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ERNAKULAM REGION (2026-27)

XII MATHEMATICS (SUBJECT CODE : 041)

ANSWER KEY

SECTION A (1 MARK EACH)

1. d
2. C
3. C
4. C
5. a
6. b
7. d
8. b
9. a
10. b
11. d
12. C
13. C
14. a
15. b
16. b
17. d
18. a
19. C
20. a

SECTION B (2 MARKS EACH)

21. Given Siny + x = log x

Differentiating Cos y $\frac{dy}{dx} = \frac{1}{x} - 1$ (1M)

$\frac{dy}{dx} = \frac{1-x}{xCos\ y}$ (1M)

OR

$Y = \tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$

$Y = \tan^{-1}\left(\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right) = \tan^{-1}\left(\cot\frac{x}{2}\right)$ (1 M)

$\tan^{-1}\left(\tan\left(\frac{\pi}{2} - \frac{x}{2}\right)\right) = \frac{\pi}{2} - \frac{x}{2}$ (1/2M)

$\frac{dy}{dx} = -\frac{1}{2}$

22. Unit vector in the direction of vector a $= \frac{\vec{a}}{|\vec{a}|}$ (1/2M)

$= \frac{3\vec{i} - 2\vec{j} + 6\vec{k}}{\sqrt{9+4+36}}$ (1M)

$= \frac{1}{7}(3\vec{i} - 2\vec{j} + 6\vec{k})$ (1/2M)

OR

Vector Area of parallelogram $= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & -2 \\ 3 & 1 & -1 \end{vmatrix} = \vec{i} - 4\vec{j} - \vec{k}$ (1M)

Area of parallelogram $= \sqrt{18} = 3\sqrt{2}$ sq units (1M)

23. Line is $x - 1 = 2y = 3z$

Equation of line Cartesian form $\frac{x-1}{6} = \frac{y}{3} = \frac{z}{2}$ (1M)

In vector form $\vec{i} + \lambda(3\vec{i} - 2\vec{j} + 6\vec{k})$ (1M)

24. Direction ratios are given by 1+2, 2-4, 3+5 ie 3, -2, 8 (1/2M)

Dividing by $\sqrt{9+4+64} = \sqrt{77}$ (1/2M)

Direction cosines are given by $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$ (1M)

25. Value of Z at (1,2) = Value of Z at (3,1) (1 M)

a +2b-5 = 3a +b-5 (1/2M)

-2a + b = 0 (1/2M)

2 a - b = 0

SECTION C (3 MARKS EACH)

26. Let P(x,y) lies on the line joining A(1,3), and B (0,0), then points A,B,P are collinear.

So, $\frac{1}{2}\begin{vmatrix} x & y & 1 \\ 1 & 3 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 0$ (1M)

$\frac{1}{2}(3x-y) = 0$ or $y = 3x$ (1/2M)

Area of Δ ABD = 3 sq units

$\frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 1 & 3 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \pm 3$ (1/2M)

$\frac{1}{2}(3\ k) = \pm 3$

$K = \pm 2$ (1/2M)

OR

Given $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & 3 \\ 5 & -2 \end{vmatrix} = -19$ (1M)

Adj $A = \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$ (1/2M)

$A^{-1} = \frac{-1}{19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix} = \frac{1}{19}\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix} = \frac{1}{19}A$ (1M)

Given $A^{-1} = K\,A$

Hence $K = \frac{1}{19}$ (1/2M)

27. Given $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$

$\frac{dx}{d\theta} = -2\sin\theta + 2\sin 2\theta$ $\frac{dy}{d\theta} = 2\cos\theta - 2\cos 2\theta$ (1M)

$\frac{dy}{dx} = \frac{dy}{d\theta} / \frac{dx}{d\theta} = \frac{2\cos\theta - 2\cos 2\theta}{-2\sin\theta + 2\sin 2\theta} = 2\left(\frac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin\theta}\right)$ (1M)

$= \frac{2\sin\frac{3\theta}{2}\sin\frac{\theta}{2}}{2\cos\frac{3\theta}{2}\sin\frac{\theta}{2}}$ (using Cos C- Cos D and Sin C - Sin D formula) (1/2M)

$= \tan\frac{3\theta}{2}$ (1/2M)

