \theta$ only for $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

✎ **My Notes**

3 Matrices

CHAPTER 3

❖ Basics and Types

- A matrix of **order** $m \times n$ has m rows and n columns: $A = [a_{ij}]_{m\times n}$.
- Two matrices are **equal** if they have the same order and equal corresponding elements.

Type	Meaning	Example
Row / Column	only one row / only one column	$[2\ \ 5]$, $\begin{bmatrix}1\\4\end{bmatrix}$
Square	$m = n$	2×2, 3×3
Diagonal	square; all non-diagonal elements 0	$\begin{bmatrix}3&0\\0&-1\end{bmatrix}$
Scalar	diagonal with all diagonal elements equal	$\begin{bmatrix}k&0\\0&k\end{bmatrix}$
Identity I	diagonal elements 1, others 0	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$
Zero O	all elements 0	$\begin{bmatrix}0&0\\0&0\end{bmatrix}$

❖ Operations on Matrices

- **Addition / subtraction:** only for matrices of the **same order**; add corresponding elements.
- **Scalar multiplication:** $kA = [k\,a_{ij}]$ (multiply every element by k).
- **Multiplication** AB is defined only when (columns of A) = (rows of B):
  - ↪ $A_{m\times n}\cdot B_{n\times p} = C_{m\times p}$, where c_{ij} = (i^{th} row of A) · (j^{th} column of B).
- Properties:
  - $A(BC) = (AB)C$, $A(B+C) = AB + AC$, $AI = IA = A$.
  - In general $AB \neq BA$ (not commutative).
  - $AB = O$ does **not** mean $A = O$ or $B = O$.

❖ Transpose, Symmetric and Skew-symmetric Matrices

- **Transpose** A' (or A^T): interchange rows and columns.
- Properties: $(A')' = A$, $(A+B)' = A' + B'$, $(kA)' = kA'$, $(AB)' = B'A'$
- **Symmetric:** $A' = A$ (i.e., $a_{ij} = a_{ji}$; elements are mirror images about the leading diagonal).
- **Skew-symmetric:** $A' = -A$ (i.e., $a_{ij} = -a_{ji}$). All diagonal elements are 0.
- Every square matrix can be written as $A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$
  - ↪ $\frac{1}{2}(A + A')$ is symmetric and $\frac{1}{2}(A - A')$ is skew-symmetric.

❖ Invertible Matrices

- A square matrix A is invertible if there is a matrix B with $AB = BA = I$. Then $B = A^{-1}$.
- The inverse, if it exists, is unique.
- $(AB)^{-1} = B^{-1}A^{-1}$ and $(A')^{-1} = (A^{-1})'$.

☞ Tips to Remember

- To show $A^2 - 5A + 7I = O$: find A^2 first, then $5A$ and $7I$, and add element-wise.
- To find A^{-1} from such an equation, multiply both sides by A^{-1}.
  - ↪ e.g. $A^2 - 5A + 7I = O \Rightarrow A - 5I + 7A^{-1} = O \Rightarrow A^{-1} = \frac{1}{7}(5I - A)$.

✎ My Notes

Infinity Think Beyond

4 Determinants

CHAPTER 4

❖ Value of a Determinant

- Every square matrix has a number called its determinant. Order 1: $|[a]| = a$.
- $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
- 3×3: expand along any row or column using signs $\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$.

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)$$

- Expand along the row/column with the most zeros to save time.

❖ Minors and Cofactors

- **Minor** M_{ij}: determinant left after deleting the i^{th} row and j^{th} column.
- **Cofactor:** $A_{ij} = (-1)^{i+j} M_{ij}$
- $|A|$ = sum of (elements of any row) $\times$ (their own cofactors).
- Sum of (elements of a row) $\times$ (cofactors of **another** row) $= 0$.

❖ Area of a Triangle and Collinearity

- Area of triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$:

$$\Delta = \frac{1}{2} \left| \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \right|$$

- Points are **collinear** $\iff$ this determinant $= 0$.

↪ Area is always positive — take the absolute value. If the area is given, use $\pm$ both signs.

❖ Adjoint and Inverse

Step 1: Find $|A|$. If $|A| = 0$, A is **singular** and A^{-1} does not exist.

Step 2: Find all cofactors A_{ij}.

Step 3: adj A = **transpose** of the cofactor matrix.

Step 4: $A^{-1} = \dfrac{1}{|A|}\operatorname{adj} A$

- 2×2 shortcut: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow \operatorname{adj} A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ (swap diagonal, change sign of the other two).
- Key result: $A(\operatorname{adj} A) = (\operatorname{adj} A)A = |A|\, I$.

