**ANSWER KEY & MARKING SCHEME**

**Class XII Mathematics (Code 041) | Session 2026–27**

**SECTION A – MCQs / Assertion–Reason (20 × 1 = 20 marks)**

**1. Answer: d) −5**

For a skew-symmetric matrix, a = −2, b = 0 and c = −3. So a+b+c = −5.

**2. Answer: b) 25**

$|5A^{-1}| = 5^3|A^{-1}| = 125/|A|$. Since $|A'|=|A|=5$, the value is 125/5 = 25.

**3. Answer: b) 45°**

$|a\times b| = |a||b|\sin\theta = 3(\sqrt{2}/3)\sin\theta = \sqrt{2}\sin\theta$. For a unit vector, $\sqrt{2}\sin\theta=1$, so $\theta=45°$

**4. Answer: c) −4**

Continuity at x=0 gives $-8 = k \lim(x\to0) \sin2x/x = 2k$. Therefore k=−4.

**5. Answer: b) tan x − cot x + C**

$1/(\sin^2x \cos^2x) = \sec^2x + \operatorname{cosec}^2x$; integrating gives tan x − cot x + C.

**6. Answer: d) 4**

The equation is $\sqrt{(1-y')}=y''$. Order = 2. On removing the radical by squaring, $(y'')^2 = 1-y'$,

so degree = 2. Thus m+n = 4.

**7. Answer: d) infinite**

If the same maximum occurs at two distinct corner points, every point on the line segment joining them also gives the same maximum value. Hence infinitely many points.

**8. Answer: a) $\hat{k}$**

$(i-j)\times(i+j)=2\hat{k}$; the corresponding unit vector in the right-handed direction is $\hat{k}$.

**9. Answer: c) π/4**

$\int_0^1 dx/(1+x^2) = [\tan^{-1}x]_0^1 = \pi/4$.

**10. Answer: d) ±√7**

$|A|=x^2-4$ and $|A^3|=|A|^3=27$, so |A|=3. Hence $x^2-4=3$, giving $x=\pm\sqrt{7}$.

**11. Answer: a) 21**

Z=11x+7y at the corner points: (0,3)→21, (3,2)→47, (0,5)→35. Minimum = 21.

**12. Answer: b) 0**

$a_{11}A_{21}+a_{12}A_{22}+a_{13}A_{23}$ is the expansion of the determinant along the first row, is 0.

**13. Answer: b) x>1**

$\cot^{-1}x = \pi/2-\tan^{-1}x$. Thus $\tan^{-1}x > \cot^{-1}x$ implies $\tan^{-1}x > \pi/4$, so $x>1$.

**14. Answer: d) 120**

A bijection from a 5-element set to itself is a permutation. Number of permutations = 5! = 120.

**15. Answer: c) y=Cx**

$y\,dx-x\,dy=0 \Rightarrow dy/y = dx/x \Rightarrow \ln|y|=\ln|x|+C \Rightarrow y=Cx$.

**16. Answer: d) $-2/x^2$**

$y=\log(x^2/e^2)=2\log x-2$ (for $x>0$); hence $y'=2/x$ and $y''=-2/x^2$.

**17. Answer: c) $\sqrt{3}$**

$|a+b|^2=|a|^2+|b|^2+2a\cdot b$ gives $1=2+2a\cdot b$, so $a\cdot b=-1/2$. Then $|a-b|^2=2-2(-1/2)=3$.

**18. Answer: a) -2**

Direction vectors are $(-2,\mu,-1)$ and $(-\mu,2,1)$. Parallel gives $-2/-\mu= \mu/2= -1/1$, hence $\mu= -2$

**19. Answer: c) A is true but R is false**

$\cos(7\pi/6)=-\sqrt{3}/2$, so $\cos^{-1}(\cos 7\pi/6)=5\pi/6$ .But principal value of $\cos^{-1}(x)$ is $[0, \pi]$

**20. Answer: c) A is true but R is false**

Direction vectors are (2,5,4) and (1,2,−3). Their dot product is 2+10−12=0, so the angle is 90°. The stated Reason is not a correct definition/explanation.