28. Given $\frac{dv}{dt} = 8\ cm^3/sec$

$V = \frac{4}{3}\pi r^3 \quad \because \frac{dv}{dt} = \frac{4}{3}\pi\, 3r^2\frac{dr}{dt}$ (1M)

$\frac{dr}{dt} = \frac{2}{\pi r^2}$ (1/2M)

Now $S = 4\pi r^2 \quad \because \frac{ds}{dt} = 8\pi r\frac{dr}{dt} = 8\pi r\frac{2}{\pi r^2} = .\frac{16}{r}$ (1M)

At r = 12, $\frac{ds}{dt} = \frac{16}{12} = \frac{4}{3}\ cm^2/sec$ (1/2M)

OR

Let r be the radius of the circular wave and A be the area

$\frac{dr}{dt}$ = 5 cm/sec , A=π r^2

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 10\pi r$

$\frac{dA}{dt}$ = 80 πcm^2/sec

29. $x \frac{dy}{dx} = y(\log y - \log x + 1)$

$\frac{dy}{dx} = \frac{y}{x}\left(log\frac{y}{x}+1\right)$ (1/2 M)

Let $\frac{y}{x}$ = v or y= v x

$\frac{dy}{dx}$ = v +x $\frac{d\,v}{dx}$ (1 M)

$\because$ v +x $\frac{d\,v}{dx}$ = v (log v +1)

x $\frac{d\,v}{dx}$ = v log v (1/2M)

$\int \frac{dv}{v\ log\ v} = \int \frac{dx}{x}$

log (log v)= log x + log C (1/2M)

Log (log v)= log cx

Log v = c x

Log $\frac{y}{x}$ = c x (1/2 M)

OR

$(x^2 - 1)\frac{dy}{dx}$ + 2xy = $\frac{2}{x^2-1}$

$\frac{dy}{dx} + \frac{2x}{x2-1} y = \frac{2}{(x^2-1)^2}$ (1 M)

If = $e^{\int \frac{2\,x\,dx}{x^2-1}} = e^{log(x^2+1)} = x^2 + 1$ (1 M)

Solution is given by $(x^2 - 1)$ y = $\int \frac{2}{x^2-1}$ dx (1/2 M)

$(x^2 - 1)$ y =log $\left|\frac{x-1}{x+1}\right|$ + C (1/2 M)

30. $\frac{x-1}{-3} = \frac{y-2}{\frac{2\lambda}{7}} = \frac{z-2}{\frac{11}{5}}$ and $\frac{x-1}{\frac{-3\lambda}{7}} = \frac{y-5}{1} = \frac{z-6}{-5}$ (11/2 M)

If the lines are perpendicular (-3)$\frac{-3\lambda}{7} + \frac{2\lambda}{7}$ - 11 =0 (1 M)

λ = 7 (1/2 M)

SECTION D (5 MARKS EACH)

32 R be the relation on N x N defined

by (a,b) R (c,d) if ad (b+ c) = bc (a +d)

(i)(a,b) R (a,b) is true since ab(b+c) = bc (a+b)

Hence R is reflexive

(ii) (a,b) R (c,d) if ad (b+ c) = bc (a +d)

(c,d) R (a,b) if cb (a+ d)= da (c+ b) which is true hence symmetric

(iii) let (a,b) R (c,d) ,then ad (b+ c) = bc (a +d)

Dividing by abcd we have $\frac{b+c}{bc} = \frac{a+d}{ad}$, or $\frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}$

And let (c,d) R (e,f) , then cf (d + e) = de (c +f)

Dividing by abcd we have $\frac{d+e}{de} = \frac{c+f}{cf}$ or $\frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c}$

Adding both and canceling common terms we have

$\frac{1}{b} + \frac{1}{e} = \frac{1}{a} + \frac{1}{f}$ or, $\frac{b+e}{be} = \frac{a+f}{af}$

Hence (a,b) R (e,f) R is transitive

R is an equivalence relation

OR

A= { x ε Z:0 ≤ x≤ 12},

Consider R = { (a,b) : a, b ε A, $|a-b|$ is divisible by 4 }

(i) (a,a) ε R is true since $|a-a| = 0$ which is divisible by 4. hence R is reflexive