❖ Results for an $n \times n$ Matrix

- $|A'| = |A|$
- $|AB| = |A|\,|B|$
- $|kA| = k^n|A|$
- $|A^{-1}| = \dfrac{1}{|A|}$
- $|\operatorname{adj} A| = |A|^{n-1}$
- $|A\,(\operatorname{adj} A)| = |A|^n$
- $|\operatorname{adj}(\operatorname{adj} A)| = |A|^{(n-1)^2}$
- $\operatorname{adj}(AB) = (\operatorname{adj} B)(\operatorname{adj} A)$
- A invertible $\iff |A| \neq 0$ (non-singular)
- A, B non-singular $\Rightarrow AB,\ BA$ non-singular

❖ Solving Linear Equations by the Matrix Method

Step 1: Write the system as $AX = B$ (coefficients in A, variables in X, constants in B).

Step 2: Find $|A|$.

Step 3: If $|A| \neq 0$: find A^{-1} and write $X = A^{-1}B$

Step 4: Equate corresponding elements to get x, y, z.

Condition	Nature	Solutions
$\|A\| \neq 0$	consistent	unique
$\|A\| = 0$ and $(\operatorname{adj} A)B \neq O$	inconsistent	no solution
$\|A\| = 0$ and $(\operatorname{adj} A)B = O$	consistent or inconsistent	infinitely many or none

✎ **My Notes**

5 Continuity and Differentiability

CHAPTER 5

❖ Functions You Should Know

- Polynomial: $x^2 + 2x + 5$
- Modulus: $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$
- Greatest integer: $[x]$ = greatest integer $\leq x$
- Signum: $\operatorname{sgn}(x) = 1\ (x > 0),\ 0\ (x = 0),\ -1\ (x < 0)$
- Trigonometric and inverse trigonometric
- Exponential e^x and logarithmic $\log x$

❖ Standard Limits

- $\lim\limits_{x\to a} \dfrac{x^n - a^n}{x - a} = na^{n-1}$
- $\lim\limits_{x\to 0} \dfrac{\sin x}{x} = 1$
- $\lim\limits_{x\to 0} \dfrac{\tan x}{x} = 1$
- $\lim\limits_{x\to 0} \dfrac{\sin^{-1} x}{x} = 1$
- $\lim\limits_{x\to 0} \dfrac{e^x - 1}{x} = 1$
- $\lim\limits_{x\to 0} \dfrac{a^x - 1}{x} = \log a$
- $\lim\limits_{x\to 0} \dfrac{\log(1+x)}{x} = 1$
- $\lim\limits_{x\to 0} (1+x)^{1/x} = e$
- $\lim\limits_{x\to \infty} \left(1 + \dfrac{1}{x}\right)^x = e$

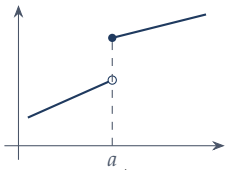
❖ Continuity at a Point

f is continuous at $x = a$ if

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$$

↪ LHL = RHL = value of the function. The graph has no break at $x = a$.

↪ To find LHL put $x = a - h$; for RHL put $x = a + h$; then let $h \to 0$.



LHL $\neq$ RHL
discontinuous at a

- Polynomials, $\sin x$, $\cos x$, e^x, $|x|$ are continuous everywhere; $\log x$ for $x > 0$; rational functions wherever the denominator $\neq 0$.
- Constant and identity functions are continuous; trigonometric functions are continuous in their domains.
- If f, g are continuous at c: $f \pm g$, fg, kf, $\frac{f}{g}$ $(g(c) \neq 0)$, $f \circ g$ and $|f|$ are continuous.
- f is continuous on an interval if it is continuous at every point of it.

❖ Finding k for a Piecewise Function to be Continuous

Step 1: Find LHL using the rule for $x < a$; find RHL using the rule for $x > a$.

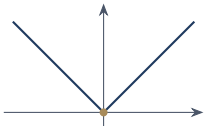
Step 2: Find $f(a)$ from the rule that includes $x = a$.

Step 3: Put LHL = RHL = $f(a)$ and solve for k.

❖ Differentiability

- f is differentiable at $x = a$ if LHD = RHD:

$$Lf'(a) = \lim_{h\to 0} \frac{f(a-h)-f(a)}{-h}, \qquad Rf'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$



$y = |x|$: corner at 0

- Differentiable $\Rightarrow$ continuous. The converse is **not** true.
  - ↪ $f(x) = |x|$ is continuous at 0 but not differentiable there (sharp corner).

❖ Standard Derivatives

- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(\cot x) = -\csc^2 x$
- $\frac{d}{dx}(\sec x) = \sec x \tan x$
- $\frac{d}{dx}(\csc x) = -\csc x \cot x$
- $\frac{d}{dx}(e^x) = e^x$, $\frac{d}{dx}(a^x) = a^x \log a$
- $\frac{d}{dx}(\log x) = \frac{1}{x}$, $\frac{d}{dx}(\log_a x) = \frac{1}{x \log a}$
- $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
- $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$
- $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$
- $\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$

❖ Rules of Differentiation

- Product: $(uv)' = uv' + vu'$ Quotient: $\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$
- Chain rule: $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ (differentiate outside, then multiply by the derivative of the inside).
- Sum/difference: $(u \pm v)' = u' \pm v'$; constant: $\frac{d}{dx}(k) = 0$.