## SECTION B – Very Short Answer (5 × 2 = 10 marks)

**21. Answer:**

For $y=3\sin^{-1}(2x)$, $2x\in[-1,1] \Rightarrow$ domain $x\in[-1/2,1/2]$; (1 mark)

range $y\in[-3\pi/2,3\pi/2]$. (1 mark)

OR

$x = \cos 2\theta$ .Then $1-x = 2\sin^2\theta$ and $1+x = 2\cos^2\theta$ . (1 mark)

$1-x/1+x = \tan^2\theta$

$\tan^{-1}\sqrt{\frac{1-x}{1+x}} = \tan^{-1}(\tan\theta) = \theta = \frac{1}{2}\cos^{-1} x$ (1 mark)

**22. Answer: 900 cm³/s**

Let side be a. $V=a^3$. $dV/dt=3a^2\, da/dt$ (1 mark)

$=3(10)^2(3)=900$ cm³/s. (1 mark)

**23. Answer: $5^{\sin x}\ln(5)\cos x + x^x(\ln x + 1)$**

Differentiate each term: $d/dx[5^{\sin x}] = 5^{\sin x}\ln 5 \cos x$; (1 mark)

$d/dx[x^x]=x^x(\ln x+1)$. (1 mark)

**24. Answer: 2b/a**

$dy/dx=(dy/dt)/(dx/dt)=(b\sec^2 t)/(a\sec t\tan t)=b/(a\sin t)$. (1 mark)

At $t=\pi/6$, $dy/dx=2b/a$. (1 mark)

**25. Answer:**

Diagonal vecor = **a** + **b** = $3i-6j+2k$ (1 mark)

So unit vector along it = $\frac{3i-6j+2k}{7}$ (1 mark)

OR:

From $|a+b+c|^2=0$ with unit vectors, $|a+b+c|^2=|a|^2+|b|^2+|c|^2+2a.b+2b.c+2a.c$ (1 mark)

implies $3+2(a\cdot b+b\cdot c+c\cdot a)=0$, so the sum is $-3/2$. (1 mark)

## SECTION C – Short Answer (6 × 3 = 18 marks)

**26. Answer: Area = 11/6 square units.**

Correct diagram Sketch should show $y=x$ and $y=x^2$ meeting at (0,0) and (1,1),

with the required region between them.. (1 mark)

The required area = $\int_0^1 x^2 dx + \int_1^2 x\,dx$ (1 mark)

$= [x^3/3]_0^1 + [x^2/2]_1^2 = 11/6.$ (1 mark)

**27. Answer: $A^{-1} = (1/7)(5I-A) = [[2/7, -1/7],[1/7, 3/7]$**

$A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$ (1/2 mark)

Then $A^2-5A+7I=O$. (1 mark)

Thus $A(5I-A)=7I$, (1/2 mark)

$A^{-1}=(5I-A)/7 = 1/7 \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ (1 mark)

**28. Answer: π/8 ln 2**

Let $I=\int_0^{(\pi/4)} \ln(1+\tan x)dx$. Replacing x by $\pi/4-x$ gives (1 mark)

$I=\int_0^{(\pi/4)} [\ln 2-\ln(1+\tan x)]dx$ (1 mark)

$= (\pi/4)\ln 2 - I$. Hence $I=\pi/8 \ln 2$. (1 mark)

OR:

t= sin x- cos x . So dt =cos x + sin x dx (1 mark)

Then $\int \frac{dt}{25-16\,t^2} = \frac{1}{40}\log\left|\frac{5+4(\sin x-\cos x)}{5-4(\sin x-\cos x)}\right|$. (1 mark)

Applying the limits from 0 to π/4, we get $I = \frac{1}{40}\log 9$ (1 mark)

**29. Answer: $x= Cy + 2y^2$**

Linear D.E: $dx/dy - x/y = 2y$. (1/2mark)

The integrating factor is 1/y, (1 mark)

Hence $x/y=2y+C$, so $x=Cy+2y^2$. (1 mark)

Then y=0 when x=0 is satisfied for every C and hence does not determine C. (1/2 mark)

OR:

Put v=y/x. Then $y'=v+xv'$. (1 mark)

The equation reduces to $\cos v \cdot dv = dx/x$. (1 mark)

Hence $\sin(y/x)=\ln|x|+C$, $x\neq 0$. (1 mark)

**30. Answer:** $=\sin^{-1}\left(\frac{x+4}{\sqrt{21}}\right)+C$

$5-8x-x^2 = 21-(x+4)^2$. (1 mark)

Therefore $\int \frac{1}{\sqrt{21-(x+4)^2}}\,dx$ (1 mark)

$=\sin^{-1}\left(\frac{x+4}{\sqrt{21}}\right)+C$ (1 mark)

**31. Answer: Maximum Z = 400 at (0,200).**

Correct graph with proper shading (1 mark)

Correct table (1 mark)

| corners | Z= x+2y |
|---|---|
| (20,40) | 100 |
| (50,100) | 250 |
| (100,0) | 100 |
| (0,200) | 400 |

Hence maximum 400 at (0,200). (1 mark)

**SECTION D – Long Answer (4 × 5 = 20 marks)**

**32. Answer: S is an equivalence relation; [3]={3,7,11}.**

Reflexive: $|a-a|=0$, divisible by 4. (1 mark)

Symmetric: if 4 divides (a−b), then 4 divides (b−a). (1 mark)