(ii) (a,b) ε R, then $|a-b|$ is divisible by 4

Hence $|b-a|$ is divisible by 4 or (b,a) ε R R is symmetric

(iii) Consider (a,b) ε R, then $|a-b|$ is divisible by 4 or a-bis divisible by 4

And (b,c)ε R, then $|b-c|$ is divisible by 4 or b-c is divisible by 4

Adding we have a-c is divisible by 4 or $|a-c|$ is divisible by 4,hence(a,c)ε R

R is transitive or R is an equivalence relation

Elements related to 1 { 1,5,9}

Equivalence class [2] = {2,6,10}

33. $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}$ dx

Let I = $\int_0^{\pi} \frac{x \sin x}{1 + \sin x}$ dx

Using the formula $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ we have

$I = \int_0^{\pi} \frac{(\pi-x)(\sin(\pi-x)}{1+\sin(\pi-x)} dx$ (1M)

Adding $2I = \pi \int_0^{\pi} \frac{\sin x}{1+\sin x} dx$ (1 M)

Multiplying by (1- sin x),

$2I = \int_0^{\pi} (sec\ x tan\ x - sec^2 x + 1) dx$ (2 M)

$= \pi$ [sec x- tan x+ x] between the limits 0 to π (1 M)

$2I = \pi(-2+\pi)$ (1 M)

$I = \frac{\pi}{2}(-2+\pi)$

OR

$$\int \frac{1}{x-x^3} dx = \int \frac{1}{x(1-x)(1+x)} dx$$

$$\frac{1}{x(1-x)(1+x)} = \frac{A}{x} + \frac{B}{1-x} + \frac{C}{1+x}$$ (1M)

Taking LCM we have 1 = A (x+1) (x-1) + B x (x-1)+ C x (x+1) (1M)

Solving A= 1 B= -1/2 C= 1/2 (1M)

$\int \frac{1}{x-x^3} dx = \int \frac{1}{x} dx + -1/2 \int \frac{1}{1-x} dx + 1/2 \int \frac{1}{x+1} dx$ (1M)

= log x + 1/2 log(1-x) + log (x+1) +C (1M)

34. Given $x\sqrt{1+y} + y\sqrt{1+x} = 0$

$x\sqrt{1+y} = -y\sqrt{1+x}$ (1M)

Squaring $x^2(1+y) = y^2(1+x)$ (1M)

$(x-y)(x+y+xy) = 0$ (1M)

x= y or $(x+y+xy) = 0$ but x can not be equal to y (1M)

$y = \frac{-x}{1-x}$

$\frac{dy}{dx} = \frac{-1}{(1-x)^2}$ (1M)

35. $\frac{x^2}{16} + \frac{y^2}{9} = 1$ figure (1 M)

Her $y = \frac{3}{4}\sqrt{16-x^2}$ (1 M)

Required area = 4 x area in the first quadrant

$= 4 \int_0^4 \frac{3}{4} \sqrt{16-x^2} dx$ ($2\frac{1}{2}$M)

= 12π sq units (1/2 M)

SECTION E (4 MARKS EACH)

36. Here $A = \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix}$ $B = \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix} \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix} = \begin{bmatrix} 30000 \\ 23000 \\ 39000 \end{bmatrix}$$

(i) The cost incurred by the organization on village X = Rs 30,000 /- (2 M)

(i) Find the cost incurred by the organization on village Y = Rs 23,000/- (2 M)

37. (i) y = 4x - $\frac{1}{2}x^2$

$\frac{dy}{dx}$ = 4- x. Hence rate of growth =4- xcm/day

(ii) For maximum height ,$\frac{dy}{dx}$ = 0, then x= 4

(iii) Y is maximum at x=4 or it will take 4 days for the plant to grow to maximum height

OR

(iv) y = 4x - $\frac{1}{2}x^2$

At x = 2, y = 6

Height of the plant after 2 days is 6 cm

38. (i) $\overrightarrow{AB} = -2\vec{i} + \vec{j} + \vec{k}$

(ii) $|\overrightarrow{AB}| = \sqrt{6}$

(iii) Unit vector along $\overrightarrow{AB} = \frac{-2\vec{i} + \vec{j} + \vec{k}}{\sqrt{6}}$

OR

(iv) Direction cosines of $\overrightarrow{AB}$ are $\frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}$

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