❖ Special Methods

(A) Implicit functions

Step 1: Differentiate both sides w.r.t. x; write $\frac{dy}{dx}$ each time you differentiate a term in y.

Step 2: Collect the $\frac{dy}{dx}$ terms on one side and solve.

(B) Logarithmic differentiation — for $y = [f(x)]^{g(x)}$ or long products

Step 1: Take log on both sides: $\log y = g(x) \log f(x)$.

Step 2: Differentiate: $\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}[g(x) \log f(x)]$.

Step 3: Multiply by y and substitute y back.

↪ For $y = u + v$ (sum of two such powers), find $\frac{du}{dx}$ and $\frac{dv}{dx}$ separately.

(C) Parametric form $x = f(t)$, $y = g(t)$

- $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$
- Second derivative: $\dfrac{d^2y}{dx^2} = \dfrac{d}{dt}\left(\dfrac{dy}{dx}\right) \div \dfrac{dx}{dt}$

❖ Logarithm Rules and Useful Substitutions

Log rules

- $\log mn = \log m + \log n$
- $\log \frac{m}{n} = \log m - \log n$
- $\log m^n = n \log m$
- $\log_a b = \dfrac{\log b}{\log a}$
- $\log e = 1,\ \log 1 = 0,\ e^{\log f(x)} = f(x)$

Substitutions for inverse-trig derivatives

Expression	Put
$a^2 + x^2$	$x = a\tan\theta$
$a^2 - x^2$	$x = a\sin\theta$ or $a\cos\theta$
$x^2 - a^2$	$x = a\sec\theta$
$\sqrt{\frac{a-x}{a+x}}$	$x = a\cos 2\theta$
$\sqrt{\frac{a^2-x^2}{a^2+x^2}}$	$x^2 = a^2\cos 2\theta$

✘ Common Mistakes

- $\dfrac{d^2y}{dx^2} \neq \dfrac{d^2y/dt^2}{d^2x/dt^2}$ — use the formula above.
- $\dfrac{d}{dx}(x^x) \neq x \cdot x^{x-1}$ — use logarithmic differentiation: answer $x^x(1 + \log x)$.

✎ **My Notes**

6 Application of Derivatives

CHAPTER 6

❖ Rate of Change

- $\frac{dy}{dx}$ is the rate of change of y with respect to x.
- If both depend on time: $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

Step 1: Write the formula connecting the quantities (area, volume, perimeter, ...).

Step 2: Differentiate both sides w.r.t. t.

Step 3: Substitute the given values and the given rate; write units (cm^2/s, cm^3/s).

↪ Circle: $A = \pi r^2$, $C = 2\pi r$. Sphere: $V = \frac{4}{3}\pi r^3$, $S = 4\pi r^2$. Cube: $V = x^3$, $S = 6x^2$.

- **Marginal cost** $MC = \frac{dC}{dx}$ and **marginal revenue** $MR = \frac{dR}{dx}$ (x = number of units).

❖ Increasing and Decreasing Functions

- $f'(x) > 0$ on an interval $\Rightarrow$ f is **strictly increasing**.
- $f'(x) < 0$ on an interval $\Rightarrow$ f is **strictly decreasing**.
- $f'(x) = 0$ throughout $\Rightarrow$ f is constant.
- Definition: f is strictly increasing on I if $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$ ("$\leq$" gives increasing).
- If f increases on part of I and decreases on another part, it is neither increasing nor decreasing on I.



e.g. $f'(x) = 3(x-1)(x-3)$ inc. on $(-\infty, 1) \cup (3, \infty)$

Step 1: Find $f'(x)$ and solve $f'(x) = 0$ (critical points).

Step 2: Mark these points on the number line; they divide it into intervals.

Step 3: Check the sign of $f'(x)$ in each interval.

Step 4: $+ \rightarrow$ increasing; $- \rightarrow$ decreasing.

❖ Word Problems on Maxima / Minima

Step 1: Draw a figure; name the variables.

Step 2: Write the quantity to be maximised/minimised (say S) in terms of **one** variable using the given condition.

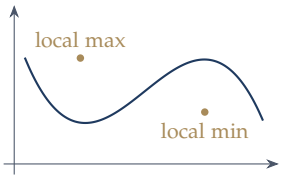
Step 3: Find $\frac{dS}{dx} = 0$ and get the critical value.

Step 4: Confirm using $\frac{d^2S}{dx^2}$ (< 0 for maximum, > 0 for minimum).