Transitive: if 4 divides (a−b) and (b−c), then 4 divides (a−c). (1½ marks)

Thus S is an equivalence relation. (1/2 mark)

For [3], take $b \in A=\{0,\ldots,12\}$ with $|b-3|$ divisible by 4: b=3,7,11 (1 mark)

OR

$f(x)=(4x+3)/(3x+4)$ is injective by equating $f(x_1)=f(x_2)$, which gives $x_1=x_2$; (2marks)

for y in the codomain, $x=(4y-3)/(4-3y)$, so f is onto. Thus f is bijective. (2marks)

f(x)=2 gives x= −5/2. (1 mark)

**33. Answer: $-(1/3)\tan^{-1}x + (2/3)\tan^{-1}(x/2) + C$**

Use partial fractions: $x^2/[(x^2+1)(x^2+4)] = y/(y+1)(y+4) = A/y+1 + B/ y+4$ (2 marks)

$=-1/[3(x^2+1)] + 4/[3(x^2+4)]$. (1 mark)

Integrating gives $-(1/3)\tan^{-1}x + (2/3)\tan^{-1}(x/2)+C$. (2 marks)

**34. Answer: Foot H=(−2,1,7); image A′=(−3,−6,10).**

Line BC has direction (1,−1,−2) and equation $r=(-j+3k)+\lambda(i-j-2k)$. (1 mark)

Let $H=(\lambda,-1-\lambda,3-2\lambda)$. Since $AH \perp BC$, $(A-H)\cdot(1,-1,-2)=0$, giving $\lambda=-2$. (2 marks)

Hence H=(−2,1,7). (1 mark)

The image is $A'=2H-A=(-3,-6,10)$. (1 mark)

OR:

Let $\lambda=(x+2)/3=(y+1)/2=(z-3)/2$. (1 mark)

Distance from P(1,3,3) gives $17\lambda^2-34\lambda=0$, (1 mark)

so $\lambda=0$ or 2. (1 mark)

Points are (−2,−1,3) and (4,3,7). (2 marks)

**35. Answer: x=0, y=5, z=3**

AB= I implies $A^{-1} = B$ (2 marks)

Therefore system of matrices AX = C gives $X= A^{-1} C$ (1 mark)

$$= BC = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$$ (2 marks)

**SECTION E – Case Study / Integrated Questions (3 × 4 = 12 marks)**

**36. Answer: Minimum at (a,b)=(1,8); minimum distance = √5.**

i) For helicopter point $(a,a^2+7)$ and soldier (3,7), distance squared $D(a)=(a-3)^2+a^4$. (1 mark)

ii) $D'(a)=2(a-3)+4a^3=2(2a^3+a-3)=0$. The real critical point is a=1. (1 mark)

iii) $D''(a)=12a^2+2>0$, so a=1 gives the minimum; $b=1^2+7=8$. Thus position is (1,8) and distance is $\sqrt{[(1-3)^2+(8-7)^2]}=\sqrt{5}$. (2 marks)

OR

iii) use the first derivative test: D′ changes from negative to positive at a=1. (2 marks)

**37. Answer:**

**i) x/1=y/2=z/(−1). ii) (2/√6, 1/√6, 1/√6). iii) Distance = 0. Collision point = (1,2,−1).**

i) For A, $r=\lambda(i+2j-k)$, so $x=\lambda$, $y=2\lambda$, $z=-\lambda$, giving $x/1=y/2=z/(-1)$. (1 mark)

ii) Direction vector of B is (2,1,1), magnitude $\sqrt{6}$;

direction cosines are $(2/\sqrt{6},1/\sqrt{6},1/\sqrt{6})$. (1 mark)

iii) $a\times b=(3,-3,-3)$, and the scalar triple product with the joining vector (3,3,0) is 0;

hence distance between the lines is 0. (2 marks)

OR

iii)Solving the two line equations gives $\lambda=1$, $\mu=-1$ and collision point $(1,2,-1)$. (2 marks)

**38. Answer: Intersection points: $(\sqrt{3},1)$ and $(-\sqrt{3},-1)$.**

**Area in the first quadrant = $\pi/3$ square units.**

(a) $x=\sqrt{3}y$ and $x^2+y^2=4 \Rightarrow 3y^2+y^2=4 \Rightarrow y=\pm1$, $x=\pm\sqrt{3}$. (1 marks)

Hence the points are $(\sqrt{3},1)$ and $(-\sqrt{3},-1)$. (1 marks)

(b) Area = A1+ A2 = $\int_0^{\sqrt{3},} x/\sqrt{3}\mathrm{d}x + \int_{\sqrt{3}}^{2} \sqrt{4-x^2}\mathrm{d}x = \pi/3$. (1+1 marks)

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