Step 5: Answer the question asked (dimensions, maximum area, ...).

❖ Maxima and Minima

Critical point: c where $f'(c) = 0$ or f is not differentiable.
Local maxima/minima occur only at critical points.



First derivative test (at a critical point c)

- f' changes $+ \to -$: local **maximum**.
- f' changes $- \to +$: local **minimum**.
- No change in sign: point of inflection.

Second derivative test

- $f'(c) = 0$ and $f''(c) < 0$: local **maximum**.
- $f'(c) = 0$ and $f''(c) > 0$: local **minimum**.
- $f''(c) = 0$: test fails — use the first derivative test.

Absolute maximum / minimum on $[a, b]$

Step 1: Find all critical points in (a, b).

Step 2: Find f at these points and at the end points a and b.

Step 3: Greatest value = absolute maximum; least value = absolute minimum.

↪ A continuous function on a closed interval always has an absolute maximum and an absolute minimum.

7 Integrals

CHAPTER 7

❖ Basic Idea

- $F(x)$ is an **antiderivative** of $f(x)$ if $F'(x) = f(x)$. Then $F(x) + C$ is also one, so there are infinitely many.
- The family of all antiderivatives is the **indefinite integral**: $\int f(x)\,dx = F(x) + C$.
- $\int [f(x) \pm g(x)]\,dx = \int f\,dx \pm \int g\,dx; \quad \int kf(x)\,dx = k\int f(x)\,dx.$
- Methods: substitution, partial fractions, by parts.

❖ Standard Integrals (add $+C$ to each)

- $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} \quad (n \neq -1)$
- $\int \dfrac{1}{x}\,dx = \log|x|$
- $\int e^x\,dx = e^x, \ \int a^x\,dx = \dfrac{a^x}{\log a}$
- $\int \sin x\,dx = -\cos x$
- $\int \cos x\,dx = \sin x$
- $\int \sec^2 x\,dx = \tan x$
- $\int \csc^2 x\,dx = -\cot x$
- $\int \sec x \tan x\,dx = \sec x$
- $\int \csc x \cot x\,dx = -\csc x$
- $\int \tan x\,dx = \log|\sec x|$
- $\int \cot x\,dx = \log|\sin x|$
- $\int \sec x\,dx = \log|\sec x + \tan x|$
- $\int \csc x\,dx = \log|\csc x - \cot x|$
- $\int \dfrac{dx}{\sqrt{1-x^2}} = \sin^{-1} x$
- $\int \dfrac{dx}{1+x^2} = \tan^{-1} x$

❖ Special Integrals

- $\int \dfrac{dx}{x^2-a^2} = \dfrac{1}{2a}\log\left|\dfrac{x-a}{x+a}\right|$
- $\int \dfrac{dx}{a^2-x^2} = \dfrac{1}{2a}\log\left|\dfrac{a+x}{a-x}\right|$
- $\int \dfrac{dx}{x^2+a^2} = \dfrac{1}{a}\tan^{-1}\dfrac{x}{a}$
- $\int \dfrac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\dfrac{x}{a}$
- $\int \dfrac{dx}{\sqrt{x^2+a^2}} = \log\left|x+\sqrt{x^2+a^2}\right|$
- $\int \dfrac{dx}{\sqrt{x^2-a^2}} = \log\left|x+\sqrt{x^2-a^2}\right|$
- $\int \sqrt{a^2-x^2}\,dx = \dfrac{x}{2}\sqrt{a^2-x^2} + \dfrac{a^2}{2}\sin^{-1}\dfrac{x}{a}$
- $\int \sqrt{x^2+a^2}\,dx = \dfrac{x}{2}\sqrt{x^2+a^2} + \dfrac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|$
- $\int \sqrt{x^2-a^2}\,dx = \dfrac{x}{2}\sqrt{x^2-a^2} - \dfrac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|$

❖ Integration by Substitution

Step 1: Look for a function whose derivative is also present.

Step 2: Put that function $= t$ and replace dx using dt.

Step 3: Integrate in t, then put back the original expression.

- Quick results: $\int \dfrac{f'(x)}{f(x)}dx = \log|f(x)| \qquad \int [f(x)]^n f'(x)\,dx = \dfrac{[f(x)]^{n+1}}{n+1}$
- Use identities first for powers and products:

- $\sin^2 x = \frac{1-\cos 2x}{2}$, $\cos^2 x = \frac{1+\cos 2x}{2}$, $\tan^2 x = \sec^2 x - 1$, $\cot^2 x = \csc^2 x - 1$
- $\sin^3 x = \frac{3\sin x - \sin 3x}{4}$, $\cos^3 x = \frac{3\cos x + \cos 3x}{4}$
- $2\sin x \cos y = \sin(x+y) + \sin(x-y)$; $2\cos x \sin y = \sin(x+y) - \sin(x-y)$
- $2\cos x \cos y = \cos(x+y) + \cos(x-y)$; $2\sin x \sin y = \cos(x-y) - \cos(x+y)$

❖ Integrals with a Quadratic $ax^2 + bx + c$

► $\int \frac{dx}{ax^2+bx+c}$ or $\int \frac{dx}{\sqrt{ax^2+bx+c}}$:
↪ Complete the square, then use a special integral.

► $\int \frac{(px+q)\,dx}{ax^2+bx+c}$ (also with $\sqrt{}$):

Step 1: Write $px + q = A\frac{d}{dx}(ax^2+bx+c) + B$.

Step 2: Compare coefficients to find A and B.

Step 3: Split into two integrals: the first by substitution, the second by completing the square.

❖ Partial Fractions (degree of numerator $<$ degree of denominator)

Form of the rational function	Partial fractions
$\frac{px+q}{(x-a)(x-b)}$	$\frac{A}{x-a} + \frac{B}{x-b}$
$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2}$
$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$
$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$
$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$

↪ **Proper** rational function: degree of numerator $<$ degree of denominator. If it is **improper**, divide first.

❖ Integration by Parts

► $\int u\,v\,dx = u\int v\,dx - \int \left(\frac{du}{dx}\int v\,dx\right)dx$

► Choose u (first function) by **ILATE**: Inverse trig, Logarithmic, Algebraic, Trigonometric, Exponential.

► If there is only one function (e.g. $\log x$, $\tan^{-1} x$), take 1 as the second function.

► Special result: $\int e^x\left[f(x) + f'(x)\right]dx = e^x f(x) + C$

❖ Definite Integrals

► If $\int f(x)\,dx = F(x)$, then $\int_a^b f(x)\,dx = F(b) - F(a)$ (Fundamental Theorem of Calculus). The value is unique; no $+C$.

► Area function: $A(x) = \int_a^x f(t)\,dt$ gives the area under $y = f(x)$ from a to x, and $A'(x) = f(x)$.

- With substitution, change the limits too.

Properties

- $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx; \quad \int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$
- $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx \qquad \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$
- $\int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx$ if $f(2a-x) = f(x)$; $= 0$ if $f(2a-x) = -f(x)$.
- $\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx$ if f is **even**; $= 0$ if f is **odd**.
  - ↪ Use $\int_0^a f(a-x)$ when $f(x) + f(a-x)$ simplifies (e.g. $\sin x \leftrightarrow \cos x$ on $[0, \frac{\pi}{2}]$): call the integral I, apply the property, add the two forms, and solve for I.
  - ↪ For $|f(x)|$: find where $f(x) = 0$ and split the interval.

✎ **My Notes**

Infinity Thi

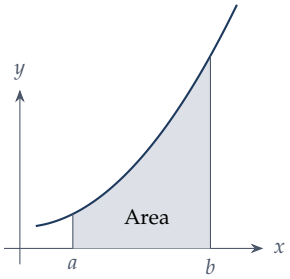
8 Application of Integrals

CHAPTER 8

❖ Area under a Curve

- Area between $y = f(x)$, the x-axis and $x = a$, $x = b$:

$$A = \int_a^b |y|\, dx$$



- Area between $x = g(y)$, the y-axis and $y = c$, $y = d$: $A = \int_c^d |x|\, dy$.
- If the curve is below the x-axis, the integral is negative — take its absolute value.

❖ Steps to Find an Area

Step 1: Draw a rough sketch of the curve and the given lines.

Step 2: Shade the required region and find the limits (points of intersection).

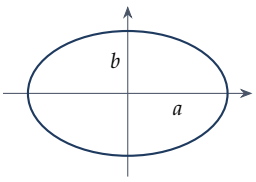
Step 3: Region between two ordinates $x = a,\ x = b \to \int y\, dx$; between $y = c,\ y = d \to \int x\, dy$.

Step 4: Use symmetry where possible (find one part and multiply).

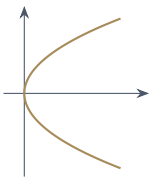
Step 5: Integrate and simplify; write "sq. units".

❖ Standard Curves

- Circle $x^2 + y^2 = a^2$: area $= \pi a^2$ (one quadrant $= \dfrac{\pi a^2}{4}$).
- Ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$: area $= \pi ab$.
  - $\hookrightarrow = 4\int_0^a \dfrac{b}{a}\sqrt{a^2 - x^2}\, dx$.
- Parabolas (vertex at origin):
  - $y^2 = 4ax$ opens right, $y^2 = -4ax$ opens left (symmetric about x-axis).
  - $x^2 = 4ay$ opens up, $x^2 = -4ay$ opens down (symmetric about y-axis).
- Ellipse cuts the axes at $(\pm a, 0)$ and $(0, \pm b)$.
- Circle with centre (h, k): $(x - h)^2 + (y - k)^2 = r^2$. Line: $ax + by + c = 0$. $y = |x|$: V-shape.



ellipse: area πab



$y^2 = 4ax$

✎ **My Notes**

9 Differential Equations

CHAPTER 9

❖ Order and Degree

- A **differential equation** involves derivatives of a dependent variable w.r.t. an independent variable. (Only one independent variable: **ordinary**; we study only these.)
- **Order** = order of the highest derivative present.
- **Degree** = power of the highest-order derivative, after the equation is made free of radicals and fractions in the derivatives.
  - ↪ Degree is **not defined** if the equation is not a polynomial in derivatives, e.g. $\frac{dy}{dx} + \sin\left(\frac{dy}{dx}\right) = 0$.
- **General solution:** contains arbitrary constants — as many as the order. **Particular solution:** no arbitrary constants (found from given conditions).
  - ↪ $y = a\sin(x+b)$ is the general solution of $y'' + y = 0$; $y = 2\sin\left(x + \frac{\pi}{4}\right)$ is a particular solution.
- To verify a solution: find the derivatives and substitute into the equation; LHS should equal RHS.

❖ Type 1: Variables Separable

Step 1: Write in the form $\frac{dy}{dx} = f(x)\,g(y)$.

Step 2: Separate: $\frac{dy}{g(y)} = f(x)\,dx$.

Step 3: Integrate both sides and add one constant C.

❖ Type 2: Homogeneous Equations

- $\frac{dy}{dx} = F(x,y)$ is homogeneous if $F(\lambda x, \lambda y) = F(x,y)$ (degree zero).

Step 1: Check homogeneity.

Step 2: Put $y = vx$, so $\frac{dy}{dx} = v + x\frac{dv}{dx}$.

Step 3: Separate the variables in v and x; integrate.

Step 4: Replace v by $\frac{y}{x}$.

↪ If the equation is in the form $\frac{dx}{dy} = F(x,y)$, put $x = vy$ instead.

❖ Type 3: Linear Differential Equations

Step 1: Write in the form $\frac{dy}{dx} + Py = Q$ (P, Q functions of x or constants).

Step 2: Find the integrating factor: $\text{I.F.} = e^{\int P\,dx}$

Step 3: Solution: $y \times \text{I.F.} = \int (Q \times \text{I.F.})\,dx + C$

- If it is linear in x: $\frac{dx}{dy} + P_1 x = Q_1$, $\text{I.F.} = e^{\int P_1 dy}$, $x \times \text{I.F.} = \int (Q_1 \times \text{I.F.})\,dy + C$.
- Useful: $e^{\log f(x)} = f(x)$; e.g. $e^{\int \frac{1}{x}dx} = x$, $e^{\int \tan x\,dx} = \sec x$.

10 Vector Algebra

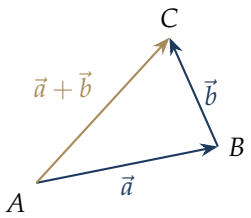
CHAPTER 10

❖ Basics

- Position vector of $P(x,y,z)$: $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$; magnitude $|\vec{r}| = \sqrt{x^2+y^2+z^2}$
- Unit vector along $\vec{a}$: $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$.
- $\overrightarrow{AB}$ = (position vector of B) − (position vector of A).
- **Direction cosines** of $\vec{r}$: $l = \dfrac{x}{|\vec{r}|}$, $m = \dfrac{y}{|\vec{r}|}$, $n = \dfrac{z}{|\vec{r}|}$, with $l^2+m^2+n^2 = 1$
- **Direction ratios:** any numbers proportional to l, m, n. The scalar components x, y, z are direction ratios.

❖ Addition and Section Formula

- **Triangle law:** $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$. Sides of a triangle taken in order add to $\vec{0}$.
- **Parallelogram law:** sum of two co-initial vectors = diagonal of the parallelogram on them.
- $\lambda\vec{a}$: magnitude becomes $|\lambda|\,|\vec{a}|$; same direction if $\lambda > 0$, opposite if $\lambda < 0$.
- Component form: $\vec{a} \pm \vec{b} = (a_1 \pm b_1)\hat{i} + (a_2 \pm b_2)\hat{j} + (a_3 \pm b_3)\hat{k}$.
- $\vec{a}$ and $\vec{b}$ are **collinear** $\iff$ $\vec{b} = \lambda\vec{a}$ (components proportional).
- Point dividing $\vec{a}$ and $\vec{b}$ in ratio $m:n$:
  - internally: $\dfrac{m\vec{b} + n\vec{a}}{m+n}$
  - externally: $\dfrac{m\vec{b} - n\vec{a}}{m-n}$
  - mid-point: $\dfrac{\vec{a}+\vec{b}}{2}$



❖ Scalar (Dot) Product

- $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta = a_1b_1 + a_2b_2 + a_3b_3$
- $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$; $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$.
- Angle: $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$. $\vec{a} \perp \vec{b} \iff \vec{a}\cdot\vec{b} = 0$.
- **Projection** of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$.
- $|\vec{a} \pm \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 \pm 2\,\vec{a}\cdot\vec{b}$.

❖ Vector (Cross) Product

- $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta\, \hat{n}$ ($\hat{n} \perp$ both $\vec{a}$ and $\vec{b}$).
- Determinant form: $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$
- $\hat{i} \times \hat{j} = \hat{k},\ \hat{j} \times \hat{k} = \hat{i},\ \hat{k} \times \hat{i} = \hat{j};\ \hat{i} \times \hat{i} = \vec{0};\ \vec{b} \times \vec{a} = -\vec{a} \times \vec{b}$.
- $\vec{a} \parallel \vec{b} \iff \vec{a} \times \vec{b} = \vec{0}$.
- Unit vector perpendicular to both: $\pm \dfrac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$.
- Area: parallelogram $= |\vec{a} \times \vec{b}|$ (diagonals given: $\frac{1}{2}|\vec{d_1} \times \vec{d_2}|$); triangle $= \frac{1}{2}|\vec{a} \times \vec{b}|$.

✎ My Notes

Infinity Think

11 Three Dimensional Geometry

CHAPTER 11

❖ Distance and Section Formula in Space

- $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$
- Point dividing AB in ratio $m:n$:
  - internally $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$; externally: replace $+$ by $-$ in each.
  - mid-point $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

❖ Direction Cosines and Direction Ratios of a Line

- If a line makes angles α, β, γ with the axes: $l = \cos\alpha,\ m = \cos\beta,\ n = \cos\gamma$; $l^2+m^2+n^2 = 1$ i.e. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.
- Direction ratios a, b, c satisfy $\frac{l}{a} = \frac{m}{b} = \frac{n}{c}$.
- From direction ratios a, b, c: $l = \frac{\pm a}{\sqrt{a^2+b^2+c^2}}$, and similarly m, n.
- Direction ratios of the line through (x_1, y_1, z_1) and (x_2, y_2, z_2): $x_2-x_1,\ y_2-y_1,\ z_2-z_1$.

❖ Equation of a Line

Line	Vector form	Cartesian form
Through point $\vec{a}$, parallel to $\vec{b}$	$\vec{r} = \vec{a} + \lambda\vec{b}$	$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$
Through two points $\vec{a}$ and $\vec{b}$	$\vec{r} = \vec{a} + \lambda(\vec{b}-\vec{a})$	$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

↪ In the Cartesian form, the denominators are the direction ratios. Make the coefficients of x, y, z equal to 1 before reading them.

❖ Angle between Two Lines

- Direction ratios a_1, b_1, c_1 and a_2, b_2, c_2:

$$\cos\theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

- Vector form: $\cos\theta = \frac{|\vec{b}_1 \cdot \vec{b}_2|}{|\vec{b}_1||\vec{b}_2|}$.
- Perpendicular: $a_1a_2 + b_1b_2 + c_1c_2 = 0$. Parallel: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (or $\vec{b}_1 \times \vec{b}_2 = \vec{0}$).
- With direction cosines: $\cos\theta = |l_1l_2 + m_1m_2 + n_1n_2|$.

❖ Shortest Distance between Two Lines

- **Skew lines:** neither parallel nor intersecting (they lie in different planes).
- Lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$:

$$d = \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

- Parallel lines ($\vec{b}_1 = \vec{b}_2 = \vec{b}$): $d = \left| \frac{\vec{b} \times (\vec{a}_2 - \vec{a}_1)}{|\vec{b}|} \right|$.
- If $d = 0$, the lines **intersect** (they are coplanar): $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 0$.
- Cartesian form:

$$d = \frac{\left\| \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} \right\|}{\sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2}}$$

Step 1: Read $\vec{a}_1, \vec{b}_1, \vec{a}_2, \vec{b}_2$ from the equations.

Step 2: Find $\vec{a}_2 - \vec{a}_1$ and $\vec{b}_1 \times \vec{b}_2$.

Step 3: Find the dot product and $|\vec{b}_1 \times \vec{b}_2|$; divide and take the absolute value.

✎ **My Notes**

12 Linear Programming

CHAPTER 12

❖ Terms

- **Objective function:** $Z = ax + by$ (to be maximised or minimised); x, y are **decision variables**.
- **Constraints:** linear inequalities in x, y; **non-negative constraints:** $x \geq 0,\ y \geq 0$.
- **Feasible region:** common region satisfying all constraints. Each point in it is a **feasible solution**.
- Points outside the feasible region are **infeasible solutions**.
- **Optimal solution:** the feasible point giving the maximum/minimum value of Z.

Key theorems

- If Z has an optimal value, it occurs at a **corner point** of the feasible region.
- Bounded region: Z has both a maximum and a minimum, each at a corner point.
- Unbounded region: a maximum or minimum may or may not exist; if it exists, it is at a corner point.

❖ Corner Point Method

Step 1: Draw each constraint as a line and shade the correct side (test with $(0,0)$).

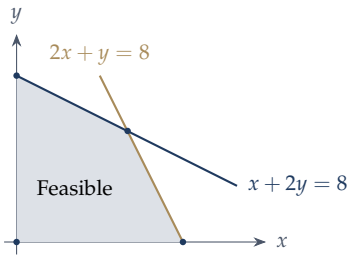
Step 2: Find the feasible region and its **corner points**.

Step 3: Find Z at every corner point.

Step 4: **Bounded** region: the largest value is the maximum, the smallest is the minimum.

Step 5: **Unbounded** region: if M is the largest value at the corners, check whether $ax + by > M$ has any point in the feasible region.

↪ No common point $\Rightarrow$ M is the maximum; otherwise no maximum. (Use $ax + by < m$ for the minimum.)



corner points marked

☞ Tips to Remember

- Read words carefully: "at least" $\rightarrow \geq$; "at most" / "not more than" $\rightarrow \leq$.
- If two corner points give the same optimal value, every point on the segment joining them is also optimal.
- Always state the final answer in words (e.g. "Maximum profit is Rs.... when ...").

✎ My Notes

13 Probability

CHAPTER 13

❖ Basic Terms and Results

- **Random experiment:** outcome cannot be predicted. **Sample space** S: set of all outcomes.
- **Equally likely:** no outcome is preferred. **Mutually exclusive:** cannot happen together ($A \cap B = \phi$).
- **Exhaustive:** $E_1 \cup E_2 \cup \cdots \cup E_n = S$.
- $P(A) = \dfrac{n(A)}{n(S)}$; $0 \le P(A) \le 1$, $P(\phi) = 0$, $P(S) = 1$, $P(\text{not } A) = 1 - P(A)$.
- **Addition theorem:** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (for mutually exclusive: $P(A \cup B) = P(A) + P(B)$).

❖ Conditional Probability

- Probability of E given that F has occurred:

$$P(E|F) = \frac{P(E \cap F)}{P(F)}, \quad P(F) \neq 0$$

- $P(S|F) = P(F|F) = 1$; $P(E'|F) = 1 - P(E|F)$.
- **Multiplication theorem:** $P(E \cap F) = P(E)\,P(F|E) = P(F)\,P(E|F)$

❖ Independent Events

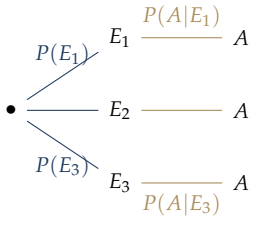
- E and F are independent if $P(E \cap F) = P(E)\,P(F)$ (equivalently $P(E|F) = P(E)$).
- If E, F are independent, so are E and F', E' and F, E' and F'.
- $P(\text{at least one of } A, B, C) = 1 - P(A')P(B')P(C')$ for independent events.
- Independent $\neq$ mutually exclusive.
  - ↪ Mutually exclusive: $P(E \cap F) = 0$. Independent: $P(E \cap F) = P(E)P(F)$.

❖ Total Probability and Bayes' Theorem

$E_1, E_2, \ldots, E_n$ form a partition of S (mutually exclusive, together cover S), and A is any event.

- **Total probability:**

$$P(A) = \sum P(E_i)\,P(A|E_i)$$



multiply along a branch, add the branches

- **Bayes' theorem:**

$$P(E_i|A) = \frac{P(E_i)\,P(A|E_i)}{\sum_j P(E_j)\,P(A|E_j)}$$

Step 1: Name the events $E_1, E_2, \ldots$ (the possible causes) and A (what is observed).

Step 2: Write $P(E_i)$ and $P(A|E_i)$ for each i.

Step 3: Find the denominator $\sum P(E_j)P(A|E_j)$ (total probability).

Step 4: Divide the required branch by this sum.

❖ Random Variable and its Probability Distribution

- A **random variable** X assigns a real number to each outcome.
- Probability distribution: list of values x_i with probabilities p_i, where $p_i \geq 0$ and $\sum p_i = 1$
- Mean (expectation): $E(X) = \sum x_i p_i$

Step 1: List all possible values of X.

Step 2: Find $P(X = x_i)$ for each value.

Step 3: Check that the probabilities add up to 1; make the table.

☞ Tips to Remember

- "Given that" / "if it is known that" → conditional probability.
- Drawing **without replacement** → events are not independent; reduce the total after each draw.

✎ My Notes

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